# The Beagle Project

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30/06/2016

## 1 Introduction

This project is still in it's early stages, and the following document simply defines the equations of the DC motor model which is used to simulate the currently implemented regulators. A more detailed derivation and definitions will be added as soon as possible.

## 2 Motor model

### Governing equations

Assuming a simple DC motor model with constant magnetic field, Newton's and Kirchoff's laws can be used to derive the governing equations

$$\frac{d^2\theta(t)}{dt^2} = \frac{1}{J}\Big(K_t i(t) - b\frac{d\theta(t)}{dt}\Big) \tag{1}$$

$$\frac{di(t)}{dt} = \frac{1}{L}\Big( - Ri(t) + V(t) - K_e\frac{d\theta(t)}{dt}\Big) \tag{2}$$

### Statespace form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{3}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{4}$$

Let

$$\mathbf{x}(t) = \begin{bmatrix}\theta(t) & \dot{\theta}(t) & i(t)\end{bmatrix}^T, \qquad \mathbf{u}(t) = V(t) \tag{5}$$

then

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b/J & K_t/J \\ 0 & -K_e/L & -R/L \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}, \quad \mathbf{C} = \mathbb{I}_{3\times 3} \tag{6}$$

**Laplace domain equivalent**

Let

$$\mathcal{L}\left\{\Theta(t)\right\}_s = \Theta(s), \quad \mathcal{L}\left\{i(t)\right\}_s = I(s), \quad \mathcal{L}\left\{V(t)\right\}_s = V(s), \tag{7}$$

then

$$s^2\Theta(s) = \frac{1}{J}\Big(K_t I(s) - bs\Theta(s)\Big) \tag{8}$$

$$sI(s) = \frac{1}{L}\Big( - RI(s) + V(s) - K_e s\Theta(s)\Big) \tag{9}$$

**transfer function from $V(s)$ to $\Theta(s)$**

$$G(s)_{U\to\Theta} = \frac{\Theta(s)}{U(s)} = \frac{K_t}{s((sL+R)(Js+b)+K_tK_e)}U(s) \tag{10}$$

Using a partial fraction decomposition, this may be written

$$G(s)_{U\to\Theta} = \frac{A}{s} - \frac{B+Cs}{s^2+Ds+E} = \frac{A}{s} + \frac{B}{s^2+Ds+E} + \frac{Cs}{s^2+Ds+E} \tag{11}$$

$$\frac{Kt}{(s*(R*b+Ke*Kt))} - \frac{(J*Kt*R+Kt*L*b)/(R*b+Ke*Kt)+(J*Kt*L*s)/(R*b+Ke*Kt)}{J*L*s^2+(L*b+J*R)*s+R*b+Ke*Kt} \tag{12}$$

**transfer function from $V(s)$ to $I(s)$**

**LS-estimation**

Using a Bilinear z-transform, it is evident that

$$\mathcal{Z}\left\{G(s)\right\}_z = \frac{a_2z^2+a_1z^1+a_0}{z^3+b_2z^2+b_1z^1+b_0} \tag{13}$$

Letting

$$\begin{cases} \theta = \begin{bmatrix} a_2 & a_1 & a_0 & b_2 & b_1 & b_0 \end{bmatrix}^T \\ \varphi(t-1) = \begin{bmatrix} u(t-1) & u(t-2) & u(t-3) & -y(t-1) & -y(t-2) & -y(t-3) \end{bmatrix}^T \end{cases} \tag{14}$$

we note that the discrete time difference equation can be written

$$y(t) = \varphi(t-1)^T\theta \tag{15}$$

## 2.1 Controllers

$$\dot{x}_1 = x_2 \tag{16}$$

$$\dot{x}_2 = (-bx_2 + K_t x_3)/J \tag{17}$$

$$\dot{x}_3 = (-K_e x_2 - Rx_3 + u)/L \tag{18}$$

with

$$V(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \tag{19}$$

$$\begin{aligned}
\frac{\partial V(x_1, x_2, x_3)}{\partial t} &= x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 && (20)\\
&= x_1x_2 + x_2(-bx_2 + K_t x_3)/J + x_3(-K_e x_2 - Rx_3 + u)/L && (21)\\
&= x_1x_2 - bx_2^2/J + (K_t/J - K_e/L)x_2x_3 - Rx_3^2/L + u/L && (22)
\end{aligned}$$