An Efficient Algorithm for Range, Range Rate Ambiguity Resolution in MPRF Pulse Doppler Radars

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Abstract— **In this paper we propose an ambiguity resolution algorithm to estimate the range and range rate of targets detected by a pulse Doppler radar using Medium Pulse Repetition Frequency (MPRF) waveform. Here we consider the case of multiple target scenario with both range and range rate ambiguous. Numerous algorithms are proposed to solve the problem of ambiguity resolution using Chinese reminder theorem, Maximum Likelihood approach incorporating a clustering algorithm. Few studies are also carried out to study the robustness of these approaches in the presence of measurement errors. But these algorithms assume at least one axis measurement is unambiguous before clustering happens. Generally the ambiguity resolution employs multiple Pulse Repetition Frequency(PRF) waveforms. In this paper we propose to solve the ambiguity resolution problem in range-range rate joint domain using novel clustering approach. The algorithm takes in to account the range migration of targets across the coherent processing interval (CPI, a batch of transmission pulses with identical waveform parameters). The correlation gates for clustering is computed using the expected measurement accuracy of range and range rate, which are functions of signal to noise ratio (SNR). The paper proposes the efficient approach based on two passes to unambiguously determine range and range rate in two dimensional space. In the first pass the coarse correlation gates are used in range direction to group unfolded range measurements from different PRF's disregarding the range rate information. In the second pass, the range and range rate unfolding is carried out on the groups formed in first pass. The second pass employs finer range and range rate correlation gates. Range migration compensation is carried out in second pass. The range rate correlation gate accounts for range rate walk through across CPIs. Simulation studies are carried out to evaluate the resolution capability of closely spaced targets, ambiguity resolution in multiple target scenarios. The ghosts generated in dense target scenario is also quantified using simulations. Constraints required to be posed for waveform synthesis is discussed in brief.**

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1. INTRODUCTION

Choice of waveform and design is an important aspect of radar system design. In this paper we restrict our attention to pulse Doppler radars. Waveforms design for pulse Doppler radars is a vast subject and study includes, the design of modulation technique to be adopted, the range of Pulse Repetition Frequency (PRF choice), the duty cycle or the pulse width, the instantaneous bandwidth of the waveform and other parameters of waveform. The waveform has direct bearing on performance of radar systems. Radar systems are mounted on various types of platforms starting from ground moving vehicles to aircraft. Radar system design increasingly becomes complex from a ground based radar to airborne radar systems. In addition modern radar systems are designed to have multifunction, multiple mode capabilities. An airborne surveillance radar is designed to have multiple modes Air to Air(A-A), Air to Sea (A-S) and Air to Ground (A-G). In each mode the target of interest is different with widely varying characteristics. Radar waveforms are designed to meet the system performance requirements in each mode. One of the basic design parameter of radar waveforms is the PRF range. There are three major types of PRF and they are Low PRF, Medium PRF and High PRF waveforms.

Low PRF waveforms offer unambiguous measurement of range up to the instrumented range of radar, but measures highly ambiguous Doppler. Here unambiguous measurement of range means, the true range of the target can be simply measured in terms of time elapsed from transmitted pulse to the return pulse from target back-scatter. In Low PRF waveforms, a single pulse is sufficient to measure the true range of targets and thus does not suffer from spurious target problems. These waveforms are generally suitable for ground based radars, where the waveform blind due to zero Doppler filtering is minimal. In airborne applications, Low PRF has limited use due to their waveform blind in Doppler. In the case of airborne radars, the mainlobe clutter spectral width depends on the antenna beamwidth in azimuth direction, the speed of platform, the antenna scan angle (in case of phased array antenna) and RF frequency of radiation [9]. The usable Doppler spectrum in LPRF waveforms after mainlobe clutter cancellation may be unacceptable and thus can not be used in airborne pulse Doppler radar to detect moving aerial objects. On the other hand High PRF waveforms measure highly ambiguous range but offer unambiguous Doppler information. Due to multiple range folds in HPRF waveforms the performance of airborne radars for tail chase aspect would be poor [1]. Medium PRF waveforms combine the advantages of HPRF and LPRF waveforms, providing ambiguous measurements of range and range rate. MPRF waveforms are a good choice minimizing the waveform blind due to Moving Target Indicator (MTI) notch compared to LPRF waveforms. Many studies have been carried out in literature to conclude MRPF waveforms are the best choice of PRF for airborne radars in air to air look down mode [1]. In case of MPRF waveforms, the true range and Doppler information has to be found from the ambiguous measurements. This is usually done by range and Doppler correlation across

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multiple MPRF waveforms [2]. This processing is generally referred to as multiple PRF ranging. The topic of this paper is the design of efficient algorithm for ambiguity resolution using multiple MPRF waveforms in a pulse Doppler radar. Ambiguity resolution, employing multiple PRF's is not straight forward if both range and Doppler are ambiguous. In such cases, the ambiguity resolution algorithm declares few spurious targets usually refereed to as "ghosts". The spurious detections, other than true target positions, arising out of ambiguous range/Doppler measurements of a target is called $T1 - T1$ ghosting and this could be addressed by proper choice of PRFs. The generation of ghosts from ambiguous measurements from two targets (in multiple target scenario), also called $T1 - T2$ ghosting, can not be fully addressed by waveform selection. It is possible to eliminate ghosts by staggering PRF set across scans employing scan to scan correlation process at tracker level. Ghosts would be present always, but would not appear in the same position for different PRF sets [7]. The number of ghosts generated by ambiguity resolution would increase remarkably for a small increase in the number of ambiguous measurements at the input. A good reference for Medium PRF waveform performance analysis is presented in [2].

Lot of studies and algorithms have been proposed to determine true target range and Doppler from ambiguous measurements of range and Doppler out of MPRF waveforms. One of the simpler methods has been presented in [4] to determine true target range from folded ranging of multiple waveforms using few integer operations for single target scenario. The method does not account for measurement inaccuracies and range migration of target during observation time across multiple PRF's. Based on the similar technique, the Doppler ambiguity resolution problem is solved in [5]. All these methods have inherent problem in the presence of realistic measurement errors producing spurious targets (ghosts). In [6], [8] a clustering algorithm is proposed for ambiguity resolution of range. The specific case is analyzed taking a multiple target scenario with target raid having same azimuth, elevation and speed, but separated by a large distance in range. A maximum likelihood (ML) based method is derived, assuming Doppler ambiguities are resolved before the ML test, to compute the likelihood of set of detections with resolved range, correctly represent the set of ambiguous range measurements. If range and Doppler are both ambiguous then the method requires to resolve one of the dimension using other methods and later apply ML test. In [8], the cluster based technique is developed for resolving range and Doppler ambiguities independently and solved as one dimensional clustering problem.

In this paper, we develop ambiguity resolution algorithm to determine the resolved range and range rate using a novel clustering algorithm, when both range and range rate are ambiguous simultaneously. The algorithm is developed for a multiple target scenario with any range and Doppler. The only assumption made here is that the targets are separated in range or Doppler to the limit of resolvability of the targets. The ambiguity resolution is carried out accounting for range migration of targets across waveforms. The resolver takes centroided ambiguous range and Doppler as input to determined resolved range and Doppler. An efficient ambiguity resolution method based on two passes has been developed.

In section 2 a brief about MPRF radar is discussed. We propose a novel clustering technique using correlation gate in section 3 followed by the discussion on ambiguity resolution algorithm in section 4. Section 5 discusses on simulation studies. We draw conclusions in section 6.

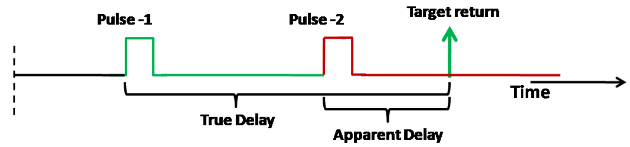

Figure 1. **Range measurement in MPRF radar**

2. Medium PRF Radar Description

The radar under consideration here is an airborne radar employing Medium PRF (MPRF) waveforms for look down mode of operation. By virtue of MPRF waveforms, both range and Doppler are ambiguous and they are resolved using multiple PRF's. Radar radiates multiple bursts with different PRFs. Each burst consisting of multiple transmission pulses with identical characteristics. The measurements carried out over multiple bursts are used to resolve the range and Doppler ambiguities. The unfolding of range and Doppler looks for correlation of measurements over different PRF's to declare the presence of target with resolved parameters. The number of different PRF's (or bursts) used in a look direction be N. The group of N bursts radiated in a look direction is called dwell in this paper. The ambiguity resolver works at dwell level and detections across dwells are handled independently. In this paper we develop ambiguity resolver in range and range rate domain considering target detections from a given look direction of antenna and do not find correlation in angle dimension across PRFs to form clusters. The presence of target is declared at certain resolved range and Doppler if the resolver finds correlation across M different PRF waveforms. Thus the detection criterion used becomes M/N. In other words, a target shall respond in minimum of M PRF's out of N different PRF's for it to be declared as target with resolved parameters. The PRF's to be used are off-line synthesized and employed during radar operation. The group of N PRF's are chosen to provide stable unfolding till the instrumented range and Doppler. Generally radars employ RF frequency agility across multiple PRFs (bursts) to decorrelate the target RCS and achieve better probability of detection. Since target Doppler frequency is dependent on RF frequency of radiation, the unfolding of Doppler measurement is carried out in range rate.

The maximum unambiguous range and range rate provided by PRF is given by Equ. 1 and Equ. 2 respectively.

$$R_u = cT/2 \tag{1}$$

where T is pulse repetition interval (PRI) or inter-pulse duration and c is speed of light.

$$\dot{R}_u = -PRF\lambda/2 \tag{2}$$

where PRF is pulse repetition frequency and λ is wavelength of transmitted RF frequency.

Fig. 1 shows the range measurement using MPRF waveform. The radar measures ambiguous range or apparent delay. Similarly the target Doppler aliasing is caused due to the use of PRF, the sampling frequency in Doppler axis, less than Nyquist frequency.

The accuracy and resolution of radar measurements [12] in range axis is given by Equ. 3 and Equ. 4. k_m is a constant depends on the merging logic used during centroiding, with typical values varying from 1 to 2. k_r is also a constant which depends on mainlobe broadening factor of pulse compressed signal due to windowing. Its typical value is 1.5 for hamming window. The range cell size dR is given by $c/(2B)$, where B is instantaneous bandwidth.

$$\sigma_r = \frac{dR}{k_m\sqrt{2(S/N)}} \tag{3}$$

$$\Delta R = 2k_r dR \tag{4}$$

The accuracy and resolution of radar in Doppler axis can be found in [12] and is reproduced here in Equ. 5 and Equ. 6 for completeness sake. Here, ToT is time on target or the burst time. The constants k_n and $k_{\dot{r}}$ depends on the centroding algorithm and windowing function used during coherent processing respectively. The computed accuracy values in range and range rate are bound by CRLB limit in the lower side. A good discussion on Cramer Rao Lower Bound (CRLB) for measurement accuracies could be found in [13].

$$\sigma_{\dot{r}} = \frac{\lambda}{2k_n ToT\sqrt{2(S/N)}} \tag{5}$$

$$\Delta\dot{R} = k_{\dot{r}}\lambda/ToT \tag{6}$$

The next important design aspect of waveforms is the modulation to be used to provide necessary range resolution with available radar peak power. The detection range performance of radar depends on average power and not the peak power. The availability of peak power is generally limited and radars are forced to transmit longer pulse durations. If radiated pulse does not use any modulation, the range resolvability of radar suffers. The problem is usually circumvented by introducing some type of modulation in the radiated pulse and performing matched filtering on the return echos (usually referred to as pulse compression). In this paper we very briefly discuss about types of modulation and other considerations to be kept in mind while choosing the type of modulation. The tool to analyze the waveform characteristic is the ambiguity function. It is defined for waveform $x(t)$ as

$$A(t, F_D) = \left|\int_{-\infty}^{\infty} x(s)\exp(j2\pi F_D s)x^*(s-t)ds\right|$$

The two parameters of the function are the time delay and the Doppler frequency shift. The matched filter is matched to $A(0,0)$. In radar scenario, the waveform which is Doppler tolerant, provides good resolution in range and Doppler is preferred. The waveform without any modulation is not preferred as its range resolution is poor and is given by $c\tau/2$, where τ is pulse width. Another kind of modulation is linear frequency modulation, which offers better range resolution compared to no modulation case and the waveform is Doppler tolerant. The main disadvantage of LFM is pulse compression sidelobes. To suppress sidelobes to required level, windowing techniques are used. LFM has another interesting effect called range - Doppler coupling due to its skewed ambiguity function. This can be compensated during ambiguity resolution. In this paper we assume radar uses LFM modulation. One of the simple modulation technique is phase coding. The popular bi-phase coding is barker code. But phase coded waveforms are not Doppler tolerant and mismatch loss [3]. But have advantage of peak sidelobe level, which depends on the length of the code. Poly phase codes, nonlinear FM are other kinds of modulation offering benefits in terms of sidelobe control and Doppler tolerance. The detailed analysis of ambiguity function and associated discussion can be found in [3].

3. Clustering Algorithm

We introduce clustering algorithm in this section.

Clustering Algorithm

In this section we describe the clustering algorithm used in ambiguity resolution algorithm of range and range rate. Let the data vector be $X = [x_1, x_2, \ldots, x_N]$, where x_i is a scalar. In this paper, x_i represents either range or range rate measurement. Thus $X \in R^{1\times N}$. We require clustering along one dimension at any given point in ambiguity resolution algorithm. Hence, here we describe the clustering algorithm for one dimensional case. In particular the given data vector has to be formed in to $\mathcal{C}$ clusters by partitioning the data vector of N elements. The problem of clustering in case of ambiguity resolution algorithm can be simplified utilizing prior information available about the principle behind generation of the data vector X. In case of ambiguity resolution, the data vector is formed by unfolding the ambiguous range or range rate measurements from all the PRFs and collating them in to the data vector. Each ambiguous measurement is result of back scatter from target or clutter point or a false alarm. Each such measurement from radar sensor has associated accuracy value. Let the accuracy of i^{th} measurement be σ_i. The clustering algorithm with prior information has following steps:

1. Sort the Data vector : In this stage we sort the data vector in ascending order. Let the sorted vector be X_o with i^{th} element x_{oi} (where o stands for ordered). The associated accuracy is σ_{oi}
2. Clustering using correlation gate : The clustering algorithm starts from the smallest element of data vector X_o. The smallest element x_{o1} is compared with the next smallest element x_{o2}. We define any two elements x_{oi} and x_{oj} similar, if $d(x_{oi}, x_{oj}) < C_g(oi, oj)$, where $d(\cdot,\cdot)$ is similarity measure and $C_g(oi, oj)$ is correlation gate. The computation of correlation gate is dependent on choice of similarity measure. There are different choices for similarity measure. For the case of Euclidean distance as similarity measure, that is, $d(x_{oi}, x_{oj}) = |x_{oi} - x_{oj}|$ (in any direction) the computation of correlation gate is given by Equ. 7. The construction of correlation gate between two samples is possible as radar sensor provides information about the measurement accuracy.

$$C_g(oi, oj) = 6\sqrt{\sigma_{oi}^2 + \sigma_{oj}^2} \tag{7}$$

Other than Euclidean similarity measure, one could use Mahalanobis distance

$$d(x_{oi}, x_{oj}) = \sqrt{(x_{oi} - x_{oj})^2/(\sigma_{oi}^2 + \sigma_{oj}^2)}$$

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For the case of Mahalanobis similarity measure, $d(x_{oi}, x_{oj})^2$ has Chi-Square distribution and the corresponding correlation

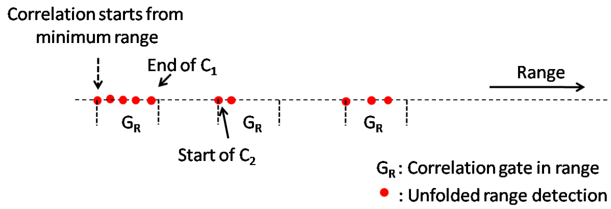

Figure 2. Formation of clusters in Range (non-overlapping)

gate becomes Chi-Squared statistic test. In our simulation, we use Euclidean distance as similarity measure.

As already pointed out the clustering starts from first smallest element. The first element is compared with the second, third and so on to evaluate the similarity measure. Elements starting from first smallest to the k_1^{th} smallest element are added to the cluster C_1, if they are similar in the sense of similarity measure $d(x_{o1}, x_{ok_1}) < C_g(o1, ok_1)$. Thus first cluster would have the smallest element to k_1^{th} smallest element, that is

$$\begin{aligned} C_1 = \{x_{oi} \text{ for } i = 1 \text{ to } k_1, &\text{such that} \\ &d(x_{o1}, x_{oi}) \leq C_g(o1, oi) \forall i \text{ and} \\ &d(x_{o1}, x_{o(k_1+1)}) > C_g(o1, o(k_1+1))\} \end{aligned} \tag{8}$$

The second cluster will start from $(k_1+1)^{th}$ smallest element and extend up to k_2^{th} smallest element. In general, the n^{th} cluster C_n will have elements up to k_n^{th} smallest element and C_{n+1} will start from $(k_n+1)^{th}$ smallest element and go up to k_{n+1}. Thus in general any cluster C_n is given by Equ. 9. This way the partition of the data vector is carried out and $\mathcal{C}$ clusters are formed. This is illustrated in Fig. 2.

$$\begin{aligned} C_n = \{x_{oi} \text{ for } i = k_{n-1}+1 \text{ to } k_n, &\text{such that} \\ &d(x_{o(k_{n-1}+1)}, x_{oi}) \leq C_g(o(k_{n-1}+1), oi) \forall i \text{ and} \\ &d(x_{o(k_{n-1}+1)}, x_{o(k_n+1)}) > C_g(o(k_{n-1}+1), o(k_n+1))\} \end{aligned} \tag{9}$$

The formation of clusters are simple if the clusters do not overlap. In case the clusters overlap, the cluster refinement is carried out to reapportion the members of the cluster. Two clusters C_n and C_{n+1} are classified as overlapping clusters if the Euclidean distance between maximum data of C_n and C_{n+1} is less than correlation gate size C_g. To resolve the cluster boundaries in case of overlapping clusters, we collate the measurements from these two clusters and perform clustering based on K-means algorithm [10] to form two refined clusters. Since that data vector is generated based on radar sensor detections, we assume here that the centroiding is performed prior to the ambiguity resolution algorithm to merge detections in multiple resolution cells from same reflector. A target could spread to multiple resolution cells due to straddle or windowing employed during processing (windowed pulse compression). It is further assumed that no two target would be detected as different targets within resolution limits of the radar defined by Equ. 4 and Equ. 6. Under these assumptions, the cluster boundary can be refined using K-means clustering logic. The cluster seeds are initialized with minimum data value from cluster C_n and maximum data value from cluster C_{n+1} [11]. A case study of how it works is presented in Fig. 3. It can be noted from Fig. 3 that the labels after

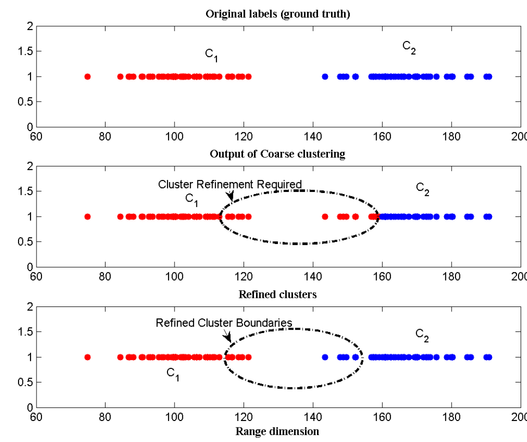

Figure 3. Formation of clusters in Range (after Refinement stage)

first level of clustering based on correlation window leads to few wrong labeling and these can be corrected by K-means clustering. If multiple consecutive clusters overlap, for instance, C_n, C_{n+1} and C_{n+2}, then two cluster at a time is handled iteratively. This would refine the cluster boundaries of all the overlapping clusters. By employing two stage clustering, the algorithm is made efficient and cluster refinement is employed where ever necessary.

The centroid position of cluster C_n with members $\{x_1^n, x_2^n, \ldots, x_P^n\}$, where P is the number of data points in C_n, is computed using each member's accuracy estimation. First consider two members x_1^n and x_2^n with associated accuracy σ_1 and σ_2 respectively. The centroid of these two point is $x_c(1,2) = ax_1^n + (1-a)x_2^n$. The value of a is computed to minimize the mean square error of the centroid and the optimum value for a is $\frac{\sigma_2^2}{(\sigma_1^2+\sigma_2^2)}$. The variance of centroid $x_c(1,2)$ with this value of a is $\frac{\sigma_1^2\sigma_2^2}{(\sigma_1^2+\sigma_2^2)}$. Next we pick the third sample and compute centroid of $x_c(1,2)$ and x_3^n with their associated accuracies. This is repeated till all the members are combined to form centroid of the cluster.

The classification error of the proposed approach is evaluated using simulations. For this purpose, we consider data vector of 100 elements with 50 elements from one cluster and the remaining from the second cluster. The cluster points are drawn from Gaussian density function of different means, but identical variance. Each data point has associated variance which is fully known. For this simulation the standard deviation of 10 is considered. The correlation gate for $\sigma_i = 10$ is given by Equ. 7 and is computed to be 85m. We examine the classification error of the proposed clustering algorithm for different mean separation between cluster density points. For given mean separation, we conduct 1000 Monte-Carlo simulation and classification error probability is computed from the ground truth and the cluster labels out of clustering algorithm. As shown in Fig. 4, the proposed algorithm is having less than 2% error for mean separation of greater than 50m. Without cluster refinement, the error reduces to less than 2% for mean separation of greater than 85m, which is our correlation window size. For ambiguity resolution, formation of clusters with right members are important and the proposed technique with refinement stage meets the requirements considering the resolution limit of the radar in measurement axis.

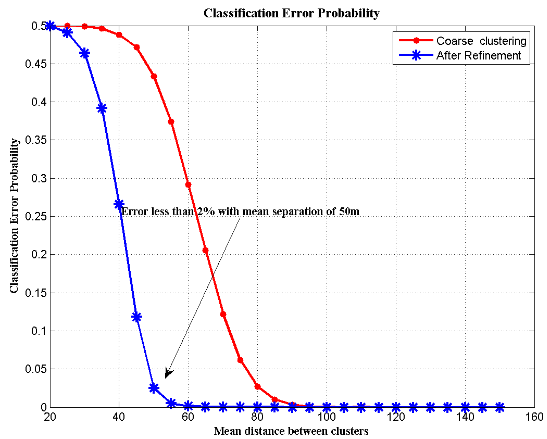

Figure 4. Classification error of proposed clustering scheme with two clusters

4. AMBIGUITY RESOLUTION ALGORITHM

In this section, we describe the proposed ambiguity resolution algorithm. The algorithm works on ambiguous range and range rate measurements from multiple PRFs. The algorithm has following steps:

1. Range unfolding and clustering with coarse range correlation window
2. Range rate unfolding and clustering in range-range rate with Range rate correlation window
3. Range migration compensation and clustering in range with fine range gate and resolved range rate
4. Declaration of final detections based on M/N criterion
5. Determination of measurement parameters and associated accuracies

As explained already in the section. 2, ambiguity resolution in range and range rate is carried out using N-PRF waveforms of a dwell and the detection is declared if the cluster has measurements from M PRFs out of N. As defined already in section 2, the look direction of antenna is same for all the PRFs in a dwell. To detect the target with resolved range and range rate, the target shall be at least visible in M PRF waveforms out of N PRFs. If this condition is not met, the target is not declared out of ambiguity resolver. Let us denote the N PRF values as $PRF_1, PRF_2, \ldots, PRF_N$ and their corresponding pulse repetition time as $T_1, T_2, \ldots, T_N$. The maximum unambiguous range of PRF's are computed as given in Equ. 1 and are represented by $R_{u1}, R_{u2}, \ldots, R_{uN}$. The maximum unambiguous range rate of PRF's are given by Equ. 2 and denoted by $\dot{R}_{u1}, \dot{R}_{u2}, \ldots, \dot{R}_{uN}$. Each PRF measures ambiguous range and range rate and let there be L_i number of measurements for i^{th} PRF. The j^{th} measurement in i^{th} PRF is a vector $Z_j^i = [r_j^i \; \dot{r}_j^i \; \sigma_{r_j^i} \; \sigma_{\dot{r}_j^i} \; PRF_i \; \lambda_i]^T$. The information on PRF, RF frequency used in i^{th} waveform, measurement accuracies in range and range rate are maintained in addition to few other information like SNR of the plot etc. (not shown here) for future use during ambiguity resolution. Let the burst time for i^{th} PRF be ToT_i. Let the maximum instrumented range or maximum unfolded range of interest be R_{max} and maximum range rate of interest be $\dot{R}_{max}$.

The ambiguity resolution algorithm is delineated here by the following steps:

1. Range unfolding stage and Clustering: The measurements from i^{th} PRF are unfolded in range as follows. The k^{th} order range unfold value of j^{th} ambiguous measurement from i^{th} PRF is represented by r_{kj}^i and is given in Equ. 10

$$r_{kj}^i = r_j^i + kR_{ui} \text{ for } j = 0 \text{ to } j = L_i \;, \quad k = 0, 1, \ldots \lceil R_{max}/R_{ui} \rceil \tag{10}$$

Here $\lceil \cdot \rceil$ represents ceiling operation. Let the number of unfolded detections of L_i measurements from i^{th} PRF, unfolded up to maximum range of R_{max}, be U_i. The unfolding operation is carried out for all measurements from the N PRFs. This forms a total of $U_i \times N$ measurements. All possible unfolded range measurements $[r_{kj}^i$, for $j = 1 \ldots L_i, k = 0 \ldots \lceil R_{max}/R_{ui} \rceil, i = 1 \ldots N]$ are sorted in the ascending order of range from smallest to largest. The j^{th} element in the ordered array be represented by r_{oj} (where o stands for ordered) with its corresponding information $(\dot{r}_{oj}, \sigma_{r_{oj}}, \sigma_{\dot{r}_{oj}}, PRF_{oj}, \lambda_{oj})$. We group these measurements in to $\mathcal{C_R}$ clusters and the grouping of measurements is based on correlation gate followed by refinement stage. Let the i^{th} cluster be C_i. The clustering algorithm is already explained in section. 3. We explain here, the major steps followed during range clustering for bringing better clarity. Given the sorted array of unfolded range detections, the clustering starts from the smallest range measurement r_{o1}. The smallest measurement is compared with the next smallest measurement r_{o2} and if their Euclidean distance is less than the correlation window $C_w(o1, o2)$ then these measurements are grouped in to first cluster. The computation of correlation window for comparison of oi and oj measurements is given in Equ. 11.

$$C_w(o1, o2) = 6\sqrt{\sigma_{r_{o1}}^2 + \sigma_{r_{o2}}^2} + \dot{R}_{max}\Delta T \tag{11}$$

where ΔT is the sum of time on target of all the bursts, that is $\Delta T = \sum_{i=1}^{N} ToT_i$. The second term in the expression accounts for range migration of targets across bursts. Since the range rate information is also ambiguous, until it is unfolded exact range migration compensation can not be carried out. This is postponed till the next stage of processing. Here to accommodate for maximum range rate of interest, we have used $\dot{R}_{max}$ to construct coarse range gate.
The process of adding measurements to first cluster continues until the distance between first and i^{th} measurement is greater than correlation window $C_w(o1, oi)$. Thus the first cluster is given by

$$C_1 = \{(r_{oi}, \cdot, \cdot, \cdot, \cdot) \text{ for } i = 1 \text{ to } (s_1 - 1), \quad d(r_{o1}, r_{oi}) \leq C_w(o1, oi) \forall i \text{ and } \quad d(r_{o1}, r_{os_1}) > C_w(o1, os_1)\} \tag{12}$$

Let us assume that the first $(s_1 - 1)$ smallest range measurements satisfy Equ. 12 and form the first cluster. Thus the set $\{r_{o1}, r_{o2}, \ldots, r_{o(s_1-1)}\}$ is the set of first $s_1 - 1$ smallest range measurements and forms the first cluster. The second cluster starts s_1 smallest range measurement. Thus, the $(k+1)^{th}$ cluster starts from s_k smallest range measurement. This is illustrated in Fig. 2. The k^{th} cluster is given by Equ. 13.

$$C_k = \{(r_{oi}, \cdot, \cdot, \cdot, \cdot) \text{ for } i = s_{k-1} \text{ to } (s_k - 1), \\ d(r_{os_{k-1}}, r_{oi}) \leq C_w(os_{k-1}, oi) \forall i \text{ and} \\ d(r_{os_{k-1}}, r_{os_k}) > C_w(os_{k-1}, os_k)\} \quad (13)$$

Next stage of processing is cluster refinement stage and is similar to what is described in section. 3. The clustering refinement considers range axis for clustering.

Clusters, thus formed after cluster refinement stage, are subjected to M/N detection validation criterion. Clusters which are formed with members coming from less than M PRFs are rejected under M/N binary integration criterion. Only those clusters which pass M/N criterion are further processed in the subsequent stages. Let the number of clusters to be processed in the next stage be $\mathcal{C}_{\mathcal{R}}$.

2. Range Rate unfolding stage and clustering in Range rate: In this stage of processing, the clusters formed in the previous stage after qualifying for M/N criterion are considered. These clusters have range unfolded (with coarse gate), but not compensated for range migration across bursts as range rate information was ambiguous in the previous stage. Unfolding of range rate information is carried out in this stage. The unfolding of range rate is similar to range unfolding carried out in the previous stage. Range rate unfolding is carried out considering one cluster at a time, which was formed in the previous step.

For Every cluster from range unfolding stage 1 : that is

For every range cluster $k = 1$ **to** $\mathcal{C}_{\mathcal{R}}$

- Consider cluster C_k. For the sake of reducing cluttering of symbols we redefine the cluster members of C_k here as

$$C_k = \{(r_i^u, \dot{r}_i, \sigma_{r_i}, \sigma_{\dot{r}_i}, PRF_i, \lambda_i) : i = 1 : N_{rk}\}$$

where N_{rk} is number of members in cluster C_k and the superscript u in r_i^u indicates range is already unfolded (unambiguous). The unfolding of range rate is done as follows:

$$\dot{r}_{ki} = \dot{r}_i + k\dot{R}_{ui}$$

where $\dot{R}_{ui} = -PRF_i\lambda_i/2$ and $k = -\left\lceil \dot{R}_{max}/\dot{R}_{ui} \right\rceil, \ldots, \left\lceil \dot{R}_{max}/\dot{R}_{ui} \right\rceil$. Thus formed unfolded range rate measurements are sorted in ascending order of range rate value. Further these unfolded range rate measurements are grouped in to clusters in range rate axis using range rate correlation gate. The clustering process is similar to what is followed in the range clustering case. The range rate correlation gate for comparison of i^{th} measurement with j^{th} is given by:

$$C_{w\dot{r}}(i,j) = 6\sqrt{\sigma_{\dot{r}_i}^2 + \sigma_{\dot{r}_j}^2} + a_{max}\Delta T$$

where, a_{max} denotes maximum target maneuver expected to be detected and tracked. The second term in the expression accounts for the range rate walk through across bursts.

If the overlap between the clusters are detected, they are subjected to cluster refinement stage as already explain in range cluster formation case. Let i^{th} Doppler cluster in k^{th} coarse range group be denoted as D_i^k. At this stage we have many clusters formed in range rate direction for a given range cluster C_k. The formed range rate clusters are qualified against M/N detection criterion, as done previously for range clusters. The qualified range rate clusters are further processed for fine range correlation and resolution. Let the number of range rate clusters be $D_{\dot{r}}^k$.

Since the clusters are formed based on range rate nearness criterion, the only thing which is pending to be verified is the nearness in range after range migration compensation. At this stage, the unambiguous range rate measurement is available and let as denote the centroid of the range rate for cluster D_i^k be $\bar{\dot{r}}_i$. Let the members of range rate cluster D_i^k be

$$D_i^k = \{(r_j^u, \dot{r}_j^u, \sigma_{r_j}, \sigma_{\dot{r}_j}, PRF_j, \lambda_j) : \text{ for } j = 1 \text{ to } N_{di}\}$$

where N_{di} is number of members in Doppler cluster D_i^k, r_j^u and $\dot{r}_j^u$ represent unambiguous range and range rate respectively. In the next step, we consider range rate cluster one at a time.

For every range rate cluster $m = 1$ **to** $m = D_{\dot{r}}^k$

– The range migration compensation is carried out on all the members of D_m^k. The range migration compensation aligns all the range measurements to the N^{th} PRF measurement time. The j^{th} member is compensated for range migration as:

$$\hat{r}_j = r_j^u + \bar{\dot{r}}_m\Delta T$$

where ΔT is the time interval between waveform PRF_j to waveform PRF_N. Once the range measurements are compensated, we cluster compensated range measurements in range direction using fine correlation gate defined in Equ. 14. The clustering process is carried out in range direction (range migration compensated) staring with sorting of range measurements in ascending order and till cluster refinement stage as already explained in detail before. Such clusters formed are once again qualified for M/N criterion before declaring final detection with the compensated cluster centroided range and range rate.

$$C_w^f(i,j) = 6\sqrt{\sigma_{r_i}^2 + \sigma_{r_j}^2 + \sigma_{\dot{r}_i}^2\Delta T^2} \quad (14)$$

We summarize here, the steps of ambiguity resolution algorithm for better clarity of readers.

1. Perform range unfolding followed by clustering in range direction with coarse range correlation gate. This forms range clusters.
2. **For** $k \to 1$: **No of range clusters**

{
    Perform range rate unfolding followed by clustering to form range rate clusters.
    For $m \to 1$: **No of range rate clusters**
    {
        Perform range migration compensation followed by clustering with fine range gate. The clusters which qualify M/N criterion form final detections.
    }
}

Thus the algorithm runs in two cascaded for loops. The centroid of each cluster is obtained as explained in section. 3.

The waveform design problem for multiple PRF ranging, is closely related to the design of ambiguity resolution algorithm. The correlation gates assumed in ambiguity resolution algorithm is crucial parameter for stable waveform design to avoid $T1 - T1$ ghosting within the region of interest. The study of resolver aids in deciding parameters M and N for M/N criterion. The study of ghosting helps in optimizing the M/N criterion. Another constraint to be placed on waveform is the range of PRF based on the number of range and range rate folds. The optimization problem of waveform selection

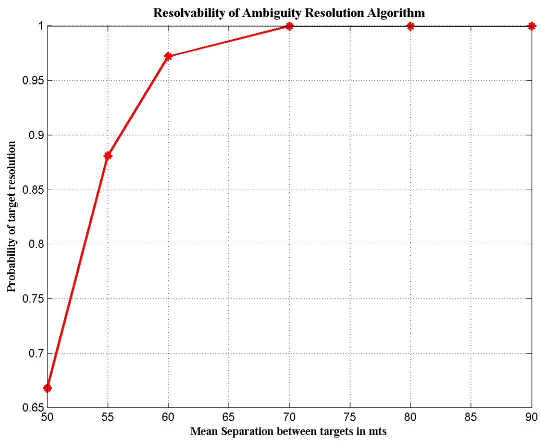

Figure 5. **Resolvability of Ambiguity Resolver with 3 targets in a beam**

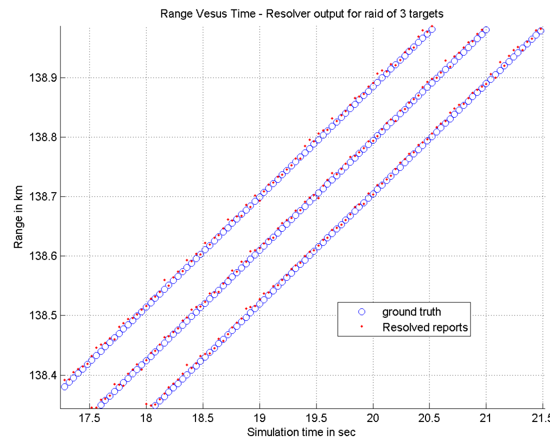

Figure 6. **Performance of resolver for target raid**

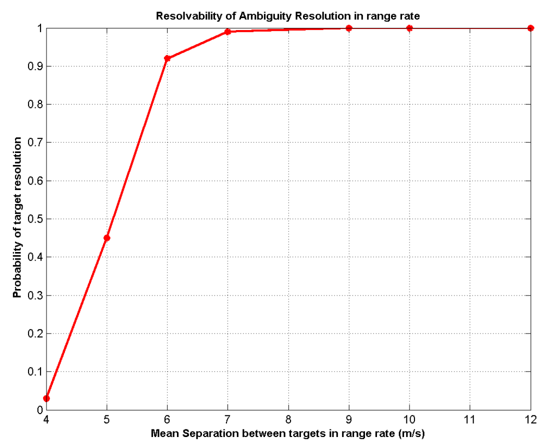

Figure 7. **Resolvability of Ambiguity Resolver in range rate axis with 3 targets in a beam**

maximizes for waveform visibility with M/N detection criterion and also ensures to avoid $T1 - T1$ ghosting in the region of unfolding.

5. Simulation

We evaluate the performance of proposed ambiguity resolution algorithm for its capability to preserve measurement resolution, handle multiple target scenario. Through simulation we study the dependency of number of ghost detections on resolver input measurement density. For the purpose of simulation, we consider range cell size of 30m. This leads to range measurement accuracy of 8m (assuming SNR of 10dB) and resolution limit of 90m. The detection criterion used is $2/4$. Thus there are 4 PRFs available for ambiguity resolution. The PRF range used is 8kHz to 12kHz with 25 to 30 range folds and 5 to 6 range rate folds. Thus both range and range rate are ambiguous. In the first experiment we consider 3 targets separated by 50m to 120m from each other to evaluate the resolvability of the proposed algorithm. 1000 Monte-Carlo trials are conducted for each case of mean separation and the number of times the targets were resolved is computed. The results are presented in Fig. 5. The resolver does not spoil the range resolution of the radar. It can be noticed that the mean separation of 70m and above can be resolved without errors. The target raid of three targets with 90m separation between them and at same range rate is considered and Fig. 6 shows the performance of resolver. It can be noticed that the resolver is able to unambiguously determine the ranges of targets. We consider a multiple target scenario with 5 targets in a beam and it is noticed in Fig. 8 that the targets are resolved properly, but the resolver produces spurious detections called ghosts. The detection criterion used was 2/4. Next we evaluate the performance of resolver for same scenario of 5 targets in every beam with detection criterion 3/5. Fig. 9 shows the result of ambiguity resolution with 3/5 criterion. It is noticed that the resolver is properly able to resolve the ambiguities in range and range rate without generating too much of ghosts, as noticed with 2/4 criterion. Thus the generation of ghosts is related to the detection criterion and proper waveform design can considerably reduce spurious measurements.

Next, we evaluate the resolvability of close by targets in range rate axis after ambiguity resolution. The time on target considered in the simulation is 10msec. Corresponding to this, the expected range rate accuracy is $1m/s$ and expected range rate resolution is $15m/s$. We consider three targets at the same range with mean separation of $5m/s$ to $20m/s$ from each other in range rate. 1000 Monte-Carlo trials are conducted for each case of mean separation and the number times the targets were resolved is computed. The results are presented in Fig. 7. The resolver does not spoil the range rate resolution of the radar. It can be noticed that the mean separation of greater than $9m/s$ and above can be resolved without errors.

We also evaluate the performance of resolver for generation of spurious plots as the input measurement density increases. The target position are considered uniformly distributed in the region of interest. The results for 2/4 and 3/5 detection criterion is tabulated in table. 1. It is noticed that 2/4 waveform performs badly as the input data density increases, while 3/5 performance stays with in acceptable limits. The improvement in the case of 3/5 is at the cost of extra radar time. All the results are obtained with 1000 Monte-Carlo trials. It is observed that $3/4$ detection criterion also reduces the ghost percentage to less than 0.1% for the case of 15 measurements for every PRF at the input of resolver. But the waveform visibility suffers. Further it is noticed in simulation that the cluster goodness factor alone not sufficient to

Table 1. Performance of resolver

Data density	2/4	3/5
2	8%	0%
4	18%	0.02%
6	29%	0.13%
8	35%	0.2%
10	41%	0.2%
15	52%	0.2%

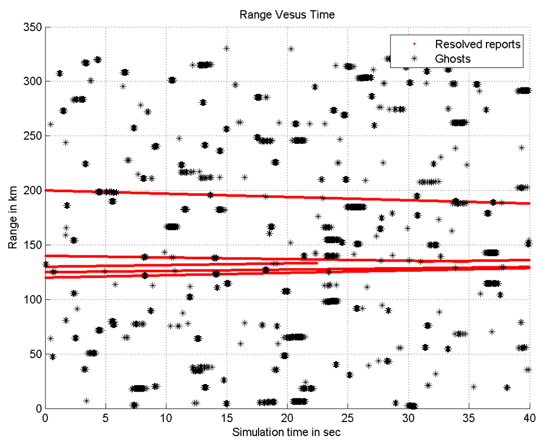

Figure 8. Performance of resolver in multiple target environment with 2/4 criterion

distinguish ghosts from targets. The cluster goodness factor (or cost factor) for targets and ghosts are shown in Fig. 10. The cost factors of target cluster has standard deviation (s.t.d) less than 20m, where as ghost has s.t.d equally probable all across.

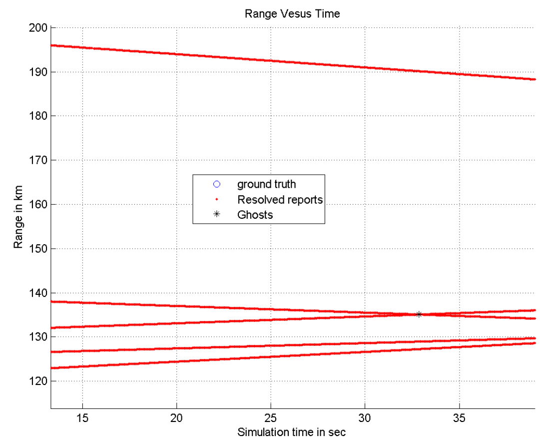

Figure 9. Performance of resolver in multiple target environment with 3/5 criterion

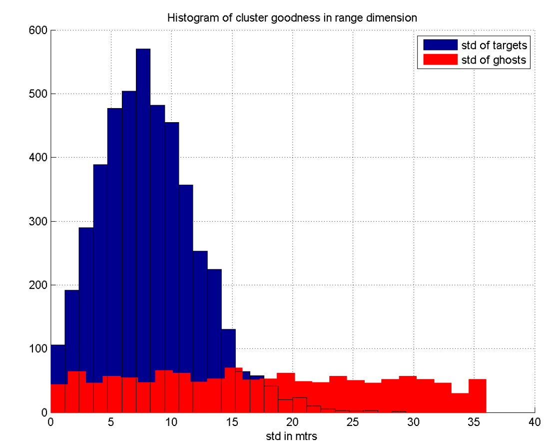

Figure 10. Cost factor for targets and ghosts

6. CONCLUSIONS

In this paper we have proposed an efficient ambiguity resolution algorithm based on novel clustering technique. The proposed method can handle both range and range rate ambiguities simultaneously, in a multiple target environment. An efficient two pass method is devised. The clustering technique exploits the all available information about data and gates are optimally constructed using measurement accuracies. The robustness of the algorithm is established through simulations in the presence of measurement noise. The ambiguity resolver does not spoil the resolution limits of the radar. Through simulation studies, it is found that 3/5 detection criterion is more suitable in terms of less ghost detections but at the cost of radar time.

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