Universidad de las Américas
Algebra I
Junio 15, 2019.

Proyecto Taller 4

$\mathcal{C} = \{(1,0,0),(0,1,0),(0,0,1)\}$, $\mathcal{B} = \{(-1,0,0),(1,1,0),(0,0,2)\}$

$T: \mathbb{R}^3 \to \mathbb{R}^3$, $T(x,y,z) = (x,0,x+z)$, $v = (1,4,5)$

a. $[v]_{\mathcal{B}} = [\alpha,\beta,\gamma] \iff v = \alpha(-1,0,0) + \beta(1,1,0) + \gamma(0,0,2)$

$$(1,4,5) = \alpha(-1,0,0) + \beta(1,1,0) + \gamma(0,0,2)$$

$$\iff \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right)$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5/2 \end{pmatrix} \to \begin{cases} \alpha = 3 \\ \beta = 4 \\ \gamma = 5/2 \end{cases}$$

Por lo tanto: $[v]_{\mathcal{B}} = [3, 4, 5/2]$

b. $w \in \mathbb{R}^3$, $[w]_{\mathcal{B}} = [1,1,1]$

$$w = 1(-1,0,0) + 1(1,1,0) + 1(0,0,2)$$
$$= (0,1,2)$$
$$w = (0,1,2) = 0(1,0,0) + 1(0,1,0) + 2(0,0,1)$$

Por lo tanto: $[w]_{\mathcal{C}} = [0,1,2]$

c. Partimos calculando $[I]_{\mathcal{B}}^{\mathcal{C}}$:

$$(-1,0,0) = -1(1,0,0) + 0(0,1,0) + 0(0,0,1)$$
$$(1,1,0) = 1(1,0,0) + 1(0,1,0) + 0(0,0,1)$$
$$(0,0,2) = 0(1,0,0) + 0(0,1,0) + 2(0,0,1)$$

$$[I]_{\mathcal{B}}^{\mathcal{C}} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Para calcular $[I]_{\mathcal{C}}^{\mathcal{B}}$, ocupamos $[I]_{\mathcal{C}}^{\mathcal{B}} = \left([I]_{\mathcal{B}}^{\mathcal{C}}\right)^{-1}$

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \quad \text{por el cálculo anterior.}$$

$$[I]_{\mathcal{C}}^{\mathcal{B}} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

d. Primero calculamos $[T]_{\mathcal{C}}^{\mathcal{C}}$

$$T(x,y,z) = (x,0,x+z)$$

$$T(1,0,0) = (1,0,1) = 1(1,0,0) + 0(0,1,0) + 1(0,0,1)$$
$$T(0,1,0) = (0,0,0) = 0(1,0,0) + 0(0,1,0) + 0(0,0,1)$$
$$T(0,0,1) = (0,0,1) = 0(1,0,0) + 0(0,1,0) + 1(0,0,1)$$

Luego: $$[T]_{\mathcal{C}}^{\mathcal{C}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Calculamos $[T]_{\mathcal{B}}^{\mathcal{B}}$ mediante el producto: $[T]_{\mathcal{B}}^{\mathcal{B}} = [I]_{\mathcal{C}}^{\mathcal{B}} [T]_{\mathcal{C}}^{\mathcal{C}} [I]_{\mathcal{B}}^{\mathcal{C}}$

$$[T]_{\mathcal{B}}^{\mathcal{B}} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1/2 & 1/2 & 1 \end{pmatrix}$$

Luego: $$[T]_{\mathcal{B}}^{\mathcal{B}} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1/2 & 1/2 & 1 \end{pmatrix}$$

e. $T(v) = T(1,4,5) = (1,0,1+5) = (1,0,6)$

$$T(v) = (1,0,6)$$

Mediante la matriz de cambio de base $[I]_{\mathcal{E}}^{\mathcal{B}}$ Tenemos:

$$[I]_{\mathcal{E}}^{\mathcal{B}} [T(v)]_{\mathcal{E}} = [T(v)]_{\mathcal{B}}$$

Como $T(v) = 1(1,0,0) + 0(0,1,0) + 6(0,0,1)$

$$[T(v)]_{\mathcal{E}} = [1,0,6]$$

Luego: $[T(v)]_{\mathcal{B}} = [I]_{\mathcal{E}}^{\mathcal{B}} [T(v)]_{\mathcal{E}}$

$$= \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$$

Es decir: $[T(v)]_{\mathcal{B}} = [-1,0,3]$

f. Calculamos el producto $[T]_{\mathcal{B}}^{\mathcal{B}} [v]_{\mathcal{B}}$

$$[T]_{\mathcal{B}}^{\mathcal{B}} [v]_{\mathcal{B}} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 5/2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 0 \\ -3/2 + 4/2 + 5/2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$$

Efectivamente se verifica la igualdad $[T]_{\mathcal{B}}^{\mathcal{B}} [v]_{\mathcal{B}} = [T(v)]_{\mathcal{B}}$