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# THE EFFECT OF EARTHQUAKE DURATION ON THE DAMAGE OF STRUCTURES

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## SUMMARY

This paper presents a theoretical analysis of the effect of duration on the damage of structures subjected to earthquakes. The earthquake excitation is modelled by a non-stationary random process whose response spectrum is probabilistically consistent with a design response spectrum specified independently of the duration. Damage is assumed to accumulate with the cyclic application of large strains in the structural members. Two types of structure are examined: one representative of a steel structure and the other reprsentative of a reinforced concrete structure. The level of expected damage is found to be a strong function of both the ductility of the response and the duration of the excitation. Results are presented for systems with linear stiffness and a particular form of softening behaviour.

## INTRODUCTION

In many parts of the world, structures must be designed to withstand damaging earthquakes without failure. For some structures, a detailed dynamic analysis is used to ensure the safety of the structure during a seismic event. In addition to a complete description of the structure, such a detailed analysis requires the specification of the anticipated nature of the ground motion. Because of the many uncertainties in predicting the precise nature of the time history of earthquake ground motion, the design response spectrum has received wide acceptance as a measure of the design input.

The design response spectrum gives the maximum response of a linear single-degree-of-freedom structure with a specified damping and natural frequency to the design earthquake. The response spectrum is a useful design tool since it allows the peak response of a structure to be estimated directly. This will be an adequate representation of the earthquake excitation to the extent that the peak response is the most important factor in the performance of the structure. However, the safety of the structure may depend upon more than just the peak response. In particular, if the structure fails due to cyclic loading, the safety of the structure will be a function of the entire history of cyclic oscillations. In order to determine the likelihood of failure of a given structure, it will be necessary to know more about the history of the response. The response spectrum alone is inadequate for this purpose.

The objective of this paper is to examine the effect of earthquake duration on the degree of damage to a simple structure when the response spectrum of the excitation alone is specified. Only by carrying out such an investigation can the adequacy of present response spectrum based design criteria be more fully assessed.

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## THE EXCITATION PROCESS

Let the earthquake acceleration be modelled by a modulated Gaussian random process $\theta(t)g(t)$, where the power spectral density of the process $S(\omega)$ and the modulation envelope $\theta(t)$ are selected to model the spectral power distribution and the non-stationary characteristics of the type of earthquake considered.

Specific forms for the modulation envelope have been suggested by a number of authors.[2–5] For the present analysis, it will be assumed that the modulation envelope $\theta(t)$ may be expressed as

$$\theta(t) = \begin{cases} 56{\cdot}25\left(\dfrac{t}{t_d}\right)^2; & 0 \leqslant t \leqslant \dfrac{2}{15}t_d \\ 1{\cdot}0; & \dfrac{2}{15}t_d \leqslant t \leqslant \dfrac{1}{2}t_d \\ \exp\left[-2{\cdot}976\left(\dfrac{t}{t_d}-\dfrac{1}{2}\right)\right]; & \dfrac{1}{2}t_d \leqslant t \leqslant t_d \end{cases} \tag{1}$$

where $t_d$ is defined as the duration of the excitation. When $t_d = 30$ sec, the modulation envelope is similar to that of the Caltech B-type artificial earthquake.[3]

The spectral power distribution is specified by means of a predetermined design response spectrum. It is assumed that the desired response spectrum is a smooth curve characterized by a given confidence level. The confidence level is the probability that the response spectrum be not exceeded. Hence, the ordinates of the power spectral density $S(\omega)$ are chosen such that

$$\Pr[|x(t)|_{\max} < \mathrm{SD}(\omega_n, \zeta);\ 0 \leqslant t \leqslant t_d] = P_s \tag{2}$$

where $\mathrm{SD}(\omega_n, \zeta)$ is the desired response spectrum ordinate for a circular natural frequency $\omega_n$ and a fraction of critical damping $\zeta$, and $P_s$ is the confidence level of the response spectrum. $x(t)$ is the response of a single-degree-of-freedom system described by

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \theta(t)g(t) \tag{3}$$

Since earthquake response spectra are usually constructed for lightly damped systems for which the response is narrow-banded, it is reasonable to assume that $g(t)$ can be represented approximately by a white-noise process with constant spectral density $S_0 = S(\omega_n)$ for each circular natural frequency $\omega_n$. This assumption has been routinely employed in earthquake response studies.[6]

To satisfy equation (2) requires the computation of the first-passage probability of a linear single-degree-of-freedom oscillator. Although the first-passage problem for such a system has not been solved exactly to date, reasonably accurate approximations have been obtained by considering the response of the randomly excited simple oscillator as a two-state Markov process.[7, 8] Herein, the approach of Mason and Iwan[8] is employed.

It may be shown that

$$P_s = \exp\left[-\int_0^{t_d} \alpha(\mathrm{SD}, S_0, \omega_n, \zeta, t)\,\mathrm{d}t\right] \tag{4}$$

where

$$\alpha(\mathrm{SD}, S_0, \omega_n, \zeta, t) = \frac{-2\nu(b,t)\log(P^*(t))}{\left[1 + \dfrac{\sigma^4(t)}{\sigma_s^4(t)}\right]\left[1 - \dfrac{\nu(b,t)}{\nu(0,t)}\right]} \tag{5}$$

and

$$P^*(t) = \frac{1}{\operatorname{erfc}\left(\dfrac{\mathrm{SD}}{\sqrt{2}\sigma(t)}\right)}\left\{\frac{1}{2}\left[1 - \sqrt{\frac{2}{\pi(1-c^2)}\frac{\mathrm{SD}}{\sigma(t)}}\right][\operatorname{erf}(y_1) - \operatorname{erf}(y_2)]\right.$$

$$\left. + \frac{c}{\pi\sqrt{1-c^2}}[\exp(-y_2^2) - \exp(-y_1^2)] + \operatorname{erfc}(y_1)\right\} \tag{6}$$

$$c = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) \tag{7}$$

$$y_1 = \frac{1}{c}\left[\frac{\mathrm{SD}}{\sqrt{2}\sigma(t)} + \frac{1}{2}\sqrt{\pi(1-c^2)}\right] \tag{8}$$

$$y_2 = \max\left[\frac{\mathrm{SD}}{\sqrt{2}\sigma(t)}, \frac{1}{c}\left[\frac{\mathrm{SD}}{\sqrt{2}\sigma(t)} - \frac{1}{2}\sqrt{\pi(1-c^2)}\right]\right\} \tag{9}$$

$v(b, t)$ is the average frequency of up-crossings of level $x = b$, where $b = \mathrm{SD}$ for this calculation. $\sigma(t)$ is the instantaneous rms response of $x$, and $\sigma_s(t)$ is the stationary rms response for the instantaneous magnitude of the excitation. Equation (4) must be solved iteratively for $S_0$.

In the analysis which follows, the NRC Regulatory Guide 1.60[9] horizontal design response spectrum for a maximum acceleration of 0·5 $g$ and a confidence limit $P_s = 84{\cdot}1$ per cent is used as the target spectrum.

## CUMULATIVE DAMAGE

When a structure is subjected to strong earthquake ground motion, large displacements can occur. The displacements may be associated with large strains in the structural members. The repeated application of large strains may cause failure in the structural members.

It is assumed that the structural members follow a prescribed failure relationship of the form

$$N\mu^s = C \tag{10}$$

where $N$ is the number of cycles to failure at a constant ductility amplitude $\mu$, and $s$ and $C$ are positive empirical constants. This is consistent with the Manson–Coffin relationship between plastic strain and low-cycle failure life.[10] Here, ductility refers to the ratio of the maximum deformation to the deformation corresponding to yielding in the members of the structure.

The simple rule proposed by Palmgren[11] and Miner[12] is applied to relate the accumulated damage of a system with varying response amplitude to a system with constant response amplitude. The rule assumes damage accumulation to be a linear function of the number of cycles of constant amplitude cyclic loading. Hence, the incremental damage due to the application of $n_i$ cycles at a ductility level $\mu_i$ is

$$D_i = \frac{n_i}{N(\mu_i)} = \frac{1}{C}n_i\mu_i^s \tag{11}$$

If the total damage is independent of the order of application of different ductility levels, the total damage for varying ductility levels is given by

$$D = \sum_i D_i \tag{12}$$

Failure occurs when $D$ reaches a value of unity. It is recognized that this damage model may give only a crude estimate of actual structural damage. However, the simplicity of the model makes it analytically attractive and it is believed that the results obtained are at least qualitatively correct.

An alternative method for evaluating structural damage in reinforced concrete structures under earthquake ground motions has been proposed by Park and Ang.[13] The damage is expressed as a linear combination of the maximum deformation and the absorbed hysteretic energy. This accounts for effects due to both the maximum response and the repeated cyclic loading.

The concept of total damage may be extended to random response by assuming that the number of peaks occurring in the response is equal to the number of cycles in the response. This assumption is reasonable for lightly damped structures where the response is a narrow-banded non-stationary cyclic process.

Let $m$ be the total number of peaks per unit time and $p(\mu; t|m)$ be the conditional probability density for the amplitude of the response given the number of peaks per unit time. The quantity $mp(\mu; t|m)$ represents the number of peaks per unit time at a level $\mu$ given $m$ peaks have occurred per unit time. Hence, the expected number of peaks per unit time at a level $\mu$ is given by

$$E[n(\mu, t)] = \int_0^\infty mp(\mu; t|m)p(m; t)\,\mathrm{d}m \tag{13}$$

where $p(m; t)$ is the time-varying probability density for the total number of peaks per unit time.

Based on the incremental damage of a deterministic response [equation (11)], the expected rate of damage accumulation may be written as

$$E\left[\frac{\mathrm{d}D}{\mathrm{d}t}(t)\right] = \int_0^\infty \frac{E[n(\mu, t)]}{N(\mu)}\mathrm{d}\mu = \frac{1}{C}\int_0^\infty \mu^s \int_0^\infty mp(\mu; t|m)p(m; t)\,\mathrm{d}m\,\mathrm{d}\mu \tag{14}$$

If the total number of peaks per unit time is assumed independent of the amplitude of the peaks, this reduces to

$$E\left[\frac{\mathrm{d}D}{\mathrm{d}t}(t)\right] = \frac{E[M(t)]}{C}\int_0^\infty \mu^s p(\mu; t)\,\mathrm{d}\mu \tag{15}$$

where $E[M(t)]$ is the expected total number of peaks per unit time. The expected total damage may be found by integrating over the duration of the excitation. Hence, the expected total damage is given by

$$E[D(t_\mathrm{d})] = E\left[\int_0^{t_\mathrm{d}} \frac{\mathrm{d}D}{\mathrm{d}t}(t)\,\mathrm{d}t\right] = \frac{1}{C}\int_0^{t_\mathrm{d}} E[M(t)]\int_0^\infty \mu^s p(\mu; t)\,\mathrm{d}\mu\,\mathrm{d}t \tag{16}$$

## EXPECTED DAMAGE FOR A LINEAR SYSTEM

The approach outlined here may be used to determine the expected damage for a simple structural system subjected to an earthquake-like excitation with a specified response spectrum, but varying duration. Consider a system which can be represented by the linear single-degree-of-freedom oscillator described by equation (3).

For a lightly damped simple oscillator, the cyclic non-stationary response is narrow-banded and the number of peaks per unit time is approximately equal to the number of zero crossings per unit time. This assumption is true for a linear system and may be extended to a non-linear system provided the appropriate effective natural frequency is used. Hence,

$$E[M(t)] = \nu(0, t) \tag{17}$$

Also, the average number of peaks per unit time occurring at a level greater than $\mu$ is approximately equal to the average number of up-crossings of the level $\mu$.

Expressing the up-crossing level in terms of the corresponding ductility ratio $\mu$, the average probability density of the peak magnitudes may be written as

$$p(\mu; t) = \frac{-1}{\nu(0, t)}\frac{\partial \nu(\mu, t)}{\partial \mu} \tag{18}$$

Substituting equations (17) and (18) into equation (16) gives

$$E[D(t_\mathrm{d})] = \frac{-1}{C}\int_0^{t_\mathrm{d}}\int_0^\infty \mu^s \frac{\partial \nu(\mu, t)}{\partial \mu}\,\mathrm{d}\mu\,\mathrm{d}t \tag{19}$$

The formulation has been carried out in terms of the ductility ratio $\mu$. This implies that yielding response behaviour is taking place. This, in turn, means that the system will behave in a non-linear manner. However, the effects of system non-linearity can often be accounted for approximately by means of a simple adjustment in the damping and natural frequency of the structure.[14] Therefore, an understanding of the linear system response is fundamental to understanding the effects of duration on damage, even for non-linear systems.

In general, the response of a yielding system will consist of two components: a narrow-banded oscillatory component near the natural frequency of the system and a low-frequency drift component. For analytical purposes, these two components can be considered separately. Studies have shown that equivalent linear systems (i.e. replacing the actual inelastic system by a linear system) can be accurately used to model the oscillatory component of the response.[15] Herein, attention is directed toward understanding the cyclic component of the response which is associated with local element failure. The collapse of the structure depends not only on the local damage, but also on the degree of drift which changes the stability of the structure. However, for purposes of local damage analysis, it is the oscillatory component of the response that is most important and this component may be estimated with sufficient accuracy using a linearized system.

Two types of structure will be considered. For a steel structure, it will be assumed that $s = 2$ and $C = 167$. These parameters are based on results reported by Yamada[16] for wide flange steel columns loaded axially to one-third of the ultimate strength and cycled in plastic bending to failure. Failure was described as fracture of the member accompanied by a loss of axial resistance.

For a reinforced concrete structure, the parameters of the failure law will be assumed to be $s = 6$ and $C = 416$. These parameters are also based on results reported by Yamada for rectangular columns loaded axially to one-third of their yield strength and cycled in plastic bending. Failure was described as the collapse of the member. The details of the design of the reinforced concrete column will clearly affect the values of the parameters $s$ and $C$, and the values used herein may not be representative of standard design practice in some countries. However, it is believed that the relationship used herein provides a valid basis for making qualitative statements about the behaviour of reinforced concrete structures.

Figures 1 and 2 show the expected total damage for a linear structure subjected to an earthquake excitation of varying duration. Curves are shown for several different levels of response ductility ratio $\mu$. The fraction of critical damping is taken to be 10 per cent for all cases shown. Figure 1 shows the results for $s$ and $C$ parameters corresponding to the reference steel structure and Figure 2 shows results for parameters corresponding to the reference reinforced concrete structure.

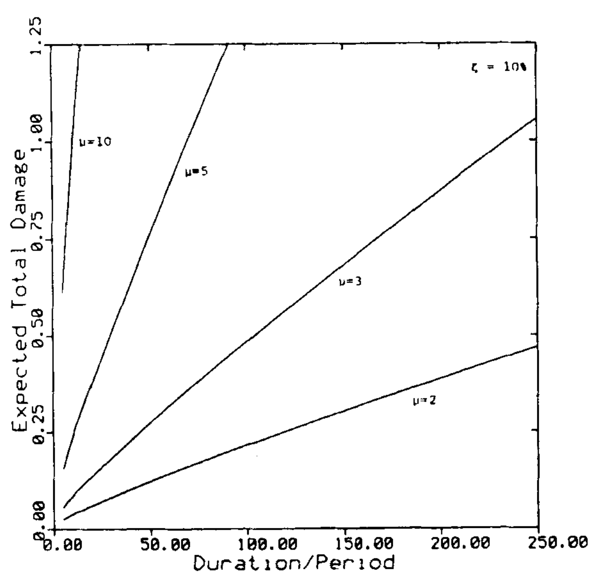


Figure 1. Expected damage versus normalized duration for representative linear steel structure

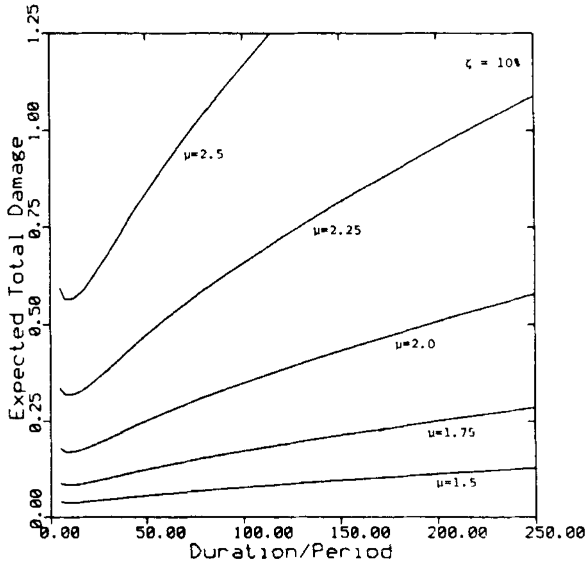


Figure 2. Expected damage versus normalized duration for representative linear reinforced concrete structure

It is clear from Figures 1 and 2 that expected damage is a strong function of both duration and ductility of response. It is assumed that no damage is associated with an excitation of zero duration. For both types of structure considered, there is an initial rapid build up of damage with increasing duration, followed by a nearly constant rate of damage accumulation corresponding to the stationary damage rate. As an illustration of the effect of duration, consider a steel structure with a period of 1 sec designed for a ductility of $\mu = 5$. For a duration of 30 secs, the expected damage as predicted by the analysis is approximately 0·51. However, for an earthquake with the identical response spectrum but a duration of 60 secs, the expected damage is predicted to be approximately 0·88. This is a significant difference which will be overlooked if duration effects are not taken into account in the design specification. The effect of duration can be much more pronounced for reinforced concrete structures.

From equation (19), it may be shown that $E[D]$ is directly proportional to $\mu^s$ for the linear system. Therefore, damage is a much stronger function of the ductility ratio for a reinforced concrete structure (higher value of $s$) than for a steel structure (lower value of $s$). This effect is clearly seen in Figures 1 and 2. The steel structure is capable of sustaining much greater levels of ductility without failure than the reinforced concrete structure. Furthermore, the expected damage in the steel structure is much less sensitive to changes in the ductility ratio than is the case for the reinforced concrete structure.

It is noted that the expected damage of the reinforced concrete structure increases extremely rapidly with duration for durations less than about 5 periods. For durations of approximately 5–25 periods, the expected damage is actually rather insensitive to duration. Then the expected damage increases nearly linearly with further increases in duration. The greatest increase in damage is associated with short durations. This is in contrast to the behaviour of the steel structure where expected damage is a more uniformly increasing function of duration.

A useful way to display expected damage information for a linear system is to plot contours of expected damage equal to unity (failure) as a function of the ductility ratio of the response and the duration of excitation, as shown in Figure 3. In this way, combinations of duration and ductility ratio for which the expected total damage is greater than or less than unity may readily be identified. From Figure 3, it is evident that the range of the allowable ductility ratio is much smaller for a reinforced concrete structure than for a steel structure.

The rapid increase in expected damage associated with short durations, along with the strong dependence upon the ductility ratio, leads to the existence of a rather sharp threshold for failure in a reinforced concrete

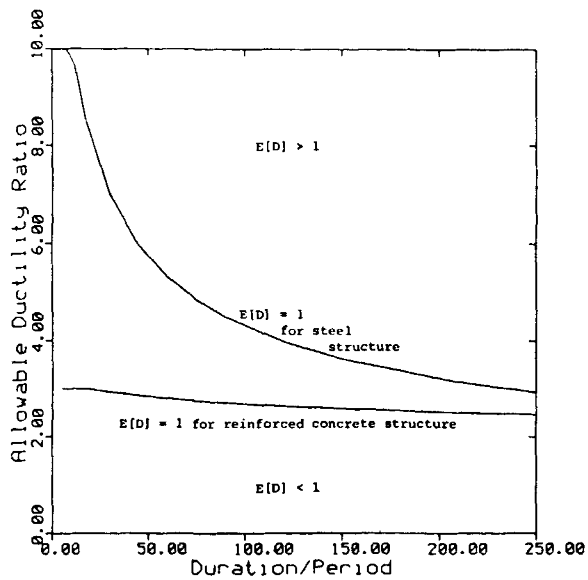


Figure 3. Contours of expected damage equal to unity versus allowable ductility ratio and normalized duration for representative linear steel and reinforced concrete structures

structure. For example, in the present case, failure will occur for nearly any moderate duration of excitation if the ductility ratio is greater than about $\mu = 2{\cdot}5$. The precise value of this threshold ductility is, of course, a function of the failure model employed in the analysis and will not be applicable to all reinforced concrete structures. However, the general nature of the functional dependence of the expected damage on ductility and duration is thought to be representative.

## EFFECT OF SOFTENING

In order to examine the effect of stiffness softening on expected damage, a simple non-linear elastic system is considered. Let the system be described by the differential equation

$$\ddot{x} + 2\zeta_e \omega_e \dot{x} + \frac{2}{\pi}\omega_n^2 x_y \tan^{-1}\left(\frac{\pi}{2}\frac{x}{x_y}\right) = \theta(t)g(t) \tag{20}$$

The normalized restoring force diagram for this system is shown in Figure 4. The restoring force is limited by the value $f_u$. The small amplitude circular natural frequency of the system is $\omega_n$. The effective 'yield level' is

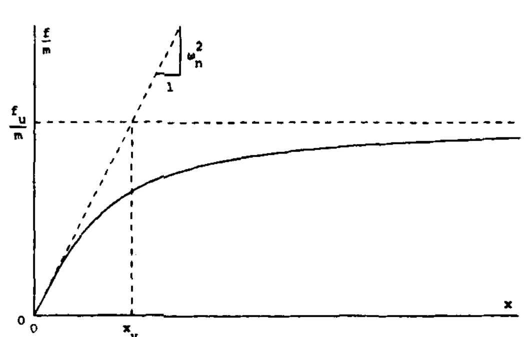


Figure 4. Softening restoring force characteristic

defined as

$$x_y = \frac{f_u}{m\omega_n^2} \tag{21}$$

This non-linear force–deflection relation is similar to the backbone curve of a typical yielding structure. A simple extension to the analysis would be to use an internal variable hysteretic model. However, no hysteretic energy dissipation will be considered herein.

The effective circular natural frequency $\omega_e$ and fraction of critical damping $\zeta_e$ which are functions of the response statistics may be determined by the method of equivalent linearization.[17] This yields

$$\omega_e = \omega_n [\pi\gamma \exp(\gamma^2) \operatorname{erfc}(\gamma)]^{\frac{1}{2}} \tag{22}$$

$$\zeta_e = \zeta [\pi\gamma \exp(\gamma^2) \operatorname{erfc}(\gamma)]^{-\frac{1}{2}} \tag{23}$$

$$\gamma = \frac{2}{\pi} \frac{x_y}{\sigma(t)} \tag{24}$$

where $\zeta$ is the small amplitude fraction of critical damping.

From equations (22)–(24) the effective circular natural frequency and fraction of critical damping will be functions of time. However, the formulation presented above may still be used to compute the expected damage for a given set of excitation and system parameters. It will no longer be possible to normalize the duration by the period of the system as in the case of the linear structure. In Figures 5–8 the expected total damage for the two structural systems is plotted as a function of the small amplitude natural frequency $\omega_n$ for two different values of duration $t_d$ and three different values of ductility ratio $\mu$. The fraction of critical damping $\zeta$ is taken to be 10 per cent, as in the previous examples.

The major effect of softening is to reduce the level of displacement response for a given excitation. This, in turn, reduces the expected level of damage for any given duration and small amplitude circular natural frequency. The effect is greatest for higher ductility ratios. Although the level of damage is reduced when softening is included, the total damage is still a function of the duration of the excitation. The strongest dependence on the duration is observed for the steel structure. As noted earlier, the expected damage of the

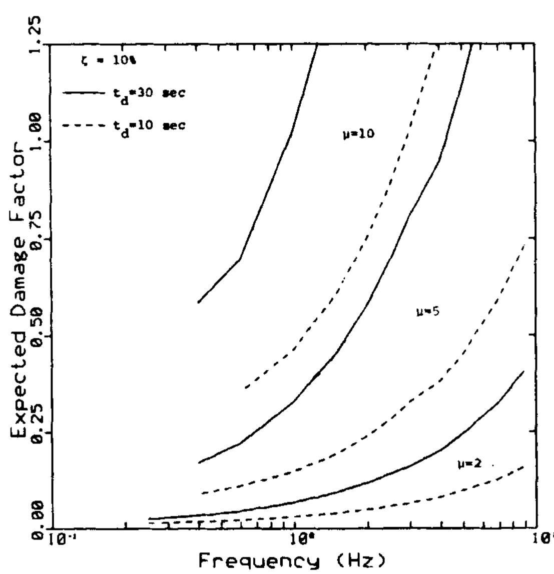


Figure 5. Expected damage versus natural frequency for representative softening steel structure

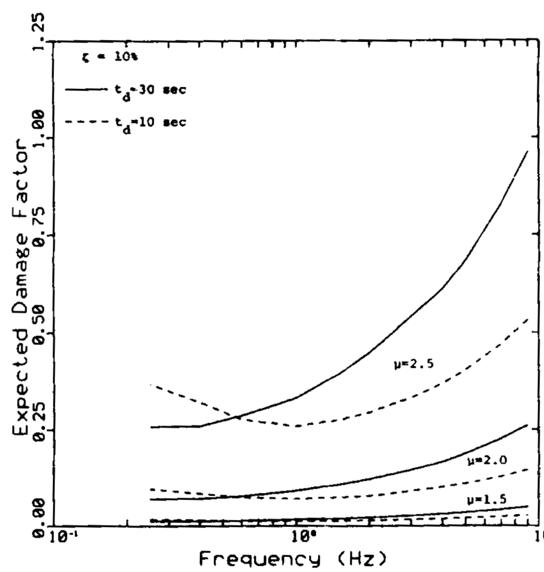


Figure 6. Expected damage versus natural frequency for representative softening reinforced concrete structure

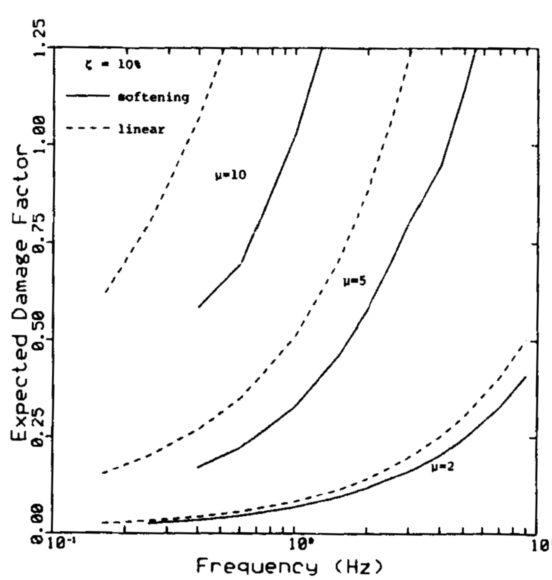


Figure 7. Expected damage versus natural frequency for representative linear and softening steel structures

reinforced concrete structure is rather insensitive to duration for durations in the range of 5–25 times the small amplitude period of the structure.

A considerable amount of computation is required to compute the expected damage contours for the non-linear system. Since the expected total damage of the softening non-linear system is less than that for the linear system, the contours for the linear system may be conservatively used in place of the contours for the non-linear system.

## CONCLUSIONS

A straightforward analytical procedure was developed to investigate the effect of earthquake duration on the damage of a structure. The results reported herein confirm that duration can have a strong effect on the

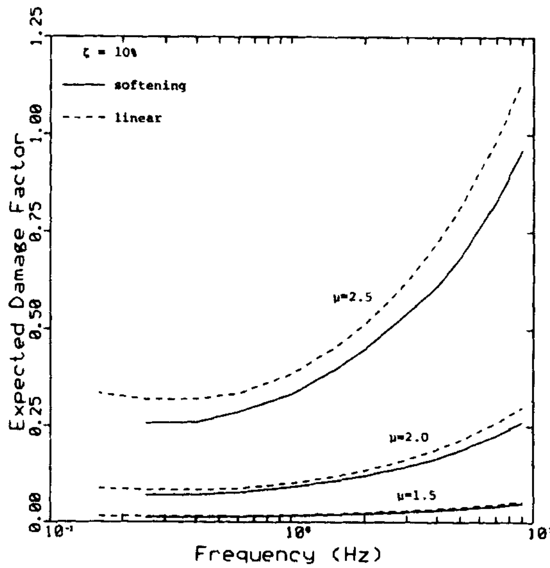


Figure 8. Expected damage versus natural frequency for representative linear and softening reinforced concrete structures

expected damage of a structure. Also, a marked difference in the behaviour of structures constructed of different materials is exhibited. This is consistent with results reported earlier in Reference 18 for a slightly different system. Use of the response spectrum alone to specify a design input ground motion ignores this effect. Although the damage model in this study was rather crude, the results may be considered qualitatively correct. It is recommended that some measure of duration be more often provided in earthquake design specifications and utilized in the analysis of structural safety.

## ACKNOWLEDGEMENTS

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