Learned transform compression with optimized entropy encoding

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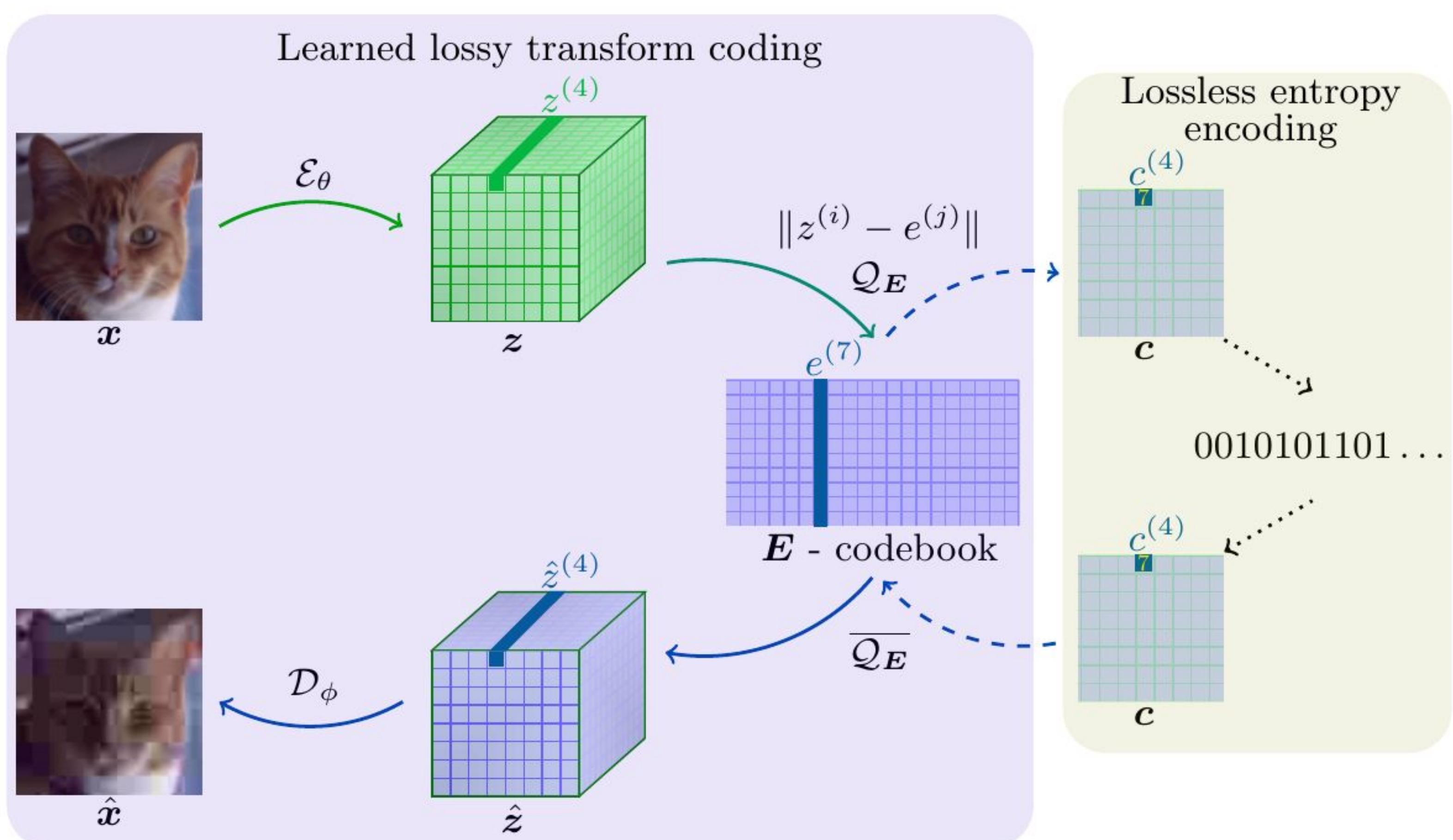
Learned transform coding with vector quantization and lossless entropy encoding



Rate-distortion trade off

Compression loss: learn encoder, decoder and quantization codebook

$$\mathcal{E}_\theta, \mathcal{D}_\phi, \mathcal{Q}_{\boldsymbol{E}} \qquad \mathcal{L} := \underbrace{\mathbb{E}_{\mu_x} d(\mathsf{x}, \hat{\mathsf{x}})}_{distortion} + \lambda \underbrace{\mathbb{E}_{\mu_c} l(\mathsf{c})}_{rate}$$

Shannon: optimal code length ≈ entropy

$$\mathbb{E}_{\mu_c} l^*(c) = -\mathbb{E}_{\mu_c} \log p_c(\mathsf{c}) = \mathbb{H}_{\mu_c}(\mathsf{c})$$

Code distribution: approximate by learned distribution

$$q_c \approx p_c \qquad -\mathbb{E}_{\mu_c} \log q_c(\mathsf{c}) \approx -\mathbb{E}_{\mu_c} \log q_c(\mathsf{c}) \qquad \mathbb{H}_{\mu_c|q_c}(\mathsf{c}) \approx \mathbb{H}_{\mu_c}(\mathsf{c})$$

Cross-entropy loss: rate as cross-entropy instead of entropy

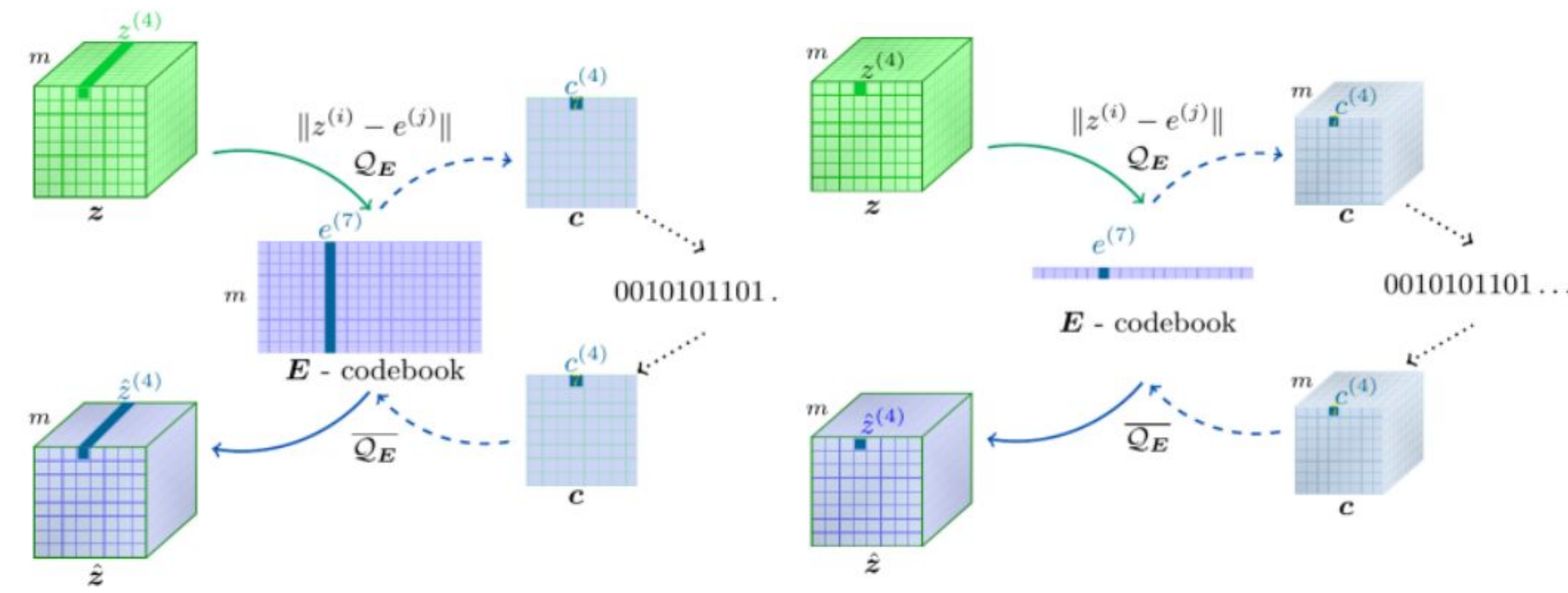
$$\mathcal{E}_\theta, \mathcal{D}_\phi, \mathcal{Q}_{\boldsymbol{E}}, \mathcal{P}_\psi \qquad \mathcal{L} := \underbrace{\mathbb{E}_{\mu_x} d(\mathsf{x}, \hat{\mathsf{x}})}_{distortion} + \lambda \underbrace{\mathbb{H}_{\mu_c|q_c}(\mathsf{c})}_{rate}$$

Quantization

Nearest neighbour search

$$\mathcal{Q}_{\boldsymbol{E}}: \quad \hat{\boldsymbol{z}}^{(i)} = \arg\min_{\boldsymbol{e}^{(j)}} \|\boldsymbol{z}^{(i)} - \boldsymbol{e}^{(j)}\| \qquad c^{(i)} = \{j : \hat{\boldsymbol{z}}^{(i)} = \boldsymbol{e}^{(j)}\}$$

Vector vs scalar quantization: message length vs flexibility



vector quantization
message: **short**
flexibility: low

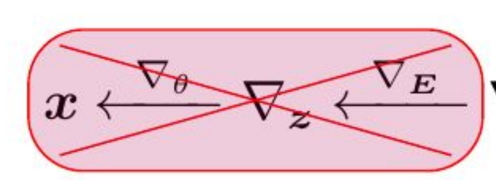
scalar quantization
message: long
flexibility: **high**

Problems

Quantization gradients: codebook and encoder not updated

$$\text{Forward:} \quad \boldsymbol{x} \xrightarrow{\mathcal{E}_\theta} \boldsymbol{z} \xrightarrow{\mathcal{Q}_{\boldsymbol{E}}} \hat{\boldsymbol{z}} \xrightarrow{\mathcal{D}_\phi} \hat{\boldsymbol{x}} \longrightarrow d(\boldsymbol{x}, \hat{\boldsymbol{x}})$$

$$\text{Backward:} \quad \cancel{\boldsymbol{x} \xleftarrow{\nabla_\theta} \nabla_{\boldsymbol{z}} \xleftarrow{\nabla_{\boldsymbol{E}}}} \nabla_{\hat{\boldsymbol{z}}} \xleftarrow{\nabla_\phi} \nabla_{\hat{\boldsymbol{x}}} \longleftarrow d(\boldsymbol{x}, \hat{\boldsymbol{x}})$$



Entropy minimization: cross-entropy objective not learning code transform

$$\mathbb{H}_{\mu_c|q_c}(\mathsf{c}) = \overbrace{D_{\mathrm{KL}}(p_c \| q_c)}^{\geq 0} + \mathbb{H}_{\mu_c}(\mathsf{c})$$

$$\mathcal{P}_\psi: \quad \min_{q_c} \ \mathbb{H}_{\mu_c|q_c}(\mathsf{c}) \approx -\frac{1}{n} \sum_i^n \log q_c(c_i), \quad c_i \sim \mu_c$$

$$\Leftrightarrow \ \min_{q_c} \ D_{\mathrm{KL}}(p_c \| q_c) + \cancel{\mathbb{H}_{\mu_c}(\mathsf{c})}$$

$$q_c \to p_c \qquad p_c \ \text{fixed}$$

Minimization of cross-entropy only with respect to the learned distribution $\mathbf{q_c}$ approximating the unknown code distribution $\mathbf{p_c}$ is suboptimal. It brings $\mathbf{q_c}$ close to $\mathbf{p_c}$ and hence reduces the number of extra bits needed due to using $\mathbf{q_c}$ instead of the true $\mathbf{p_c}$ but it does not encourage low entropy of the true code distribution.

Solutions

Push-forward mesure & soft quantization: *hard and soft cross-entropy*

1) $$\mu_c[\mathsf{c} \in \boldsymbol{A}] = \mu_c[\mathcal{T}_{\boldsymbol{E},\theta}(\mathsf{x}) \in \boldsymbol{A}] = \mu_x[\mathsf{x} \in \mathcal{T}^{-1}_{\boldsymbol{E},\theta}(\boldsymbol{A})] \qquad \mathcal{T}_{\boldsymbol{E},\theta} = \mathcal{Q}_{\boldsymbol{E}} \circ \mathcal{E}_\theta$$

$$\mathcal{Q}_{\boldsymbol{E}}, \mathcal{E}_\theta \to \mu_c \to \mathbb{H}_{\mu_c}$$

2) $$p_c(c = j) = \begin{cases} 1 & \text{if } \hat{\boldsymbol{z}} = \boldsymbol{e}^{(j)} \\ 0 & \text{otherwise} \end{cases} \qquad h_{ce} = -\frac{1}{n} \sum_i^n \log q_c(c_i), \quad c_i \sim \mu_c$$

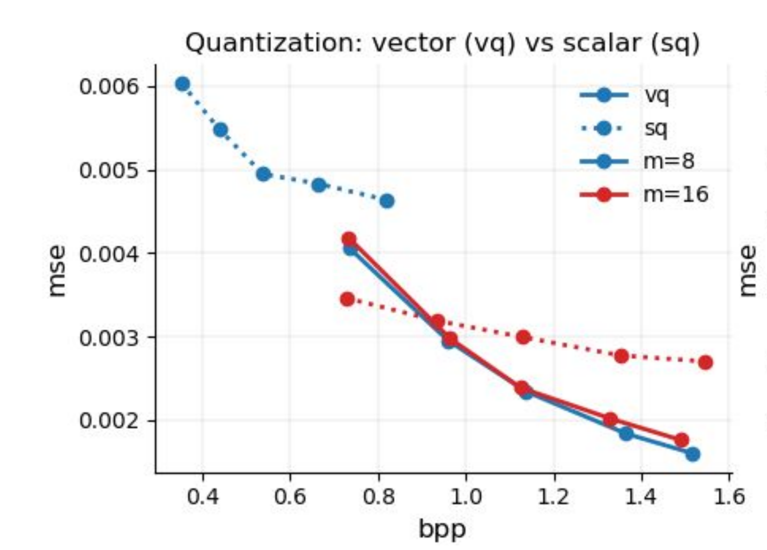
$$\hat{p}_c(c = j) = \frac{\exp(-\sigma \|\boldsymbol{z} - \boldsymbol{e}^{(j)}\|)}{\sum_j^k \exp(-\sigma \|\boldsymbol{z} - \boldsymbol{e}^{(j)}\|)} \qquad s_{ce} = -\frac{1}{n} \sum_{i,j}^{n,k} \hat{p}_c(c_i = j) \log \mathrm{sg}[q_c(j)]$$

3) $$\tilde{\boldsymbol{z}} = \sum_j^k \hat{p}_c(c = j) \, \boldsymbol{e}^{(j)} \qquad \hat{d}(\boldsymbol{x}, \hat{\boldsymbol{x}}) = d(\boldsymbol{x}, \mathcal{D}_\phi[\mathrm{sg}(\hat{\boldsymbol{z}} - \tilde{\boldsymbol{z}}) + \tilde{\boldsymbol{z}}])$$

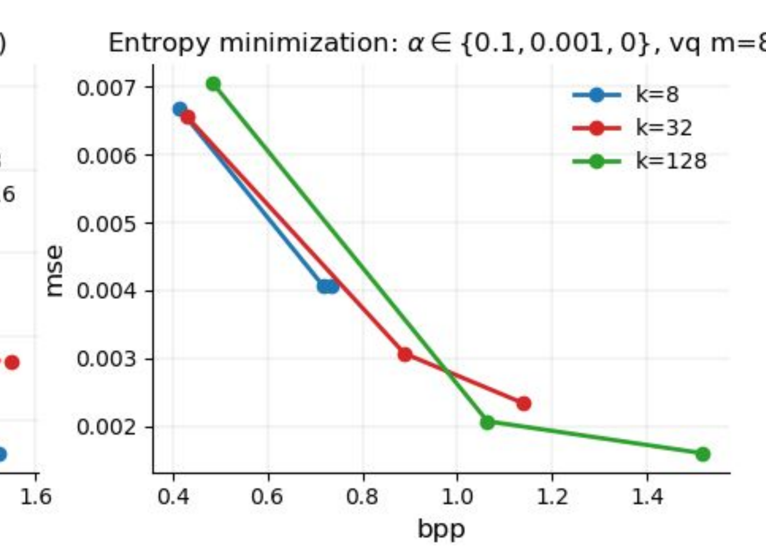
$$\arg\min_{\mathcal{E}_\theta, \mathcal{Q}_{\boldsymbol{E}}, \mathcal{D}_\phi, \mathcal{P}_\psi} \ \frac{1}{n} \sum_i^n \hat{d}(\boldsymbol{x}_i, \hat{\boldsymbol{x}}_i) + \alpha \, s_{ce}(\boldsymbol{c}_i) + \beta \, h_{ce}(\boldsymbol{c}_i), \quad \boldsymbol{x}_i \sim \mu_x$$

Proof of concept experiments

CIFAR: vector vs scalar quantization, effect of soft cross-entropy term



solid / dashed lines - vector / scalar quantization; blue / red - m = 8 / 16 channels in latent representation; points from left to right - increasing number of codebook words k ={8, 16, 32, 64, 128}; α=0, β=1, σ=1



all lines vector quantization with m=8 channels in latent representation; blue / red / green - k = 8 / 32 / 128 codebook words; points from right to left increasing strength of soft cross-entropy α = {0, 0.001, 0.1}, β=1, σ=1

References

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