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and

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Since $E_2 = (E_1 + E_2) - E_1$,

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Distinguish variables and the parameter notationally: Set $t = x_3$.

$$\begin{array}{rcrcl} x_1 & & = & 9t - 3 & \qquad E_1 + 2(E_2 + E_1) \\ & x_2 & = & t - 1 & \qquad E_2 + E_1 \end{array}$$

Conclusion: $(x_1, x_2, x_3) = (9t - 3, t - 1, t)$ is a solution **for all** t.

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for some t.

$(*)$ is called the **general solution** of the system.