MATH 307 — Quiz #1

Instructions: You have 50 minutes to solve all three multipart problems. All answers should be exact – no decimal approximations. No calculators, notes, or other aids. Hand in only your solution booklet. There are 21 points available, plus 1 bonus point, but I'll record the grade out of 20.

1.

(a) (2 points) Compute $|z|^2$, where $z = \dfrac{5i}{(1-i)(2-i)(3-i)}$.

(b) (2 points) Express $\operatorname{Log}\left(\dfrac{1}{4} - \dfrac{1}{4}i\right)$ in the form $x + iy$. Here, Log denotes the **principal branch** of the logarithm.

(c) (2 points) Express $z = (\sqrt{3} - i)^6$ in the form $re^{i\theta}$.

(d) (2 points) Let $w = e^{\bar{z}^2/2}$. Express $\operatorname{Re}(w)$ and $\operatorname{Im}(w)$ in terms of $x = \operatorname{Re}(z)$ and $y = \operatorname{Im}(z)$.

(e) (2 points) Find all values of $(-8i)^{1/3}$. Express them in the form $x + iy$.

2. Sketch the sets in the complex plane, each on its own set of axes. Label points, radii, angles, etc. so that your meaning is unambiguous.

(a) (2 points) $A = \{z : |z| < |z+1|\}$

(b) (2 points) $B = \left\{z = re^{i\theta} : 1 \le r \le 2 \text{ and } -\dfrac{\pi}{3} \le \theta \le \dfrac{\pi}{3}\right\}$

(c) (2 points) $C = \{e^{i\pi/6} z^2 : z \in B\}$

(d) (1 point) $\boxed{\star\star \text{ Bonus } \star\star}$ $D = \{z : z^2 \in B\}$ $\boxed{\star\star\text{Bonus}\star\star}$

3. (5 points) Let $\sqrt{\cdot}$ be the branch of the square root defined by

$$\sqrt{re^{i\theta}} = \sqrt{r}\, e^{i\theta/2} \qquad \text{for} \qquad \theta \in [\pi, 3\pi).$$

For which z does $\sqrt{z^2} = z$ hold?

Hint: Write $z = re^{i\psi}$ with $\psi \in [-\frac{\pi}{2}, 3\pi)$. When computing z^2, consider the cases $\psi \in [-\frac{\pi}{2}, \frac{\pi}{2})$ and $\psi \in [\frac{\pi}{2}, \frac{3\pi}{2})$ separately. In each case, identify a k such that $2\psi + 2\pi k$ belongs to $[\pi, 3\pi)$.