Search Go DICTIONARY | CLOSE WINDOW

Variance of a discrete random variable

Noun Phrase. (Probability.)

The variance of a discrete random variable is a measure of the dispersion of the variable. The variance measures how different the values of the random variable tend to be from the expectation, or mean, of the random variable. The more the values tend to differ from the expectation, the greater the variance.

The variance of a random variable is simply the variance of the probability distribution of the random variable. So, for discrete random variables, the variance is the probability-weighted mean of the squared deviations of the different values from their mean.

Let's look at two examples.

Example 1.

Seventy-six percent of the students at a particular University of California campus are undergraduates, and the other 24% are graduates. If we choose a student at random from the campus and code the student as a "1" if the student is an undergraduate and as a "0" if the student is a graduate, then the student's standing is a random variable -- we'll call it X -- which can take on the value 0 or the value 1.

The probability distribution of X is as follows:

Value x of X	0	1
$P(X=x)$	0.76	0.24

The expectation of X is $E(X)=(0)(0.76)+(1)(0.24)=0.24$.

The variance of X is computed by

$$\begin{aligned} Var(X) &= (0-E(X))^2 P(X=0)+(1-E(X))^2 P(X=1) \\ &= (0-0.24)^2(0.76)+(1-0.24)^2(0.24) \\ &= 0.1824. \end{aligned}$$

Example 2.

Let Y be the random variable which gives the number that appears when a fair die is rolled.

The probability distribution of Y is as follows:

Value y of Y	1	2	3	4	5	6
$P(Y=y)$	1/6	1/6	1/6	1/6	1/6	1/6

The expectation of Y is

$$E(Y) = (1)\left(\frac{1}{6}\right)+(2)\left(\frac{1}{6}\right)+(3)\left(\frac{1}{6}\right)+(4)\left(\frac{1}{6}\right)+(5)\left(\frac{1}{6}\right)+(6)\left(\frac{1}{6}\right)=\frac{21}{6}=3.5.$$

The variance of Y is then

$$\begin{aligned} Var(Y) &= (1-3.5)^2\left(\frac{1}{6}\right)+(2-3.5)^2\left(\frac{1}{6}\right)+(3-3.5)^2\left(\frac{1}{6}\right)+(4-3.5)^2\left(\frac{1}{6}\right)+(5-3.5)^2\left(\frac{1}{6}\right)+(6-3.5)^2\left(\frac{1}{6}\right) \\ &\approx 2.92. \end{aligned}$$

Definition.

Let X be a discrete random variable that takes on the values $x_1, x_2, \ldots, x_n$ (and only those values). The *variance* of X, often denoted $Var(X)$, is defined to be

$$Var(X) = E\left[(X-E(X))^2\right] = \sum_{i=1}^{n}(x_i - E(X))^2 P(X=x_i),$$

where $E(X)$ is the expectation of X.

The *standard deviation* of X is the positive square root of its variance ,

$$\sqrt{Var(X)}.$$

Some Comments.

- The notion of variance of a discrete random variable agrees with the notion of variance of a population. The difference between the two lies in the fact that the variance of a discrete random variable involves the probabilities of the values of the random variable, while the population variance involves the relative frequencies of the population values. Such a comparison may also be made between the notions of expectation of a random variable and sample mean or population mean.

- The variance of the discrete random variable X may also be computed by the equivalent formula $Var(X) = E(X^2) - (E(X))^2$. This formula often leads to easier computation than the formula given in the definition. To see that the two formulas are equivalent, note that

$$\begin{aligned} Var(X) &= E\left[(X-E(X))^2\right] && \text{[by the definition of } Var(X)\text{]} \\ &= E\left[X^2 - 2XE(X) + (E(X))^2\right] \\ &= E(X^2) - 2E(X)E(X) + (E(X))^2 && \text{[by properties of the expectation } E(X)\text{]} \\ &= E(X^2) - (E(X))^2. \end{aligned}$$

Properties of the variance.

- Let a be any real number, and let X be a random variable. Then

$$Var(aX) = a^2 Var(X)$$

and

$$Var(X \pm a) = Var(X).$$

This second equation is a consequence of the fact that the deviations from the expectation do not change if you add (or subtract) a constant to all the values of the random variable.

- For any random variable X, we have $Var(X) = 0$ if, and only if, X is constant.
- If X and Y are two independent random variables, then

$$Var(X+Y) = Var(X) + Var(Y).$$