Math 367 – Tutorial #2

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September 21-23, 2021

1. Find equations for the tangent plane and the normal line...
   (a) ...to $z = e^{2x^2+3y}$ at $(x, y) = (3, -6)$;
   (b) ...to $y = \sqrt{x_1^2 + 3x_2^2 + x_3^2}$ at $\mathbf{x} = (6, 2, 1)$.
2. Use linear approximation to estimate...
   (a) $f(3.05, -6.1)$, where $f(x, y) = e^{2x^2+3y}$;
   (b) $f(6.1, 1.9, 0.9)$, where $f(\mathbf{x}) = \sqrt{x_1^2 + 3x_2^2 + x_3^2}$.

   What is the relative error percentage in your approximation?

$$\text{relative error percentage} = \left| \frac{f(\mathbf{x}) - L(\mathbf{x})}{f(\mathbf{x})} \right| \times 100\%$$

   Note that you computed the relevant linear approximations (tangent planes) in problem 1.
3. Without doing any calculations, find an equations of the normal line and tangent plane to the sphere S_r of radius r,

$$S : x^2 + y^2 + z^2 = r,$$

   at an arbitrary point $(a, b, c) \in S$.
4. The surface S defined by

$$-x^2 - y^2 + z^2 = 1$$

   is a two-sheeted hyperboloid. Find equations of the tangent planes to S at the points $(2, 2, \pm 3)$.