MATH 311 – WINTER 2018 – LAB 1

(1) Let

$$U := \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : 21x - 7z = 0 \right\}.$$

(a) Find a matrix A such that $U = N(A)$.

(b) By solving the system $A\mathbf{x} = \mathbf{0}$, find a matrix B such that $U = C(B)$.

(2) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 4 \\ -7 \\ -10 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(a) Write $\mathbf{b}_j$ as a linear combination of the columns of A, if possible.

(b) Prove that each column of A is in the span of the other two.

(3) Let

$$U := \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x^2 + y^2 = z^2 \right\}.$$

(a) Prove that U contains $\mathbf{0}$.

(b) Prove that for all $\mathbf{u} \in U$ and for all $t \in \mathbb{R}$, $t\mathbf{u} \in \mathbb{R}$.

(c) Find vectors $\mathbf{u}_1, \mathbf{u}_2 \in U$ such that $\mathbf{u}_1 + \mathbf{u}_2 \notin U$. Conclude that U is not a subspace of $\mathbb{R}^3$.

(4) Let $\mathbf{v} \in \mathbb{R}^n$ and let

$$U := \{\mathbf{x} \in \mathbb{R}^n : \mathbf{v} \cdot \mathbf{x} = 0\}.$$

(a) Prove, from the definition, that U is a subspace of $\mathbb{R}^n$.

(b) Find a matrix $V \in \mathbb{R}^{1\times n}$ such that $U = N(V)$. Conclude, again, that U is a subspace of $\mathbb{R}^n$.

(5) Let

$$U = \left\{ \begin{bmatrix} x+y \\ x-y \\ 3x \end{bmatrix} : x, y \in \mathbb{R} \right\}.$$

(a) Find a matrix $A \in \mathbb{R}^{3\times 2}$ such that $U = C(A)$.

(b) Find a column vector $\mathbf{v} \in \mathbb{R}^3$ that is orthogonal to both columns of A.

(c) Show that

$$\mathbf{v}^T A = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

(Hint: Interpret the columns of $\mathbf{v}^T A$ as dot products.)

(d) Use (c) to show that

$$\mathbf{v}^T A\mathbf{x} = \begin{bmatrix} 0 \end{bmatrix},$$

for all $\mathbf{x} \in \mathbb{R}^2$. Conclude that $C(A)$ is a subset of $N(\mathbf{v}^T)$.

(e) Interpret $N(\mathbf{v}^T)$ as a plane in $\mathbb{R}^3$, passing throgh the origin. Find an equation for it, in the form

$$ax + by + cz = 0$$

.

(f) By solving the system $\mathbf{v}^T\mathbf{x} = \mathbf{0}$, find a matrix $B \in \mathbb{R}^{3\times 2}$ such that

$$N(\mathbf{v}^T) = C(B).$$

(g) ($*$) Prove that $C(A) = C(B)$.