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Expectation

Noun. (Probability.)

The *expectation* (or *mean* , or *expected value*) of a random variable is the mean of the probability distribution of the random variable.

So, for a discrete random variable, the expectation is a weighted mean of the values that the variable may take on, with the weights being the probabilities.

Just as the mean of a probability distribution is a measure of central tendency for the distribution, the expectation of a random variable is a measure of the central tendency of the values that the random variable takes on.

The expectation of the random variable X is usually denoted by $E(X)$.

Example 1:

Let D be the random variable giving the number obtained when a fair die is rolled. We specify the probability distribution of D in the table below, which contains the possible values of D and the probabilities that D takes on those values:

Value of D	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

So

$$E(D) = (1)\left(\frac{1}{6}\right)+(2)\left(\frac{1}{6}\right)+(3)\left(\frac{1}{6}\right)+(4)\left(\frac{1}{6}\right)+(5)\left(\frac{1}{6}\right)+(6)\left(\frac{1}{6}\right) = 3.5.$$

Note a couple of things about this particular expectation. First, it is not a possible value of D. (It is impossible to obtain a 3.5 in a roll of a die.) This is not a problem, and in fact it motivates the idea of using the term *expectation* rather than *expected value*, since the expectation may not be a value that the random variable takes on. Second, the expectation in this case is equal to the arithmetic mean of the possible values of D, that is, the arithmetic mean of 0, 1, 2, 3, 4, 5, and 6. This is because D takes each of these values with equal probability.

Example 2:

Let S be the random variable giving the sum of the numbers when two, fair dice are rolled. The probability distribution of S is given in the table below, which contains the possible values of S and the probabilities that S takes on those values:

Value of S	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

So

$$E(S) = (2)\left(\frac{1}{36}\right)+(3)\left(\frac{2}{36}\right)+(4)\left(\frac{3}{36}\right)+(5)\left(\frac{4}{36}\right)+(6)\left(\frac{5}{36}\right)+(7)\left(\frac{6}{36}\right)$$
$$+(8)\left(\frac{5}{36}\right)+(9)\left(\frac{4}{36}\right)+(10)\left(\frac{3}{36}\right)+(11)\left(\frac{2}{36}\right)+(12)\left(\frac{1}{36}\right) = 7.$$

Note that the expectation of the sum of the numbers on the dice is equal to the sum of the expectations of the numbers on the individual dice , that is,
$7 = E(S) = E(D+D) = E(D)+E(D) = 3.5+3.5$.

We have the following general definition.

Definition of expectation of a discrete random variable:

Suppose that the probability distribution of the discrete random variable X is given by the following table:

Value of X	x_1	x_2	x_3	...	x_j	...	x_k
Probability	p_1	p_2	p_3	...	p_j	...	p_k

Then the *expectation* of X is given by

$$E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \ldots + x_k p_k = \sum_{i=1}^{k} x_i p_i \quad .$$

Properties of the expectation:

- If a is a constant and X a random variable, then

  $$E(a) = a$$

  and

  $$E(aX) = a\,E(X).$$

- If X and Y are two random variables (independent or not), then

  $$E(X+Y) = E(X)+E(Y).$$

  As a special case, for any constant a,

  $$E(X+a) = E(X)+a.$$

- If X and Y are two independent random variables, then

  $$E(XY) = E(X)E(Y).$$

See also:

- Population mean