Using the Solow model to study Climate Change

Deadlines:
-Round 1 (spreadsheet simulations): 15 April, 08:30
-Round 2 (pdf report): 6 May 08:30
Both rounds must be submitted via Canvas

Use the student ANRs for naming the files. E.g., EGI_2022_A1_0123_32355_2322.pdf (.xlsx) is the first assignment, submitted by the students with ANRs 0123, 32355 and 2322.

Table. Student information

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Instructions: The assignment consists of two rounds. In the first round you are asked to numerically simulate the model. In the second round you are asked to submit a report with the interpretation of the numerical results. This second-round report consists of the questions marked with **[R2]** in this file, they should be answered here [as indicated below] and should be submitted as a .pdf.

The points for each question are provided in the rubric of Round 2 in Canvas.
The spreadsheet is not graded, but **the Round 1 submission is a pre-requisite for the Round 2 report. If you do not submit the spreadsheet with your calculations by the Round 1 deadline, your Round 2 Report will not be graded. Your Round 1 submission should demonstrate that you attempted to perform ALL the required simulations. Incomplete Round 1 submissions will receive no feedback.**

We model the inter-relation between economic activity and climate change at a global scale. A by-product of economic activity is the emission of Green House Gases (GHG). Emissions increase the concentration of GHG in the atmosphere, which results in higher temperatures. In turn, the climate change unraveled by global warming has detrimental effects for economic activity; we refer to the latter as damages.

The model consists of two modules. An economic module which follows from the Solow model with technological progress and models the dynamic evolution of output and GHG emissions. This is complemented with a climate module that models the relationship between GHG concentrations and temperature. From the economic module we obtain GHG emissions that feed the GHG concentrations in the climate module. From the climate module we obtain temperature which in turn determines the damages suffered by the economy in the form of lost output.

Economic module

$Z(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}$	Material output	(1)
$A(t+1) = A(t)e^{g}$	Productivity	(2)
$L(t+1) = L(t)e^{n}$	Population	(3)
$Y(t) = [1 - D(t)]Z(t)$	GDP: Output net of damages	(4)
$C(t) = (1 - s_K - s_m)Y(t)$	Consumption	(5)
$K(t+1) - K(t) = s_K Y(t) - \delta K(t)$	Physical capital accumulation	(6)
$m(t) = \chi \left[1 - \frac{s_m}{\omega}\right]^{\varepsilon} Z(t)$	GHG emissions	(7)

Climate module

$M(t+1) = M(t) + m(t)$	Accumulation of GHG in the atmosphere	(8)
$T(t) = 1 + \frac{M(t)^{\kappa}}{1000}$	Temperature increase (relative to 1800)	(9)
$D(t) = 1 - \left[\frac{1}{1+\theta_1 T(t)^{\theta_2}}\right]$	Damages as a proportion of Z	(10)

Where:

$s_K \in (0, 1)$	investment rate in physical capital
$s_m \in [0,\ \omega]$	investment rate in emission reduction
$s_K + s_m < 1$	
$n > 0$	population growth rate
$g > 0$	rate of technological progress
$\chi > 0$	emission intensity without investment in emission reduction
$1/\omega > 0$	efficiency of investment in emission reduction
$\varepsilon > 0$	sensibility of emissions to investment in emission reduction
$\kappa > 0$	sensibility of temperature to GHG concentrations
$\theta_1 > 0$ and $\theta_2 > 0$	sensibility of damages to temperature

You have access to an excel file with multiple spreadsheets.

All the relevant parameter values and initial values are provided in the excel file

It is your task to perform different simulations of the Solow model with climate detailed above (Round 1). Based on these simulations you will analyze the economic implications of climate change and climate policy scenarios (Round 2).

Each simulation is to be performed on a separate sheet of the excel file (see the sheet labels)

Present all your answers using 4 decimals.

**Note: For the computations of growth rates use the exponential formulation. For 'a' variable* X *this is:*

$$X(t+1) = X(t)e^{g_X(t+1)},$$

where $g_X(t+1)$ *is the growth rate of variable* X *between year* t *and year* $t+1$.

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a. **[R2] Use equation (7) to explain why s_m is a measure of the effort to reduce emissions (i.e., abatement effort).**

Equation 7 states: $m(t) = \chi \left[1 - \frac{s_m}{\omega}\right]^{\varepsilon} Z(t)$. *$s_m$ is a measure of the effort to reduce emissions as an increase in s_m decreases GHG, emission, i.e. the first order condition of $m(t)$ with respect to s_m should be negative. This can be done as follows:*

$$F.O.C._{wrt\, s_m} : m'(t) = -[\varepsilon\chi \left[1 - \frac{s_m}{\omega}\right]^{\varepsilon - 1} Z(t) \left[\frac{1}{\omega}\right]]$$

Since $\varepsilon > 0, \frac{1}{\omega} > 0, \chi > 0, Z(t) > 0, s_m \in [0,\ \omega]$ *(which means that* $\frac{s_m}{\omega} \leq 1$ *and thus* $\left[1 - \frac{s_m}{\omega}\right]^{\varepsilon - 1}$ *is also larger than 0, the F.O.C is less than 0 (due to the negative sign at the front). Hence, as we increase s_m we would observe a decrease in the value of $m(t)$. Finally, we can call it an effort as it decreases the amount of GDP available for consumption.*

b. **[R2] Is there free lunch when it comes to abatement effort s_m? If not, to what is this economy 'sacrificing' in order to increase s_m?**

There is no free lunch as it decreases the amount of output available consumption. The derivative of $C(t)$ towards s_m is $-Y(t) < 0$. Consumption is often used as a proxy of utility or welfare. Thus, increase s_m might be against the interest of the economy.

- Use the information above and the parameters in the excel file to simulate the population, L, and productivity, A, up to 2100 in the corresponding columns of the three simulation sheets (sim1, sim2, sim3). That is, the series of population and productivity are the same across simulations. Take as given the initial levels (i.e., the 2020 values) of A and L as provided in the excel file.

Simulation 1 - Baseline scenario [*no abatement effort*]

[Perform this simulation using the sheet labeled sim1]

You will start with the simulation of a baseline scenario where there is no abatement effort.

Computing initial values

- Throughout this simulation we assume that the abatement effort s_m is equal to zero for all the years between 2020 and 2100. Complete the series of s_m accordingly (*column G*)

In what follows the goal is to obtain the series of all the remaining endogenous variables ($Z,\ m,\ T,\ D,\ y,\ c,\ K,\ M$) up to 2100.

Here you are presented with a 5-steps procedure to produce the dynamic simulation of the model. These steps are meant as a guide.

- Step 1: Use equation (1) to compute the 2020 value of Z. Use this and the initial level of abatement to compute the level of emissions m in 2020 according to equation (7).

- Step 2: Use the 2020 value of GHG concentrations, M, to compute the 2020 value of the temperature increase with respect to 1800, T, according to equation (9). Use this and equation (10) to compute the 2020 damages, D.

- Step 3: Compute the 2020 levels of GDP per capita, y, (*column M*) and consumption per capita, c, (*column O*).

- Step 4: Use the dynamic equation (6) to compute the immediately subsequent value of physical capital. Use the dynamic equation (8) to compute the immediately subsequent value of the atmospheric concentrations of GHG (M).

- Step 5: Repeat Steps 2 to 3 to calculate the immediately subsequent value for each of the other variables ($Z,\ m,\ T,\ D,\ y,\ c$). Repeat Step 4 to calculate the immediately subsequent value of physical capital and of GHG concentrations (K, M). Use this procedure to complete the series up to 2100 for the endogenous variables $Z,\ m,\ T,\ D,\ y,\ c,\ K,$ and M.

- Compute the yearly growth of GDP per capita, $\hat{y}$, (*column N*) and consumption per capita, $\hat{c}$, (*column P*).

c. **[R2] Suppose for a moment (only for this question) that climate plays no role in the model; that is, think of the Solow model with technological progress covered in the first half of the course. Given the parameter values proposed, what would be the growth rate of GDP per capita in steady state? Explain**

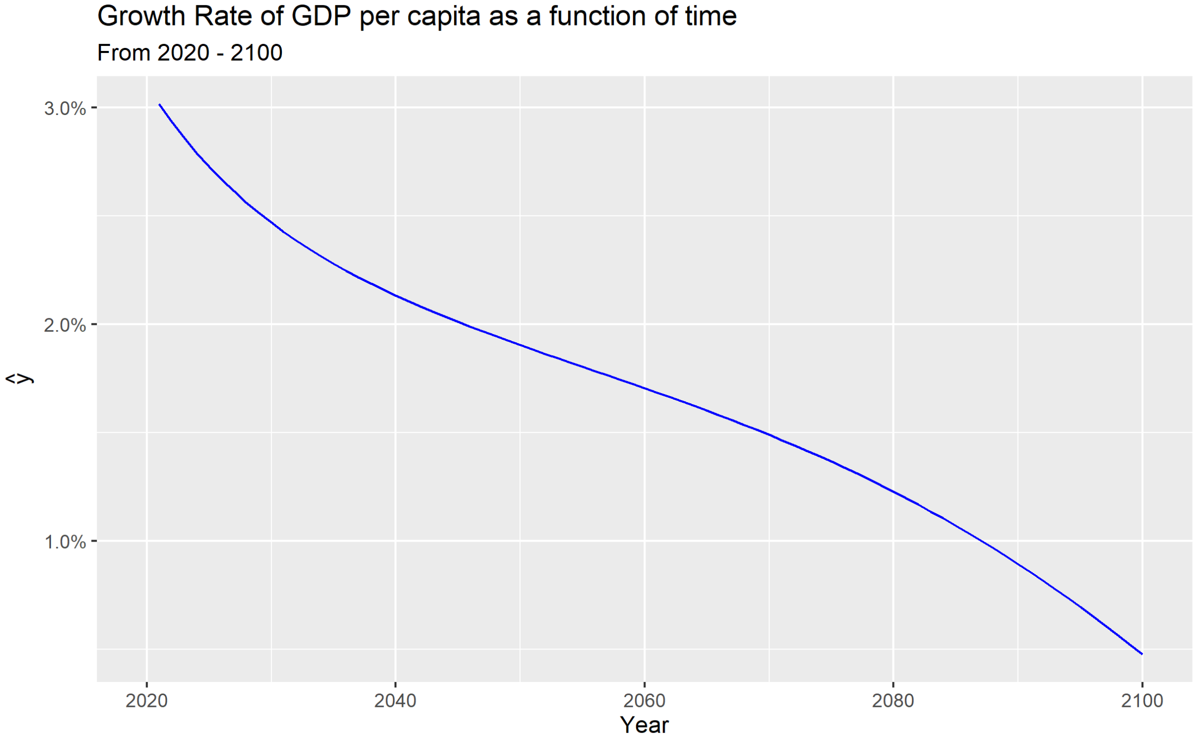
Since the long run steady state value of $y(t) = A(t)(\frac{s}{n+g+\delta})^{\frac{\alpha}{1-\alpha}}$ under these conditions (this was found by solving for the steady state in efficiency units and multiplying this by $A(t)$), the growth rate in steady state can be found, which is:

$$\hat{y}^* = \hat{A} = g > 0.$$

Hence, the long run growth rate of GDP per capita is g, which has a value of 0.02.

d. **[R2] Present a line chart of the growth rate of GDP per capita, computed in *column N*, as a function of time for the 2021-2100 period.**

Figure 1:



e. **[R2] Describe and explain what you observe in the figure. To what do you attribute this time-path of the growth rate of GDP per capita between 2021 and 2100?**

The trend of the GDP per capita growth rate in the "classical" Solow-growth model is that the curve is convex and approaches the long-run growth rate, in our case 2%. This effect dominates up to around 2040 as it looks like the model without climate damage. However, climate change is destroying GDP more severely as there are more GHG emissions through higher material output and higher temperatures. This negative effect on GDP growth outweighs the effect of "classical" GDP growth. We see that the GDP growth rate turns concave and falls below the "classical" long-run GDP growth rate with a decreasing and accelerating rate.

f. **[R2] Compare the last decade of the yearly growth rate of GDP per capita computed in *column N* with the long-run growth rate that you would have obtained in the model without climate. Do these rates differ? If so, to what do you attribute the difference?**

The last value (2100) has a growth rate of 0.00476, or around 0.5%. This is ¼th of the value found in question c. This difference can once again be attributed to the damages decreasing GDP caused by climate change. This *effect is aggravated by there being no abatement efforts. The reduction of GDP growth rates can be attributed to a growing $D(t)$. Material output continuously increase GHG accumulated in the atmosphere. This causes a gradual increase in temperatures which in turn increases damages. This causes a continuous negative effect on GDP per capita growth rates. Hence, the growth rate that is below the long run growth rate of an economy without a climate parameter.*

Simulation 2 – Maximum abatement

[Perform this simulation using the sheet labeled sim2]

- According to equation (7), if s_m is equal to ω emissions are fully abated. Throughout this simulation we assume that the abatement effort s_m is equal to ω for all the years between 2020 and 2100. Complete the series of s_m accordingly (*column G*).
    - Apply the same procedure used in Simulation 1 (Steps 1-5) to complete the series for all the remaining endogenous variables ($Z,\ m,\ T,\ D,\ y,\ c,\ K,\ M$) up to 2100.
- Compute the yearly growth of GDP per capita, $\hat{y}$, (*column N*) and consumption per capita, $\hat{c}$, (*column P*).

g. **[R2] Compare the growth rate of GDP per capita and consumption per capita. Describe and explain the observed relationship between these two growth rates.**

The GDP per capita growth rate is identical to the consumption per capita growth rate in every year. This is logical as $c(t) = (1 - s_K - s_m)y(t)$, thus consumption per capita is just GDP per capita multiplied by a constant. Taking logs and derivatives we find that the growth rates must be equal as s_K and s_m are constants.

The growth rates of GDP per capita and consumption are equal to each other in every period. This is because consumption is a constant fraction of output, as s_m and s_K remain constant.

h. **[R2] Start from the capital accumulation equation and the assumption that s_m is constant at a level that fully abates emissions. Use your knowledge of the Solow model to derive analytical expressions (function only of parameters, initial levels of damages and technology, and time) for the growth rate and the level of GDP per capita in steady state, $\hat{y}^*(t)$ and $y^*(t)$. Include a sufficiently detailed step-by-step of your derivations.**

Let us look at the absolute value of $y(t)$ and $\hat{y}(t)$ ($z(t)$ denotes $\frac{Z(t)}{L(t)}$ and $\hat{z}(t) = \frac{\dot{z}(t)}{z(t)}$).

$$y(t) = [1 - D(t)] * z(t)$$

$$\hat{y}(t) = \frac{-\dot{D}(t)}{1 - D(t)} + \hat{z}(t)$$

Now we calculate $D(t)$ and $\dot{D}(t)$. We know that $s_m = \omega$, thus $m(t) = \chi\left[1 - \frac{s_m}{\omega}\right]^{\varepsilon} Z(t) = [1-1]^{\varepsilon} Z(t) = 0$. Hence, $M(t)$ is equal to $M(t=0)$, $T(t) = 1 + \frac{M(t=0)^{\kappa}}{1000}$ for all periods and $D(t) = D(t) = 1 - \left[\frac{1}{1+\theta_1\left(1+\frac{M(t=0)^{\kappa}}{1000}\right)^{\theta_2}}\right]$.
Thus, $D(t)$ is always equal to its initial value, let us denote it by D_0. As $D(t)$ is constant, we also know that $\dot{D}(t) = 0$, hence $\hat{y}(t) = \hat{z}(t)$. We also know that $y(t) = [1 - D_0] * z(t)$.
Let us focus on finding the equilibrium value of $z(t)$ and its growth rate.
Similarly, to the Solow growth model with technology growth we can rewrite Z in efficiency units, find the growth rate of capital in efficiency units and find an expression of capital accumulation with $Z(t)$:

$$\tilde{z}(t) = \tilde{k}(t)^{\alpha}$$

$$\dot{\tilde{k}} = \left(\frac{\dot{K}}{AL}\right) = \frac{\dot{K}}{AL} - (n+g)\tilde{k}$$

$$\dot{K} = sY(t) - \delta K(t) = s\big(1 - D(t)\big)Z(t) - \delta K(t) = s(1 - D_0)Z(t) - \delta K(t)$$

We can plug the third equation into the second one one and then the first equation into the newly created $\frac{Z(t)}{A(t)L(t)}$:

$$\begin{aligned}\dot{\tilde{k}} = \left(\frac{\dot{K}}{AL}\right) &= \frac{s_K(1 - D_0)Z(t) - \delta K(t)}{AL} - (n+g)\tilde{k} \\ &= \frac{s_K(1 - D_0)Z(t)}{AL} - (n+g+\delta)\tilde{k} \\ &= s_K(1 - D_0)\tilde{z}(t) - (n+g+\delta)\tilde{k} \\ &= s_K(1 - D_0)\tilde{k}(t)^{\alpha} - (n+g+\delta)\tilde{k}\end{aligned}$$

In the steady state the growth rate of capital in efficiency units should be 0. Thus:

$$s_K(1 - D_0)\tilde{k}^*(t)^{\alpha} - (n+g+\delta)\tilde{k}^* = 0 \Leftrightarrow \tilde{k}^* = \left(\frac{s_K(1 - D_0)}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

We can find the steady state value of material output in efficiency units:

$$\tilde{z}^*(t) = \tilde{k}(t)^{*\alpha} = \left(\frac{s_K(1 - D_0)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

And the steady state value of material output per capita:

$$\begin{aligned}z^*(t) = \tilde{z}^*(t) * A(t) &= \left(\frac{s_K(1 - D_0)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} * \ A_0\, e^{g(t-2020)} \\ &= \left(\frac{s_K(1 - D_0)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} * \ A_0\, e^{g(t-2020)}\end{aligned}$$

With this expression we can also find the growth rate of the steady state value of material output per capita. As every variable in the fraction is a constant we just need to consider the growth rate of technology, which is g:

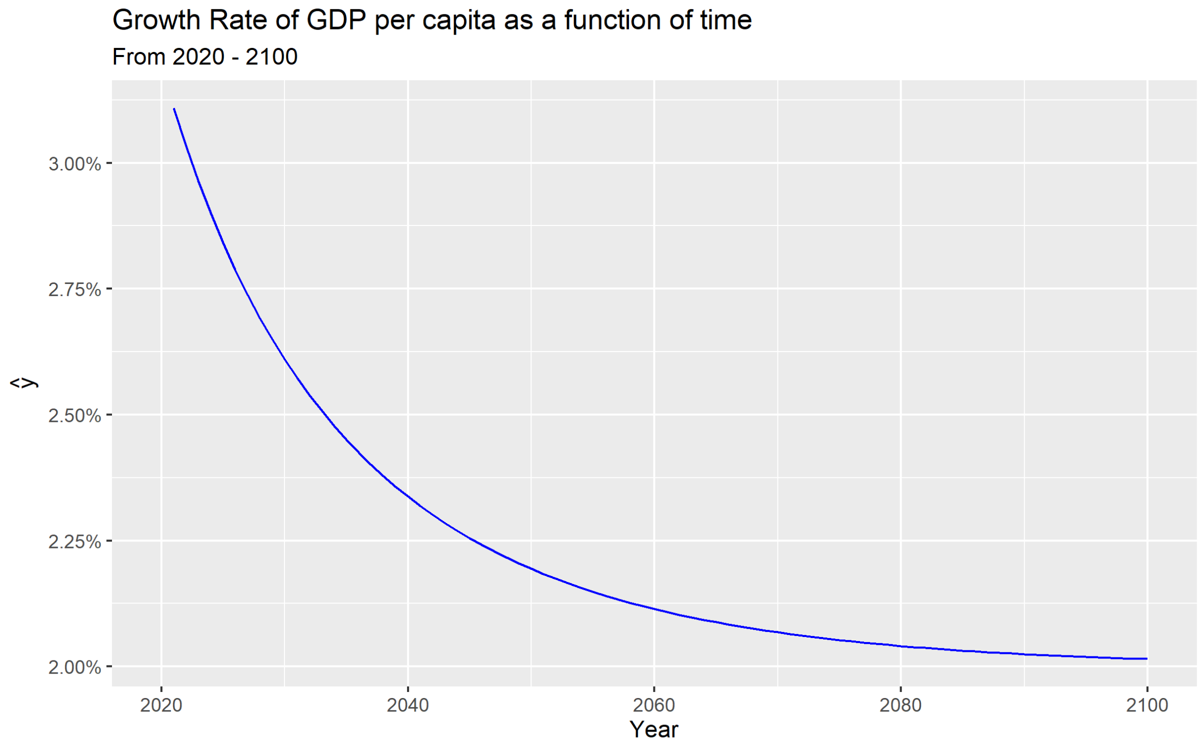
$$\hat{z}(t) = g$$

Now using the first two equations we can finally calculate the steady state value and growth rate of GDP per capita:

$$y(t) = [1 - D_0] * z(t) = [1 - D_0]\left(\frac{s(1 - D_0)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} * \ A_0\, e^{g(t-2020)}$$

$$\hat{y}(t) = \hat{z}(t) = g$$

i. **[R2] Present a line chart of the growth rate of GDP per capita, computed in *column N*, as a function of time for the 2021-2100 period.**



j. **[R2] Describe and explain what you observe in this figure; link your explanation to your answers to literals c and h.**

Due to the full abatement, there is no destruction in GDP through climate change. Thus, GDP per capita growth converges to its "classical" long-run value of technological progress, in our case 2%, as explained in c). The GDP per capita growth is the growth rate of output in efficiency units plus technology growth. Overtime, capital in efficiency units falls due to depreciation and we get closer to the growth rate of output in efficiency units being 0 In h) we showed it mathematically.

Simulation 3 – Incremental abatement

[Perform this simulation using the sheet labeled sim3]

According to equation (7), if s_m is equal to ω emissions are fully abated. In this simulation we assume that the economy starts from a low level of abatement effort. This effort is gradually increasing over time. For any year t between 2020 and 2100 the abatement effort is governed by the following equation

$$s_m(t) = \frac{\omega}{1+e^{\eta(2020-t)}} \tag{11}$$

with $\omega > 0$ and $\eta > 0$ (values provided in the excel file)

k. **[R2] Use equation (11) to show that**

1. **The abatement effort is increasing over time**
2. **In the long run the economy approaches to full abatement**

For the first statement we check whether the derivative of $s_m(t)$ towards time is positive:

$$\dot{s_m}(t) = -\omega\left(1+e^{\eta(2020-t)}\right)^{-2} * \left(-\eta e^{\eta(2020-t)}\right) = \frac{\omega\eta e^{\eta(2020-t)}}{(1+e^{\eta(2020-t)})^2} > 0$$

For the second statement we check what $s_m(t)$ converges to if t goes to infinity:

$$\lim_{t\to\infty} \frac{\omega}{1+e^{\eta(2020-t)}} = \lim_{t\to\infty} \frac{\omega}{1+\ e^{\eta*(-\infty)}} = \lim_{t\to\infty} \frac{\omega}{1+\ 0} = \frac{\omega}{1} = \omega$$

- Complete the series of s_m according to equation (11) (*column G*).

  - Apply the same procedure used in Simulation 1 (Steps 1-5) to complete the series for all the remaining endogenous variables ($Z,\ m,\ T,\ D,\ y,\ c,\ K,\ M$) up to 2100.

Evaluation of results

[Perform the following exercises in the sheet labeled 'evaluation']

You will start with the evaluation of the simulations 2 and 3 relative to the baseline (simulation 1) based on the yearly outcomes for consumption per capita.

- [Optional]: Link *columns C-E* in the *evaluation* sheet to the series of consumption per capita that you obtained in the three simulations: i.e., Link *col C* to *col M in sim1*, *col D* to *col M in sim2*, *col E* to *col M in sim 3*.

- For each year between 2020 and 2100 compute difference between consumption per capita under simulation 2 and under simulation 1. We refer to this as the *Current Value* (CV) difference.

- For each year between 2020 and 2100 compute the *Present Value* (PV), in terms of year 2020, of the difference between consumption per capita under simulation 2 and under simulation 1. *Use the continuous compounding formulation of the PV, where the PV of a variable* X *in year* $2020 + t$ *is*: $PV[X(t)] = e^{-rt}X(t)$; where $r > 0$ is the discount rate; its value is provided in the excel file.

- For each year between 2020 and 2100 compute the *Current Value* (CV) of the difference between consumption per capita under simulation 3 and under simulation 1.

- For each year between 2020 and 2100 compute the *Present Value* (PV) of the difference between consumption per capita under simulation 3 and under simulation 1. Use the same formula and discount rate as in the comparison between simulations 2 and 1.

l. **[R2] Based on the results of the CV of the difference in consumption per capita, is consumption per capita under full abatement (simulation 2) always higher than under no abatement (simulation 1)? If not, when is consumption per capita higher under full abatement? Explain your results based on the short-run/long-run costs and benefits of a higher abatement effort.**

Initially consumption is higher in simulation 1. However, starting in 2055 consumption is higher in simulation 2 Abatement in simulation 2 is a constant fraction that is subtracted from consumption. In early years, climate damage has very little effect on GDP and thus on consumption. However, as GHG accumulate in the atmosphere and temperatures rise, damage impacts GDP more and more, thus also decreasing consumption up to the point that the negative effect of climate change outweighs the negative effect of abatement. The benefits of abatement, the mitigation of climate damage, only pay of in the long-run.

m. **[R2] Would your results change if you consider the PV instead? Explain.**

No. When transforming the current value of consumption into the present value, both the consumption with and without abatement are discounted by the same factor. Thus, if one is larger than the other it will remain as such, even when discounting. The only difference is that in the long-run the absolute difference in consumption will be *smaller when transforming it into the present. Thus, the benefit that abatement brings in the long-run might appear smaller.*

Now you will evaluate the simulations based on the aggregation of the yearly outcomes. For this consider the following measure, that we will refer to as the *Cumulative Present Value* (CPV):

$$CPV[X(T)] = \sum_{t=0}^{T} PV[X(t)] \tag{12}$$

The formula above implies that for given T the CPV sums the PVs of all the preceding years up to and including year $2020 + T$ itself: i.e., the sum of the PVs from the initial year 2020 up to year $2020 + T$.

n. **[R2] Interpret the following statement: If a decision maker (e.g., the government, voter,…) uses the CPV as described in (12) to evaluate evolution of a variable X, then T defines how many years into the future this decision maker cares about.**

The discounted value of X (the present value) is the worth of future consumption considering that it is worth less than current consumption. Thus, if the decision maker considers this measure of future consumption in their decision making it means that they do not only take into account the present consumption, but also all future consumption up to T.

In what follows we refer to T in (12) as the "planning horizon"

- For each year between 2020 and 2100 compute the CPV up to that year. For example, the value of the CVP in 2050 requires using $T = 30$ in (12).

o. **[R2] Use equation (12) and your results for the PV of the difference in consumption per capita between simulation 2 and 1, and between simulation 3 and 1, to complete the following table [*use three decimals*]**

Table 1: CPV

T	$year$	$CPV[c_2 - c_1]$	$CPV[c_3 - c_1]$
20	2040	-7.981	-4.785
40	2060	-9.453	-7.660
60	2080	-2.586	-7.116
80	2100	14.652	-1.499

Where the c_i denotes consumption per capita under simulation i.

p. **[R2] Describe and explain the evolution of each of the two CPVs in Table 1 (sim2-sim1 and sim3-sim1) as the planning horizon, T, is extended. According to these results, would myopic decision makers (i.e., decision makers with a relatively short planning horizon T) support high abatement efforts s_m? Explain.**

Up to and including 2084 for simulation 2 vs 1 and in all years for simulation 3 vs 1 the cumulative present value of is negative. We established in l that the benefits of abatement only outweigh the costs in the long-run. Thus, if a myopic decision maker only considers consumption in early periods, they disregards *the latter period where abatement pays off. Thus, they will decide against abatement.*

q. **[R2] Describe and explain the differences between each of the two CPV columns in Table 1; consider the differences in the time path of abatement effort s_m between simulations 2 and 3 in your answer.**

Up to the year 2068 the cumulative present value of simulation 2 is lower than in simulation 3. After that, simulation 2 quickly becomes much more attractive than simulation 3. This can be explained as in simulation 3 (1) abatement costs are low in the beginning and are gradually phased in and (2) the additional damages caused by the delayed abatement are only realized in latter periods. Thus, when a decision maker only considers the early periods they will choose the policy option of simulation 3. However, in the long-run the abatement costs in simulation 3 catch up with the ones in simulation 2 (even though they never fully catch up). Furthermore, now the additional damages due to lower abatement are realized (e.g. $D(t = 2080)$ is around 14.3% in simulation 3 compared to only 5.8% in simulation 2). This causes a much lower consumption in simulation 3, lowering the cumulative present value of simulation 3 below that of simulation 2 if years after 2068 are considered.

Taking stock (critical assessment of the model)

r. **[R2] What are the main merits of model that we developed in this assignment to study the economic implications of climate change and climate policy?**

The main economic dynamic which the model captures is the trade-off in climate policy between long-term damages from climate change, but short-term costs of mitigating it. The model is even more useful compared to the model from assignment 1 as abatement effort is variable over time, reflecting that climate policy is not a one-time choice, but a repeating decision.
It does not only reflect the developments of economic variables such as GDP and consumption, but it can also explain processes in political economics and political decision making by incorporating cumulative present value of consumption and time horizons. In such, it addresses even topics from behavioral economics, in this case the present bias.
Finally, the model is split into an economic and a climate module. While climate processes are difficult to model, by including a climate model that describes the relationship between greenhouse gases, temperature and damages we can implement more insight from climatology, e.g. how exactly GHG concentration and temperature increase are connected.

s. **[R2] What are the main limitations of model? Propose a direction in which you would extend/alter the model to deal with these limitations.**

The model does not capture several aspects how climate change affects economic performance: (1) Climate change does not "only" destroy material output, thus decreasing output available for consumption and investment. One *could also argue that it leads to an increased depreciation rate as higher temperature and extreme weather events destroy capital more rapidly. This can be modelled by making δ an increasing function of T. (2) Climate change may also have a negative impact on human impact through worse public health and physical strain of higher temperatures. Again, we can include the mincer specification and make ψ a decreasing function of T and/or exogenies technological progress A and define θ or λ a decreasing function of T. Furthermore, different types of economic activities might release different amounts of GHG. Without making too large changes to the model one could think that either the output used on consumption or the output used on savings/investments release more GHG. One can argue for both and the question which one is harmful could be answered empirically. This question is interesting as it would also change the golden saving rate and the equilibrium capital stock of the economy. One might also include "green" (clean) capital is less productive, but "brown" (polluting) capital, each with different saving rates and productivities. The composition of capital would determine the pollution of the economy. One could argue whether those sectors work as substitutes or as complements (i.e. will we always need some fossil fuels or can we fully substitute them with renewable energies. These would allow for climate change mitigation which still produces output.*

On a different note, if one would want to fully delve into the political economics of climate change, one could add an election module which periodically decides on the abatement effort. Voters could vary by how much they are impacted by climate change and by their myopia. The heterogeneity in impact could also be linked to "green" and "brown" capital.

Finally, pro forma it is important to mention that not all damages from climate change is reflected in a decrease in output. These includes loss in biodiversity or destroying local cultures as regions become uninhabitable. While these are difficult to include in models, it is important to consider that even if output remains unaffected by climate change, we might still loose social welfare.