The Crazy-Cab Project

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Project Status

This document describes the crazy-cab project with the intention of exploring motion-planning and control of the unmanned ground vehicle (*UGV*) control object. A comprehensive dynamical model has been derived, complete with a working Simulink implementation and a discrete time state-space formulation. The system identification is still rather crude with inertial parameters based on platonic solids, and the friction model has yet to be identified based on ground truth data. Currently, I am working on implementing a driver in *ROS* to control the car and developing the Kinect 2 signal processing to access this data. The plan is then to implement a dynamical filter for state estimation and some robust means of motion planning in combination with *MPC*.

Project Overview

In **Section 1**, a general mode of the *UGV* is developed. The considered hardware poses many interesting challenges in this respect on account of being very small, at $2.3 \times 1.4 \times 1.0$ $[cm]$. Consequently, the car only supports (i) back-wheel drive and (ii) rigid steering (sideways- and forward translation with simultaneous rotation). This implies any steering requiring at lest one wheel-pair being in a drifting state, often times with both wheel-pairs being in a drifting state, requiring (iii) a very accurate friction model. In addition, the car implements an *IMU* with a tri-axis accelerometer and tri-axis gyroscope. Consequently, the model should preferably implement (iv) the car attitude rate and acceleration in the body frame, while (v) depicting the position and velocity in the global frame. Finally, (vi) the centre of mass does not align with the centre of rotation in the *UGV* and also needs to be be modelled. To recapitulate, the points which need to be taken into consideration are.

(i) Back-wheel drive

(ii) Rigid steering

(iii) Friction

(iv) Inclusion of the attitude rate and acceleration in the body frame

(v) Depiction of position and velocity in the global frame

(vi) Offset between centre of mass and centre of rotation

In **Section 2**, the parameters of the model are identified using a special method of clothoid fitting developed for the small *UGV*. The inertial parameters of the rigid-body are presented, parameters in the motor dynamics are derived and the wheel moment of inertia is computed.

In **Section 3**, the motion planning of the *UGV* *will be* developed the property of differential flatness through an optimisation based approach. *Not written*

In **Section 4**, the control of the *UGV* *will be* presented, both with simple *PID* and *MPC* control.*Not yet begun*

In **Section 5**, the state estimation with Kinect 2 cameras *will be* presented using an UKF formulation running the non-linear model. *Not yet begun*

1 Car dynamics

As we have access to three dimensional gyroscopic- and accelerometer data, a force-based model is developed using the Newton-Euler equations with the PWM duty cycles of the motors as input signals. The purpose of this model is to accurately describe the UGV when in a state of sideways- and forward translation with simultaneous rotation. For this purpose, we also explore friction models general enough to accurately depict a "drifting" state of the UGV. For future reference, we let $\mathbf{p}$ [m] denote the position of the UGV centre of mass in a global Cartesian coordinate system, $\mathbf{q}$ denotes the Cayley-Dickinson constructed quaternion in it's Hamilton form, used to describe an $SO(3)$ rotation from the body to the inertial coordinate system. We also denote an angular offset of the two electric motors by θ_i [rad], such that $\dot{\theta}_i$ is the angular speed of each motor. Finally, the frequency duty cycles of the pulse-width modulated (PWM) signals to the two motors are denoted d_i (see Table 1)

Table 1: Definitions of the signals in the UGV system.

Signal	Notation	Dimension	Unit
Position	$\mathbf{p} = \begin{bmatrix} x & y & z \end{bmatrix}^T$	$\in \mathbb{R}^{3\times 1}$	$[m]$
Attitude	$\mathbf{q} = \begin{bmatrix} q_w & \mathbf{q}_v \end{bmatrix}^T = \begin{bmatrix} q_w & q_x & q_y & q_z \end{bmatrix}^T$	$\in \mathbb{R}^{4\times 1}(\mathbb{H})$	$[unitless]$
Angular rate	$\boldsymbol{\omega} = \begin{bmatrix} x & y & z \end{bmatrix}^T$	$\in \mathbb{R}^{3\times 1}$	$[rad/s]$
Wheel angle	$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T$	$\in \mathbb{R}^{2\times 1}$	$[rad]$
Duty cycle	$\mathbf{d} = \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T$	$\in [0,1]^{2\times 1}$	$[unitless]$

Basis vectors in the respective coordinate systems are written in bold font with a hat $\hat{\cdot}$, where the sub-indexing $\cdot_{\mathcal{G}}$ refers to a vector in the global coordinate system. The inertial system, sub-indexed $\cdot_{\mathcal{I}}$, is centred at a position $\mathbf{p}$ in the global frame, in the quadcopter centre of mass. Finally, the body frame, sub-indexed $\cdot_{\mathcal{B}}$, is defined with origin in the centre of mass and, rotated by from the inertial frame. This rotation is defined as an isometry with fixed origin in a three-dimensional Euclidean space, $SO(3)$, sub-indexed with $\cdot_{\mathcal{IB}}$ if rotating vectors from the the inertial or global frame to the body frame. Conversely, maps from the body frame to the global or inertial frames are denoted $\cdot_{\mathcal{BI}}$ (see Figure 2).

We define the vectors from the body origin to the centre of mass, $\boldsymbol{\Delta}_{cm}$ $[m]$, from the body origin to the i wheels, $\boldsymbol{\Delta}_i$ $[m]$, and from the body origin to the point where the drag force is applied, $\boldsymbol{\Delta}_d$ $[m]$. In the force diagram, these vectors may be identified as

$$\boldsymbol{\Delta}_{cm} = \begin{bmatrix} l_{cm} \\ 0 \\ h \end{bmatrix}^T, \quad \boldsymbol{\Delta}_1 = \begin{bmatrix} 0 \\ -w \\ 0 \end{bmatrix}^T, \quad \boldsymbol{\Delta}_2 = \begin{bmatrix} 0 \\ w \\ 0 \end{bmatrix}^T, \quad \boldsymbol{\Delta}_3 = \begin{bmatrix} l \\ w \\ 0 \end{bmatrix}^T, \quad \boldsymbol{\Delta}_4 = \begin{bmatrix} l \\ -w \\ 0 \end{bmatrix}^T, \quad \boldsymbol{\Delta}_d = \begin{bmatrix} l_d \\ 0 \\ h \end{bmatrix}^T \tag{1}$$

Now, assume each of two motors $i \in [1, 2]$, generate a torque

$$\tau_i^w = I^w \ddot{\theta}_i^w \qquad [N \cdot m] \tag{2}$$

in the $\hat{\mathbf{y}}_{\mathcal{B}}$-direction where I^w $[kg \cdot m/s^2]$ denotes the wheel moment of inertia and $\ddot{\theta}_i^w$ $[rad/s^2]$ denotes the angular acceleration of the rotor shaft. This results in a thrust accelerating the car

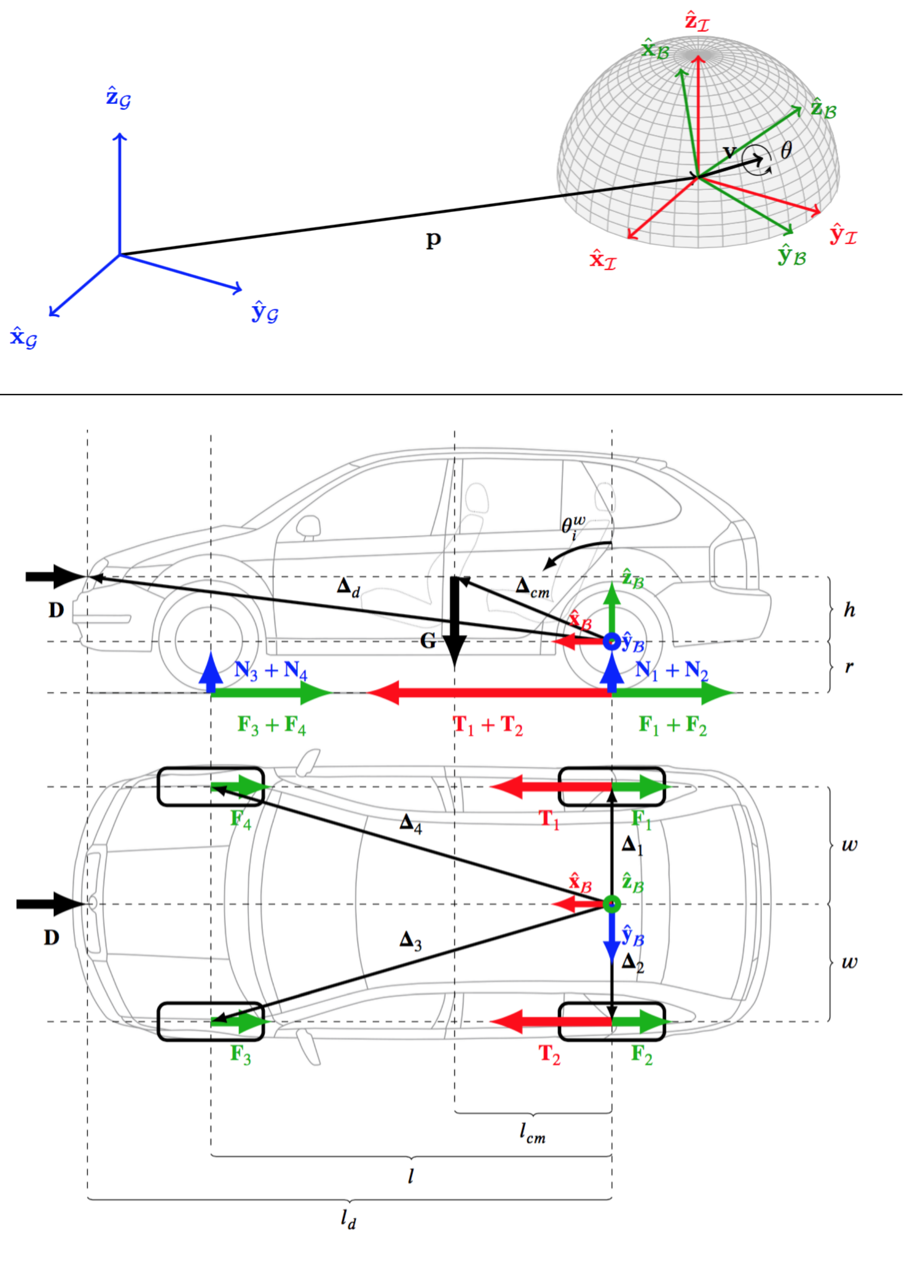

Figure 1: *Top:* The global (blue), inertial (*red*) and body (*green*) coordinate system used to describe the *UGV* rotation and position. *Bottom:* The forces and torques of the toy car.

in the $\hat{\mathbf{x}}_\mathcal{B}$-direction

$$T_i^w = \frac{\tau_i^w}{r} = \frac{I^w}{r}\ddot{\theta}_i^w \qquad [N] \tag{3}$$

Furthermore, consider reactive forces caused by the friction in each wheel i, here modelled as a vector, $\mathbf{F}_i \in \mathbb{R}^3$ $[N]$, residing in the $\hat{\mathbf{x}}_\mathcal{B}\hat{\mathbf{y}}_\mathcal{B}$-plane. FInally, we define the normal forces on each wheel as $\mathbf{N}_i$ $[N]$ confined to the $\hat{\mathbf{z}}_\mathcal{B}$ direction. The total forces in the body frame is then

$$\mathbf{F}_\mathcal{B} = \mathbf{R}_{\mathcal{B}\mathcal{I}}^T(-mg\hat{\mathbf{z}}_\mathcal{G}) + (T_1^w + T_2^w)\hat{\mathbf{x}}_\mathcal{B} - \mathbf{F}_1 - \mathbf{F}_2 - \mathbf{F}_3 - \mathbf{F}_4 - \mathbf{D} + (N_1 - N_2 - N_3 - N_4)\hat{\mathbf{z}}_\mathcal{B} \qquad [N] \tag{4}$$

Similarly, the total torques of the system may be written

$$\boldsymbol{\tau}_\mathcal{B} = [\boldsymbol{\Delta}_{cm}]_\times \mathbf{G} + \sum_{i=1}^{2}[\boldsymbol{\Delta}_i]_\times \mathbf{T}_i + \sum_{i=1}^{4}[\boldsymbol{\Delta}_i]_\times(\mathbf{F}_i + \mathbf{N}_i) + [\boldsymbol{\Delta}_d]_\times \mathbf{D} \qquad [N \cdot m] \tag{5}$$

Using the above definitions, we may then form the dynamical model of the car by the Newton-Euler equations

$$\begin{bmatrix}\mathbf{F}_\mathcal{B}\\ \boldsymbol{\tau}_\mathcal{B}\end{bmatrix} = \begin{bmatrix} m\mathbf{I} & -m[\boldsymbol{\Delta}_{cm}]_\times \\ m[\boldsymbol{\Delta}_{cm}]_\times & \mathbf{I}_\mathcal{B} - m[\boldsymbol{\Delta}_{cm}]_\times^2\end{bmatrix}\begin{bmatrix}\mathbf{a}_\mathcal{B}\\ \boldsymbol{\alpha}_\mathcal{B}\end{bmatrix} + \begin{bmatrix} m[\boldsymbol{\omega}_\mathcal{B}]_\times^2\boldsymbol{\Delta}_{cm} \\ [\boldsymbol{\omega}_\mathcal{B}]_\times(\mathbf{I}_\mathcal{B} - m[\boldsymbol{\Delta}_{cm}]_\times^2)\boldsymbol{\omega}_\mathcal{B}\end{bmatrix}. \tag{6}$$

With this model, we may devise controllers and simulate the system. However, there two significant question marks which need to be straightened out, and any input would be greatly appreciated.

1.1 Friction model

At this stage, we use a very simple model to represent the friction forces based solely on viscous and sticky friction, components which depend on the speed of the car in the body frame. At each wheel, this speed is given by $\mathbf{v}_i = \dot{\mathbf{p}}_\mathcal{B} + [\boldsymbol{\Delta}_i]_\times\boldsymbol{\omega}_\mathcal{B}$. As the friction is restricted to the $\hat{\mathbf{x}}_\mathcal{B}\hat{\mathbf{y}}_\mathcal{B}$-plane, we simply let

$$\mathbf{F}_i(\mathbf{v}_i) = \begin{bmatrix}\gamma_x \mathrm{sign}(\mathbf{v}_i\hat{\mathbf{x}}_\mathcal{B}) + \gamma_c\mathbf{v}_i\hat{\mathbf{x}}_\mathcal{B}\\ \gamma_y \mathrm{sign}(\mathbf{v}_i\hat{\mathbf{y}}_\mathcal{B}) + \gamma_c\mathbf{v}_i\hat{\mathbf{y}}_\mathcal{B}\\ 0\end{bmatrix} \qquad [kg \cdot m/s^2] \tag{7}$$

for some $\gamma_i \in \mathbb{R}^+$. With this friction and rigid body model, we may simulate the system which yields the expected behaviour when starting with an initial body rate with $\omega_z(t=0) < 0$ and $d_1(t), d_2(t) \equiv$ constant with $d_1(t) > d_2(t)$. The simulation was carried out on plane ground, but the model can encompass slanted ground as well.

2 System identification

In this section, we define the system parameters based on well known results if inertia in platonic solids, a simple regression to find the motor parameters and a clothoid based method of identifying the friction parameters.

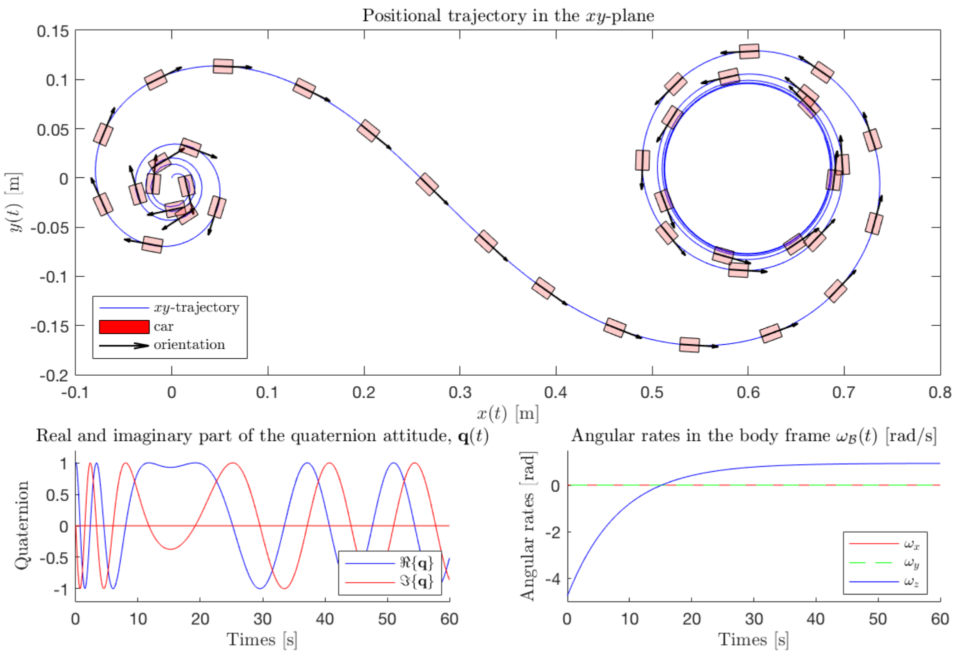

Figure 2: Simulated open-loop response in the xy-plane with $\mathbf{p}(t=0) = \mathbf{0}$, $\dot{\mathbf{p}}(t=0) = \mathbf{0}$, $\mathbf{q}(t=0) = [1,0,0,0]^T$, $\omega_{\mathcal{B}}(t=0) = [0,0,-4.5]^T$ with $\mathbf{d} = [0.5, 0.4]^T$.

2.1 Identification of motor parameters

If we consider the wheel as a solid homogenous cylinder with radius R, length L, mass M, volume V and density ρ, we may form a mass element $dm = \rho dV = 2\pi\rho L r dr$. The wheel's moment of inertia around the rotational axis is then

$$I_w = \int_0^M r^2 dm = 2\pi\rho L \int_0^R r^3 dr = 2\pi L \Big[\frac{M}{\pi R^2 L}\Big]\frac{R^4}{4} = \frac{1}{2}MR^2 \qquad [kg \cdot m^2] \tag{8}$$

As $R = 0.002$ $[m]$ and the wheel weight $M \approx 0.5$ $[g]$ we conclude that the wheel moment of inertia is $I_w \approx 1.0 \cdot 10^{-9}$ $[kg/s^2]$.

TODO: Identify motor parameters

2.2 Identification of rigid-body parameters

To get a rough estimate of the moment of inertia tensor of the rigid body, we assume that the car may be approximated by homogenous cuboid with dimensions $2l_{cm} \times 2w \times 2h$. The moment of inertia tensor may then be computed as

$$I_{\mathcal{B}} \approx \frac{m}{3}\begin{bmatrix} h^2 + l_{cm}^2 & 0 & 0 \\ 0 & w^2 + l_{cm}^2 & 0 \\ 0 & 0 & w^2 + h^2 \end{bmatrix} = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 5.0 & 0 \\ 0 & 0 & 4.2 \end{bmatrix} \cdot 10^{-7} \qquad [kg \cdot m^2] \tag{9}$$

given the measured $m = 0.005$ $[kg]$ and physical dimensions $l_{cm} = 0.01$ $[m]$, $w = 0.007$ $[m]$ and $h = 0.005$ $[m]$.

TODO: Identify friction parameters through clothoid fitting with constant PWM signals

2.3 Identification summary

In summary, the identified parameters of the model are given in Table 2, where it should be noted that the friction coefficient may vary greatly depending on the surface.

Table 2: Identified parameters in the UGV system.

Parameter	Value	Unit
l_d	0.023	$[m]$
l	0.02	$[m]$
l_{cm}	0.01	$[m]$
w	0.007	$[m]$
r	0.002	$[m]$
h	0.005	$[m]$
m	0.005	$[kg]$
g	9.81	$[kg \cdot m/s^2]$
γ_x	-	$[kg/s]$
γ_y	-	$[kg/s]$
γ_c	-	$[kg/s]$
I_w	$1.0 \cdot 10^{-9}$	$[kg \cdot m^2]$
$I_{\mathcal{B}}$	$\begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 5.0 & 0 \\ 0 & 0 & 4.2 \end{bmatrix} \cdot 10^{-7}$	$[kg \cdot m^2]^{3\times 3}$

3 Differential flatness and motion-planning

4 Control

4.1 Torque Vectoring

4.2 Model Predictive Control

5 State Estimation