MATH 307 — Worksheet #7

1. Find the Laurent expansion of
$$f(z) = \frac{1}{z(z^2 + 1)}$$
valid in the given region.

   (a) $0 < |z| < 1$

   (b) $|z| > 1$

2. Find the Laurent expansions of
$$f(z) = \frac{1}{1 + z^2} + \frac{1}{3 - z}$$
valid in the given region.

   (a) $|z| < 1$

   (b) $1 < |z| < 3$

   (c) $|z| > 3$

3. Find the poles of the $f(z)$. For each such pole, a, determine:

   - $\text{ord}_a f(z)$,
   - $\text{res}_a f(z)$,
   - the annuli of convergence of the Laurent expansions of $f(z)$ around a.

   (a) $f(z) = \dfrac{e^z(z - 3)}{(z - 1)(z - 5)}$

   (b) $f(z) = \dfrac{e^z - 1}{z}$

   (c) $f(z) = \dfrac{e^z - 2}{z}$

   (d) $f(z) = \dfrac{\cos z}{1 - z}$

(e) $f(z) = \dfrac{z^2 - 1}{\cos(\pi z) + 1}$

4. Find and classify the singularities of $f(z)$ (removable, pole of order n, essential singularity).

(a) $f(z) = \sin \dfrac{1}{z}$

(b) $f(z) = \csc \dfrac{1}{z}$