Sovereign Debt: A Bayesian Signaling Approach

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1. **Central Research Question**

   Sovereign debt has been a tool used by many countries to grow their economy and smooth consumption to alleviate domestic shocks. Most of the literature studies sovereign debt through models in which a borrowing country agrees to a contract with the lending country. Across time, both countries lend to one another. These types of models assume perfect information amongst the two countries and the current financial state of the country only applies if they are at risk of default. My model introduces asymmetric information, through uncertainty as to what the reserves are of the country. While the IMF reports reserve levels for many countries, these are conducted from a survey and therefore there is some sort of uncertainty as to what these reserve levels are. In this paper I will exploit this and build a model to illustrate how developing countries post default have to maintain some sort of luck in auditing of their reserves to improve the stability of their reserves, and therefore their country. Mainly, I seek to answer the following question: Can a financially constrained post default country still develop their economy through external sovereign debt?

2. **Literature Review**

   This model is an applied extension of Espinosa Ray (2022). This contract theory model studies a general principal agent problem where the agent uses Bayesian Persuasion to influence the principal. This mechanism is applied to this paper in a sovereign debt context. In doing so, I have made significant extensions to the paper by making this model dynamic through an evolution of θ and q (to be explained later).

An extensive amount of research into sovereign debt has focused on a general lending relationship that occurs between two countries. If default occurs, the country faces a penalty for some time periods and then at some time period, the country can just enter the market again as if nothing happened. In Arellano (AER 2008), this is modeled through a probability parameter λ. The model I propose gives better context to how the transition from being a defaulted country back to a "normal" country occurs and the struggles and risk the previously defaulted country must endure when trying to build back up their nation post default. In our model, Sovereign debt is necessary for the borrowing country however the distrust of the lending country, built by the default, leads to an asymmetric information problem. This asymmetric information problem has two layers. One is the reserve level of the country (an indication of the ability to pay back a loan) and the other is of the investment the borrowing country puts in place. Despite the borrowing country having a perfect investment with a guaranteed positive return, the asymmetric information leads to them struggling to build themselves back up.

3. **Model Setup**

In this signaling game, there is a lending country (principal) and a borrowing country (agent). There are two possible types of domestic reserves, θ^L, θ^H where $\theta^L = 0 < \theta^H = 1$. The borrowing country at is type θ^L.

This borrowing country has an opportunity each time period to fund a project with guaranteed return $1 + 2r$. If given funding by the lending country, this outcome will be achieved. If no funding is given, the project has to be liquidated and the borrowing country gets a return $1 - r$. This project is known to the developing country but not known by the lender. Note that this is important as full information in this project would lead to the country getting funding every period regardless of their signal realization. This project will strictly be used to increase the reserves hence,

$$\theta^L_{t+1} = \theta^L_0 + rI_{t+1} \text{ where } I_0 = 0 \text{ and } I_{t+1} = \begin{cases} I_t + 2 & \text{if received funding} \\ I_t - 1 & \text{if no funding} \end{cases}$$

The lending country makes a choice each period whether to lend to this country or not. If the lending country believes the country is of type θ_H they offer a risk free rate ensuring the borrowing country receives the full $1+2r$ return. If the lending country believes the country to be of type θ^L, they will still offer a lending contract however at a risk premium such that the net return to investment is $1-r-\gamma$, where $\gamma > 0$ and small. This risk premium is not explicitly derived in this problem however this excessive risk premium could be due to the lenders previous experience participating with θ^L type countries. If the country is believed to be θ^L by the lending country, the borrowing country chooses not to receive the funding as they are better off just liquidating the investment. Hence the payoffs for the lending country are,

$$U = \begin{cases} 0 & \text{if } \theta^i = \theta^L \\ 1 & \text{if } \theta^i = \theta^H \end{cases}$$

The lending country has prior beliefs about the current state of the world (his outside option) and the borrowing country. Over time, the belief about the current state of the world don't change (this will be an extension later on) however the belief about the borrowing country changes due to previous lending actions. The beliefs are characterized below.

$$\mathbf{P}(\text{outside option} = \theta^H) = p \qquad \mathbf{P}(\text{borrowing C} = \theta^H) = q = \begin{cases} 0 & \text{if } p + \alpha A_t < 0 \\ p + \alpha A_t & \\ 1 & \text{if } p + \alpha A_t > 1 \end{cases}$$

where α is a parameter of the lending countries sensitivity to previous trading periods and A_t is a dynamic variable that encompasses the previous trading actions. Specifically,

$$A_{t+1} = \begin{cases} A_t + 1 & \text{if received funding} \\ A_t - 1 & \text{if no funding} \end{cases}$$

The borrowing country chooses an auditing firm to send a signal to the lending country each period as to what their current reserves are. The audit cannot be altered ex-post. We assume there are a multitude of auditing firms associated with different levels of precision. The borrowing country has to pick what type of auditing firm to employ each period.

$$x_t = \theta_t^i + \sigma_t^i \epsilon$$

where σ_t^i is a choice of the borrowing country and $\epsilon \sim \mathcal{N}(0, 1)$. Again, the borrowing country has no ability to alter x_t once the signal is realized. Note that a choice of $\sigma_t^L = 0$ (perfect signal) ensures the borrowing country will never receive funding. We also assume that $\sigma^i > \underline{\sigma}$ to eliminate perfect signaling technology (no perfect audit). The lending country is aware of this signal design and therefore creates a decision rule of lending. There will only be lending if the beliefs are such that,

$$\mathbf{P}(i = H|x) = \frac{q\frac{1}{\sigma_t^H}\phi\left(\frac{x-\theta_t^H}{\sigma_t^H}\right)}{q\frac{1}{\sigma_t^H}\phi\left(\frac{x-\theta_t^H}{\sigma_t^H}\right) + (1-q)\frac{1}{\sigma_t^L}\phi\left(\frac{x-\theta_t^L}{\sigma_t^L}\right)} \geq p$$

This decision rule can be simplified using the properties of normal distributions. This results in a retention regime, $X \in [x_t^-, x_t^+]$, characterized by the solutions to the following equation.

$$(\sigma^2 - (\sigma_t^L)^2)x^2 + 2x((\sigma_t^L)^2\theta_t^H - (\sigma_t^H)^2\theta_t^L) + ((\sigma_t^H)^2(\theta_t^L)^2 - (\sigma_t^L)^2(\theta_t^L)^2 + 2A(\sigma_t^H)^2(\sigma_t^L)^2) \geq 0$$

where $A = \ln(\beta\sigma_b/\sigma_g)$. Furthermore, we assume the lending country has an opportunity to walk away from lending if $\theta_0^L = x^-$. At this point, the game is over.

The borrowing country wants to maximize the probability of receiving funding therefore each period they pick σ_t^L such that,

$$\sigma_t^L \in \arg max_{\sigma_t^L \geq \underline{\sigma}} \int_{x_{t-1}^-}^{x_{t-1}^+} \frac{1}{\sigma_t^L}\phi\left(\frac{x-\theta_t^L}{\sigma_t^L}\right) dx$$

A country of type H, will optimally pick $\sigma_t^H = \underline{\sigma}$ every period. Note here we assume that the country has an information disadvantage in regards to X. This assumption does not significantly affect our general results.

The goal of the borrowing country is to achieve $\theta_t^L = \theta^H$. In this case, the probability of receiving funding is maximized and the country can then continue to grow their reserves. In expectation, this goal is not achievable however with some lucky pulls of ϵ, this goal is achievable. The goal of this paper is to study how "lucky" this financially constrained country must be in order to achieve financially soundness, and thus almost surely guarantee low borrowing rates.

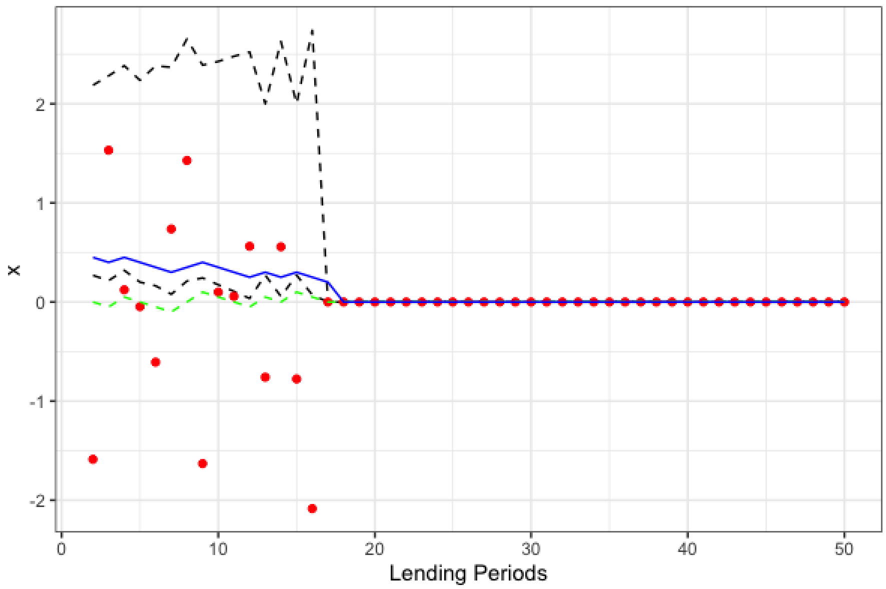
4. **Simulation Results**

For this simulation, I used the following parameters.

$$\theta_0^L = 0 \quad \theta^H = 1 \quad \underline{\sigma} = 0.5 \quad p = 0.5 \quad r = 0.05 \quad \alpha = 0.05$$

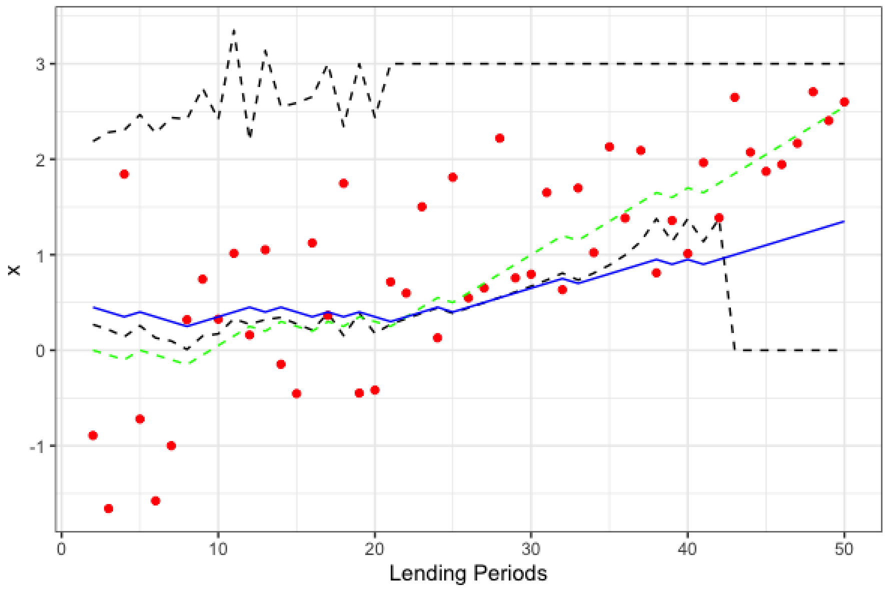
First, I want to visualize the two equilibria that can occur from this model. The first is the failure of the borrowing country to grow. This equilibrium occurs if the borrowing country has "bad luck" in auditing outcomes. If this occurs multiple time in the beginning stages of lending, then the borrowing country will find themselves cutoff from the world economy. In this equilibrium, the lending country becomes more and more pessimistic about the borrowing countries ability to repay until $\underline{x} = \theta^L$. At this point, the lending is completely cutoff. The figure below characterizes the evolution of lending periods leading to the cutoff.

In this figure, the dashed black lines represent the retention regime put in place by the principal each period, the green dotted line represents the evolution of θ, the blue line represents the evolution of q, and the red dots represent the auditing outcome each lending period.



As we can see, the couple rounds of bad luck by the borrowing country, due to auditing outcomes, deteriorate the beliefs of the borrowing country as well as the true reserves of the borrowing country. This eventually leads to the cutoff which occurs around $t = 16$.

The other possible equilibrium is one in which the borrowing country "gets lucky" in terms of auditing. This particular simulation, given by the figure below, is partically interesting as there is a lot of bad luck in the early time periods which almost lead to a terminal relationship (around $t = 8$). However, the borrowing country then gets a lot of good luck and starts to grow their reserves. This figure also points out a key feature of this model in that once a borrowing country starts to grow their reserves, lending becomes easier to obtain. Around $t = 25$, we can lending is approved at the risk-free rate almost every period leading to sustained growth through this investment.



It is often helpful to step away from the asymmetric information case and look at what would occur under full information. Under full information, the investment would be known by the lending country as well as the true reserve level of the borrowing country. Despite $\theta_0 = \theta^L$, the lending country will always lend as this investment has a zero probability of failing, therefore there is no reason to ever say no to lending. Even in the equilibrium in which the borrowing country overcomes the starting reserve level, there is still an opportunity for the funding to not be given. The case of full information will always be more efficient than the case under asymmetric information.

Next, I want to run this simulation a number of times to generate some empirical findings. Specifically I want to find the average length of lending as well as the average ending reserve level. Additionally, I want to find the probability of maintaining a relationship after 50 lending periods. These statistics are given in the table below.

Average Length of Lending	12.99
Average Ending Reserve Level	1.04
$\mathbf{P}$(maintain for $t = 50$)	0.423

While on average, the borrowing country ends up with a ending reserve level of more than 1, the probability it maintains a relationship is roughly 42%. These statistics make more sense when you consider the figure showing the successful equilibrium. As a country continues to grow their reserve level, lending becomes more frequent and the borrowing country becomes less and less reliant on the "luck" of the audit. When this occurs, the country will experience big growth in reserves which typically amount to more than $\theta = 2$.

A nice feature of this model is we have this very intuitive property baked into the problem without explicitly setting this up to occur. The real feature of the model occurs in the early time periods. The first couple lending periods are the hardest for the borrowing country to secure lending. If they can overcome this, they are set and will almost surely reach the good equilibrium. This only occurs with a probability of 42% however. So on average, the borrowing country will take this risky investment and be forced to liquidate, due to not having full funding and then this will make it harder to get lending in the next period.

5. **Further Work to be Done**

The foundation of this model is solid, however a bit limited by parameters and the investment technology is a bit too ad-hoc. Additionally, the q updating technology, while easy for simulation, is also ad-hoc. I am planning to dive deeper into these two parts of the model and flesh them out. I am also working to generalize this model as much as I can and derive some closed form solutions. While this is a daunting task right now, I think changing the investment technology as well as the q updating in a certain fashion should make this generalization possible.

The main focus of this paper is to study the struggle a borrowing country faces when they try to secure lending post default. To give the borrowing country the "best shot" at overcoming this starting disadvantage, I made the investment project have a guaranteed positive return if funding was given. This, however is obviously not true in the real world. Work can be done on this section of the model (admittedly this is the part of the model I don't like the most) to better flesh out the investment technology. If done properly, my results would not change however the justification for my investment technology as well as the excessive risk premium would be better.