Learned transform compression with optimized entropy encoding

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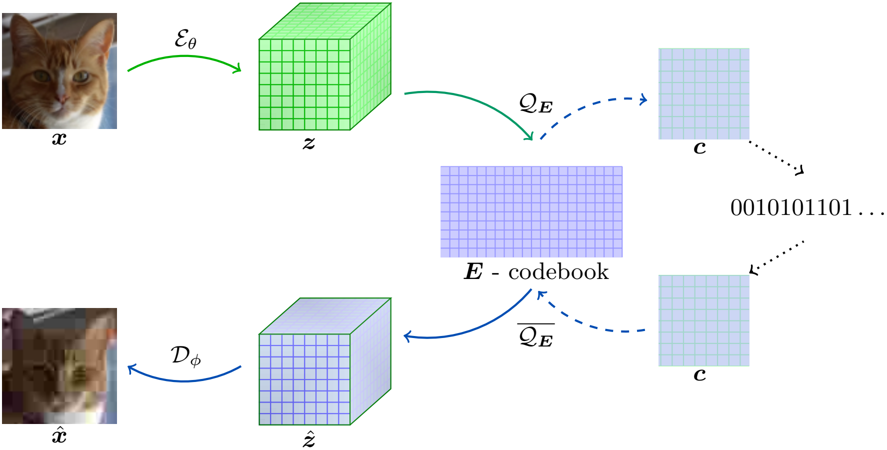
Transform coding with vector quantization



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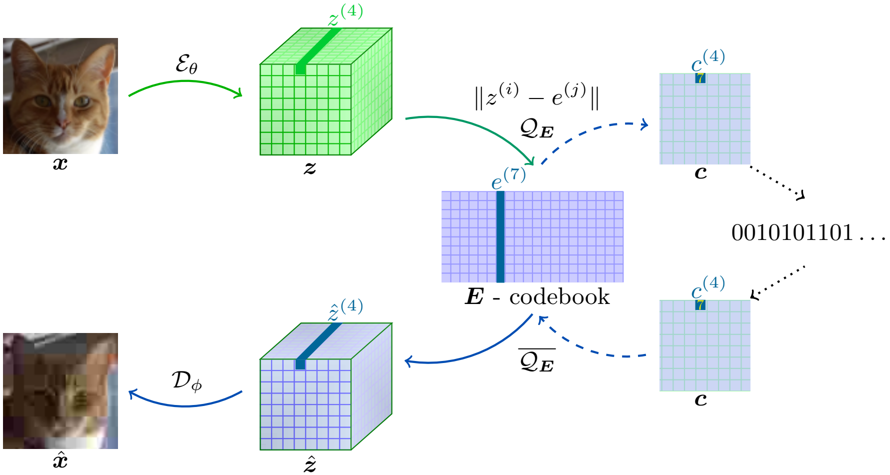

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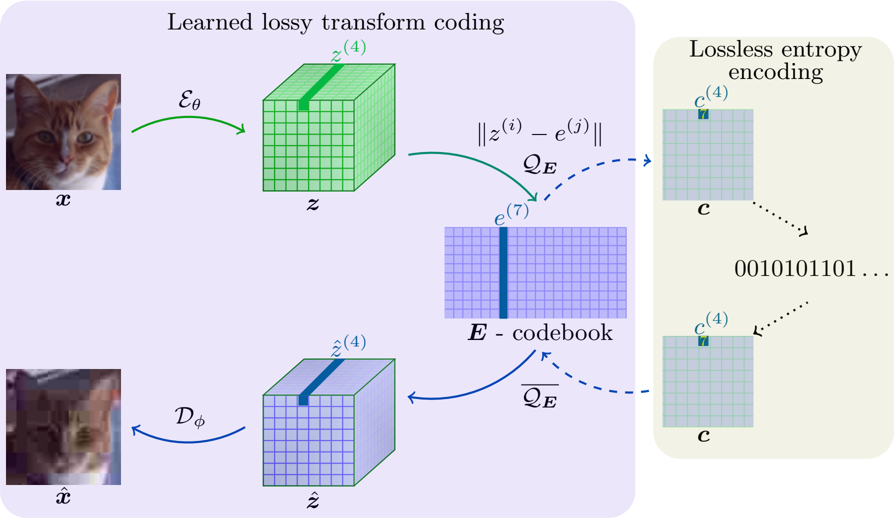
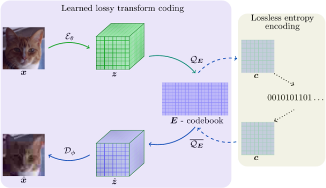


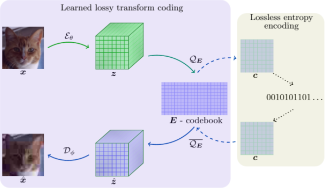
End-to-end optimized compression



$$\mathcal{E}_\theta, \mathcal{D}_\phi, \mathcal{Q}_{\boldsymbol{E}}$$

$$\mathcal{L} := \underbrace{\mathbb{E}_{\mu_x} d(\mathtt{x}, \hat{\mathtt{x}})}_{distortion} + \lambda \underbrace{\mathbb{E}_{\mu_c} l(\mathtt{c})}_{rate}$$

End-to-end optimized compression



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Shannon: $l^*(c) = -\log p_c(c) \implies \mathbb{E}_{\mu_c} l^*(c) = -\mathbb{E}_{\mu_c} \log p_c(\mathsf{c}) = \mathbb{H}_{\mu_c}(\mathsf{c})$

$$\int_A p_c \, \mathrm{d}\# = \sum_{\boldsymbol{a} \in \boldsymbol{A}} p_c(\boldsymbol{a}) = \mu_c(\boldsymbol{A}) \qquad \log p_c = ??? \quad \mathbb{H}_{\mu_c}(\mathsf{c}) = ???$$

$$q_c \approx p_c \qquad -\mathbb{E}_{\mu_c} \log q_c(\mathsf{c}) \approx -\mathbb{E}_{\mu_c} \log q_c(\mathsf{c}) \quad \mathbb{H}_{\mu_c|q_c}(\mathsf{c}) \approx \mathbb{H}_{\mu_c}(\mathsf{c})$$

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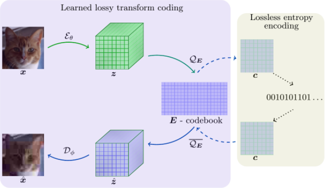

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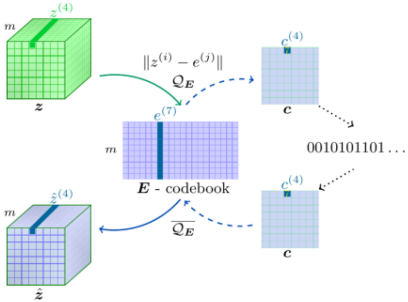
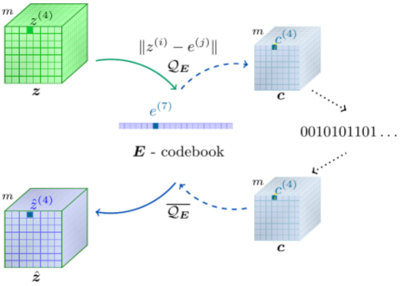
Vector quantization

Vector quantization



message length: d^2

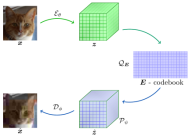
Scalar quantization



message length: $d^2 m$

$$\mathcal{Q}_{\boldsymbol{E}}: \quad \hat{\boldsymbol{z}}^{(i)} = \arg\min_{\boldsymbol{e}^{(j)}} \|\boldsymbol{z}^{(i)} - \boldsymbol{e}^{(j)}\| \qquad c^{(i)} = \{j : \hat{\boldsymbol{z}}^{(i)} = \boldsymbol{e}^{(j)}\}$$

Model learning - problems



Problem $\mathcal{Q}_{\boldsymbol{E}}$: quantization non-differentiable

Forward: $\boldsymbol{x} \xrightarrow{\mathcal{E}_\theta} \boldsymbol{z} \xrightarrow{\mathcal{Q}_{\boldsymbol{E}}} \hat{\boldsymbol{z}} \xrightarrow{\mathcal{D}_\phi} \hat{\boldsymbol{x}} \longrightarrow d(\boldsymbol{x}, \hat{\boldsymbol{x}})$

Backward: $\boxed{\xcancel{\boldsymbol{x} \xleftarrow{\nabla_\theta} \nabla_{\boldsymbol{z}} \xleftarrow{\nabla_{\boldsymbol{E}}}}} \nabla_{\hat{\boldsymbol{z}}} \xleftarrow{\nabla_\phi} \nabla_{\hat{\boldsymbol{x}}} \longleftarrow d(\boldsymbol{x}, \hat{\boldsymbol{x}})$

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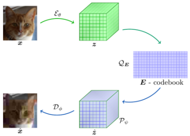

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Problem $\mathbb{H}_{\mu_c}$: entropy not minimized

$q_c \approx p_c: \quad \mathbb{H}_{\mu_c|q_c}(\mathsf{c}) \geq \mathbb{H}_{\mu_c}(\mathsf{c}) \qquad \mathbb{H}_{\mu_c|q_c}(\mathsf{c}) = \overbrace{D_{\mathrm{KL}}(p_c \| q_c)}^{\geq 0} + \mathbb{H}_{\mu_c}(\mathsf{c})$

$$\min_{q_c} \ \mathbb{H}_{\mu_c|q_c}(\mathsf{c}) \ \Leftrightarrow \ \min_{q_c} \ D_{\mathrm{KL}}(p_c \| q_c) + \cancel{\mathbb{H}_{\mu_c}(\mathsf{c})}$$

$q_c \to p_c \quad p_c$ fixed

Push-forward measure & soft-quantization

1) $\mu_c[\mathtt{c} \in \boldsymbol{A}] = \mu_c[\mathcal{T}_{\boldsymbol{E},\theta}(\mathtt{x}) \in \boldsymbol{A}] = \mu_x[\mathtt{x} \in \mathcal{T}^{-1}_{\boldsymbol{E},\theta}(\boldsymbol{A})]$ $\quad \mathcal{T}_{\boldsymbol{E},\theta} = \mathcal{Q}_{\boldsymbol{E}} \circ \mathcal{E}_\theta$

$$\mathcal{Q}_{\boldsymbol{E}}, \mathcal{E}_\theta \to \mu_c \to \mathbb{H}_{\mu_c}$$

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2) $p_c(c = j) = \begin{cases} 1 & \text{if } \hat{\boldsymbol{z}} = \boldsymbol{e}^{(j)} \\ 0 & \text{otherwise} \end{cases}$ $\quad h_{ce} = -\frac{1}{n} \sum_i^n \log q_c(c_i), \quad c_i \sim \mu_c$

$$\hat{p}_c(c = j) = \frac{\exp(-\sigma \| \boldsymbol{z} - \boldsymbol{e}^{(j)} \|)}{\sum_j^k \exp(-\sigma \| \boldsymbol{z} - \boldsymbol{e}^{(j)} \|)} \qquad s_{ce} = -\frac{1}{n} \sum_{i,j}^{n,k} \hat{p}_c(c_i = j) \log \mathrm{sg}[q_c(j)]$$

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3) $$\tilde{\boldsymbol{z}} = \sum_j^k \hat{p}_c(c = j)\, \boldsymbol{e}^{(j)} \qquad \hat{d}(\boldsymbol{x}, \hat{\boldsymbol{x}}) = d(\boldsymbol{x}, \mathcal{D}_\phi[\mathrm{sg}(\hat{\boldsymbol{z}} - \tilde{\boldsymbol{z}}) + \tilde{\boldsymbol{z}}])$$

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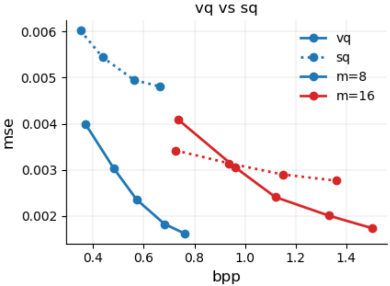
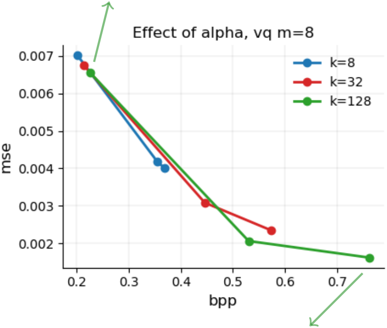
$$\boxed{\arg\min_{\mathcal{E}_\theta, \mathcal{Q}_{\boldsymbol{E}}, \mathcal{D}_\phi, \mathcal{P}_\psi} \frac{1}{n}\sum_i^n \hat{d}(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_i) + \alpha\, s_{ce}(\boldsymbol{c}_i) + \beta\, h_{ce}(\boldsymbol{c}_i), \quad \boldsymbol{x}_i \sim \mu_x}$$

Proof of concept - experiments

$\mathcal{E}_\theta, \mathcal{D}_\phi$: CNN, stride-2 down-/up-sampling, 64 kernels size 3-4, 10 residual blocks with skip connections

$$\mathcal{P}_\psi : q_c(\mathsf{c}) = \prod_i^{d^2} q_{c_i}(\mathsf{c}_i), \quad q_{c_i} = q_{c_j}$$





ADAM, one cycle cosine schedule

$\sigma = 1, \ \beta = 1$ Imagenet32

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