Pan-Private Uniformity Testing

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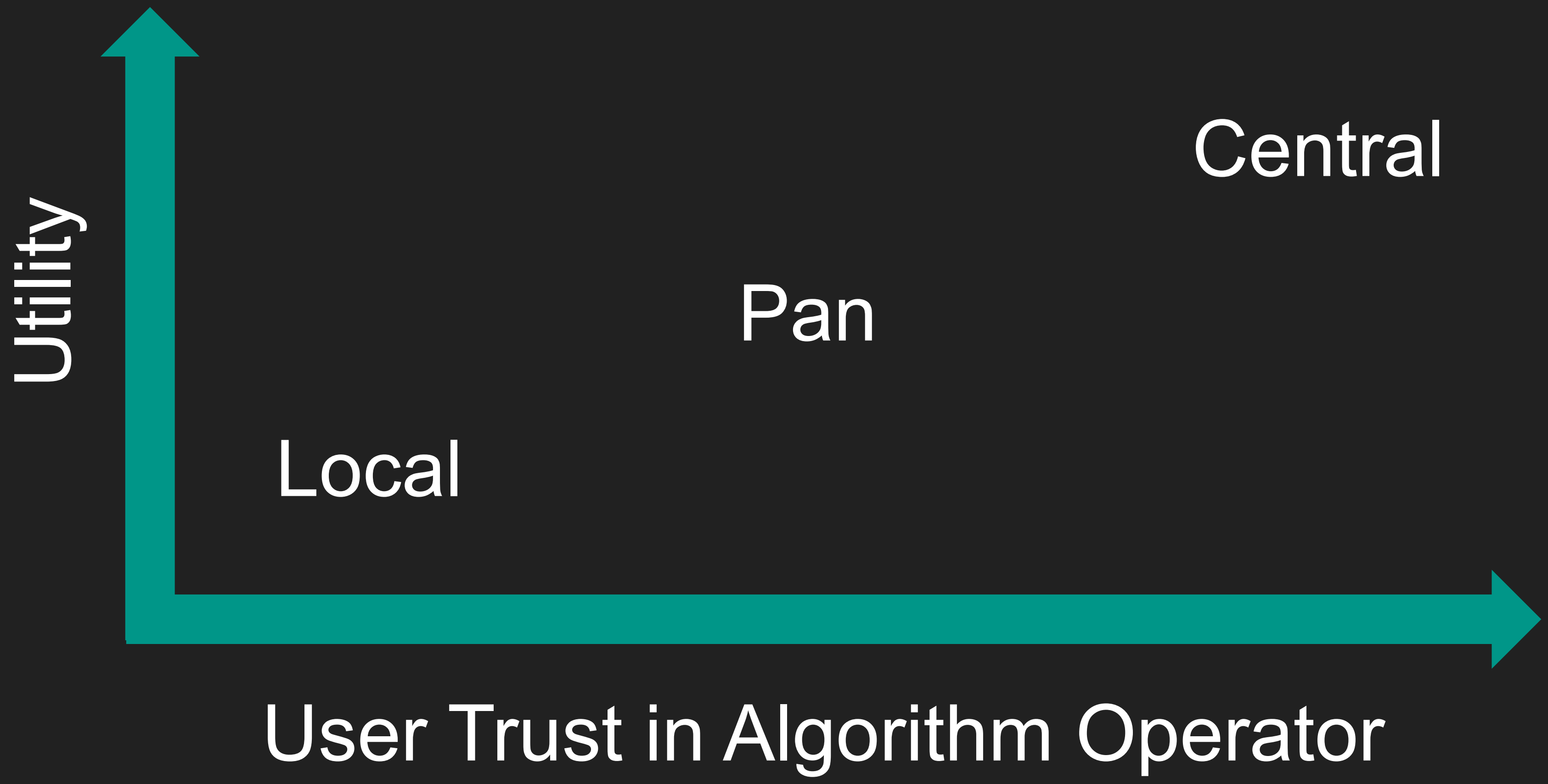
Models of Differential Privacy

Central

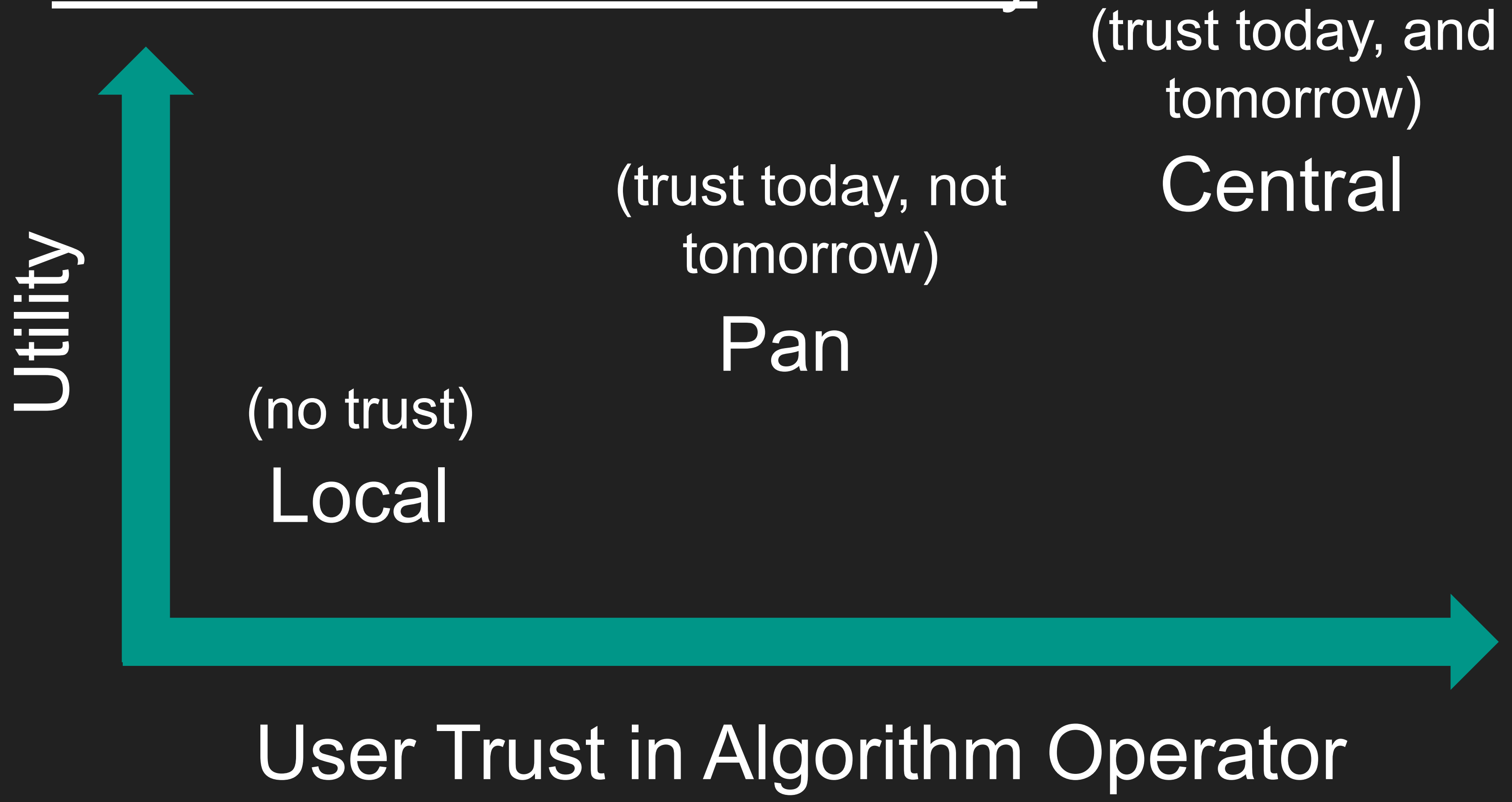
Pan

Local

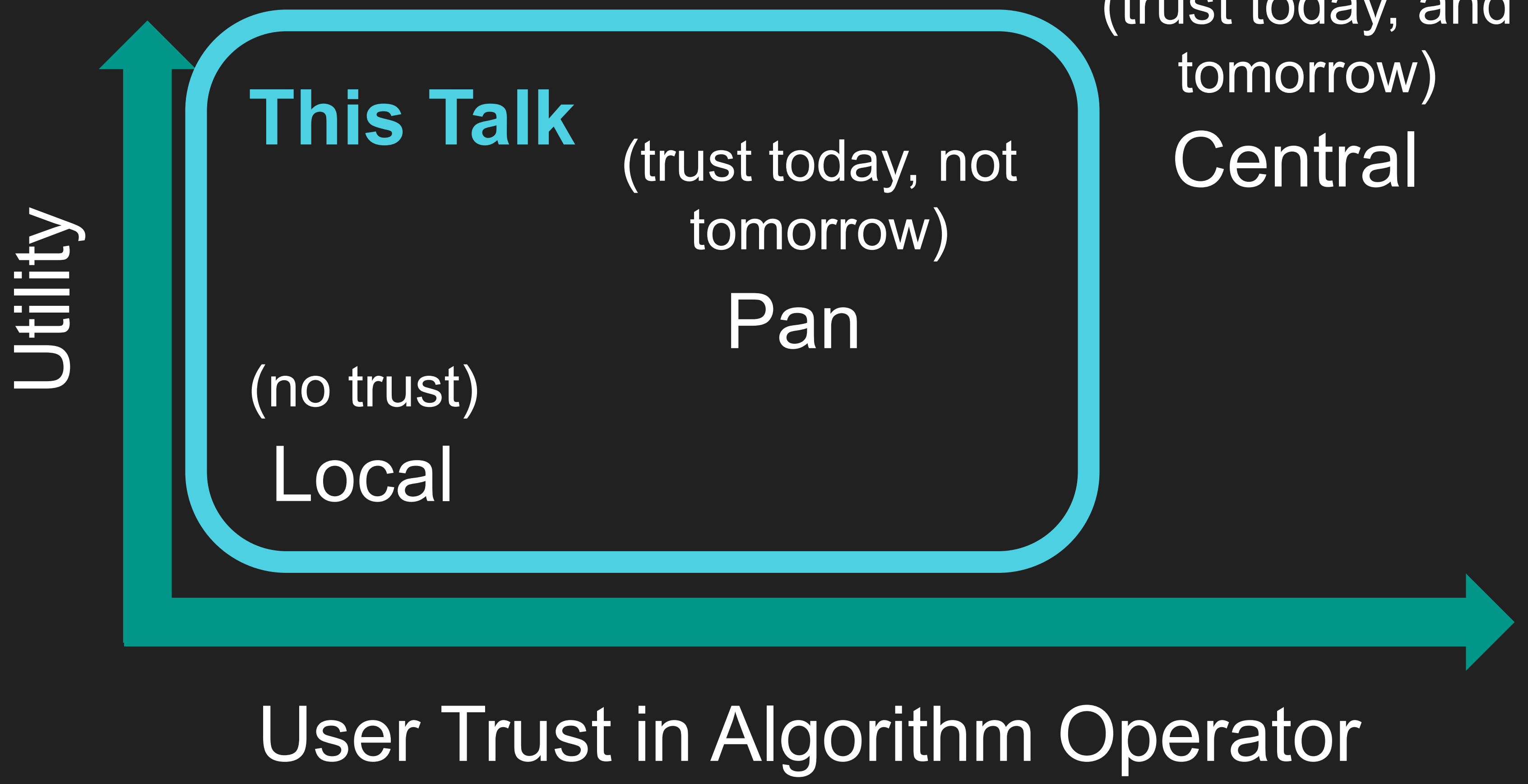
Models of Differential Privacy



Models of Differential Privacy



Models of Differential Privacy



Outline

1. Local Privacy Basics
2. Pan-Privacy Basics
3. Result 1: Connecting Local and Pan-Privacy
4. Result 2: Pan-Private Uniformity Testing

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1. **Local Privacy Basics**

2. Pan-Privacy Basics

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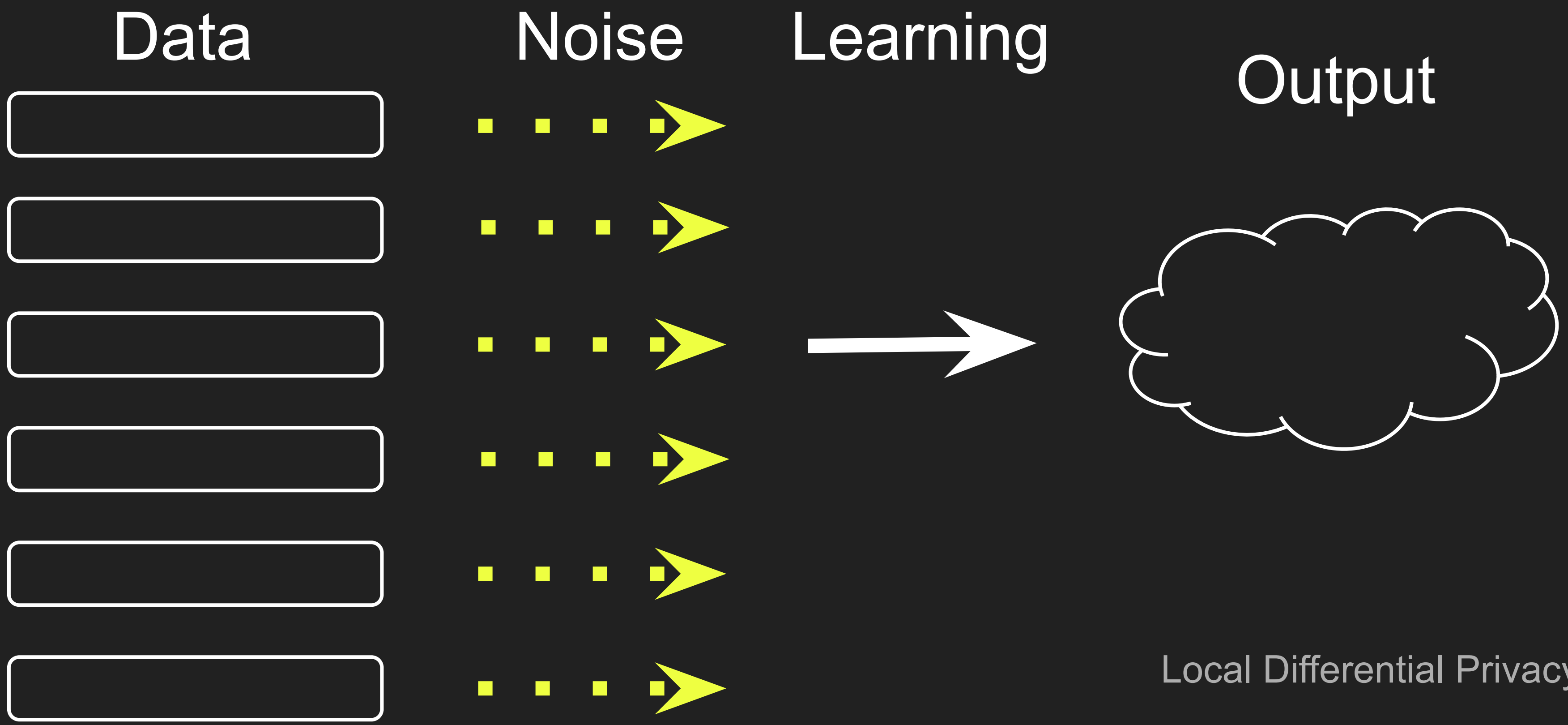
Local DP Learning From Data



Local Differential Privacy

Local DP in Words

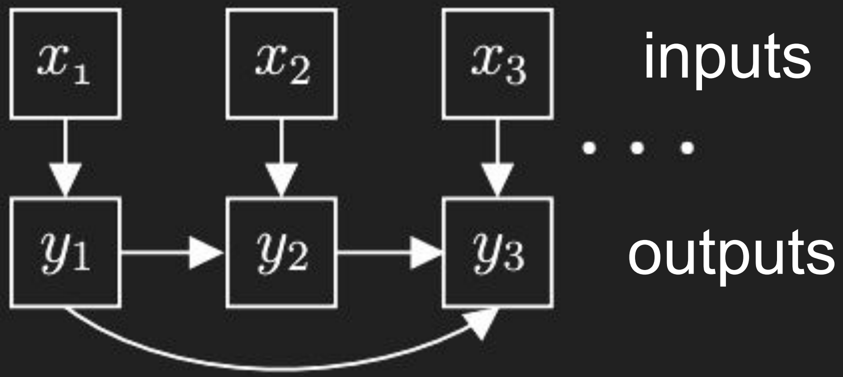
Distributed database, users keep their data

Protocol $\boldsymbol{A}$ learns about the data through public communication with users

Users send responses through *randomizers* $\boldsymbol{R}$, differentially private functions of one datum

Types of LDP Interactivity

Definition: Protocol A is *sequentially interactive* [DJW13] if all users speak once (possibly in multiple rounds).



Local DP in Math

Definition: Sequentially interactive protocol $\mathcal{A}$ is (ε,δ)-locally differentially private (LDP) if all randomizers are (ε,δ)-randomizers.

$$(P[R(x) \text{ in } Y] \leq e^{\varepsilon} P[R(x') \text{ in } Y] + \delta)$$

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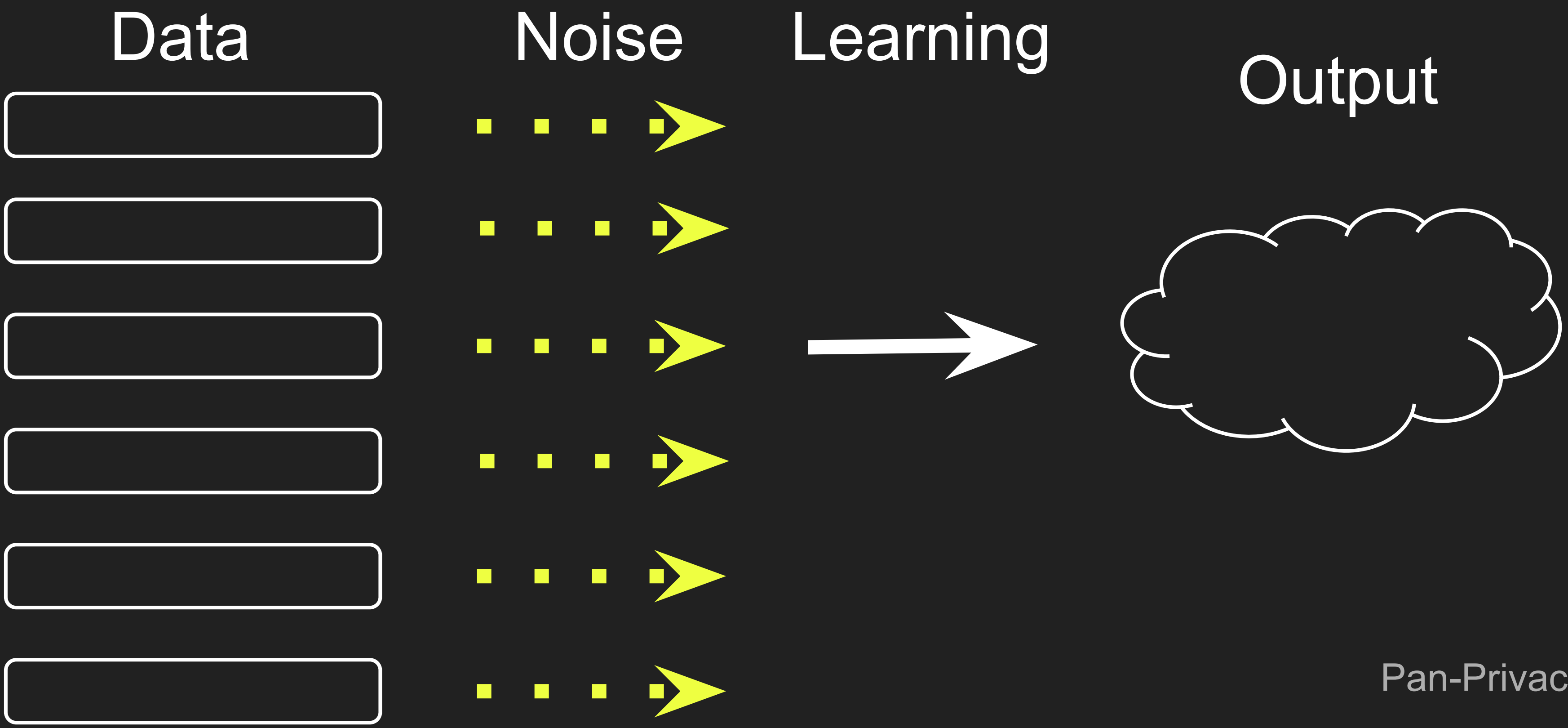
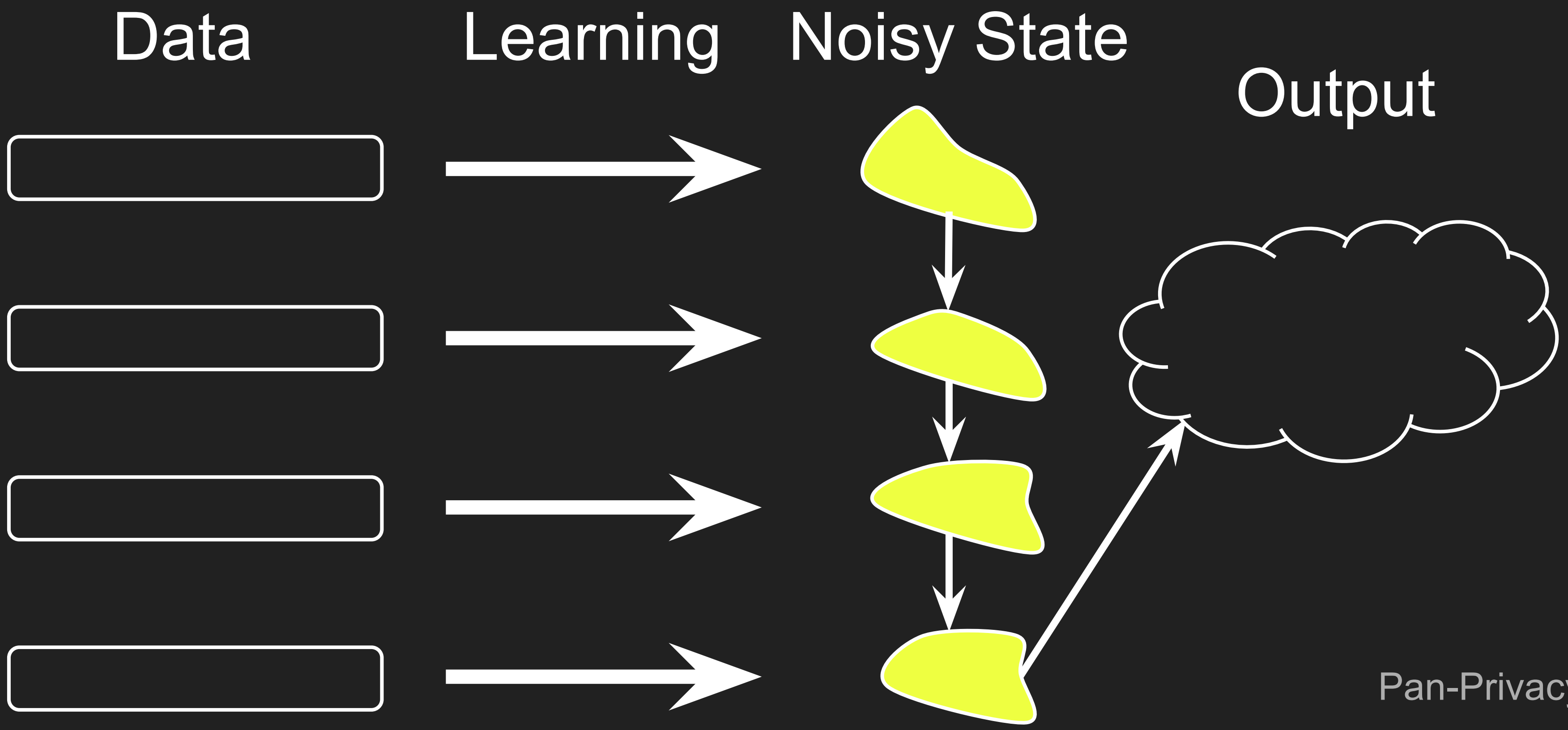
Local DP Learning From Data



Pan-Private [DNPRY10] Learning From Data



Pan-Privacy in Words

Data arrives in a stream, one element at a time

Algorithm $\mathcal{A}$ sees element, updates internal state, continues

Adversary sees (any) one internal state and final output, and this view must be a differentially private function of the stream

See data (easier than local), private intermediary state (harder than central)

Pan-Privacy in Math

Definition: Streams S and S' are neighbors if they differ in at most one stream element. Protocol A is (ε,δ)-pan private against one intrusion if, for all neighboring S and S', times t, internal state subsets $\mathcal{I}$, and output subsets $\mathcal{O}$,

$$P[I(S_{\leq t}) \text{ in } \mathcal{I}, O(S_{\leq t} \circ S_{>t}) \text{ in } \mathcal{O}] \leq$$

$$e^{\varepsilon} P[I(S_{\leq t}') \text{ in } \mathcal{I}, O(S_{\leq t}' \circ S_{>t}') \text{ in } \mathcal{O}] + \delta.$$

Why Pan-Privacy?

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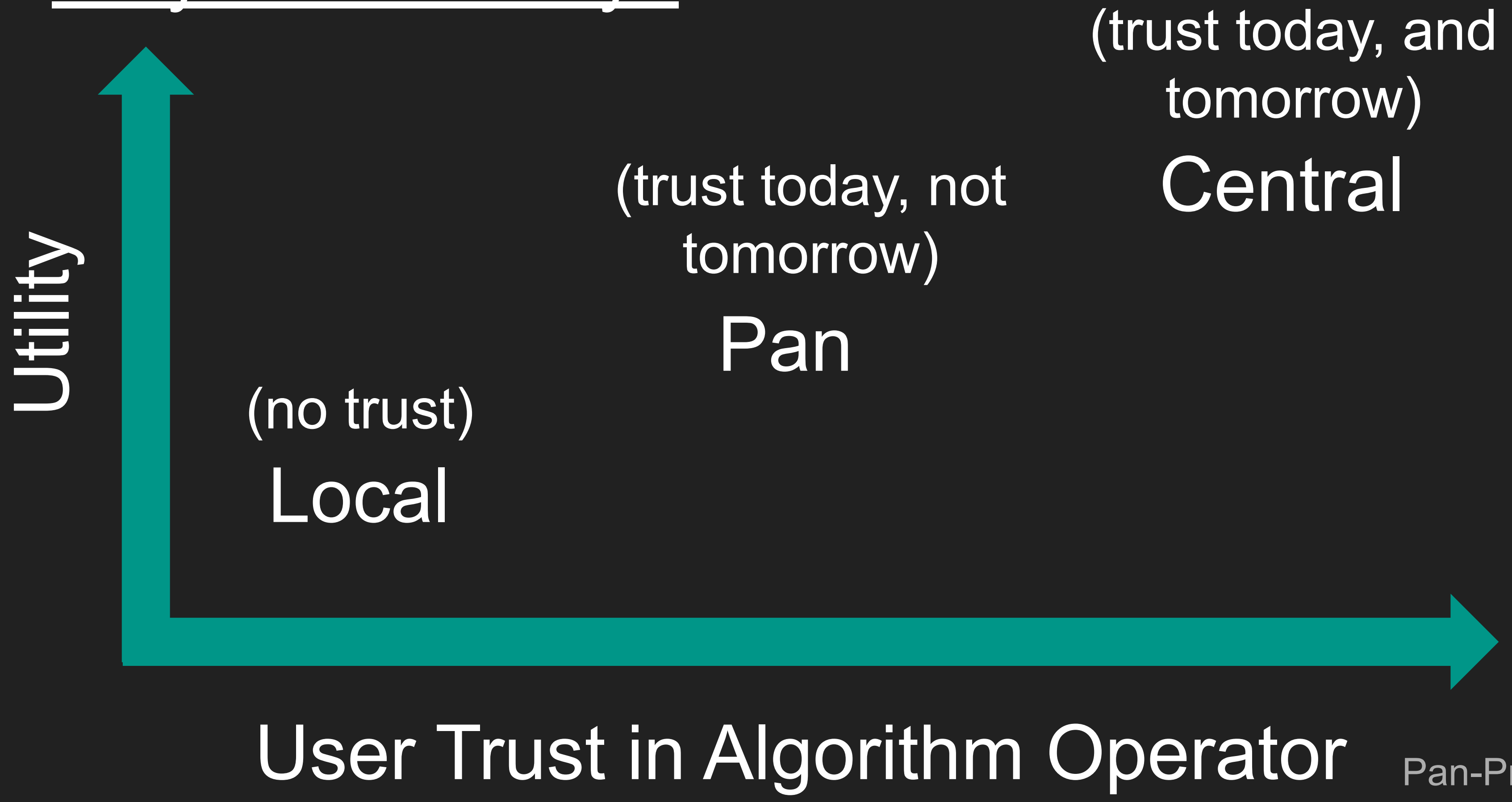
Most useful when user trusts operator today, but wants to “future-proof” their data

Examples: worried about government subpoena or operator ownership changes

If user trusts the operator today, privacy of intermediate state protects against future intrusions

Why Pan-Privacy?



Q: Does the one-intrusion assumption matter?

Q: Does the one-intrusion assumption matter?

A: Yes

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Result 1: Pan- vs. Local

Theorem: Any algorithm A_P that is ε-pan-private against two intrusions can be converted into an identical sequentially interactive ε-LDP protocol A_S, and vice-versa.

Result 1: Pan- vs. Local

Theorem: Any algorithm A_P that is ε-pan-private against two intrusions can be converted into an identical sequentially interactive ε-LDP protocol A_S, and vice-versa.

So if you need privacy against multiple intrusions, may as well use local privacy.

Result 1: Pan- vs. Local

Proof Sketch

Local to pan: run a local protocol and maintain transcript as internal state.

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Proof Sketch

Local to pan: run a local protocol and maintain transcript as internal state.

Pan to local: adversary sees two internal states, can “diff” them. So must randomize whenever update internal state. Randomize every state ≈ sequential interactivity.

Q: Is single-intrusion pan-privacy meaningful?

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A: We suggest yes

Why Single-Intrusion Pan-Privacy?

Single-intrusion pan-privacy suffers when a user contributes data between intrusions (“diff” attack)

Users most worried about giving data to an operator that’s already compromised

For users who trust operator today, single-intrusion pan-privacy is useful (and more private than central)

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Result 2: Pan-Private Uniformity Testing

Uniformity testing: algorithm receives samples from unknown distribution p over $[k]$ and must distinguish $p = U_k$ from

$||p - U_k||_{TV} \geq \alpha$ w.p. $\geq 2/3$

Result 2: Pan- Uniformity Testing

	Previous Work	This Work
Without Privacy	$\Theta(k^{1/2})$ [CDVV14]	
ε-DP	$\Theta(k^{1/2})$ [ASZ18]	
ε-Pan Privacy		
SI ε-LDP		
NI ε-LDP	$\Theta(k)$ [ACFT19]	

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Result 2: Pan- Uniformity Testing

Theorem: ε-pan-private uniformity testing has sample complexity

$$O\left(\frac{k^{2/3}}{\alpha^{4/3}\varepsilon^{2/3}} + \frac{\sqrt{k}}{\alpha^2} + \frac{\sqrt{k}}{\alpha\varepsilon}\right)$$

$$\Omega\left(\frac{k^{2/3}}{\alpha^{4/3}\varepsilon^{2/3}} + \frac{\sqrt{k}}{\alpha^2} + \frac{\sqrt{k}}{\alpha\sqrt{\varepsilon}} + \frac{1}{\alpha\varepsilon}\right)$$

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Key idea: split difference between central and local approaches

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Central [CDK17, ADR18, ASZ18]: uses “fine” statistic

Looks at sample counts for all k elements and measures departure from expected count under uniform distribution

Need to add noise to each count to be pan-private

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Central [CDK17, ADR18, ASZ18]: uses “fine” statistic

Looks at sample counts for all k elements and measures departure from expected count under uniform distribution

Need to add noise to each count to be pan-private. Can get pan- $\mathbf{O}(k^{3/4})$ like this...but can we do better?

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Central [CDK17, ADR18, ASZ18]: uses “fine” statistic

Maybe pan- should use a coarser statistic?

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

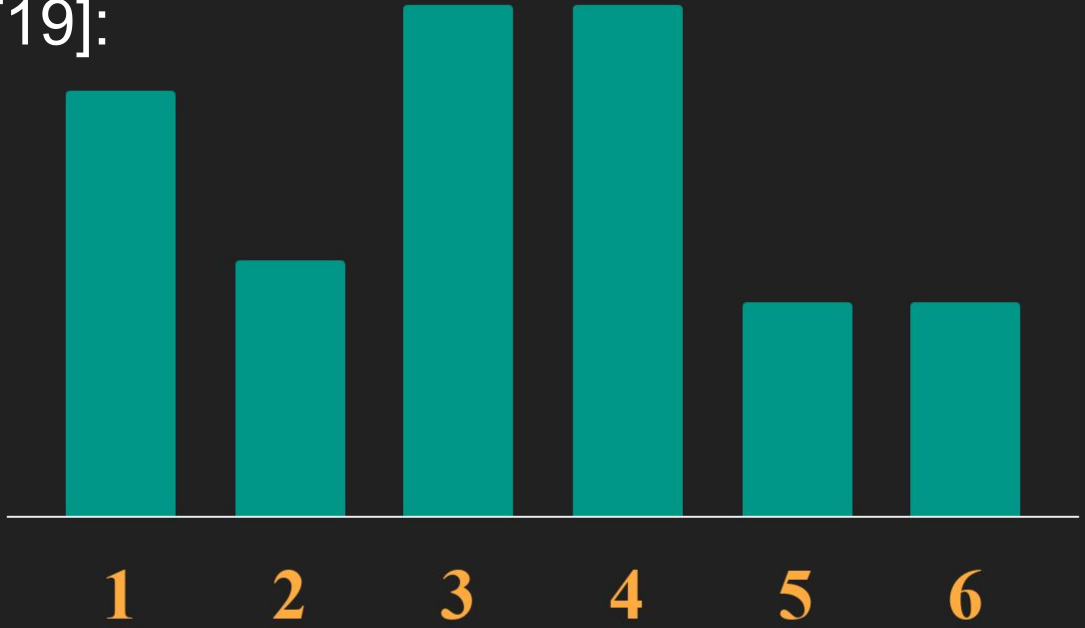
Local [ACFT19]: uses coarse statistic

Randomly halves domain, now uniformity testing over **[2]**

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Local [ACFT19]:



Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Local [ACFT19]:

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Local [ACFT19]: uses coarse statistic

Small response domain: good for local!

But sacrifices a lot of testing distance: α to $\alpha/k^{1/2}$… so end up using $\mathbf{O}(k)$ samples

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Local [ACFT19]: uses coarse statistic

Maybe pan- should maintain a finer statistic?

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

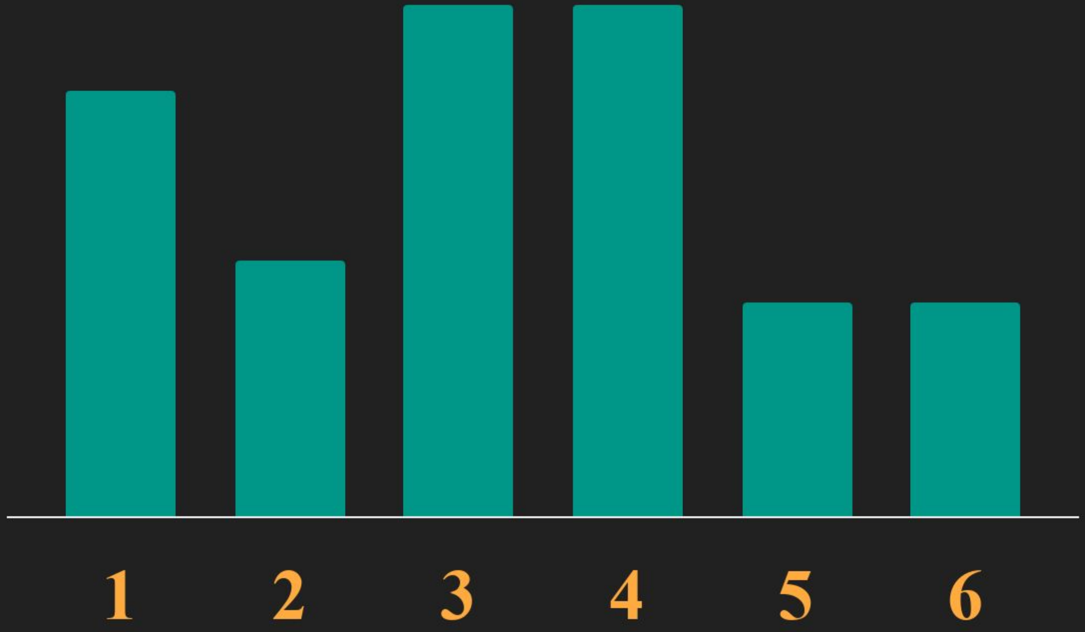
Pan: coarser than central, finer than local

Randomly partition domain into n equal-size groups, now uniformity testing over $[n]$

Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Pan:



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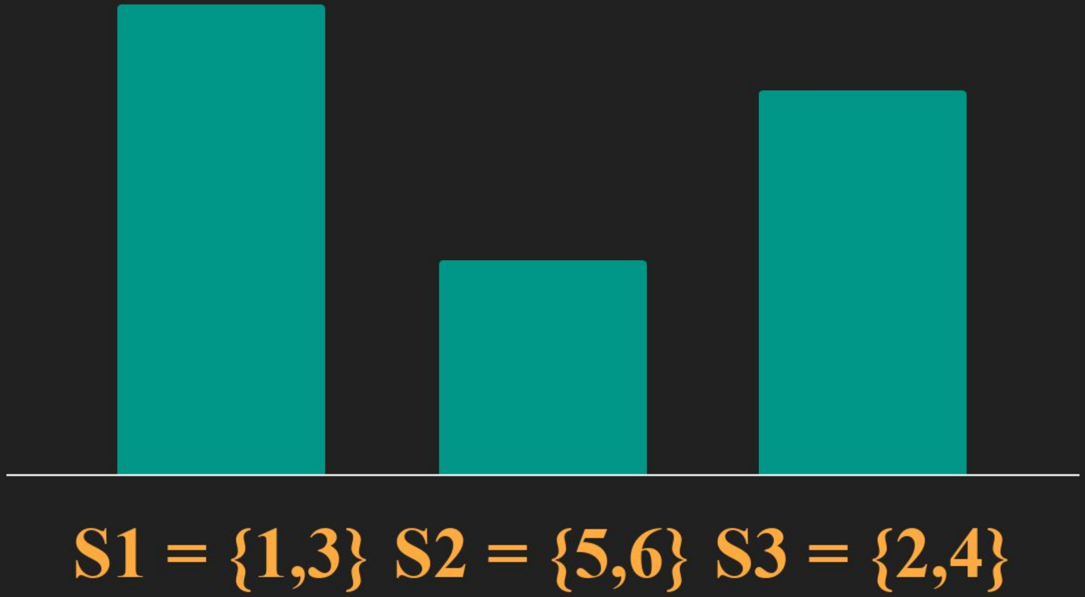
Upper Bound Sketch

Pan:



Result 2: Pan- Uniformity Testing

Upper Bound Sketch

Pan: coarser than central, finer than local

Testing distance change is α to $\alpha(n/k)^{1/2}$

Pick $n = \Theta(k^{2/3}\varepsilon^{4/3}/\alpha^{4/3})$ to trade off coarse (not too much noise per bin) and fine (preserve testing distance)

Result 2: Pan- Uniformity Testing

Theorem: **ε**-pan-private uniformity testing has sample complexity

$$O\left(\frac{k^{2/3}}{\alpha^{4/3}\varepsilon^{2/3}} + \frac{\sqrt{k}}{\alpha^2} + \frac{\sqrt{k}}{\alpha\varepsilon}\right)$$

$$\Omega\left(\frac{k^{2/3}}{\alpha^{4/3}\varepsilon^{2/3}} + \frac{\sqrt{k}}{\alpha^2} + \frac{\sqrt{k}}{\alpha\sqrt{\varepsilon}} + \frac{1}{\alpha\varepsilon}\right)$$

Result 2: Pan- Uniformity Testing

Lower Bound Sketch

Adapts information theory lower bound from [DGKR19] for uniformity testing under memory restrictions

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Adapts information theory lower bound from [DGKR19] for uniformity testing under memory restrictions

Main contribution: replacing memory restriction with privacy restriction

Result 2: Pan- Uniformity Testing

Theorem: **ε**-pan-private uniformity testing has sample complexity

$$O\left(\frac{k^{2/3}}{\alpha^{4/3}\varepsilon^{2/3}} + \frac{\sqrt{k}}{\alpha^2} + \frac{\sqrt{k}}{\alpha\varepsilon}\right)$$

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Takeaways

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- Pan-privacy is appropriate when user trusts algorithm operator today but maybe not tomorrow
- Pan-privacy against more than one intrusion is equivalent to sequentially interactive local privacy
- Pan-privacy against a single intrusion trades off both utility and privacy between central and local models
    - $\Theta(k^{1/2})$, $\Theta(k^{2/3})$, and $\Theta(k)$ uniformity testing bounds

Open Questions

- Uniformity testing:
    - close gap between pan upper and lower bounds
    - fully interactive locally private lower bound?

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- What about **(ε,δ)**-pan-privacy?

Open Questions

- Uniformity testing:
  - close gap between pan upper and lower bounds
  - fully interactive locally private lower bound?

- What about (ε,δ)-pan-privacy?

- How powerful is pan-privacy in general?

References

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