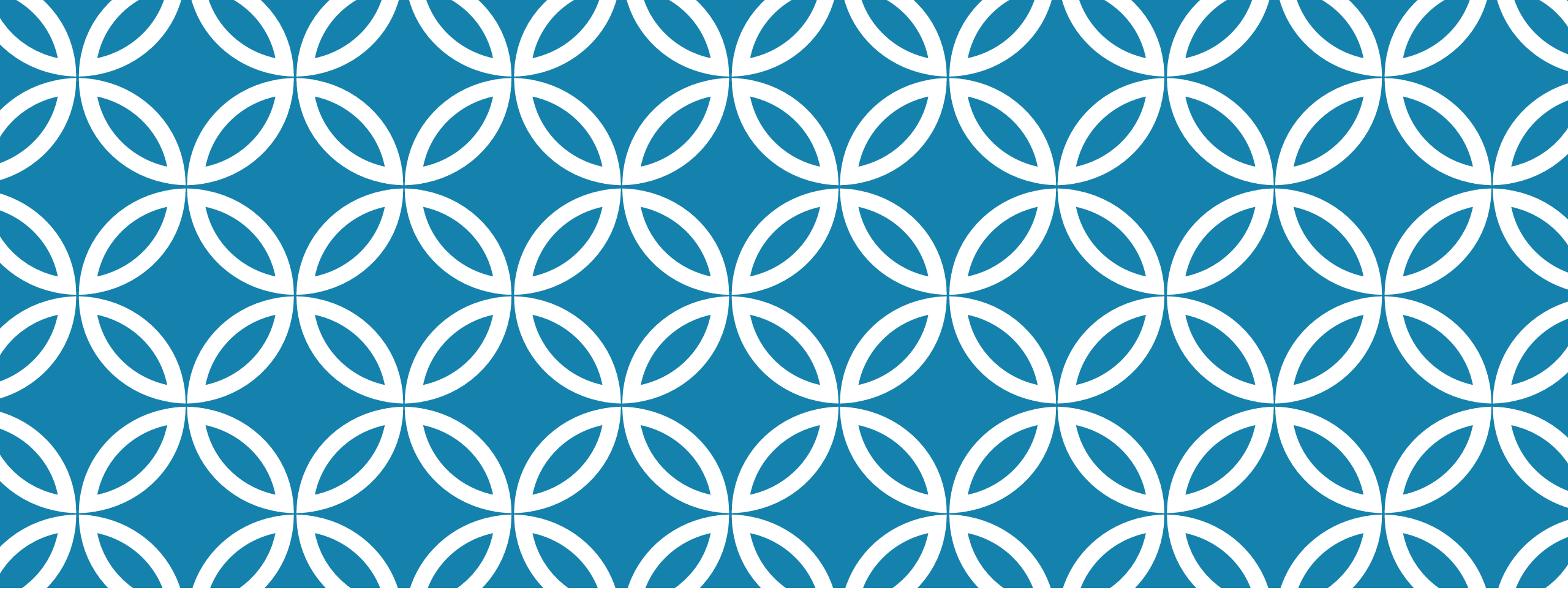

COMBINATORIAL AUCTIONS

@markgritter

#minnebar 11

4/23/2016

WHY AUCTIONS?

If you're an economist:

- Auctions are *efficient*: the person who values the item most gets it

If you're a game designer:

- Auctions are *fun*: many games use auction mechanics

If you're a seller or marketplace:

- Auctions are *competitive*: they produce more revenue compared to private sales

If you're a buyer:

- Auctions help with *price discovery*

WHY AUCTIONS?

If you're an economist:

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SOMETIMES

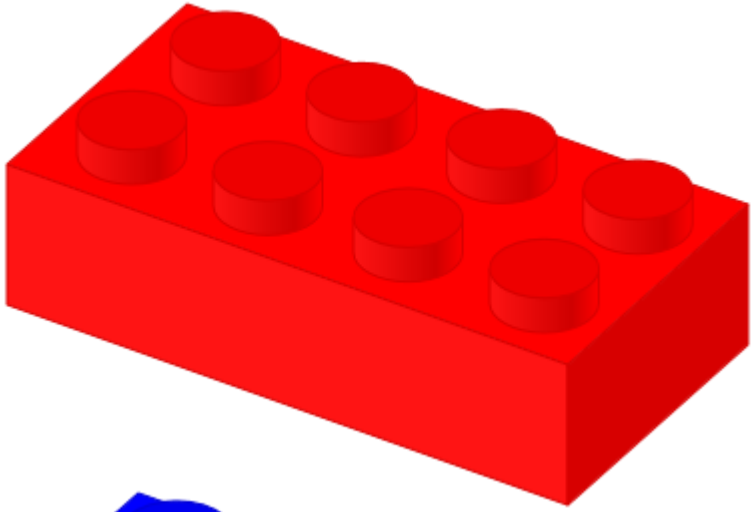
ENGLISH AUCTION

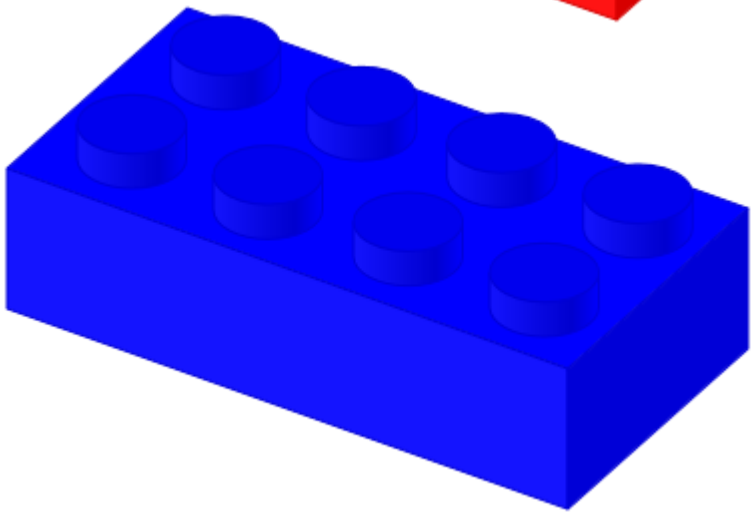
Ascending: price increases on each bid

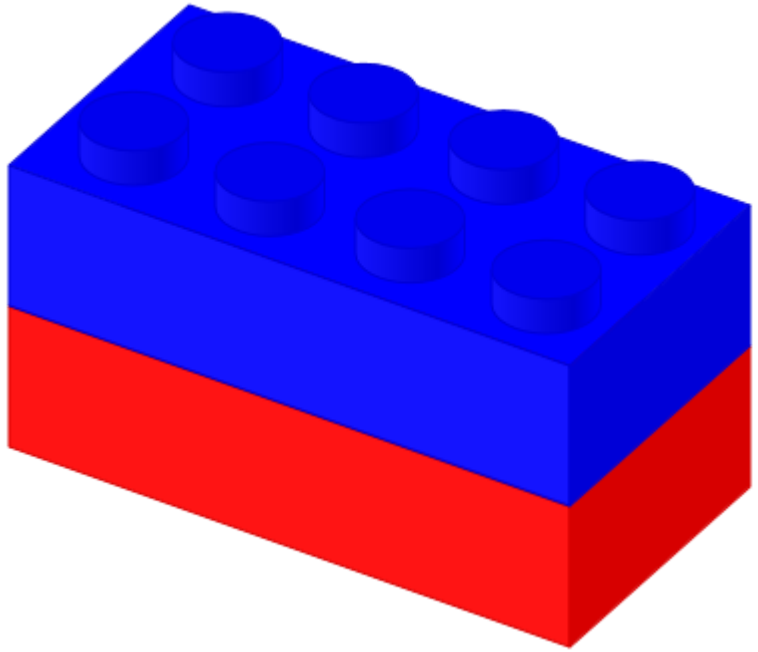
First-price: the highest bidder pays the amount of his or her bid

Open bid: bids and bidder identities are revealed (usually)

AUCTION #1 (MULTI-PART ENGLISH AUCTION)







=$5

WHAT WENT WRONG?

WINNER'S CURSE

Maybe you don't know the market value of the item you're bidding on.

- Economists say this only occurs "if the winner fails to account for the winner's curse when bidding."

GAME-PLAYING

Bidders have little incentive to reveal their true valuation.

Bidders may collude to lower the final purchase price.

- Directly: negotiate offline and have the winner bid (maybe with a kickback to the other player)
- Indirectly: bidding in such a way as to discourage competition

Any effort spent trying to win the auction is complete waste, from the economist's point of view

- Maybe not the game designer's…

EXPOSURE

If you need two items, you have to bid on one of them first.

We can solve this with *combinatorial* auctions.

- Instead of bidding for just one item at a time, bid for *packages* of items.
- All-or-nothing bids, you either receive the whole package or your bid is rejected.

ECONOMICS DETOUR

Goods are *substitutes* when they can be used for the same purpose.

- The two blue bricks are substitutes.
- Having more of product X makes you desire less of product Y.

Goods are *complements* when they have joint demand.

- Red bricks and blue bricks are complements.
- Demand for product A is decreased when price of good B goes up ("negative cross elasticity of demand")

COMPLEMENTS AND SUBSTITUTES IN REAL LIFE

Every airplane landing at MSP needs to be paired with a takeoff, and every takeoff needs to be paired with a landing slot elsewhere.

Every truck which travels to Duluth must also leave Duluth; it's better if it's full.

Any cellular provider which operates in Minneapolis should offer service in St. Paul as well. But which frequency gets picked doesn't matter.

VICKREY AUCTION

Sealed-bid: only one bid, submitted privately

Second-price: the winner pays the amount of the second-highest price

Dominant strategy is to bid your true value.

- Any less, and you might lose--- but don't pay less if you win.
- Any more, and you might pay too much if you win.

REVENUE EQUIVALENCE THEOREM

Any auction mechanism* that results in the same allocation of goods to bidders, also has the same revenue for the seller.

- Seller's revenue from Vickrey and English auctions are the same!
- The winner is paying the *opportunity cost* for the item, i.e., the value it would have for its second-best use.

* beware of economists bearing theorems. In this case it is any "Bayesian-Nash incentive compatible" mechanism.

VICKREY-CLARKE-GROVES (VCG) AUCTION

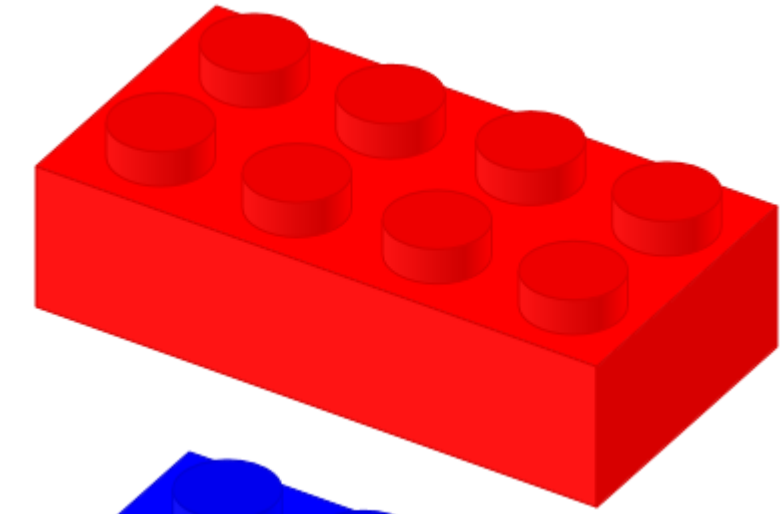
Generalized second-price, sealed-bid

1. Bidders submit values for all possible packages of goods for sale.

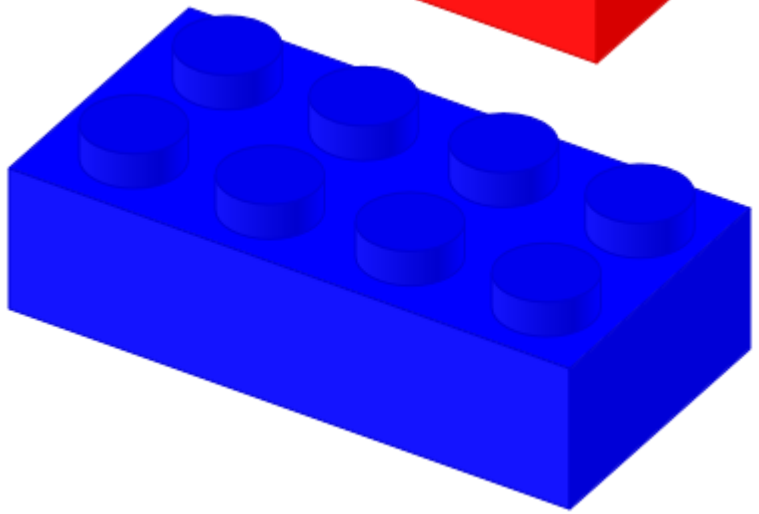
2. Auctioneer determines the value-maximizing combination of bids, subject to the goods available.

3. Each winner pays the opportunity cost of their bid
   - Take the value of the auction without the winner
   - Subtract the value of the other items in the winning coalition

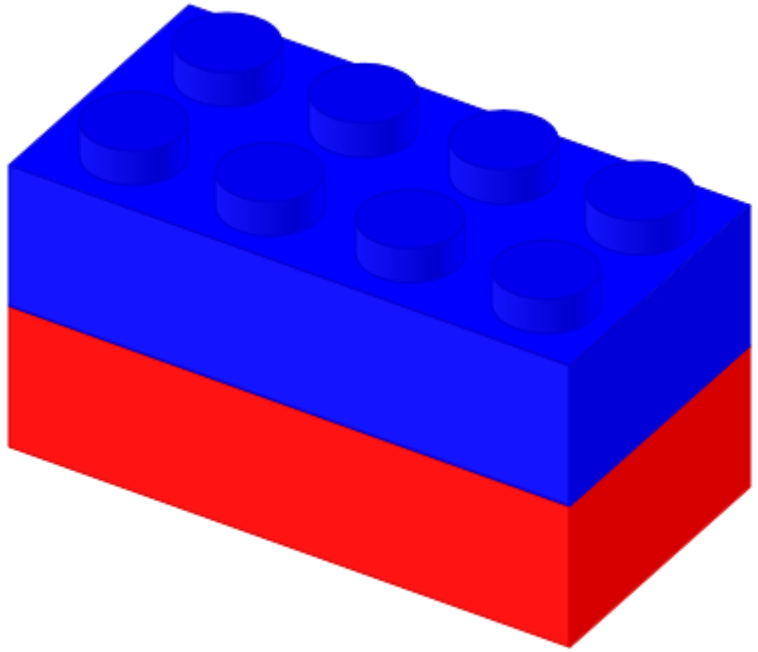
AUCTION #2 (VCG)



x1



x2



=$5

A's bid	Value
None	
1 Blue	
2 Blue	
1 Red	
1 Red, 1 Blue	
1 Red, 2 Blue	

B's bid	Value
None	
1 Blue	
2 Blue	
1 Red	
1 Red, 1 Blue	
1 Red, 2 Blue	

MATH

$v_n(x)$ Bidder n's reported value for package x

$\bar{x}$ Vector of available goods

x_n Goods assigned to bidder n, subject to $\sum_n x_n \leq \bar{x}$

$$x^* = \arg\max_{x_1,x_2,\ldots,x_N} \sum_n v_n(x_n)$$

Value-maximizing allocation

$$a_n = \max\{\textstyle\sum_{k\neq n} v_k(x_k) \mid \sum_{k\neq n} x_k \leq \bar{x}\}$$

Value maximum without bidder n

$$p_n = a_n - \sum_{k\neq n} v_k(x_k^*)$$

Payment by bidder n

SOME THEOREMS*

Truthful reporting is a dominant strategy, just like in the single-good model.

VCG is the unique direct revelation mechanism (under certain assumptions about values) for which:

- No payment is made by or to losing bidders
- Truthful reporting is dominant
- Items are efficiently distributed

THE TRUTH

Nobody ever uses VCG. Why not?

COMPLEX BIDS

VCG asks bidders to provide a means to evaluate every combination of items.

Even with complex bidding languages this could be very complicated, cost a lot, and require trust in the auctioneer.

PRICE DISCOVERY

When values are uncertain, bidders receive no guidance from the mechanism about what is affordable.

- May spend a lot of time determining values for items where they are bound to be outbid anyway.
- No way to discover new opportunities not originally anticipated.

COMPUTATIONAL COMPLEXITY

Determining a winner is NP-hard.

- We can use an integer-linear program to solve it, but even approximate solutions are hard.
- Plus approximations ruin the nice theoretical properties.

$$x^* = \underset{x_1, x_2, \ldots, x_N}{\arg\max} \sum_n v_n(x_n) \qquad \sum_n x_n \leq \bar{x}$$

SELLER REVENUES CAN BE VERY LOW

Bidder 1 wants two spectrum licenses only, for $2m

Bidder 2 wants one license for $2m

Bidder 3 wants one license for $2m

Bidder 2 and 3 win: each pays $0

- Auction is worth $2m without them, minus $2m for other winning bid
- This is not acceptable to Congress.

SELLER REVENUES ARE NON-MONOTONIC

Bidder 1 wants two spectrum licenses for $2m

Bidder 2 wants one license for $2m

Winner pays $2m, despite less competition!

(All examples from Ausubel and Milgrom, “Combinatorial Auctions” chapter 1, 2006.)

LOSERS CAN COLLUDE

Bidder 1 wants two spectrum licenses for $2m

Bidder 2 wants one license for $0.5m

Bidder 3 wants one license for $0.5m

Bidder 1 wins and pays $1m.

- But if bidders 2 and 3 increase their bids, they win and pay $0 as in the original example.

SHILL BIDDING

Bidder 1 wants two spectrum licenses for $2m

Bidder 2 wants one license for $0.5m, or two for $1m

Bidder 1 wins, but if bidder 2 submits a shill bid as in the previous examples, he wins both licenses for $0.

LIMITED BUDGETS

Bidder A values one license at $1m, or two for $2m

Bidder B wants one license for $800k

Bidder C will pay either $1.1m or $0 for one license, depending on unknown factors.

If A cannot bid $2m because he has a $1.2m budget, then he has no dominant strategy.

- If C bids $1.1m, A must bid more than $800k to win a license
- If C bids $0, A must bid $400k for the single license and $1.2m for two

GAME THEORY!

VCG is in the *core* of the multiplayer "transferable utility" game when goods are substitutes for all bidders.

- But the whole point was to handle goods that are complements

core = "feasible allocations that cannot be improved upon by a subset of the players", i.e., cannot be beaten by another coalition

ASCENDING PROXY AUCTION

1. Bidders (or their proxies) make initial bids on packages.
2. Auctioneer selects the collection of bids which maximizes revenue and publishes the result.
3. Bidders can respond with a new bid.
4. Repeat until no new bids
5. Each winning bidder pays the amount of their bid

STILL NOT POPULAR IN PRACTICE

First-price, iterative, sealed-bid: only provisional winning bids are revealed

- Permits price discovery!
- $0 bids are rejected by the bidder, so bidding leads to higher revenues than VCG.

Still too complicated! Remember, the auctioneer's problem is NP-hard and now has to be done once per round.

Alternative: make *bidders* come up with a better allocation.

SIMULTANEOUS ASCENDING AUCTION

U.S. PCS spectrum auctions starting in 1994 worked this way:

1. Multiple items for sale, each with its own price

2. Every round, bidders may increase price on any of the items.

3. Auction ends only when no item increases in price.

English auction, but run in parallel.

SUCCESS! SORT OF.

The spectrum auctions mainly achieved their goals.

- Large amount of revenue raised
- Bidders were able to put together packages that made sense, even though the bidding language didn't explicitly support it.
- Easy winner-determination problem

But, a fair amount of collusive behavior still occurred, so the result was probably not efficient.

CODE BIDDING IN ACTION

	Marshalltown, IA 283 E		**Rochester, MN 378 D**		Waterloo, IA 452 E		
Round	McLeod	USWest	McLeod	**USWest**	AT&T	McLeod	USWest
24	56,000					287,000	
…			…	…			
46				568,000			
52			689,000				
55				723,000			
58			795,000				
59				875,000			**313,378**
60						345,000	
62			963,000				
64		**62,378**		1,059,000			
65	69,000						
68					371,000		

Cramton and Schwartz, "Collusive Bidding in the FCC Spectrum Auctions", 2002

CLOCK AUCTION

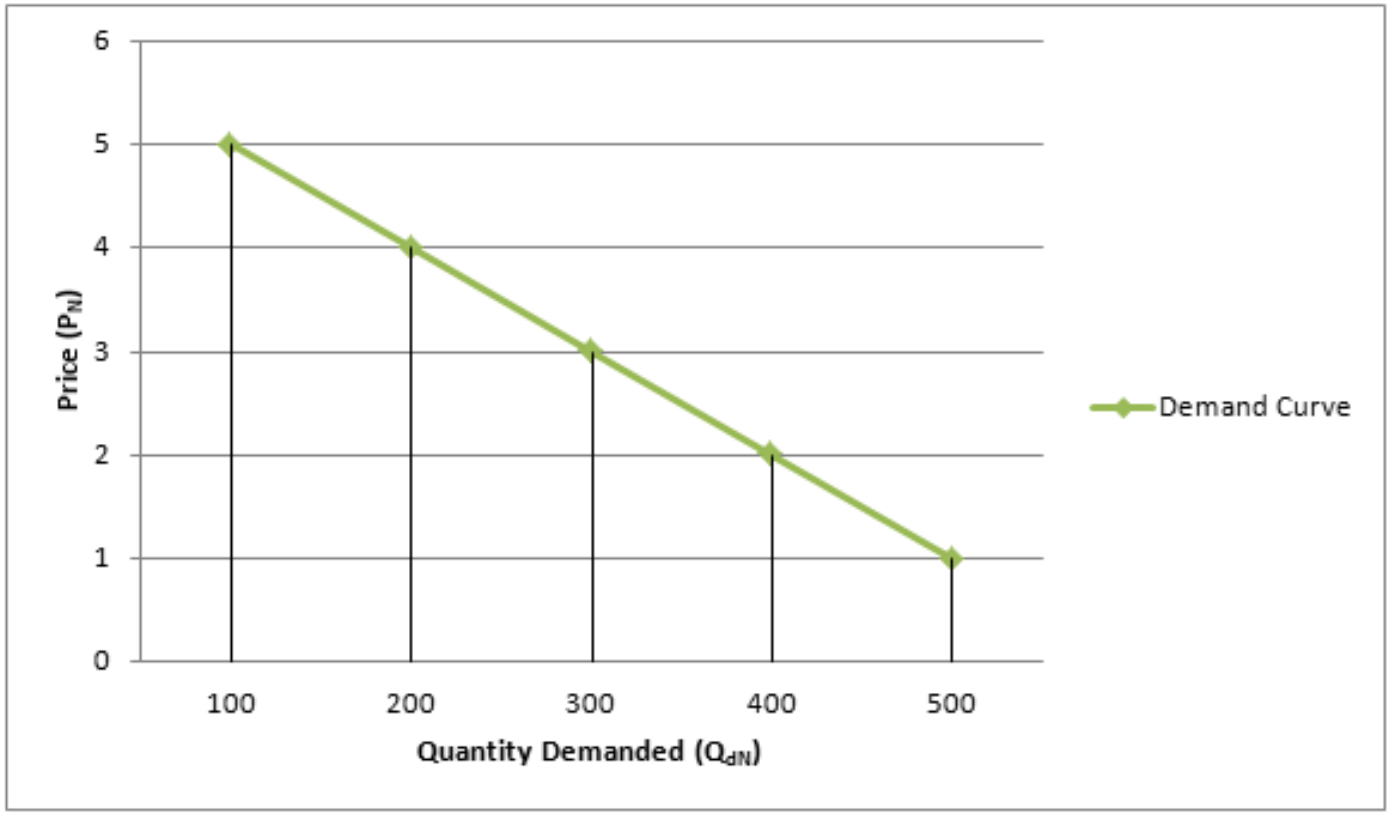
Useful when there are many similar or divisible goods.

1. In each round, the auctioneer announces unit prices
2. Bidders indicate how many units they want at that price.
3. Auctioneer increases prices until demand equals supply

Very little information leakage--- bidders only see prices, not bids.

AUCTION #3


Skittles
BITE SIZE CANDIES
Artificial & Natural Flavors
ORiginAL
Grape • Lime • Lemon
Strawberry • Orange
www.skittles.com


6
5
4
3
2
1
0
Price (P_N)
100
200
300
400
500
Quantity Demanded (Q_{dN})
Demand Curve

COMBINATORIAL AUCTIONS IRL

Truckload transportation

- Reverse auction, shipper is looking to buy.
- But, the winner gets a contract, not individual truck loads at time of auction.
- Carriers achieve some efficiencies by building packages, but most bids are still for a single lane.

COMBINATORIAL AUCTIONS IRL

Electricity markets

- Clock auctions with a final proxy round have been used.

Transport for London bus routes

- Study seems to show *negative* cost synergies between routes.

Mars, Incorporated industrial procurement

COMBINATORIAL AUCTIONS, NOT YET IRL

"Combinatorium":

- VCG auction-based multiplayer game
- Prototyped with 6 players over 10 rounds
- Bid for goods that fulfill randomly-assigned contracts, and on "victory points", but limited bid language.
- Winning players successfully predicted which good packages were achievable--- price discovery!

COMBINATORIAL AUCTIONS, NOT YET IRL

On-demand labor apps have a problem

- Even high pay rates ($20/hour) are not effective at retention, because real rate is low due to overhead
- Could reverse package auctions drive greater efficiency?

Airline landing slots

- This whole field of study was jump-started by airline deregulation
- Still assigning slots via non-auction mechanisms.

THANKS FOR PARTICIPATING!

Twitter: @markgritter

My real job: storage for virtualization!

9 8 7 6 5

4 3 2 1