MATH 311 – WINTER 2018 – LAB 2

(1) Critique the following incorrect/nonsensical statements.

(a) If $N(A) = \{0\}$ then A is linearly independent.

(b) Vectors $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are linearly dependent when all of their linear combinations are trivial.

(c) $\operatorname{span}(\mathbf{a}_1, \ldots, \mathbf{a}_n)$ is a linear combination of $\mathbf{a}_1, \ldots, \mathbf{a}_n$.

(d) A vector $\mathbf{b}$ belongs to the column space of A if and only if $A\mathbf{x} = \mathbf{b}$ has a unique solution.

(e) $\mathbf{b}$ is a column space of A if and only if $A\mathbf{x} = \mathbf{b}$ has a solution.

(f) The columns of $A \in \mathbb{R}^{m\times n}$ form a basis of $\mathbb{R}^n$ if $N(A)$ is the zero subspace of $\mathbb{R}^m$ and $C(A)$ spans $\mathbb{R}^n$.

(2) Are the vectors

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} -3 \\ 2 \\ 1 \\ -5 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 4 \\ -2 \\ -1 \\ 6 \end{bmatrix}$$

linearly independent? If not, write one of the $\mathbf{a}_j$ as a linear combination of the other two.

(3)

$$A = \begin{bmatrix} -1 & 2 & 4 & 3 \\ 2 & 0 & 4 & -2 \\ 1 & 3 & 11 & 2 \\ 0 & -1 & -3 & -1 \end{bmatrix}.$$

(a) Identify the pivot columns of A. Write the nonpivot columns of A as as linear combinations of its pivot columns.

(b) Find a basis of $N(A)$.

(4) Let $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3 \in \mathbb{R}^m$ and let

$$\mathbf{b}_1 = \mathbf{a}_1, \quad \mathbf{b}_2 = \mathbf{a}_1 + \mathbf{a}_2, \quad \mathbf{b}_3 = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3.$$

(a) Prove:

$$\operatorname{span}(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3) = \operatorname{span}(\mathbf{b}_1, \mathbf{b}_1, \mathbf{b}_1).$$

(b) Prove that $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are linearly independent if and only if $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$ are.

(5) Prove the statement or give a counterexample

(a) A and $\operatorname{rref}(A)$ have the same column space.

(b) A and $\operatorname{rref}(A)$ have the same nullspace.