physics 222 midterm study notes

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January 2019

Contents

1 Chapter 21: Electric Charge and Electric Fields

1.1 Properties of Electric Charge

- Electric Charge is an **inherent physical property** of certain subatomic particules that is resple for electrical/magnetic phenomena
- Charge is represented by the the symbols **q** or **Q**
- The SI unit of charge is the **coulomb (C)**
- There are two types of charge, positive and negative.
- Most (but not all) fundamental particles have an electric charge
- **Charged particles:** electrons, muons, protons, quarks
- **Uncharged particles:** neutrons, neutrinos, photons, Higgs boson
- charge is conserved (neither created or destroyed
- opposite charges attract; similar ones repel.

1.2 Fundamental Unit of Electric Charge

Quantity Name	Value
e	1.602E^{-19}C
charge of an electron	-e
charge of a proton	e
charge of an object	e(N$_{protons} - N_{electrons}$)

1.3 Conductors and Insulators

Conductors: Electric charges move freely. Material has high conductivity.

Insulators: Electric charges do not move freely. Material has low conductivity

Charge spreads out locally on a conductor but is localized on an insulator.

1.4 Charging by Direct Contact

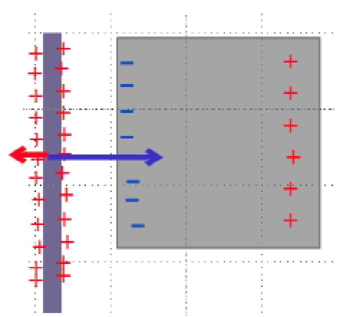
Objects can be charged by rubbing them together because some substances hold onto their electrons tighter than other substances.

When placed in direct contact, some objects will lose electrons become positively charged, while others will gain electrons and become negatively charged.

Material	Relative charging with rubbing
rabbit fur	+ + + + + +
glass	+ + + + +
human hair	+ + + +
nylon/wool	+ + +
silk	+ +
paper	+
cotton	−
wood	− −
amber	− − −
rubber	− − − −
PVC	− − − − −
Teflon	− − − − − −

Charges objects obtain when rubbed together – more symbols mean more charge

1.5 Example of Charge Polarization in Conductors



Despite the conductor having no net charge, the charged wand is attracted to it. Why?

>> Attractive force between wand and negative side is greater than the repulsive force between wand and positive side.

$$F \propto \frac{q_1 q_2}{d^2}$$

(preview of the Coulomb force)

Why?

>> Distance to the unlike charge is smaller than distance to like charge.

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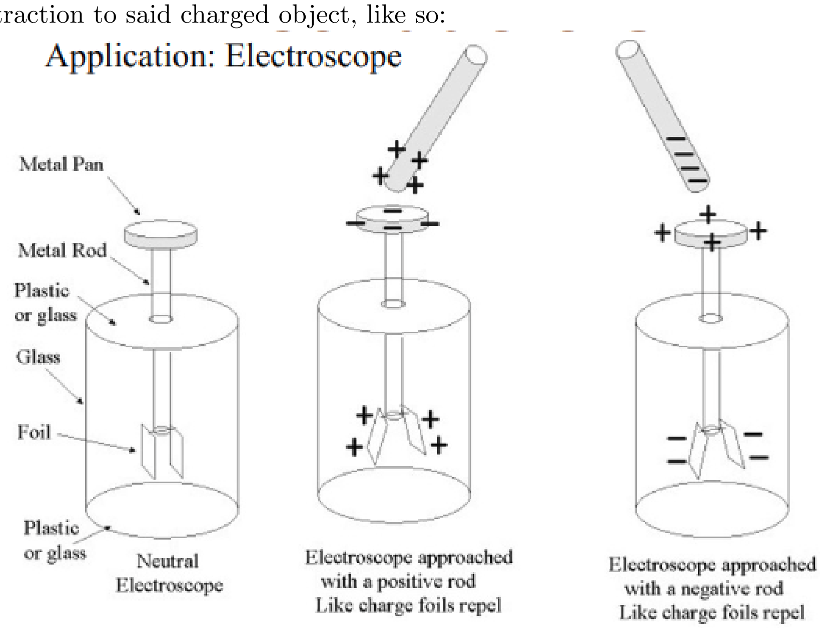
polarization.png

A real-world example of how charge polarization in conductors might be useful is that of an Electroscope. In the presence of charged objects, the gold leaf attached to the central metal stem of the electroscope moves outwards, due to its attraction to said charged object, like so:



1.6 Charging by induction

In an insulator, electrons are not free to move. How then, do insulators become polarized?
Normally, electrons are symmetrically distributed about the nucleus. In the presence of a charge, electrons move toward or away from a charge, distorting the symmetry. Now, on average, electrons are a different distance away from the charge than protons. Although the total number of protons and electrons within the object itself has not changed (unlike polarization within a charged object), they now (on average) lie at different distances from the charged object.

1.7 Coulomb's Law

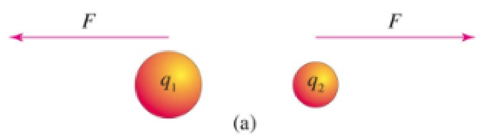
1.7.1 Conceptual Definition

The force on a charge due to another charge is proportional to the product of the charges, and inversely proportional to the separation squared. In mathematical terms:

$$F \propto \frac{q_1 q_2}{r^2}$$

1.7.2 Conceptual Example

Suppose you have two objects (with charges q_1 and q_2, respectively) at a given distance of separation, r. We can visualize this like so:



(a)

$$F \propto \frac{q_1 q_2}{r^2}$$

1. **If q_1 doubles, what happens to the force on each object?** The force on both objects doubles.
2. **If the charge is the same, but the distance doubles, what happens to the force?** It decreases by a factor of four.
3. **If the charge is the same but the distance is halved, what happens to the force?** It increases by a factor of four.

1.7.3 Mathematical definition

In order to make proportionality an equation, we need to include the const k:

$$F = \frac{k(q_1 q_2)}{r^2}$$

In air:

$$k = 8.98E^9 Nm^2/C^2$$

1.7.4 Constant values

Quantity Name	Value
Coulomb Constant (k)	$1/(4\pi\epsilon_0)$
Permittivity of free space (ϵ_0)	$8.85E^{-12}\ F * m^2$
Coulomb's electrical constant (k_e)	$8.99E_9\ Nm^2C^{-}2$

1.7.5 Vector form of Coulomb's Law

The force on particle 1 by particle 2 can be noted by:

$$F_{1,2} = \frac{k q_1 q_2}{r_{1,2}^2}\hat{r}$$

where is the unit vector that points from particle 1 to particle 2.

1.8 Superposition

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} + \vec{F}_{4,1}$$

$$= \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2}\hat{r}_{14}$$

1.9 Conceptual Definition

We quantify electrical fields by the force they exert on the particles around them. For an electrical field "E", at location (x, y, z) woud be defined as:

$$E(x, y, z) = \frac{F_{onq}}{q}$$

To calculate the force exerted on a point by and electrical field, you would use the alternate form:

$$F = qE$$

The SI unit used to measure electrical fields is **Newtons.**

1.10 Electric Field on a Single Point Charge

In general, the electric field of a single point charge q is:

$$E = k\frac{q}{r^2}\hat{r}$$

The direction of electric field is defined to be in the direction of the force experienced by a positive test charge placed in the field.

1.11 Inverse Square Law

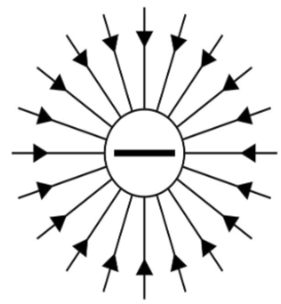
The electric field follows the inverse square law- as distance doubles, the e field decreases by a factor of 2^2, or 4.

$$E = k\frac{q}{r}\hat{r}$$

1.12 Electric Field Lines

Electric field lines are often used to visualize the electric field. They are not measurable quantities. Electric field lines are not electric field vectors.

Electric field lines of a positive point charge



Electric field lines of a negative point charge

Electric field lines are drawn so that the density of electric lines (i.e. lines per unit area) is proportional to the magnitude of E:

$$E \propto \frac{Number of field lines}{A}$$

A is perpendicular to the field lines.

To review:

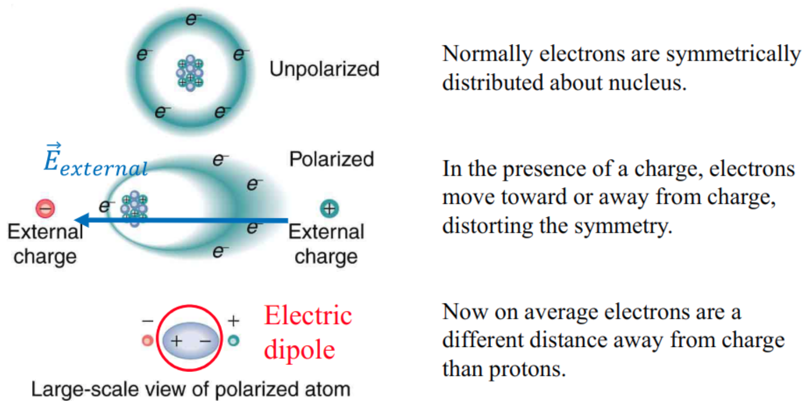
- Lines emanate from a positive charge and terminate on a negative charge (though some lines extend to infinity).
- Lines are closer together where the field is stronger.
- For straight field lines, E is in the direction of the field line.
- For curved fied lines, E is tangent to the field line.
- No two electric field lines cross each other.

1.13 Electric Dipole

Two opposite charges with a slight separation between them is an electric dipole. A dipole can either be:

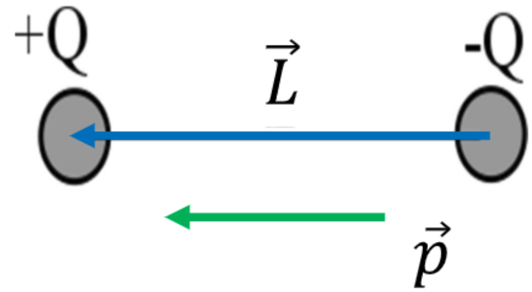
- Induced by an external electric field
- Be permanent and naturally occurring

1.13.1 Charge-Induced Electric Dipoles



1.13.2 Dipole Moment

Consider two equal charges, +Q and -Q, separated by a distance L.

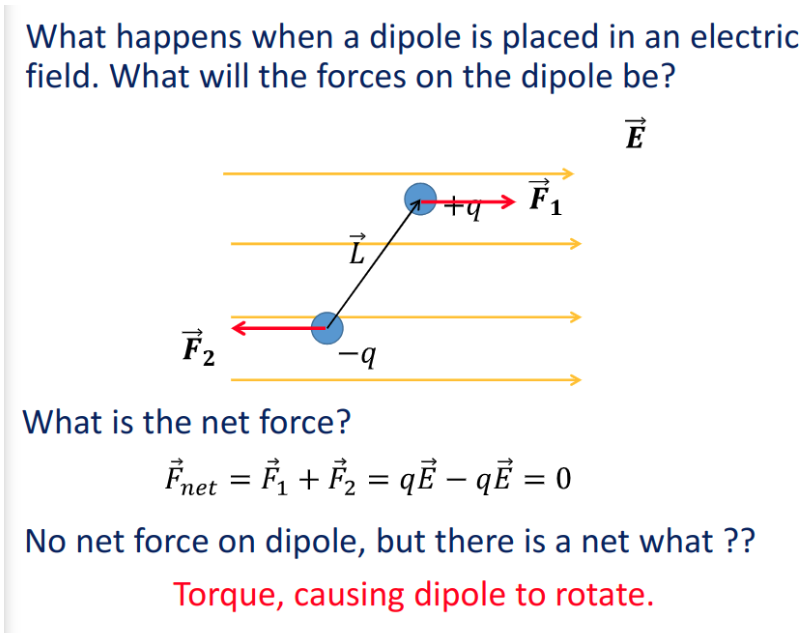


1. Define a vector $\vec{L}$ to point from the negative to the positive charge.
2. Define a vector $\vec{p} = Q\vec{L}$. $\vec{p}$ is called the **dipole moment**.

Units of $\vec{p}$ are C*m.

Note that: Even though $\vec{p}$ points in the same direction as $\vec{L}$, it does need to be the same length as the vector, as the two vectors represent different physical properties.

1.13.3 Torque



+x

So what are the torques about the center of the Electric Dipole?

$$\tau_1 = \frac{L}{2} F_1 Sin(\theta)$$

$$\tau_2 = \frac{L}{2} F_2 Sin(\theta)$$

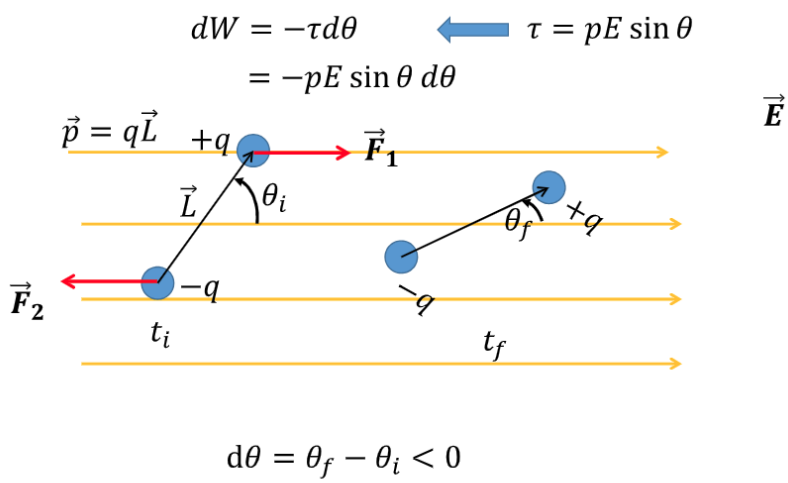
$$\tau_{net} = \tau_1 + \tau_2 = lqESin(\theta) = pESin(\theta)$$

$$\vec{\tau} \ = \vec{P} \times \vec{E}$$

The torque will rotate the dipole such that it tends to align with $\vec{E}$

1.13.4 Work done by $\vec{E}$ on dipole

Consider an electric field that rotates a dipole from time t_i to t_f. The electric field does work on the dipole in rotating it. The amount of work done rotating the dipole an incremental angle $d\theta$ is represented by:



1.13.5 Potential Energy of a dipole moment

Tbe work done by $\vec{E}$ changes the potential energy of the dipole. Positive work done by $\vec{E}$ decreases the dipole's Potential Energy; negative work done by $\vec{E}$ increases the dipole's potential energy.

$$dU = -dW <== dW = -pEsin(\theta)d\theta$$

$$= -(-pEsin\theta d\theta)$$

$$= pEsin\theta d\theta$$

Then, we integrate dU:

$$U = \int dU$$

$$= \int pE\sin\theta d\theta$$

$$= -pEcos\theta + U_0$$

where U_0 is some arbitrary integration constant. The integration constant "U" is most conveniently defined to be 0J when dipole is rotated 90 degrees from $\vec{E}$, so, when possible, we set U_0 to equal 0.

In summary:

- U is at its minimum value when $\vec{p}$ abnd $\vec{E}$ are aligned.
- U is at its maximum value when $\vec{p}$ and $\vec{E}$ are opposite.
- A dipole favors a state where potential energy is minimal.