On X-ray spectral-timing methods

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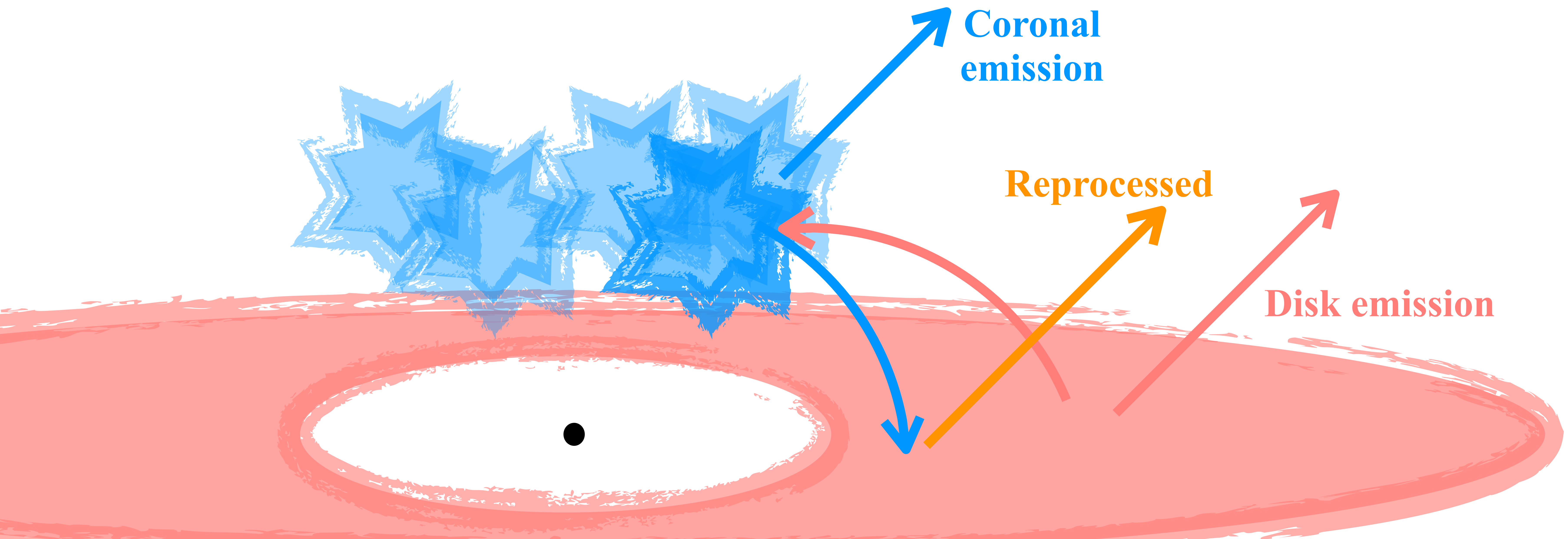
"Compact objects in 3D – steps towards X-ray polarimetric-spectral-timing" Lorentz Center Workshop

Why Timing?

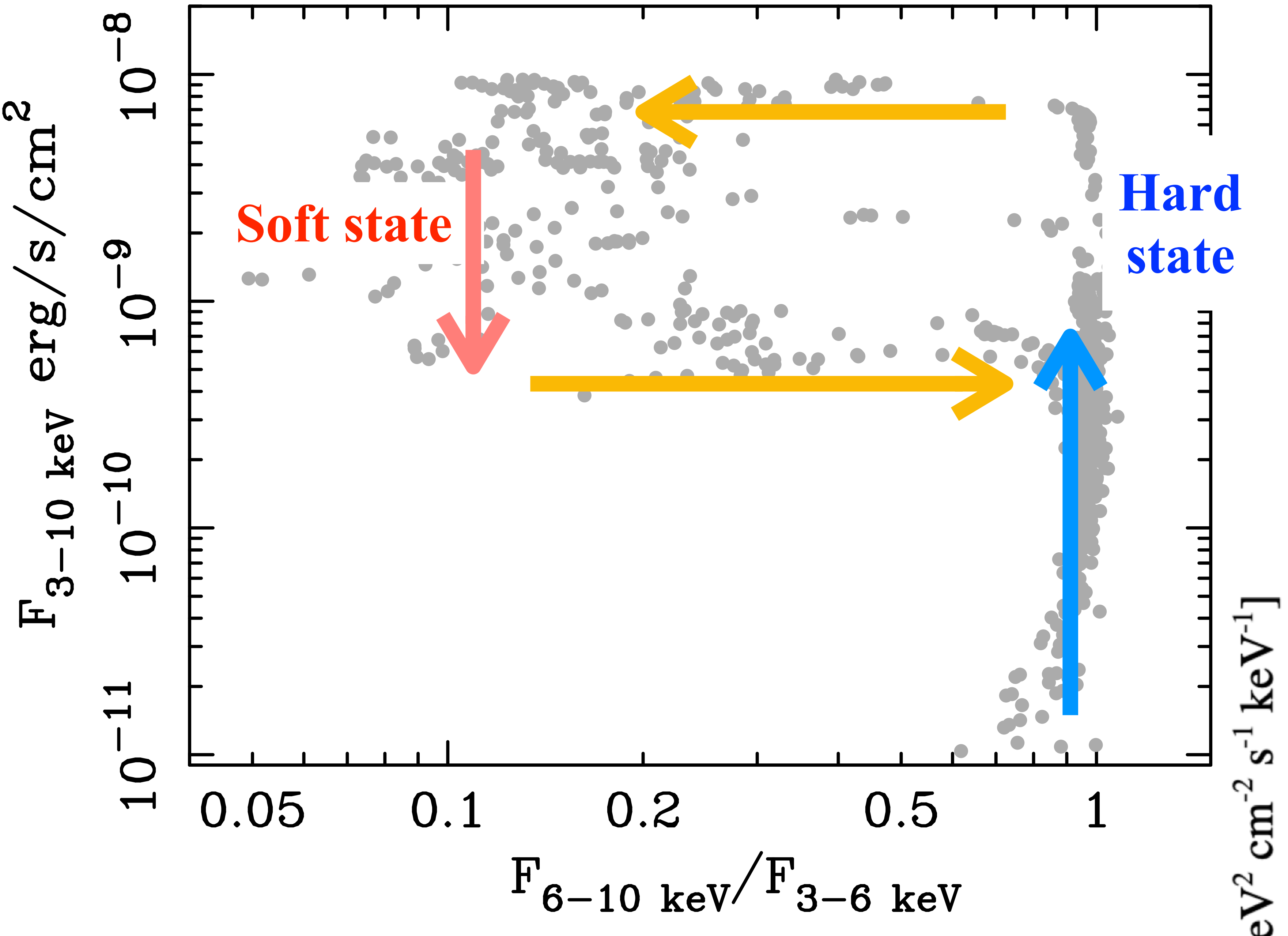
Timing adds a complementary dimension:

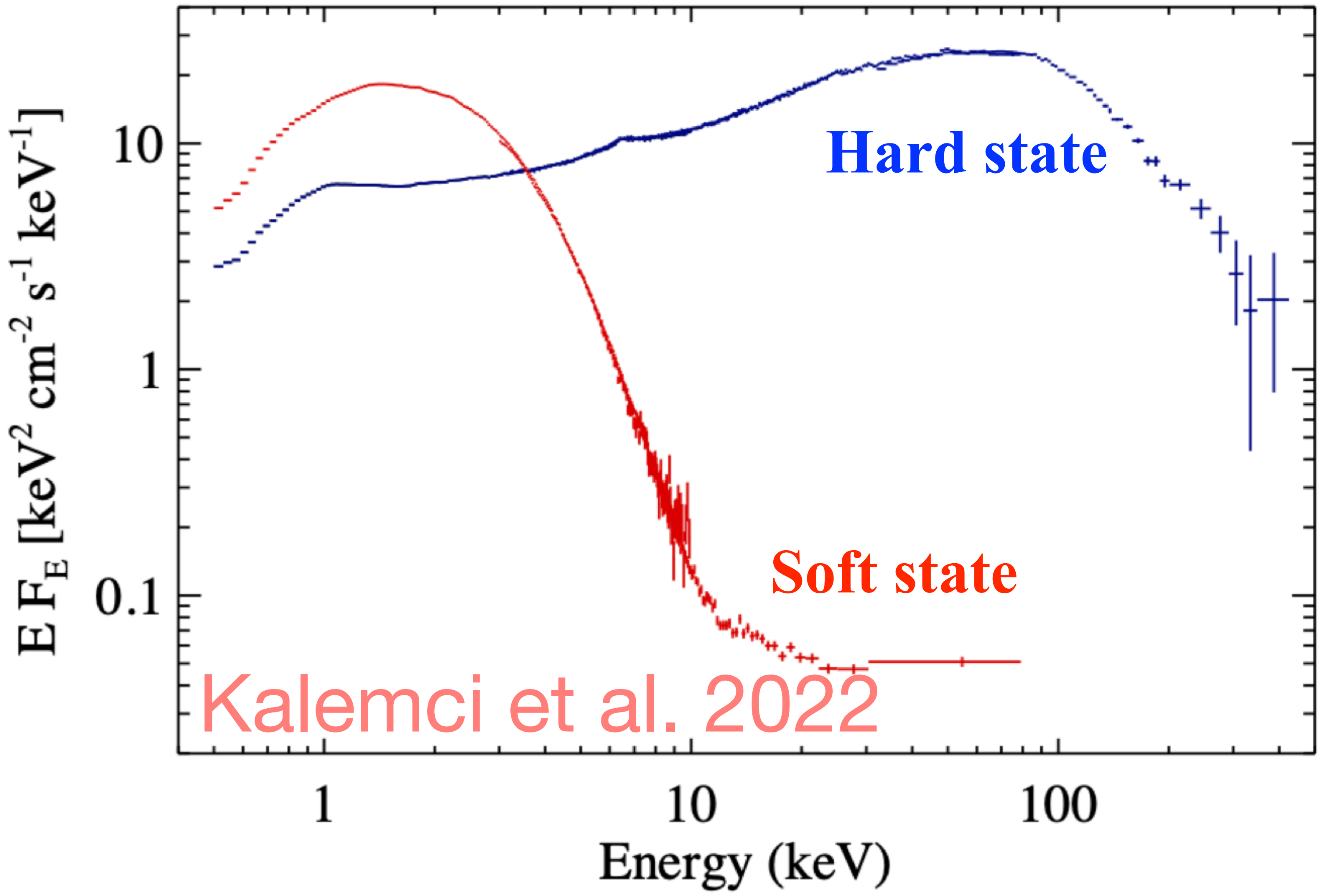
Tells where spectral components are produced and how they are related

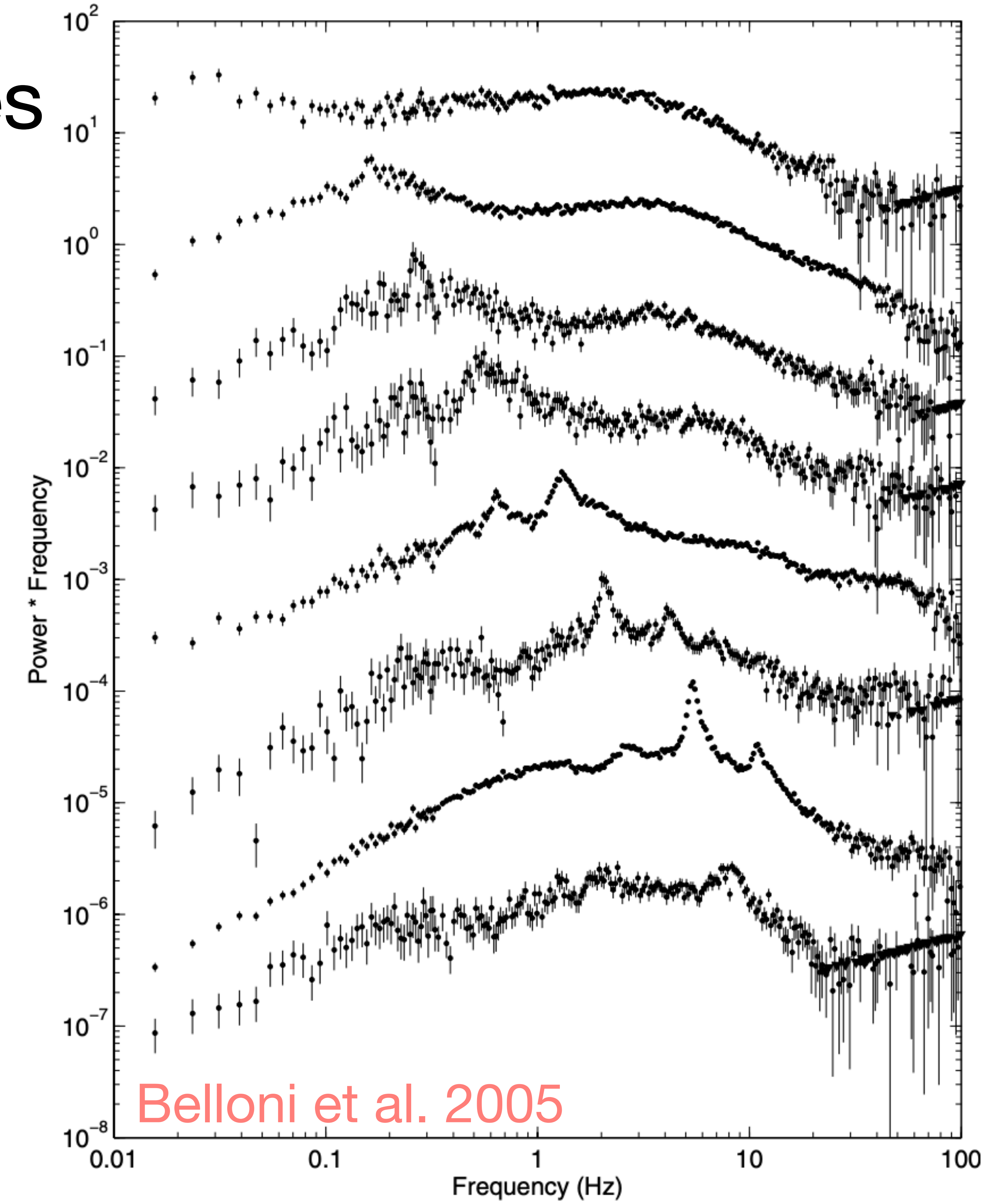


Black Hole X-ray binaries

Different states ➔ different physical properties







Fourier Analysis

Why?

- Direct access to variability on different timescales
- Clear identification of variability components and characteristic timescales
- Averaging over variability cycles improves S/N

But…

- Meaningful for stationary processes
- Rapid variations (e.g. of QPOs) may be misinterpreted
- Handle with care! Subject to many biases

van der Klis 1989; Belloni & Bhattacharya 2022; Bachetti & Huppenkothen 2023

Tools to study variability (ranked by "common usage")

1) Power Spectrum

$$P(\nu_i) = A\,|X(\nu_i)|^2 \rightarrow$$ How much variability

2) Cross Spectrum

$$CS(\nu_i) = X^*(\nu_i)Y(\nu_i) \rightarrow$$ How variability propagates

3) Bispectrum

$$B(\nu_i, \nu_j) = X(\nu_i)X(\nu_j)X^*(\nu_i + \nu_j) \rightarrow$$ Phase coupling

see Kavitha's talk and tutorial

Physical Timescales

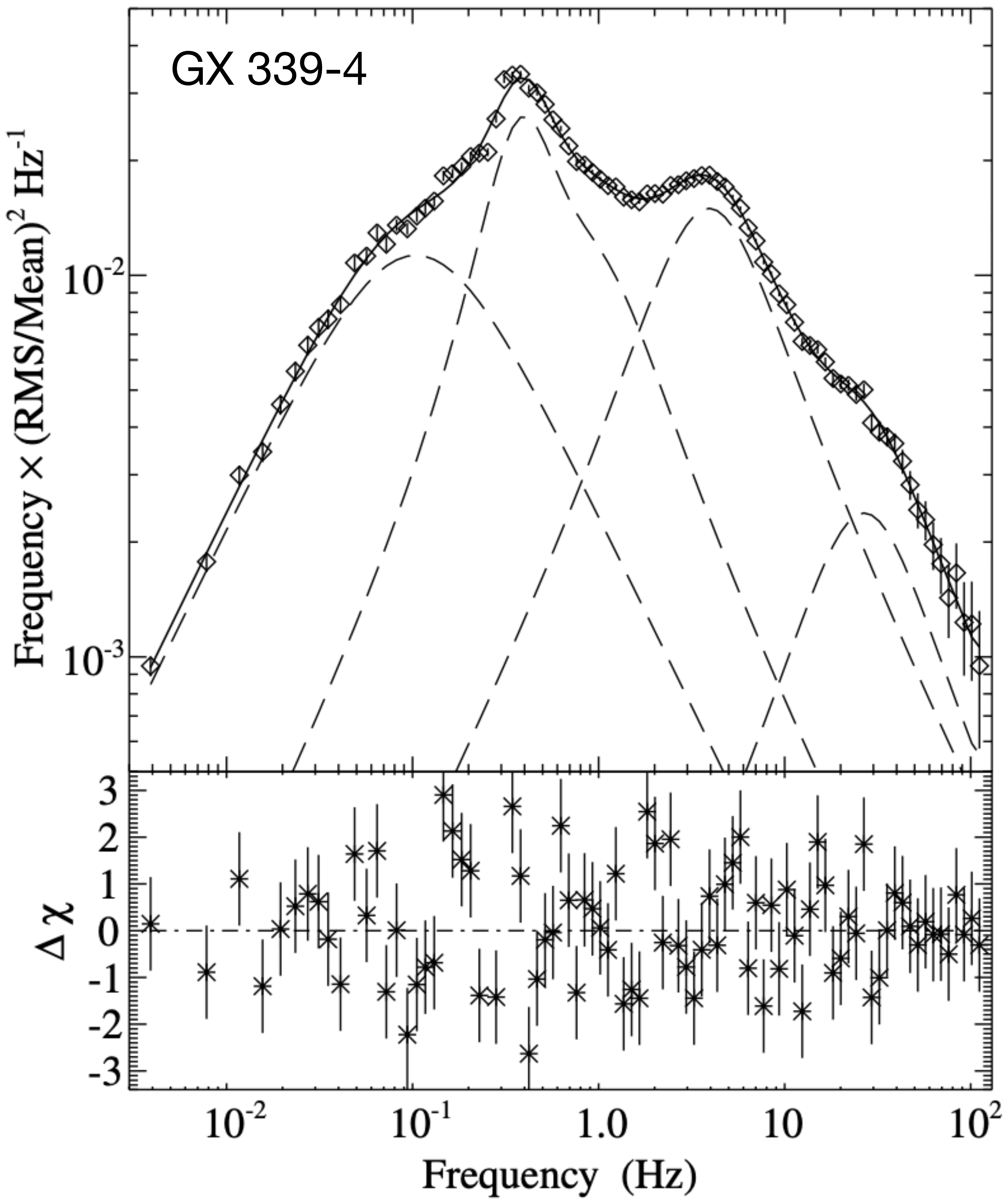
Dynamical	$t_{dyn} \approx 3.3\ (R/10\ R_g)^{3/2}$ ms	➔	Orbital motions
Thermal	$t_{th} = t_{dyn}/\alpha$	➔	Heating/cooling
Viscous	$t_{visc} = t_{th}/(H/R)^2$	➔	Mass Accretion
Light-crossing	$t_{lc} = R/c$	➔	Reverberation

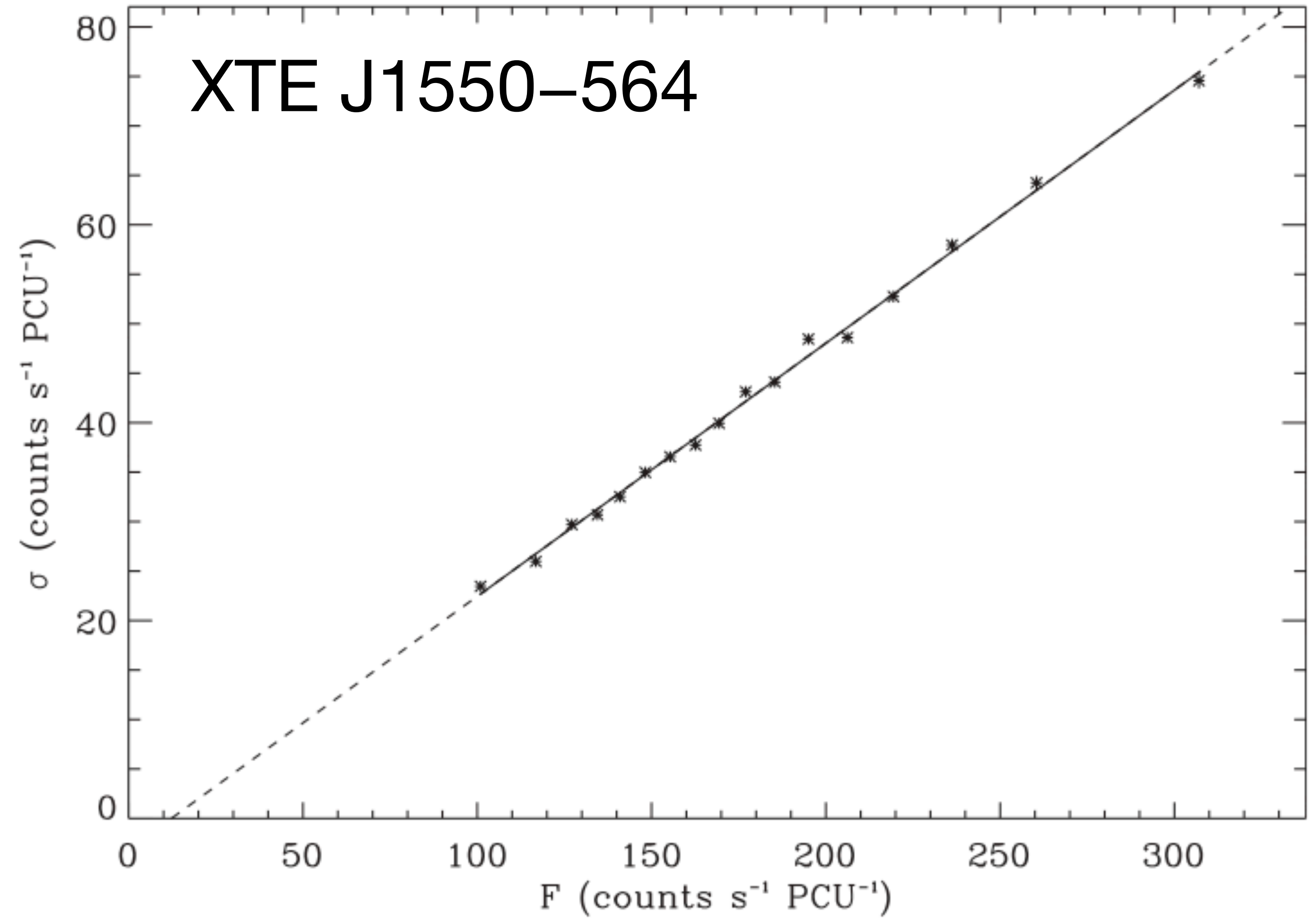
Orders of magnitude for a standard disc (assuming H/R~0.01, α~0.1, and M_{BH}=10 $M_\odot$)

	t_{lc}	t_{dyn}	t_{th}	t_{visc}
$10\ R_g$	0.5 ms	3 ms	0.03 s	10 s
$100\ R_g$	5 ms	0.1 s	1 s	minutes
$1000\ R_g$	50 ms	3 s	30 s	hours

Power spectra: broad band noise

- From mHz to 100s of Hz
- Rms-flux relation
- Break frequencies
- Lorentzian components





Nowak 2000;
Belloni et al. 1997;
Belloni & Psaltis 2002
Uttley & McHardy 2001;
Uttley et al. 2005;
Heil et al. 2012

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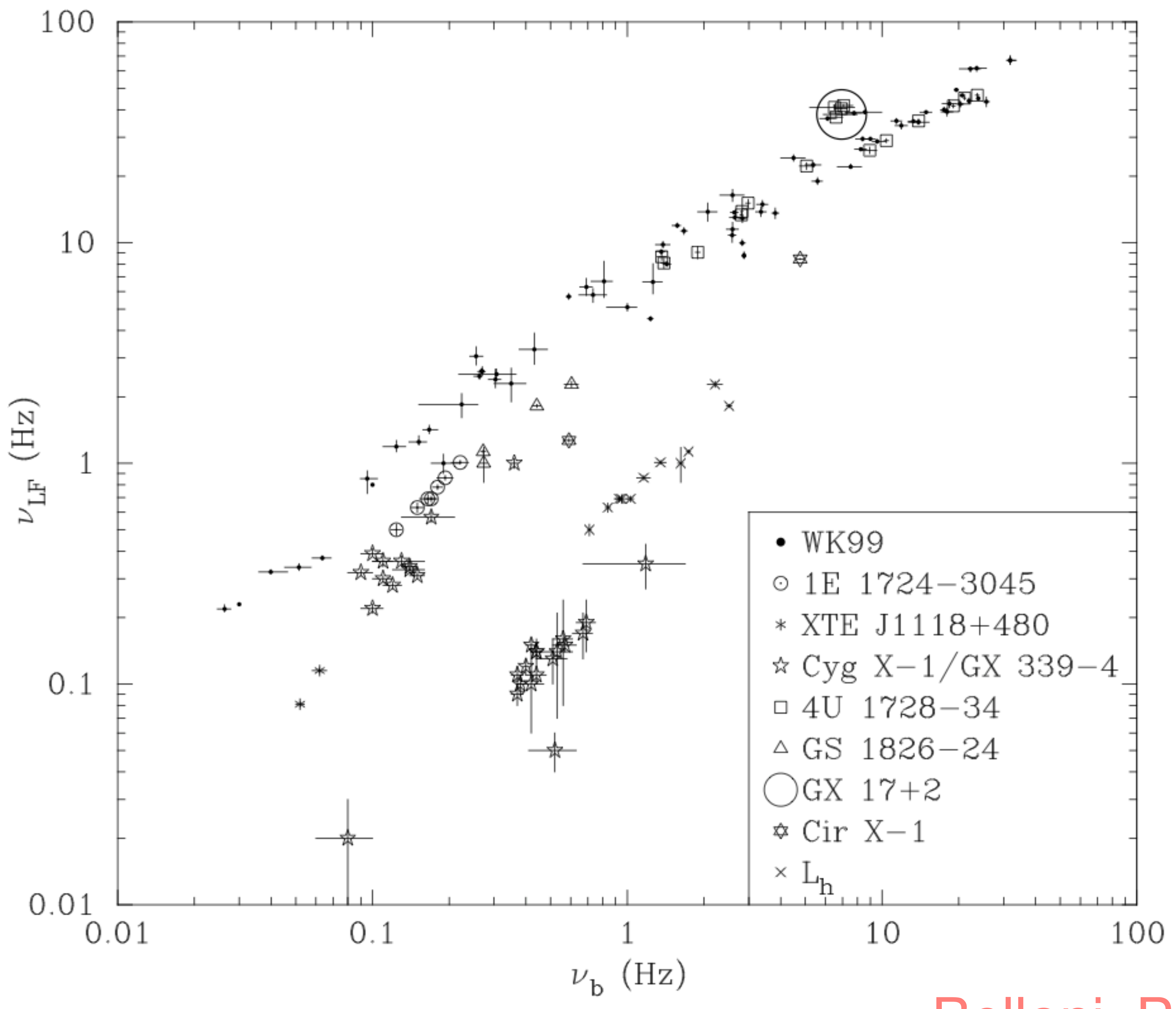
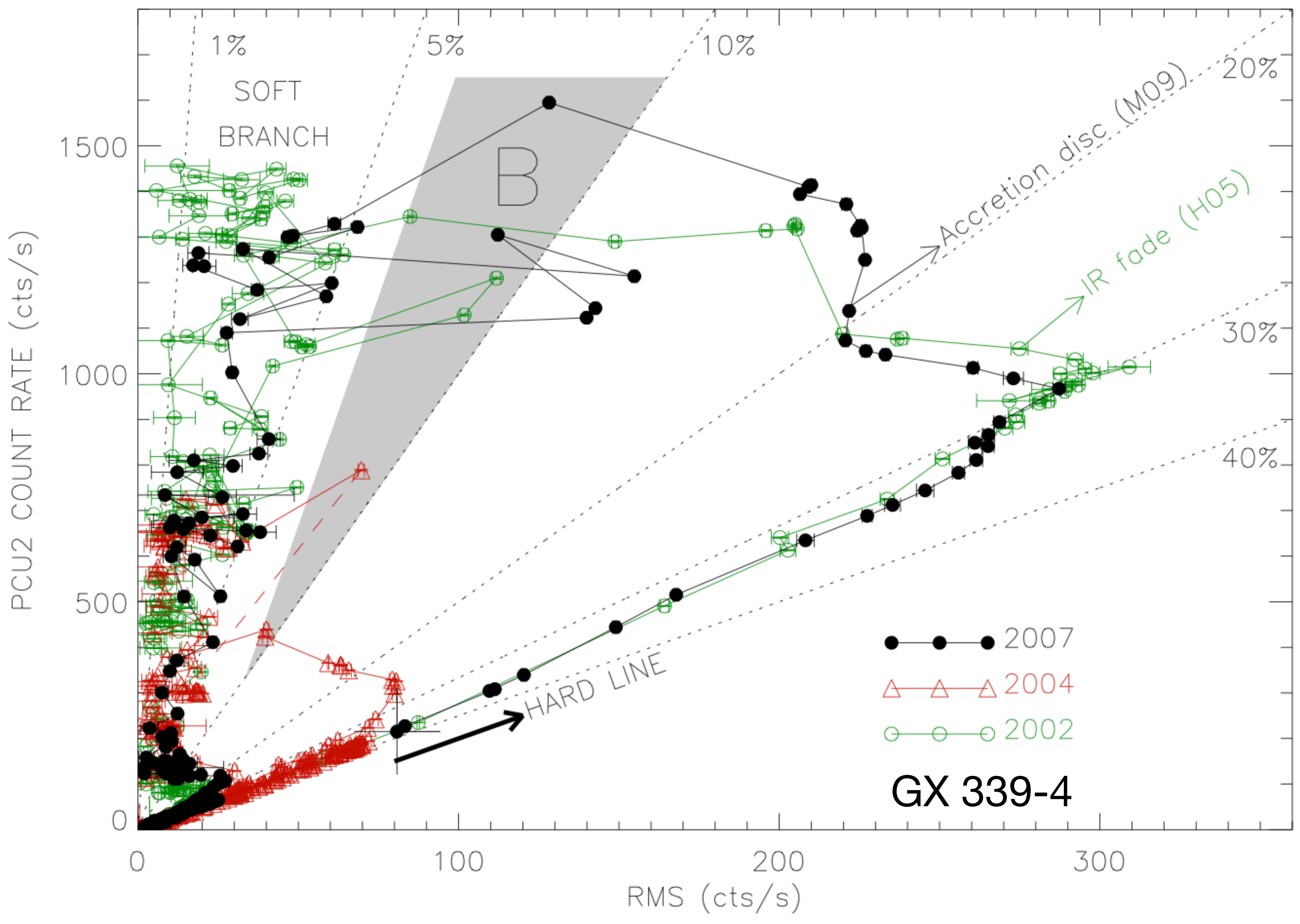


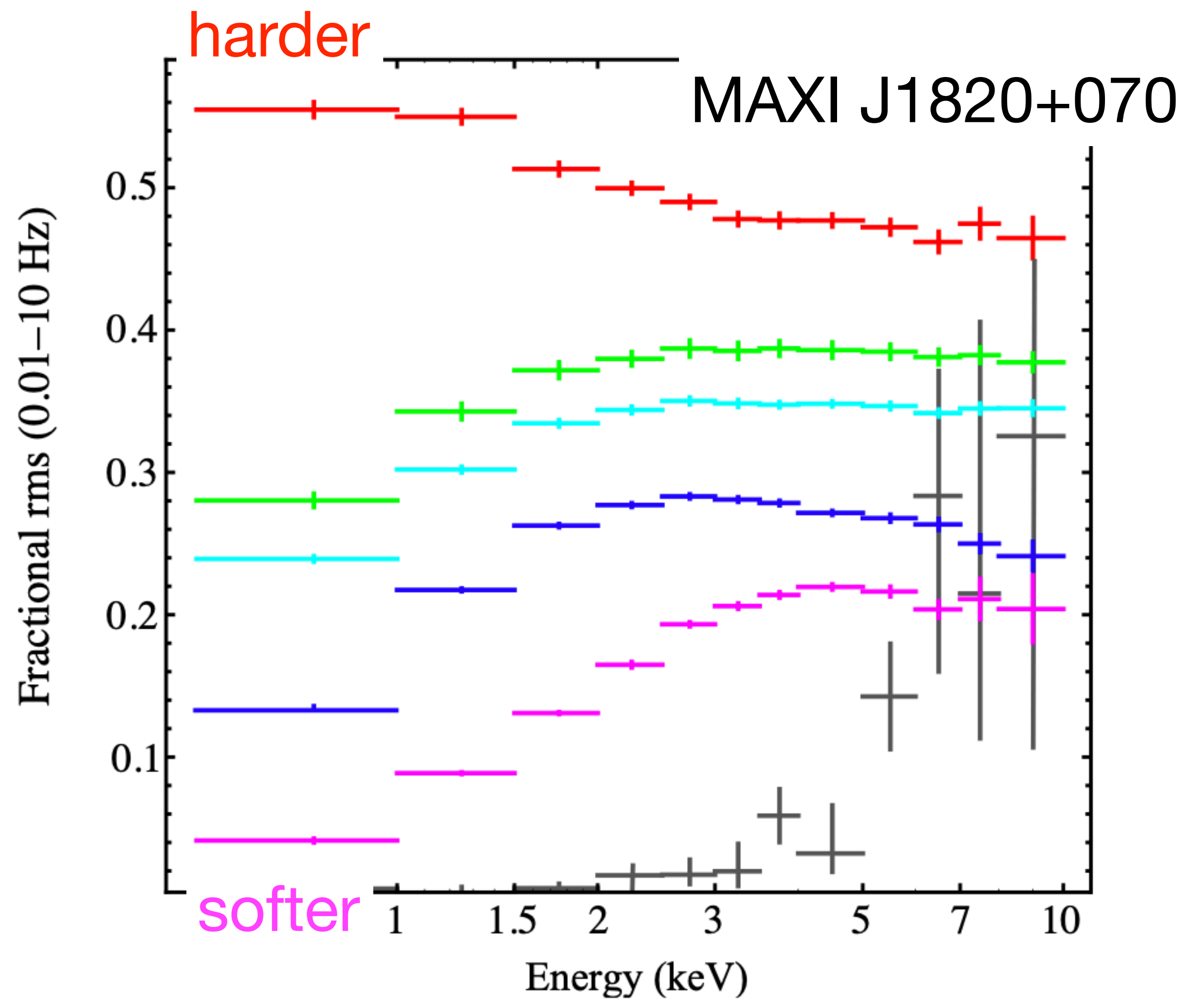
Belloni, Psaltis & van der Klis 2002

Power spectra: energy- and state- dependence

- Hard state more variable
- Variability drops in disc-dominated bands



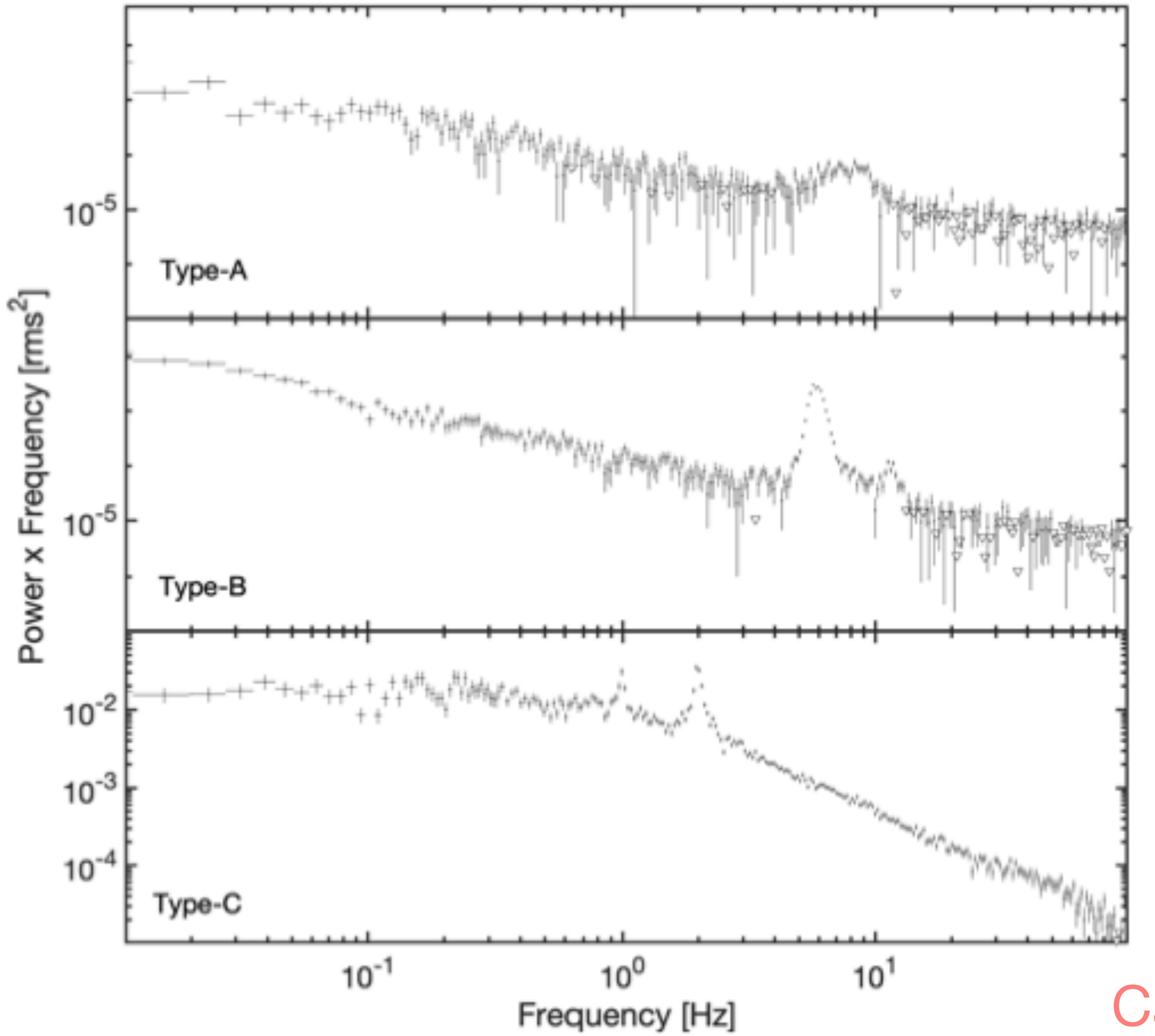
Muñoz-Darias et al. 2011
Heil et al. 2012



Axelsson & Veledina 2021;
Gierlinski & Zdziarski 2005;
Wilkinson & Uttley 2009;
De Marco et al. 2015;

Power spectra: QPOs

- Type A, B, C
- Likely geometric origin
- Also state-dependent



$Q \lesssim 3$

$Q \sim 5 - 7$

$Q \sim 5 - 15$

Casella, Belloni & Stella 2005
Motta et al. 2011

Power spectra: QPOs

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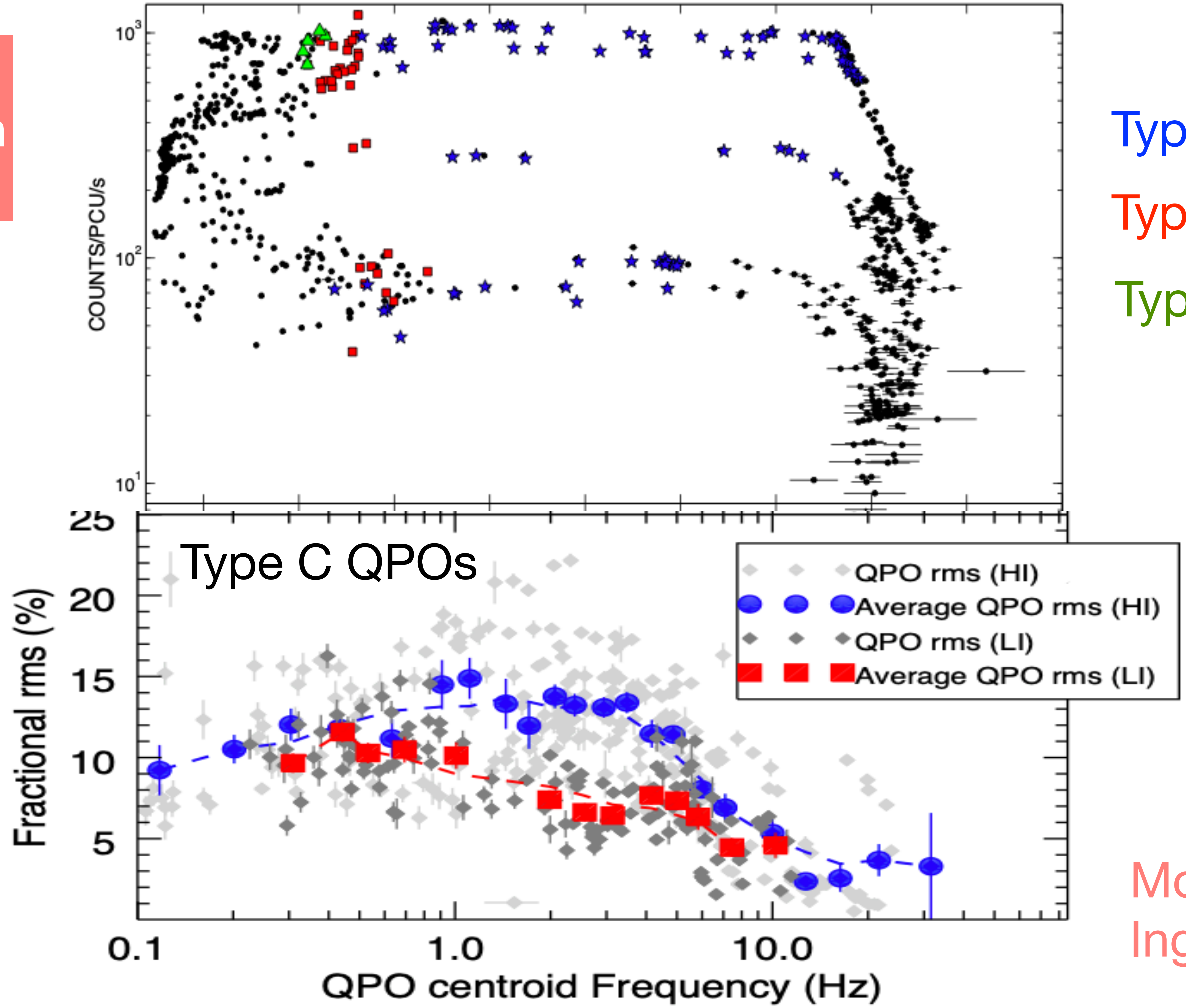

Type C: Hard state; HIMS

Type B: SIMS (transition)

Type A: SIMS-Soft state

Motta et al. 2011; 2015; Ingram & Done 2011

Power spectra: QPOs

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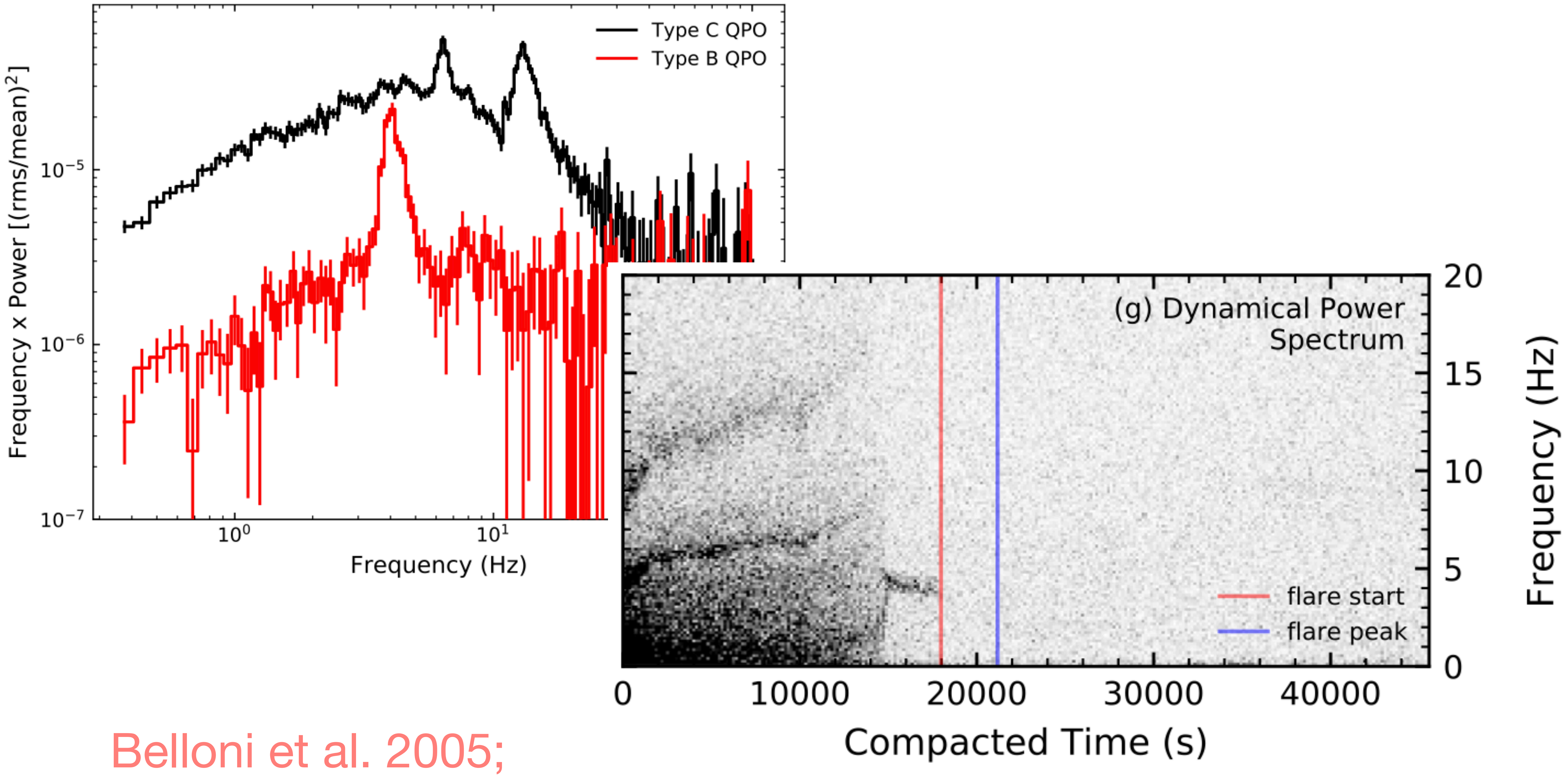
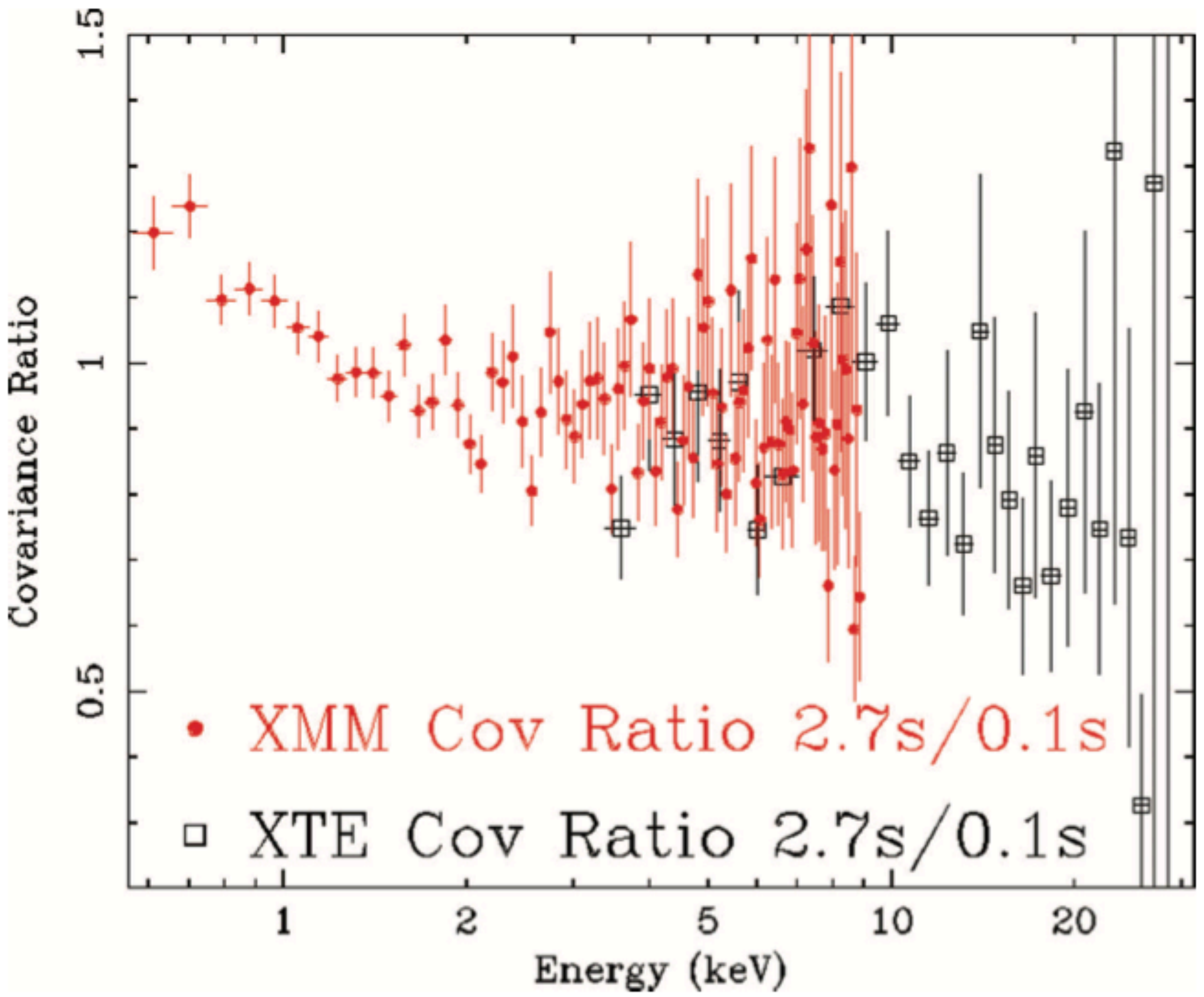
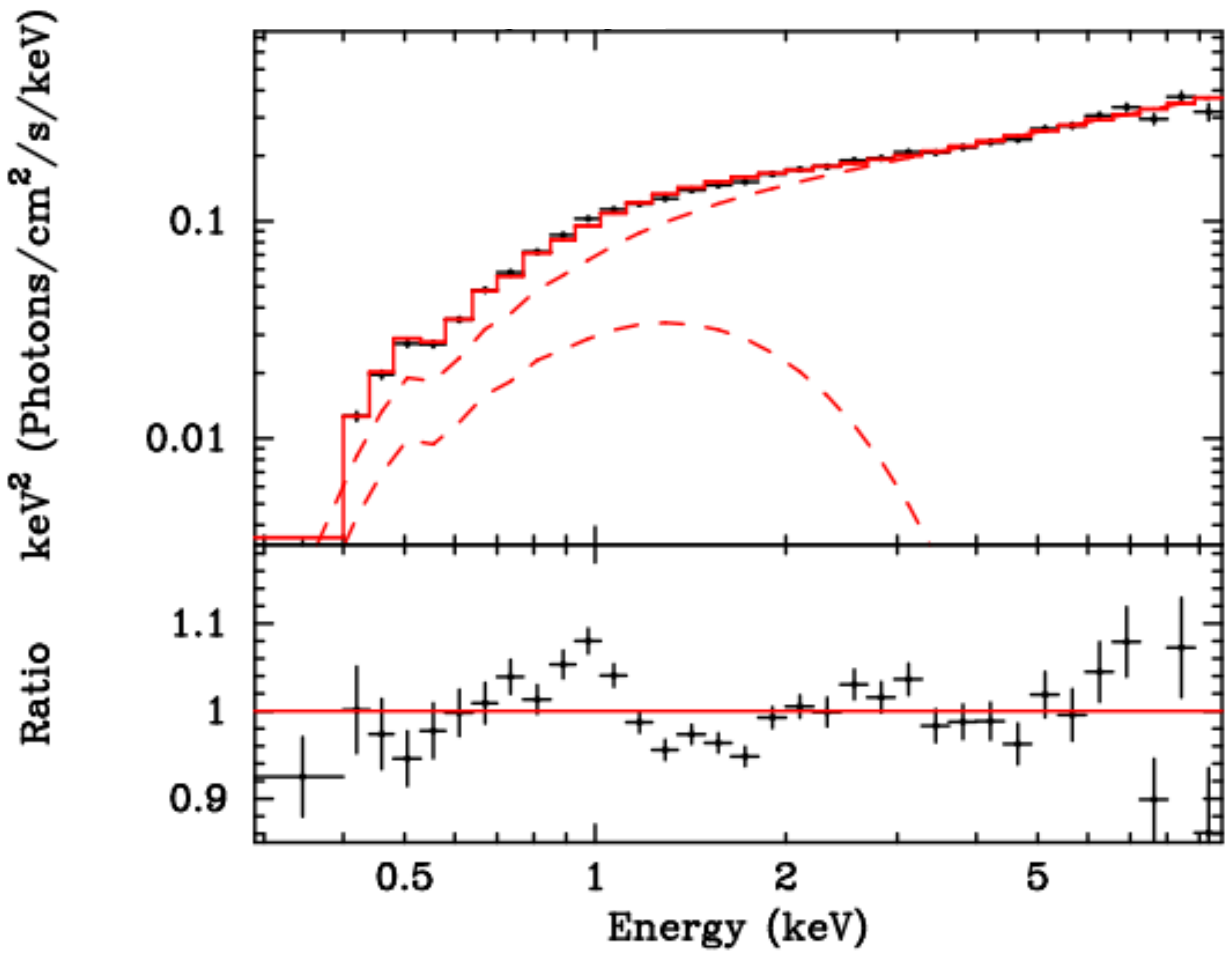


Belloni et al. 2005;
Homan et al. 2020
Stiele & Kong 2023

Cross spectra: covariance

- The disc in the hard state is not "passive"

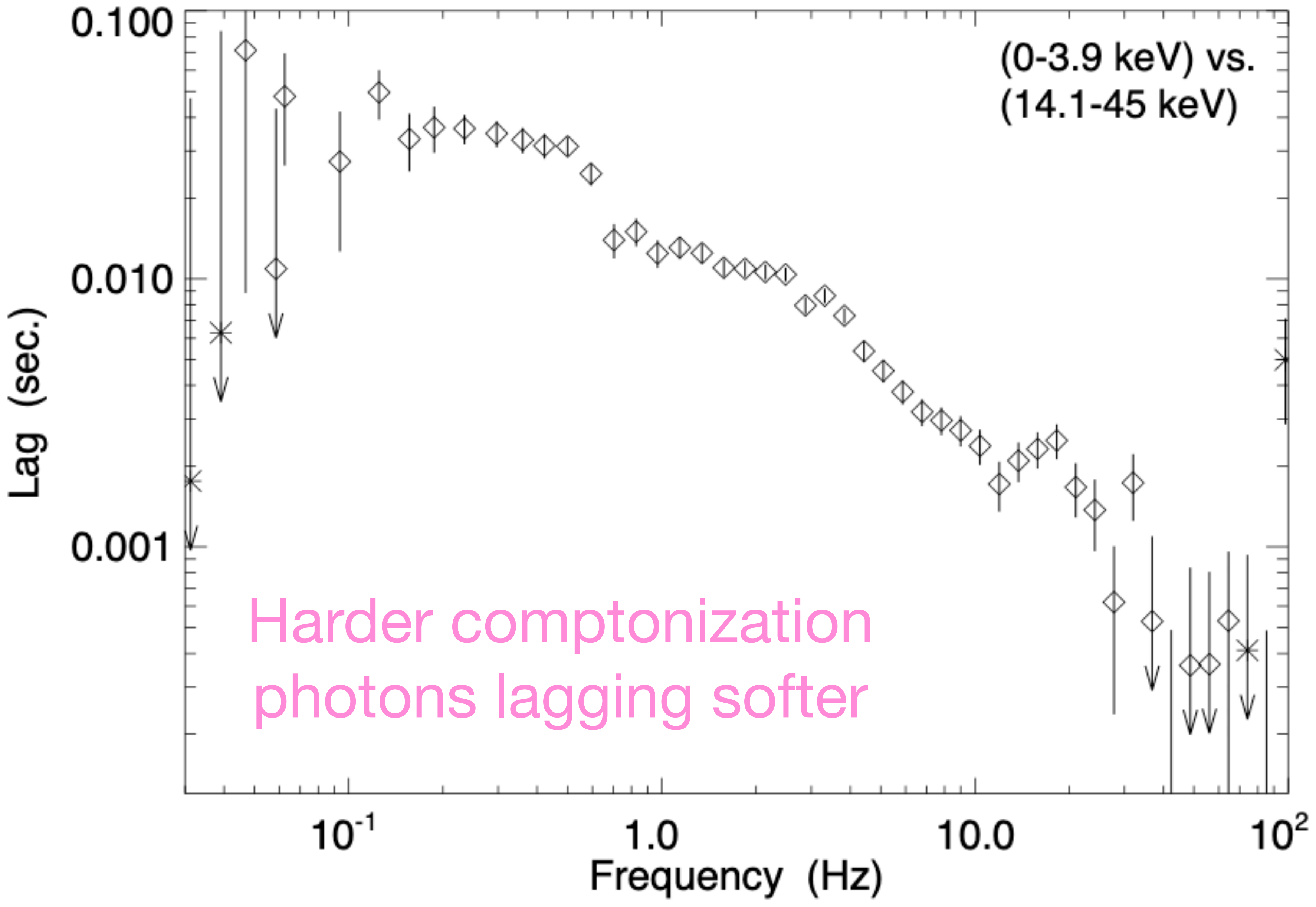




Wilkinson & Uttley 2009; De Marco et al. 2015; 2021

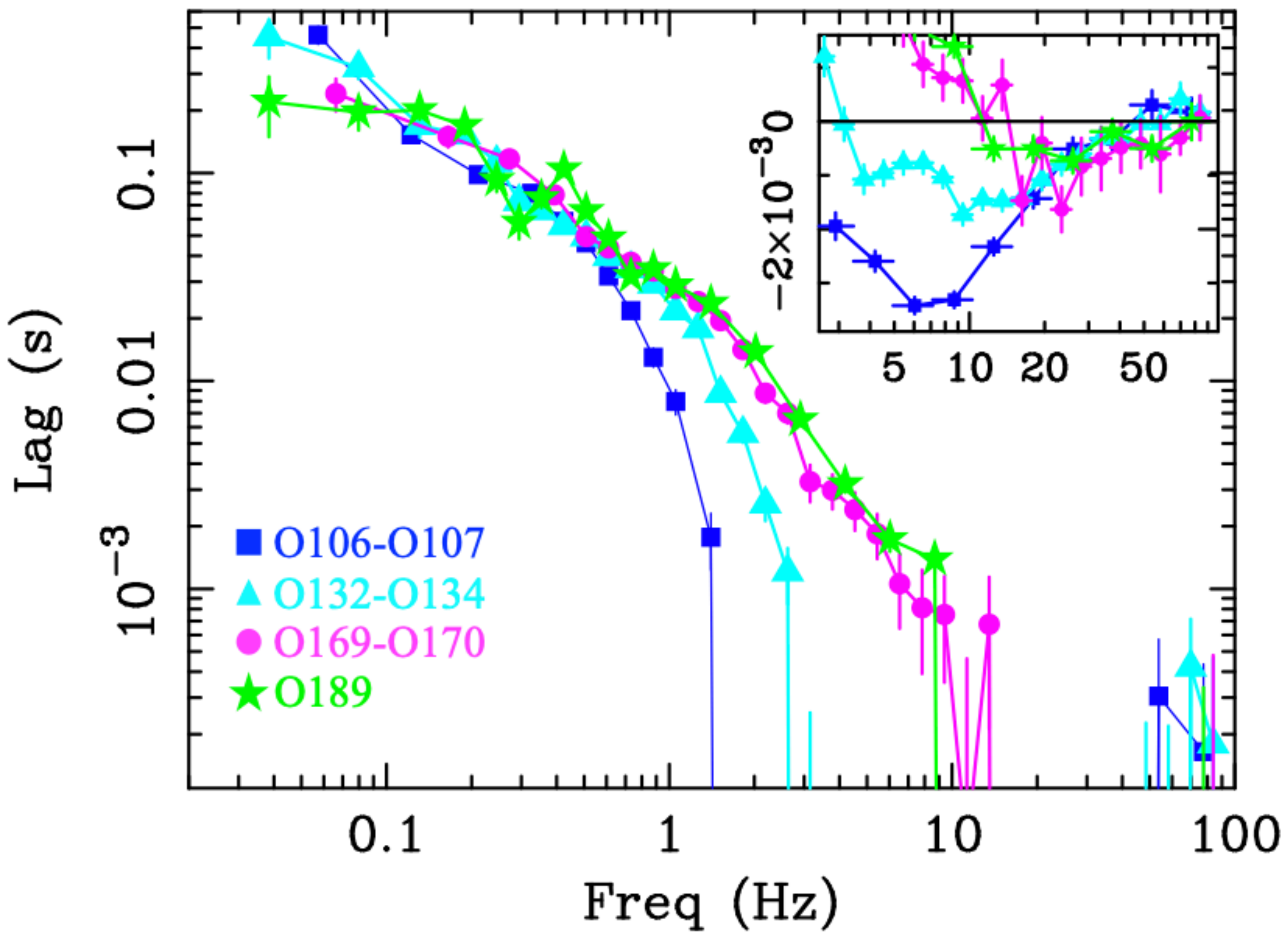
Cross spectra: time lags

- Frequency- and energy-dependence
- Low frequencies hard lags
- High frequencies soft lags
- State-dependent



Nowak et al. 1999

Disc leading at low frequencies, lagging at high frequencies



De Marco et al. 2021

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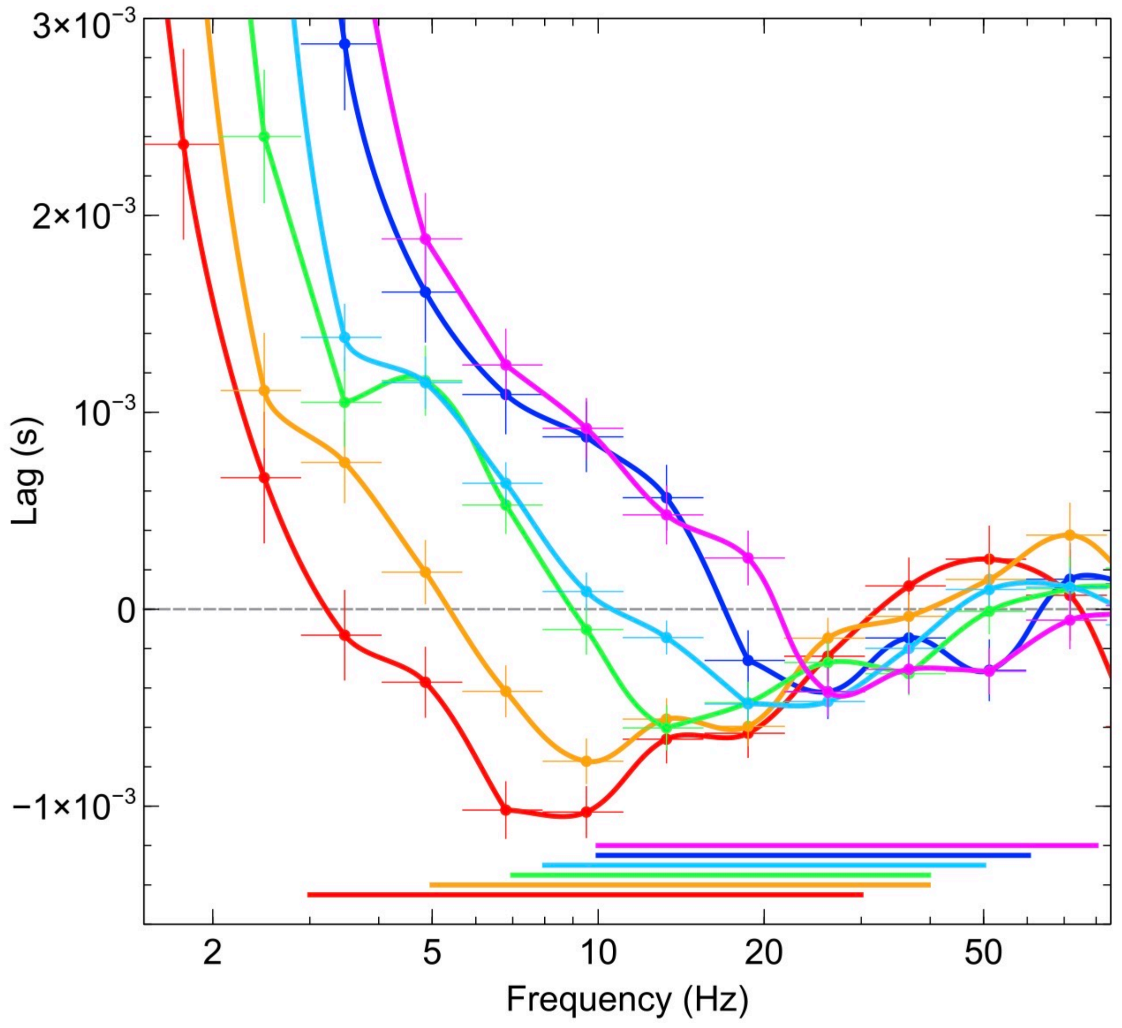
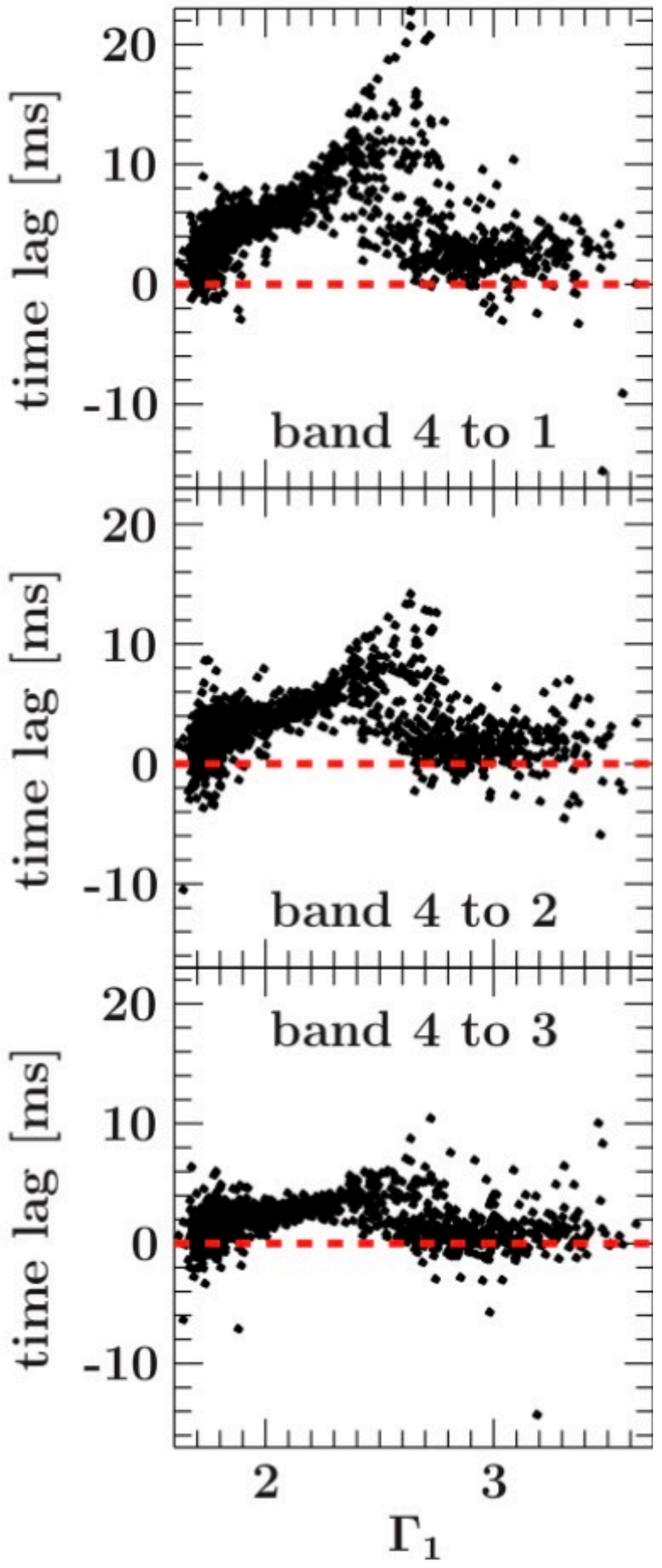
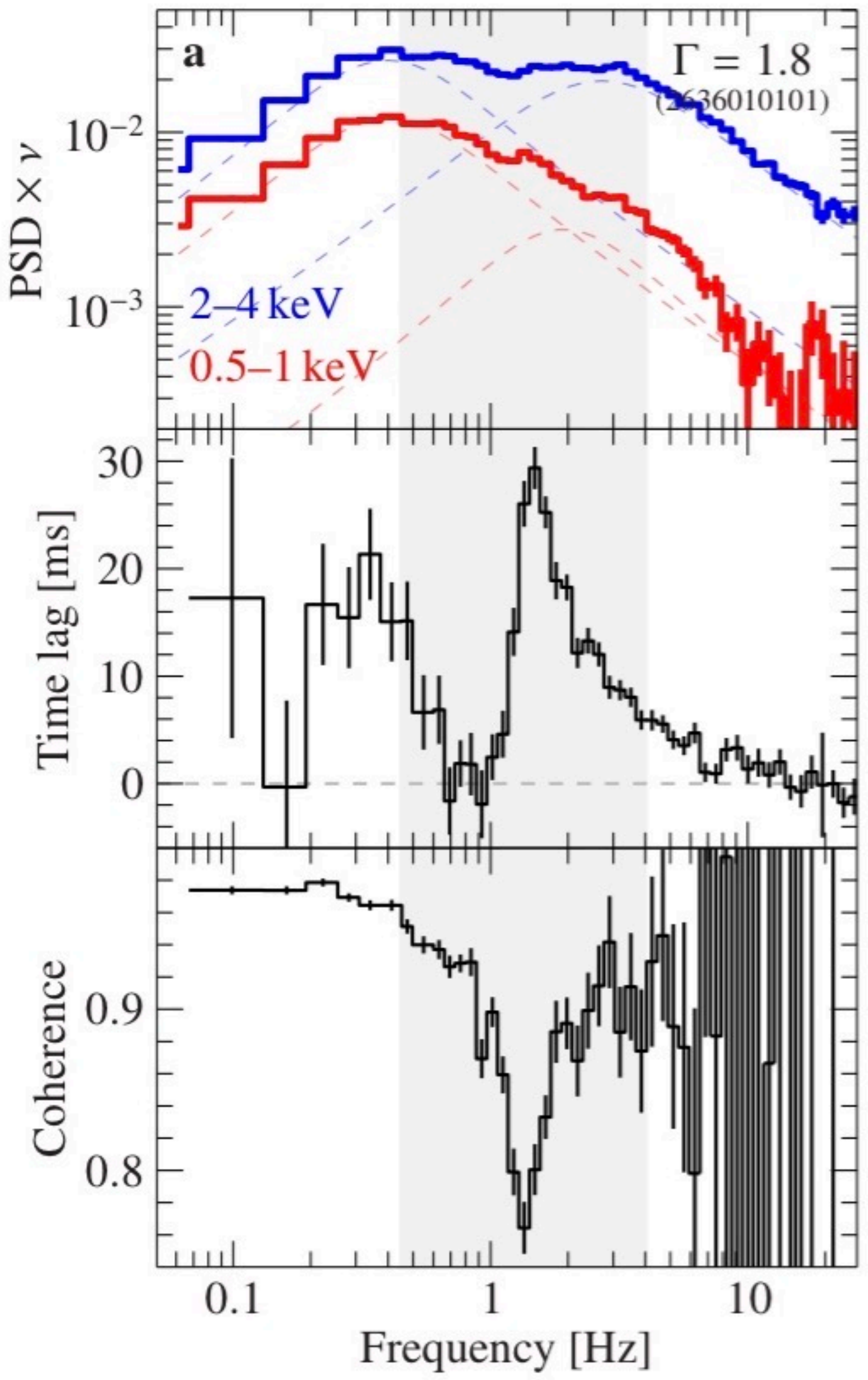
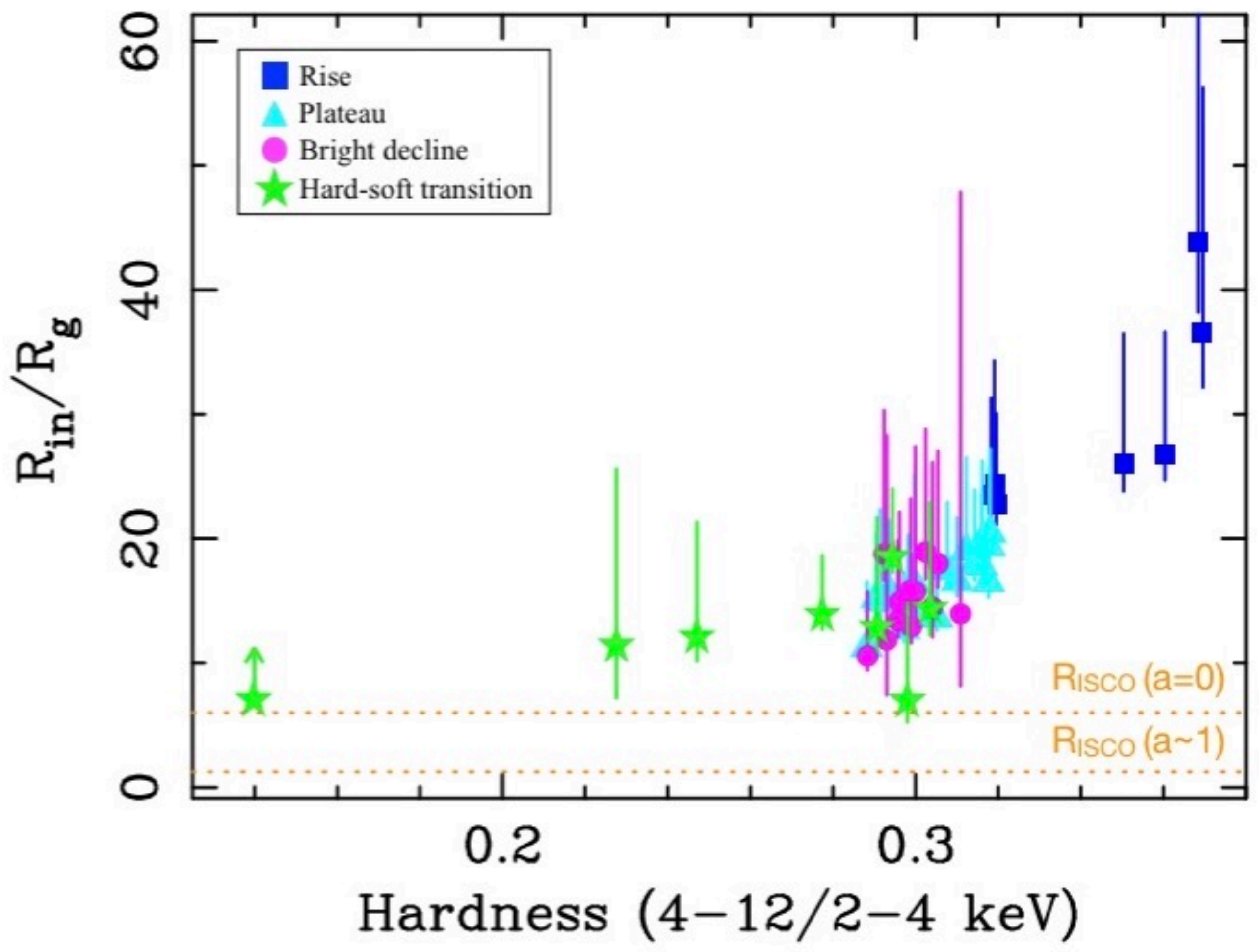






Pottschmidt et al. 2001
Grinberg et al. 2014;
Reig et al. 2017;
Kara et al. 2019;
De Marco et al. 2021;
König et al. 2024



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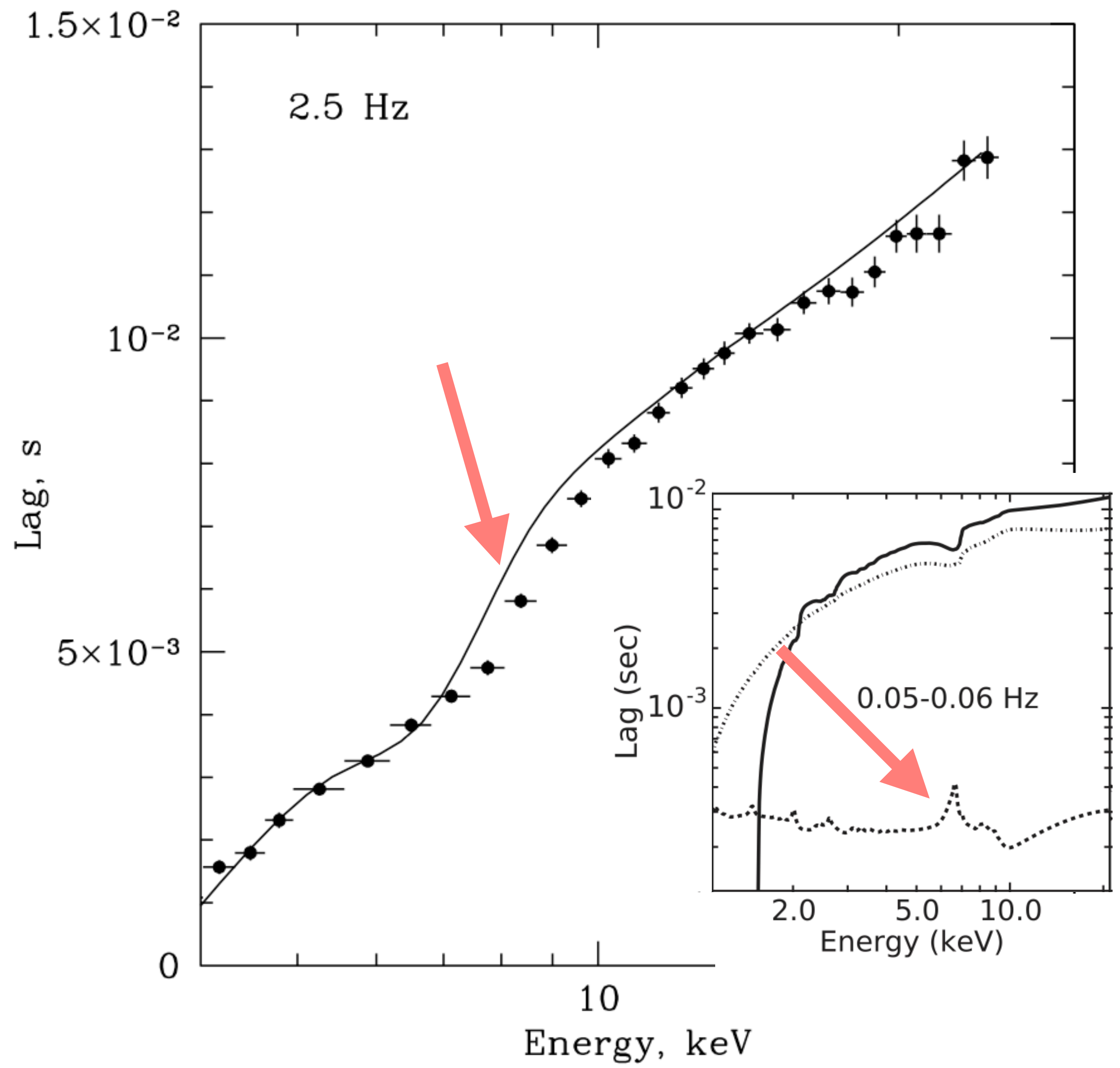
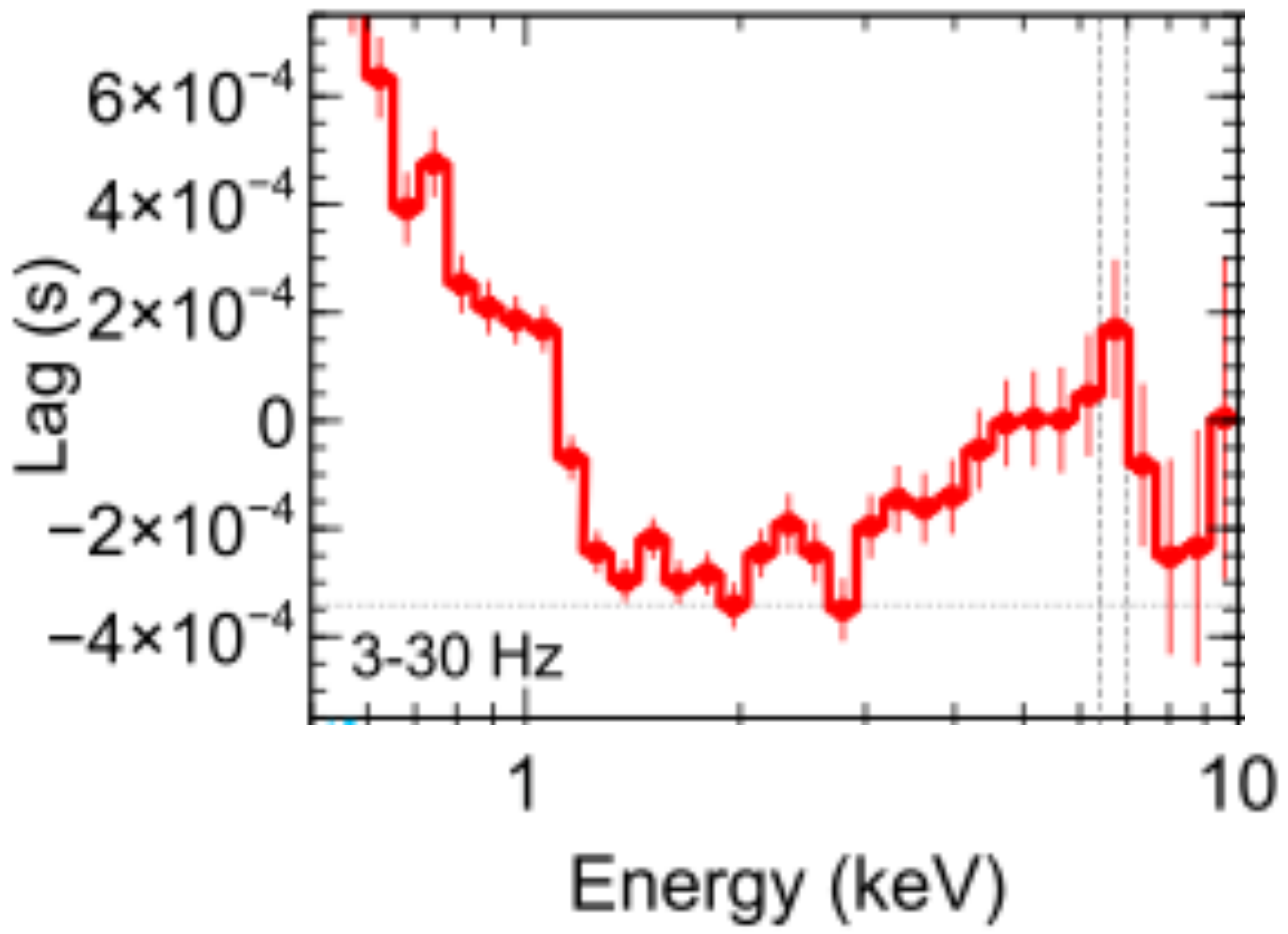
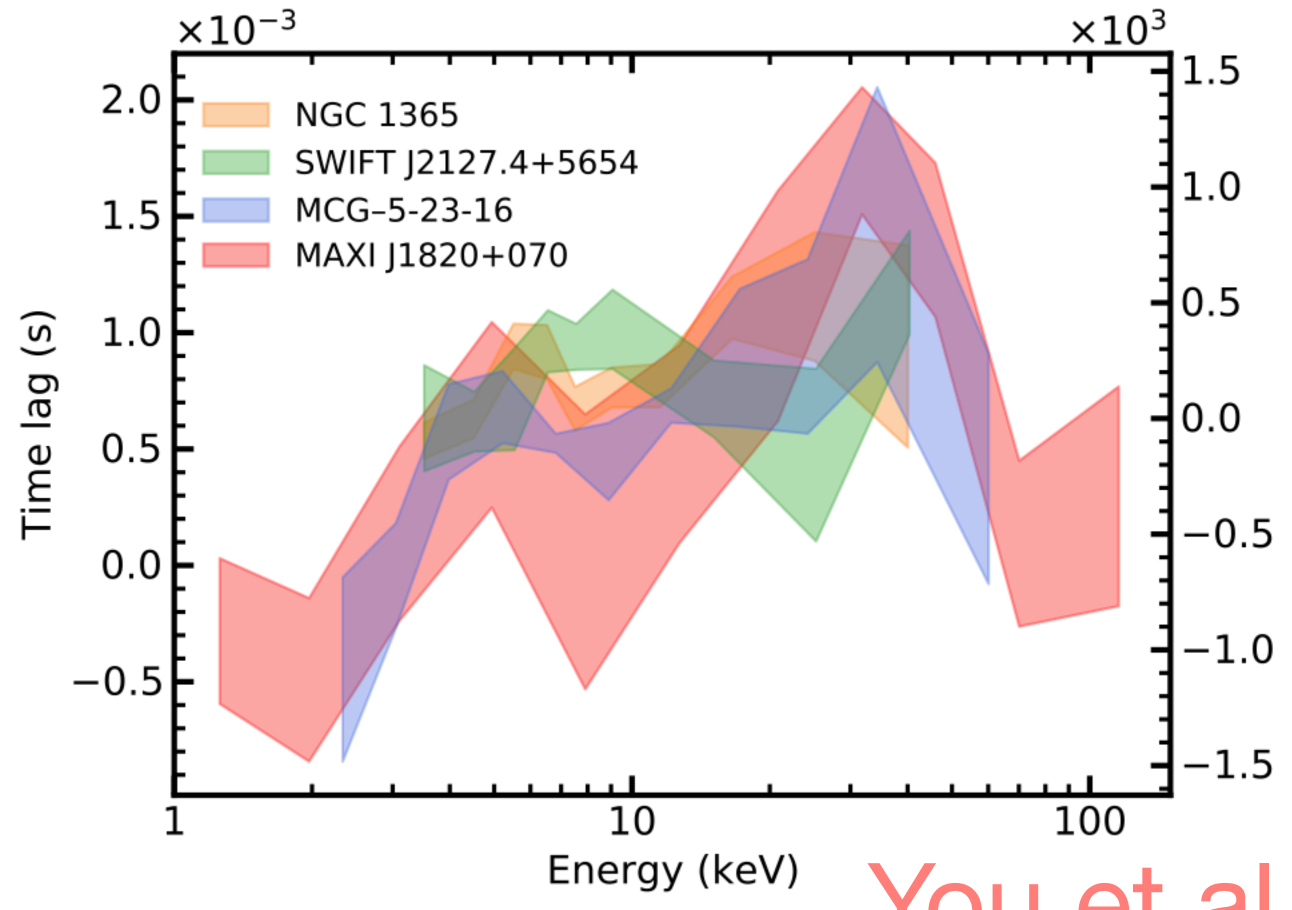


Kotov et al. 2001
Mastroserio et al. 2018, 2019



Kara et al. 2019

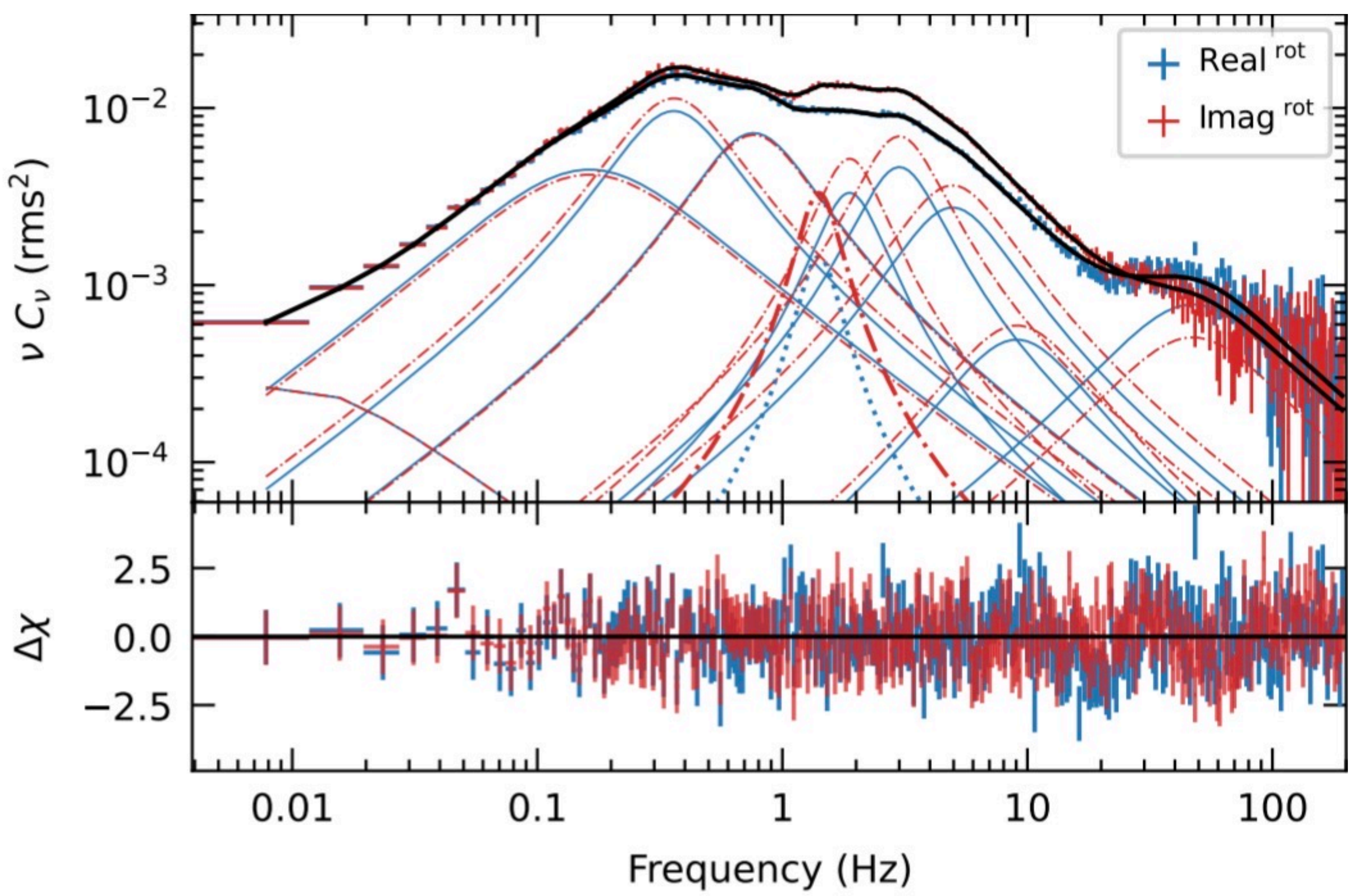


You et al. 2025

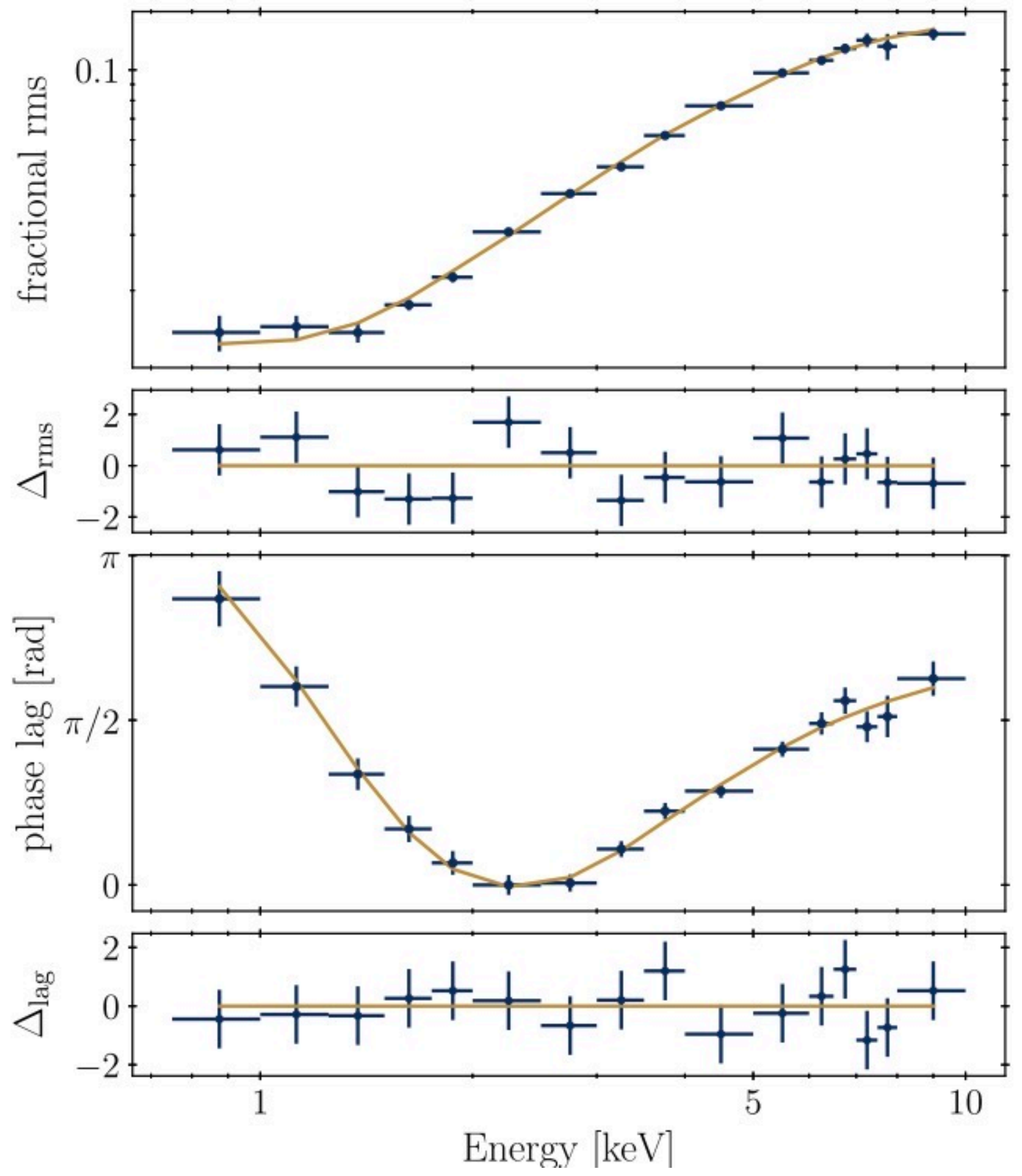
Combined cross- and power-spectral analysis

- Imaginary QPOs
- Spectral-timing models
- Multi-dimensional modelling

see Matteo Lucchini and Mariano Mendez tutorials



Fogantini et al. 2025; Bellavita et al. 2025; Mendez et al. 2023



Bellavita et al. 2022
Lucchini et al. 2025
Mastroserio et al. 2018, 2019

Limitations

- Biases (red-noise leakage, aliasing) can distort results
- Gaps in the lightcurves limit access to very slow timescales and worsen spectral resolution

- Assumption of stationarity may not be valid (accretion flow can evolve quickly) ➔ smearing, loss of transient signals, etc.
- Averaging over segments to improve S/N misses temporal evolution

- Spectral-timing needs photons and intrinsic variability: limited application to soft states, biased towards brighter and more variable sources/states

- Cross spectra miss non-linear interactions

e.g. van der Klis 1989; Vaughan & Nowak 1997; Uttley et al. 2002; 2014

Limitations

- Different physical geometries can produce similar Fourier observables (e.g. vertically extened corona vs. truncated disc)
- Physical processes mix up (e.g. propagation, reverberation, pivoting) preventing a simple interpretation

- Instrumental effects (redistribution matrix, dead time, etc.) can imprint "fake" features in spectral-timing products

- Models describing time-averaged spectra should be adapted to fit variability spectra

e.g. Bachetti & Huppenkothen 2018; Ingram et al. 2019; Uttley et al. 2014; De Marco et al. 2021

Hands-on

Learning Goals

1) Compute power spectra, cross-spectra, lag-frequency spectra, lag-energy spectra, coherence, covariance spectra

2) Understand what physical information these products give

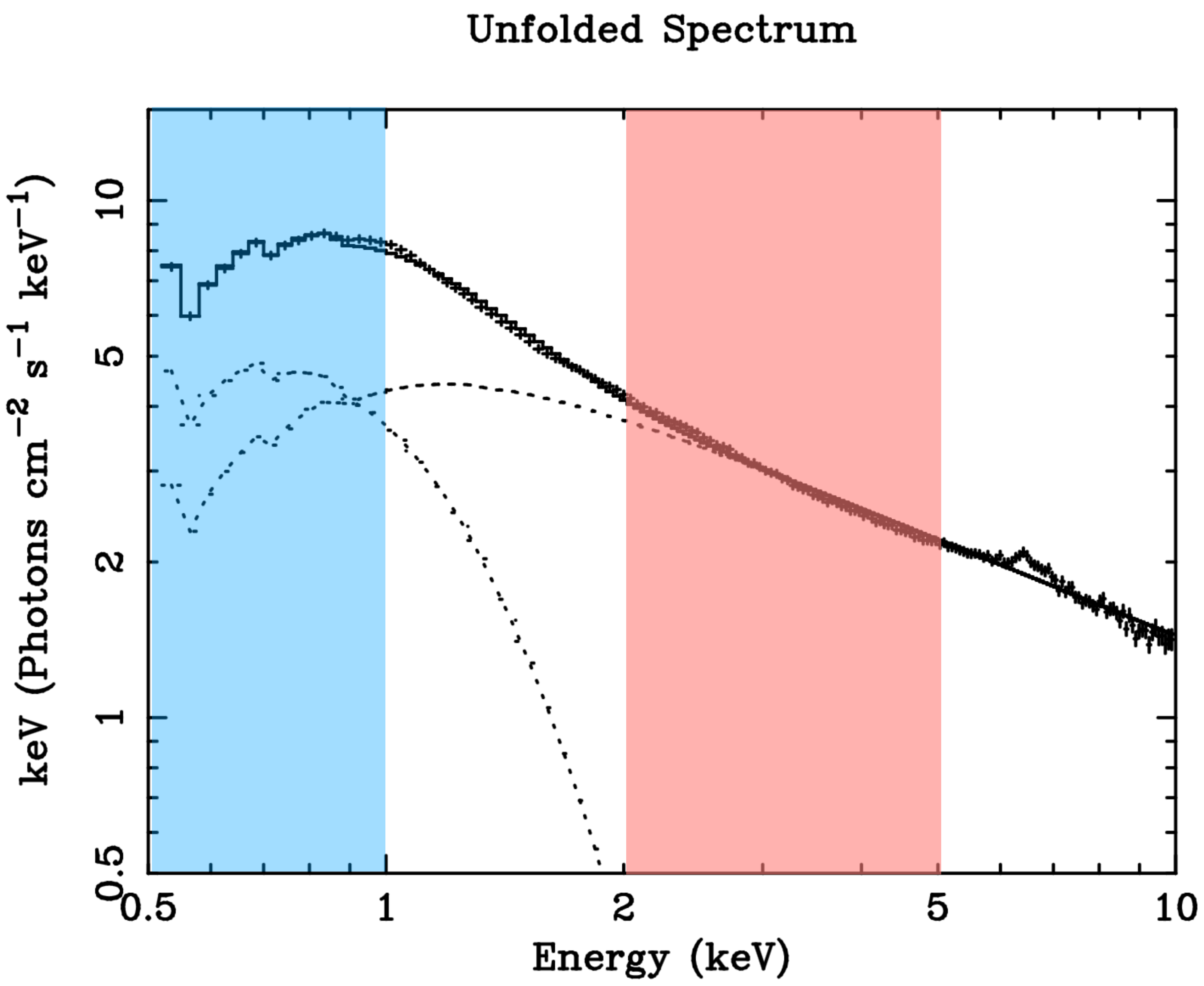
3) Know how not be fooled by data!

Tool: Stingray https://docs.stingray.science/en/stable/index.html

https://docs.stingray.science/en/stable/api.html#

The lightcurves

Choice of energy bands:



Nicer observation
MAXI J1820+070

The power spectrum

Information: Distribution of variability power over different timescales

Different frequencies probe different physical scales

Variability power in different energy bands

Noise vs. intrinsic source variability

Identify periodicities or quasi-periodicities

Fractional rms to determine the accretion state of the source

…

The power spectrum

$P(\nu_i) = A\,|DFT(\nu_i)|^2 \rightarrow$ "Periodogram" (estimate of intrinsic power spectrum $\mathscr{P}(\nu)$)

$$P(\nu) \sim \frac{\mathscr{P}(\nu)}{2}\chi_2^2$$

$$\langle P(\nu)\rangle = \mathscr{P}(\nu)$$

$$\sigma_{P(\nu)}^2 = \mathscr{P}^2(\nu)$$

inconsistent
estimator of the true
power spectrum

The power spectrum

$P(\nu_i) = A\,|DFT(\nu_i)|^2 \rightarrow$ To reduce the scatter we average periodograms:

$$\bar{P}(\nu) = \frac{1}{MW}\sum_{j=1}^{M}\sum_{i=1}^{W} P_j(\nu_i) \rightarrow \qquad \bar{P}(\nu) \sim \frac{\mathscr{P}(\nu)}{2MW}\chi^2_{2MW}$$

$$\langle P(\nu)\rangle = \mathscr{P}(\nu)$$

$$\sigma^2_{P(\nu)} = \frac{\mathscr{P}^2(\nu)}{MW}$$

$M =$ number of segments

$W =$ number of frequencies per bin

$$\bar{P}(\nu) \xrightarrow[MW\rightarrow\infty]{} \mathscr{N}\left(\mathscr{P}, \frac{\mathscr{P}^2}{MW}\right)$$

effectively for $MW > 50\ldots$

e.g. van der Klis 1989; Papadakis et al. 1993

The power spectrum

$P(\nu_i) = A\,|DFT(\nu_i)|^2 \rightarrow$ "Periodogram" (estimate of intrinsic power spectrum $\mathscr{P}(\nu)$)

Normalization:

- "Absolute" $[(counts\ s^{-1})^2\ Hz^{-1}] \rightarrow P_{noise} = 2\bar{x}$
- "Leahy" $[counts\ s^{-1}\ Hz^{-1}] \rightarrow P_{noise} = 2$ $\Rightarrow P_{signal}/P_{noise} \propto \bar{x}$
- "Fractional" $[(rms/mean)^2\ Hz^{-1}] \rightarrow P_{noise} = \dfrac{2}{\bar{x}}$

…where $\bar{x}$ is the mean count rate

The (averaged) cross-spectrum

Information: Measures linearly correlated variability(*) amplitude in two bands

Suppresses uncorrelated or not-linearly correlated variability (including noise)

Tells which band varies first (relative delays)

Measures coherence (how close to linear, correlated variability is)

Different frequencies probe different physical scales

…

(*) $x(t)$ and $y(t)$ are linearly correlated if a linear transform exists, such that $\Rightarrow Y(\nu) = H(\nu)X(\nu)$ where, $Y(\nu)$ and $X(\nu)$ are the Fourier transforms, and $H(\nu)$ is the impulse response or transfer function. This implies that the relative phase between Y and X is constant.

The cross-spectrum

$R(\nu) = DFT$ of reference band (e.g. soft) lightcurve

$C(\nu) = DFT$ of channel band (e.g. hard) lightcurve

Definition:

$$CS(\nu_i) = C^*(\nu_i)R(\nu_i) = (\tilde{C}^* + C^*_{noise})(\tilde{R} + R_{noise}) = \tilde{C}^*\tilde{R} + \ uncorrelated\ noise\ terms$$

$$\overline{CS}(\nu) \xrightarrow[MW\to\infty]{} \tilde{C}^*(\nu)\tilde{R}(\nu)$$

The average cross-spectrum preserves coherent variations between the two lightcurves

The cross-spectrum

The cross-spectrum allows us to estimate phase and time lags, coherence, and covariance:

$$\gamma^2(\nu) = \frac{|\overline{CS}(\nu)|^2 - n^2}{\overline{P}_C(\nu)\overline{P}_R(\nu)}$$

$$\phi(\nu) = \arctan\frac{Im[\overline{CS}(\nu)]}{Re[\overline{CS}(\nu)]}$$

$$\overline{CS}(\nu)$$

$$Cov(\nu) = \sqrt{\frac{\Delta\nu_j(|\overline{CS}(\nu)|^2 - n^2)}{\overline{P}_R(\nu) - P_{R,noise}}}$$

$$\tau(\nu) = \frac{\phi(\nu)}{2\pi\nu}$$

Time lags: dilution

Dilution: each band contains contribution from different components

Assumption → no disc in channel band

$\phi_0 = 0$

Channel $\rightarrow c(t) = c_{pow}$ $\quad\Rightarrow C(\nu) = |C_{pow}|e^{i\phi_0} = |C_{pow}|$

Reference $\rightarrow r(t) = r_{pow} + r_{rev} = r_{pow}(1+f)$ $\quad\Rightarrow R(\nu) = |R_{pow}|(e^{i\phi_{pow}} + fe^{i\phi_{rev}})$

$$CS(\nu) = |R_{pow}C_{pow}|(e^{-i\phi_{pow}} + fe^{-i\phi_{rev}}) \rightarrow \phi = \arctan[-\frac{\sin(\phi_{pow}) + f\sin(\phi_{rev})}{\cos(\phi_{pow}) + f\cos\phi_{rev}}]$$

$$(\phi_{pow} \sim 0,\ \phi_{rev} < 0) \quad \phi = \arctan[\frac{f\sin(\phi_{rev})}{1 + f\cos(\phi_{rev})}] \xrightarrow[\phi_{rev}\rightarrow 0]{} \phi_{rev}\frac{f}{1+f}$$

Uttley et al. 2014