MATH 311 — WINTER 2018 — LAB 4

1. Find the least-squares solution of $A\mathbf{x} = \mathbf{b}$, where:
   (a) $A = \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 8 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
2. Consider the dataset
$$(0,0),\ (1,1),\ (2,3),\ (3,6).$$
   (a) Find the least-squares line for these data, and the associated sum of squared errors (SSE).
   (b) Find the quadratic equation,
$$y(x) = \beta_0 + \beta_1 x + \beta_2 x^2,$$
   minimizing the SSE.
   Suggestion: Find a matrix X such that
$$X \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$
3. Consider the following data, collected from

   `https://data.worldbank.org/indicator/SP.POP.TOTL`,

   describing the total human population of planet earth.

Year	Population ($\times 10^9$)
1970	3.7
1980	4.4
1990	5.3
2000	6.1
2010	6.9

   Find the least-squares line for the associated logarithmic plot (t vs. $\log_{10}(\text{pop})$).
4. Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$
   Note that the columns of A are *not* linearly independent.
   (a) Find the projection, $\widehat{\mathbf{b}}$, of $\mathbf{b}$ onto $C(A)$.
   (b) Solve the system
$$A\widehat{\mathbf{x}} = \widehat{\mathbf{b}}.$$
   (c) Find the unique solution that belongs to $C(A^T)$.
   (d) Can you identity the minimal solution, i.e., the one with smallest length?