# MATH 307 — Quiz #1

| Question: | 1 | 2 | 3 | Total |
|---|---|---|---|---|
| Points: | 10 | 6 | 5 | 20 |
| Score: | | | | |

1.

(a) (2 points) Compute $|z|^2$, where $z = \dfrac{5i}{(1-i)(2-i)(3-i)}$.

(b) (2 points) Express $\operatorname{Log}\left(\dfrac{1}{4} - \dfrac{1}{4}i\right)$ in the form $x + iy$. Here, Log denotes the principal branch of the logarithm.

(c) (2 points) Express $z = (\sqrt{3} - i)^6$ in the form $re^{i\theta}$.

(d) (2 points) Let $w = e^{\bar{z}^2/2}$. Express $\operatorname{Re}(w)$ and $\operatorname{Im}(w)$ in terms of $x = \operatorname{Re}(z)$ and $y = \operatorname{Im}(z)$.

(e) (2 points) Find all values of $(-8i)^{1/3}$. Express them in the form $x + iy$.

2. Sketch the sets in the complex plane, each on its own set of axes. Label radii, angles, etc. so that your meaning is unambiguous

(a) (2 points) $A = \left\{ z = re^{i\theta} : 1 \le r \le 2 \text{ and } -\dfrac{\pi}{3} \le \theta \le \dfrac{\pi}{3} \right\}$

(b) (2 points) $B = \{e^{i\pi/6} z^2 : z \in A\}$

(c) (2 points) $C = \{z : z^2 \in A\}$

3. (5 points) Let $\sqrt{\cdot}$ be the branch of the square root defined by

$$\sqrt{re^{i\theta}} = \sqrt{r}e^{i\theta/2} \qquad \text{for} \qquad \theta \in [\pi, 3\pi).$$

For which $z$ does $\sqrt{z^2} = z$ hold?

Hint: Write $z = re^{i\psi}$ with $\psi \in [-\frac{\pi}{2}, 3\pi)$. When computing $z^2$, consider the cases $\psi \in [-\frac{\pi}{2}, \frac{\pi}{2})$ and $\psi \in [\frac{\pi}{2}, \frac{3\pi}{2})$ separately. In each case, identify a $k$ such that $2\psi + 2\pi k$ belongs to $[\pi, 3\pi)$.