The systems

$$\begin{array}{rcrcrcrl}
x_1 & - & 2x_2 & - & 7x_3 & = & -1 & \qquad E_1 \\
-x_1 & + & 3x_2 & + & 6x_3 & = & 0 & \qquad E_2
\end{array}$$

and

$$\begin{array}{rcrcrcrl}
x_1 & - & 2x_2 & - & 7x_3 & = & -1 & \qquad E_1 \\
 & & x_2 & - & x_3 & = & -1 & \qquad E_2 + E_1
\end{array}$$

have the same solutions:

Let $\vec{x} = (x_1, x_2, x_3)$.

$\vec{x}$ a solution of E_1 and $E_2 \Longrightarrow \vec{x}$ a solution of (E_1 and) $E_1 + E_2$.

Since $E_1 = (E_1 + E_2) - E_2$,

$\vec{x}$ a solution of $E_1 + E_2$ and $E_2 \Longrightarrow \vec{x}$ a solution of E_1 (and $E_1 + E_2$)

Similarly, the systems

$$\begin{array}{rcrcrcrl} x_1 & - & 2x_2 & - & 7x_3 & = & -1 & \qquad E_1 \\ & & x_2 & - & x_3 & = & -1 & \qquad E_2 + E_1 \end{array}$$

and

$$\begin{array}{rcrcrcrl} x_1 & & & - & 9x_3 & = & -3 & \qquad E_1 + 2(E_2 + E_1) \\ & & x_2 & - & x_3 & = & -1 & \qquad E_2 + E_1 \end{array}$$

have the same solutions.

Psychological step: Stop thinking of x_3 as a variable. Rather, think of it as a **parameter**.

Distinguish variables and the parameter notationally: Set $t = x_3$.

$$\begin{array}{rrcrl} x_1 & & = & 9t - 3 & \qquad E_1 + 2(E_2 + E_1) \\ & x_2 & = & t - 1 & \qquad E_2 + E_1 \end{array}$$

Conclusion: $(x_1, x_2, x_3) = (9 - 3t, t - 1, t)$ is a solution **for all** t.

In particular, the system has **infinitely many solutions**.

Analyze the above argument: All solutions have the form

$$(x_1, x_2, x_3) = (9 - 3t, t - 1, t), \qquad (*)$$

for some t.

$(*)$ is called the **general solution** of the system.