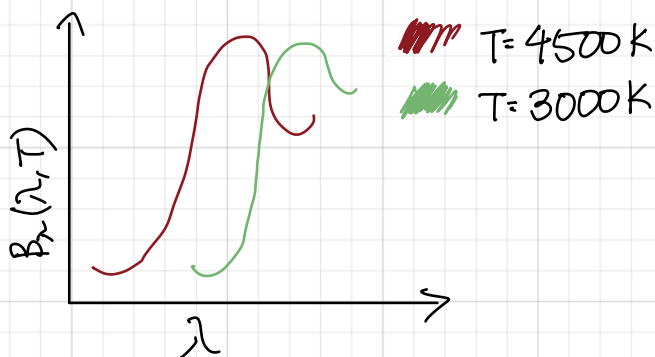# Rayleigh-Jeans + Wien + Planck

BB Radiation

→ an ideal object that absorbs all incident of light & emits radiation based on its temperature.

emitted spec ß described by its spectral radiance

$$↳ B_\lambda (\lambda, T)$$

how much power ß emitted at each λ.

→ As temperature ↑'s, curve's peak shifts to shorter λ's (inverse relationship)

T = 4500 K

T = 3000 K

$B_\lambda (\lambda, T)$

λ

> Hence, Hotter objects glow bluer

First predicted by
Wilhelm Wien
→ derived Displacement Law in 1893

$$\lambda_{max} T = constant$$

or

→ Before Wien came Stefan (1879) & Boltzmann (1884)

> predicted total emitted power increases strongly
with Temperature ($\propto T^4$)

>> not with peak color shift

→ Wien's Displacement Law Equation

$$\lambda_{peak} = b/T$$

* b = "constant of proportionality"
or
wien's constant

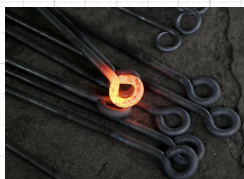$$b = 2.897771 \times 10^{-3} \, m \cdot k$$

or

$$b \approx 2898 \, \mu m \cdot k$$

→ Example of Wien's Displacement Law

> piece of metal heated by a blow torch first becomes
"red hot" $\xrightarrow{to}$ orange-red ⟶ White hot

> Determining colors of stars by its temp.
Using Orion Constellation

Betelgeuse T≈3800k

Rigel T≈12100k

Bellatrix T≈2200k

Mintaka T≈31800k

→ Wien's Displacement Law gave a _scaling rule_
  ⇓ meaning
  > If temp is changed, spectrum shifts on λ axis
  > Implies the form mathematically
     $P(\lambda,T) = \frac{1}{\lambda^5} f(\lambda T)$
  ⇓ hence
  > λ^{-5} is _forced_ by thermodynamics + scaling
  > unknown piece is $\underline{f(\lambda,T)}$ ("shape" of function)

  ↳ used adiabatic expansion/compression of radiatio in a
  cavity (a piston-cylinder through experiment), where
  λ scales with cavity size & temp scales in a
  linked way
  ✱ at this point, curve shape must be function of λ,T
  BUT, didn't know _the function yet_
            ✓ so

→ all about Radiation Approx (1896)
  > proposed a specific form of unknown function
  by bringing in molecular-kinetic / Maxwell-Boltzmann
  reasoning
  ✩ H's argument
  > Radiation energy is controlled by microscopic emitters/absorbers
                     ("molecules/oscillators" in the walls)

> energy should follow a ==Boltzmann factor==
$$\propto e^{-\alpha/(\lambda T)}$$
⇓ proposed
$$P(\lambda, T) = \frac{c}{\lambda^5} \exp\left(-\frac{\alpha}{\lambda T}\right)$$

✦ Why this was convincing at the time

→ matched short-λ (high-$\tilde{\nu}$) part of observed spectrum very well
    ↳ could be measured!

✦ Why it failed

→ Ruben & Kurlbaum measurements in IR/long-λ region showed Wien's exponential drop-off was too steep at long λ's
    > underpredicted IR Radiation)

    ⇓ fixed by Planck!

Max Planck in 1900
    presented his Law of
        B.B. Radiation

First quantized energy!

→ Planck changed one deep assumption about how matter exchanges energy with radiation
  > In Wien's Approx, IR tail behaved more like:
    $B_\lambda(T) \propto T/\lambda^4$
    (Rayleigh-Jeans Behavior)
    >> not exponential decay

        ☑ hence
  * Planck needed a formula that has
      > Wien's Behavior at short λ.

      > Rayleigh-Jeans Behavior at long λ

→ Planck's Approach
  ↳ Change Statistical Assumption

> Modeled the cavity walls as containing tiny electromagnetic oscillators ("resonators") of freq. ν.

> Observed the classical problem
    >> R-J says oscillator energy are continuous
        (classical equipartition theorem)
    ⇓ leads to
    UV catastrophe (blows up at high ν)
> Planck thought oscillators cannot take arbitrary energies
    ☑ instead
    comes in packets (quantized!)
        $E_n = nh\nu$ (n = 0, 1, 2, ...)

⭐ How Planck fix Wien's failure

→ After quantization, probability of occurring energy at level E_n, follows Boltzmann Statistics

$$P_n \propto e^{-E_n/kT} = e^{-nh\nu/kT}$$

⇓ compute average oscillation energy

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} (nh\nu) e^{\{-nh\nu/kT\}}}{\sum_{n=0}^{\infty} e^{\{-nh\nu/kT\}}} = \frac{h\nu}{e^{\{h\nu/kB\}} - 1}$$

→ Final Planck's Radiation Law

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\{hc/\lambda kT\}} - 1}$$

→ Why Panck didn't settle here immediately

  > Planck himself didn't initially present as "birth of QM"
  > Radiation Field itself was still classically described

  ⇓ hence, after Planck

→ lots of Physicists still wanted a fully classical explanation
→ Planck's derivation didn't yet feel like a new "mechanics" but an odd statistical assumption

Rayleigh–Jeans Law (1905)

$$B_\nu(T) = \frac{2\nu^2 kT}{c^2}$$

  > matched experiments in IR/microwave Region
  > failed at high ν or low λ

$$B_\nu \propto \nu^2 \rightarrow \infty \quad \text{UV catastrophe}$$

> UV catastrophe is a **theoretical catastrophe**
> ⮡ realized when classical theory was pushed to
> its logical endpoint

→ Why UV catastrophe was "discovered" after Planck's Law
> After Planck, scientists like R-J wanted to derive the
classical spectrum cleanly using:
>> Standing wave modes in a cavity)
>> equipartition principle

R-J showed clearly _why_ it failed

Review
Wien's Approx

$$B_\lambda(\lambda, T) = \frac{C_1}{\lambda^5} \exp\left(-\frac{C_2}{\lambda T}\right)$$
$$C_1 = 2hc^2 \qquad a = hc/k$$
*k = Boltzmann Constant

RJ Law
$$B_\lambda(\lambda, T) = \frac{2ckT}{\lambda^4} \implies_{\text{which}} B_\lambda \propto \frac{1}{\lambda^4} \to \infty$$
$$\text{is UV catastrophe}$$

Planck Law
$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Plank's assumption for $I = av^3 (e^{bv/T} - 1)$

① Radiation emitters & absorbers in the BB to be harmonically oscillating electric charges ("resonators") in equilibrium with electromagnetic radiation in a cavity.

② Total energy of those resonators whose freq. is v consisted of N indivisble "energy elements", each of magnitude hv

⇓ Thus

energy is quantized

↙ meaning

only certain discrete values were allowed for a resonator energy

2 Application of Energy Quantization
① Blackbody Radiation
② Photoelectric Effect