(1) Let $f(z)$ be a function satisfying the following properties:

- $f(z)$ is analytic on $\mathbb{C} - \{0, 1\}$,
- $f(z)$ has a simple pole at $z = 0$,
- $f(z)$ has a removable singularity at $z = 1$.

Which of the following must hold? [Present list of checkboxes.]

(a) $\lim_{z \to 0} |f(z)| = \infty$

(b) $\lim_{z \to 1} |f(z)|$ exists and is finite.

(c) The Taylor expansion of $f(z)$ around $z = \frac{2}{3}$ has radius of convergence $\frac{1}{3}$.

(d) $\lim_{z \to 0} z^2 f(z) = 0$

(e) $\int_{|z-\frac{1}{2}|=1} f(z)\, dz \neq 0$

(f) $f(z)$ has a Laurent expansion convergent on $|z| > 0$.
[ans: a, b, d, f]

(2) `$a = list_random(2, 3, 5, 6, 7); $b = random(2, 9);`

The power series

$$f(z) = \sum_{n=0}^{\infty} \frac{z^{2n}}{(n + \texttt{\$b})\texttt{\$a}^n}$$

has radius of convergence $R =$ [ ] [ans: $\sqrt{\texttt{\$a}}$].

(3) `$a = list_random(2, 3, 5, 6, 7); $b = random(2, 9);`

Let

$$f(z) = \frac{\sin(\texttt{\$b}z)}{(e^{\texttt{\$b}z} - 1)(z^4 + z^2)}.$$

$f(z)$ has a pole of order [ ] [ans: 2] at $z = 0$ and simple poles at $z = \pm$ [ ] [ans: i or $-i$].

(4) Let

$$f(z) = \sum_{n=0}^{\infty} a_n \left(z - \frac{2}{\pi}\right)^n$$

be the Taylor expansion of $\sin \frac{1}{z}$ around $z = \frac{2}{\pi}$.

Then $a_0 =$ [ ] [ans: 1] and $a_1 =$ [ ] [ans: 0].

Its radius of convergence is $R =$ [ ] [ans: $1/\pi$].

(5) Let

$$f(z) = \sum_{n=-\infty}^{\infty} a_n \left(z - \frac{2}{\pi}\right)^n$$

be the Laurent expansion of

$$f(z) = \frac{2z}{z^2 + 1}$$

convergent in $|z - i| > 2$.

Then $a_{-1} = \boxed{}$ [ans: 1] and $a_{2020} = \boxed{}$ [ans: 0].