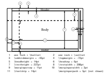
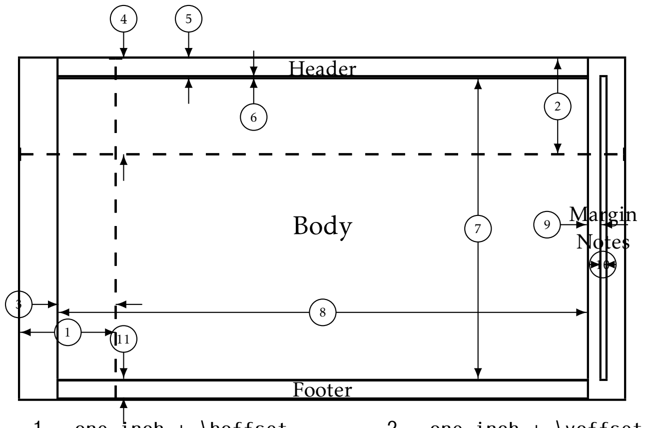
cairo.thws
Center for Artificial Intelligence
Technical University of Applied Sciences Würzburg-Schweinfurt

Magda Gregorová
magda.gregorova@thws.de

Probability basics

November 23, 2025





1	one inch + \hoffset	2	one inch + \voffset
3	\oddsidemargin = -43pt	4	\topmargin = -72pt
5	\headheight = 14pt	6	\headsep = 0pt
7	\textheight = 227pt	8	\textwidth = 398pt
9	\marginparsep = 11pt	10	\marginparwidth = 3pt
11	\footskip = 14pt		\marginparpush = 5pt (not shown)

Outline

Blabla

1. σ-algebra $\mathscr{S}$ - non-empty collection of subsets closed under complement and countable unions
   - if $A \in \mathscr{S}$, then $A^c \in \mathscr{S}$
   - if $A_i \in \mathscr{S}$ for $i \in I$ (countable index set), then $\bigcup_{i \in I} A_i \in \mathscr{S}$
   - if $A \in \mathscr{S}$ and $A \in \mathscr{S}$, then $A \cup B \in \mathscr{S}$
   - in consequence $\emptyset \in \mathscr{S}$ and $S \in \mathscr{S}$
2. $(S, \mathscr{S})$ forms a measurable space in this context called the sample space.
3. Probability measure is the same as probability distribution or probability law
4. More generally a positive measure on $(S, \mathscr{S})$ is a function $\mu : \mathscr{S} \to [0, \infty]$ satisfying non-negativity and countable additivity. A probability measure is a positive measure with total measure equal to 1.
5. The triplet $(S, \mathscr{S}, \mu)$ is a measure space. Probability space is a special case of a measure space where the total measure is 1.
6. Any finite positive measure μ on the sample space $(S, \mathscr{S})$ can be re-scaled into a probability measure as $\mathbb{P}(A) = \mu(A)/\mu(S)$, $A \in \mathscr{S} \Rightarrow$ link to energy models.

Probability space

Probability space $(S, \mathscr{S}, \mathbb{P})$:

- measurable space $(S, \mathscr{S})$
  - S - sample space
  - $\mathscr{S}$ - σ-algebra on S - collection of subsets
- probability measure $\mathbb{P}$ - real-valued function on sample space $(S, \mathscr{S})$ s.t.:
  - non-negativity: $\mathbb{P}(A) \geq 0$ for all $A \in \mathscr{S}$
  - countable additivity: countable disjoint $\{A_i : i \in I\} \in \mathscr{S} \Rightarrow \mathbb{P}(\bigcup_{i \in I} A_i) = \sum_{i \in I} \mathbb{P}(A_i)$
  - normalization: $\mathbb{P}(S) = 1$

Note: any finite positive measure μ on $(S, \mathscr{S}) \Rightarrow$ prob. measure $\mathbb{P}(A) = \mu(A)/\mu(S)$.

Positive measure

Positive measure

Positive measure on $(S, \mathscr{S})$ **- function** $\mu : \mathscr{S} \to [0, \infty]$ **s.t.:**

- $\mu(\emptyset) = 0$
- countable additivity: countable disjoint $\{A_i : i \in I\} \in \mathscr{S} \Rightarrow \mu(\bigcup_{i\in I} A_i) = \sum_{i\in I} \mu(A_i)$
- $\Rightarrow$ measure space $(S, \mathscr{S}, \mu)$

Note: if $\mu(S) < \infty \Rightarrow ((S, \mathscr{S}, \mu))$ finite measure space.
if $\mu(S) = 1 \Rightarrow ((S, \mathscr{S}, \mu))$ probability (measure) space.

Important:

adslfkj
ad;lfkj

Random variables

cairo.thws
Center for Artificial Intelligence
Technical University of
Applied Sciences
Würzburg-Schweinfurt

Random variable $X : S \to T$ - measurable function S to T

- $(S, \mathscr{S}, \mathbb{P})$ - probability space
- $(T, \mathscr{T})$ - another measurable space
- for outcome $s \in S$, X takes value $x = X(s) \in T$ - realization of r.v. X
- pre-image of $x \in T$: $X^{-1}(x) = \{s \in S : X(s) = x\} \in \mathscr{S}$
- pre-image of $B \in T$: $X^{-1}(B) = \{s \in S : X(s) \in B\} \in \mathscr{S}$