SENSOR DATA FUSION

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CONTENTS
Slide structure

INTRODUCTION IN DATA FUSION

INTRODUCTION IN DATA FUSION
Why ?

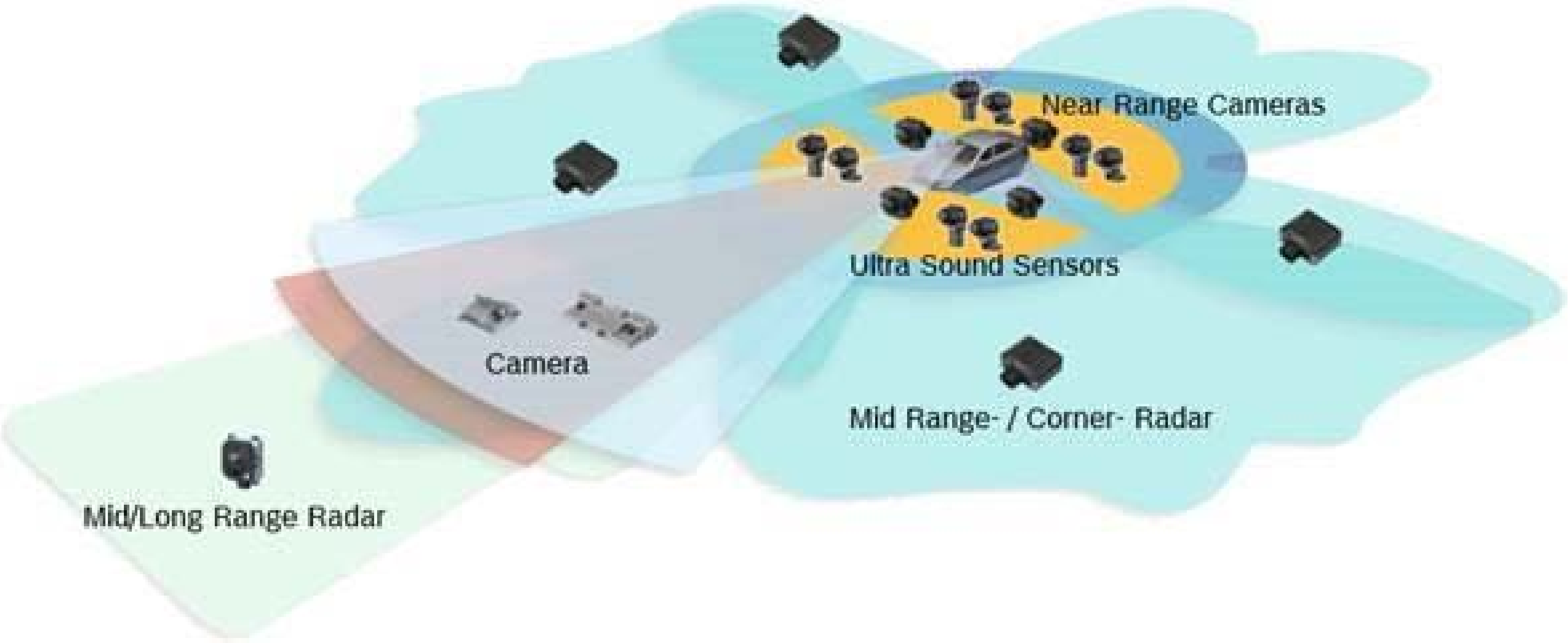
- Each sensor has its uncertainty (error in measurement)
  - i.e. GPS – give position with an accuracy of 3 meter

- Each sensor has its drawbacks
  - i.e. GPS – does not work well in tunnels

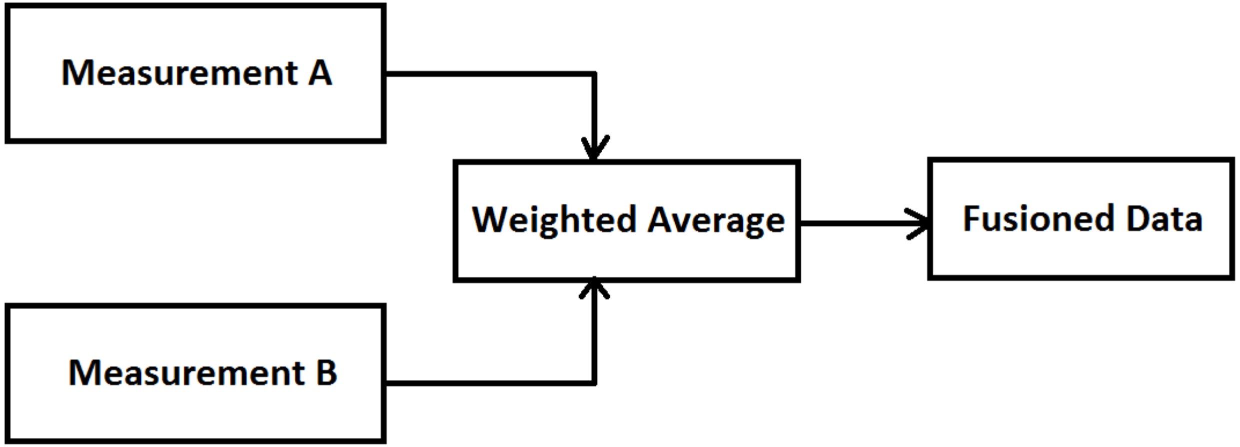
    IMU (inertial measurement unit)– accumulate error due to the integration process

- Each estimation method does not meet exactly the process
  - i.e. A theoretical model works perfect, but the process has variation due to mechanical issues

INTRODUCTION IN DATA FUSION
Why ?

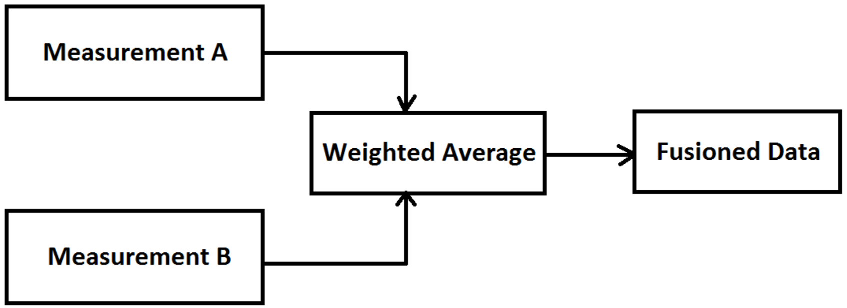


INTRODUCTION IN DATA FUSION
How ?



- Which are the weights ?
- Why we want this fusion ?

INTRODUCTION IN DATA FUSION
How ?



- Why we want this fusion ?

Uncertainty

Some “good” weights

- Which are the weights ?

Greater weight to better data

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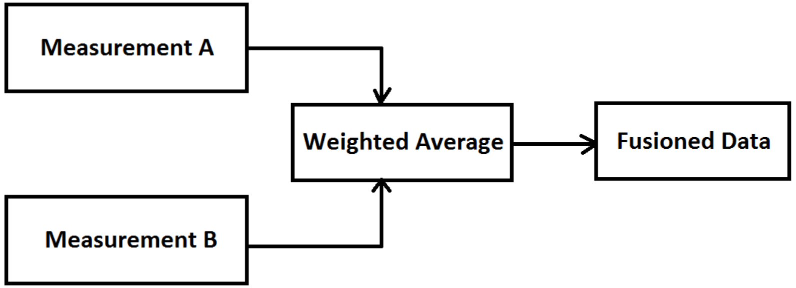
INTRODUCTION IN DATA FUSION
Example



- Estimated relative position
- Accumulated error

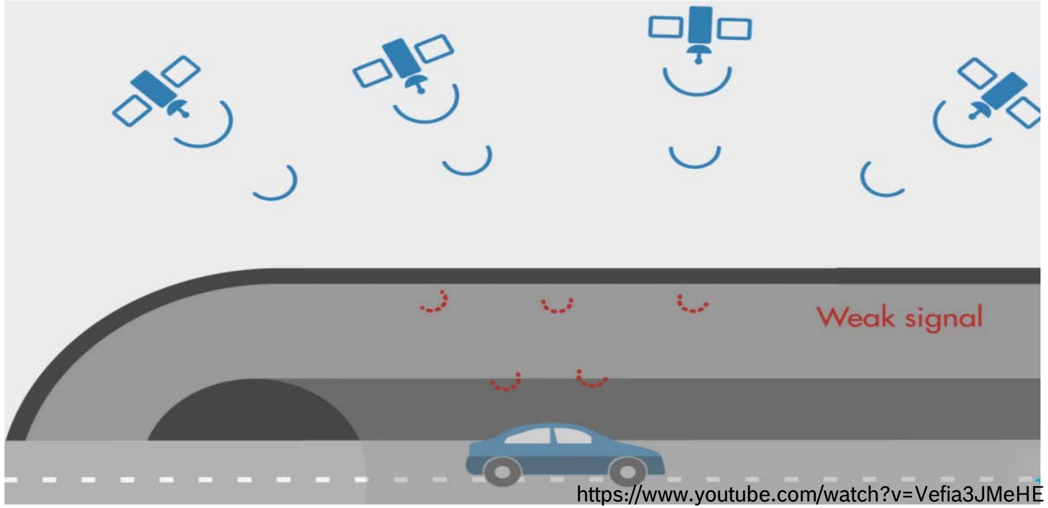
A

- Absolute position
- Noisy

B

Which one you trust ?

INTRODUCTION IN DATA FUSION
Example



Good GPS – use A only for correction, position is based on B

Bad GPS – use A exclusively to predict from the other sensors

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PRELIMINARY NOTIONS

Preliminary Notions
Random Variable

Def.: A function which link a random event with a number.

$$X : E \rightarrow R$$

Example: Roll two dices. One roll represents an event. The random number associated with this event is the sum of the number of dices. This sum is a random variable.

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Preliminary Notions
Expected Value

Take as random values the exam grades of all students from Control Engineering at math

$$E[X] = \frac{\sum_{i=1}^{N}(x_i)}{N}$$

What we expect to be the grade of a student?

Average of the students grades

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Preliminary Notions
Variance

Take as random values the exam grades of all students from Control Engineering at math

$$Var(X) = E[(\overline{x} - X)^2] = \frac{\sum_{i=1}^{N}(\overline{x} - x_i)^2}{N}$$

$$[Var] = [X]^2$$

How far from an average student are the best and the worst student

Preliminary Notions
Standard Deviation

Take as random values the exam grades of all students from Control Engineering at math

$$\sigma(X) = \sqrt{Var(X)}$$

$[\sigma] = [X]$

More convenient

How far from an average student are the best and the worst student

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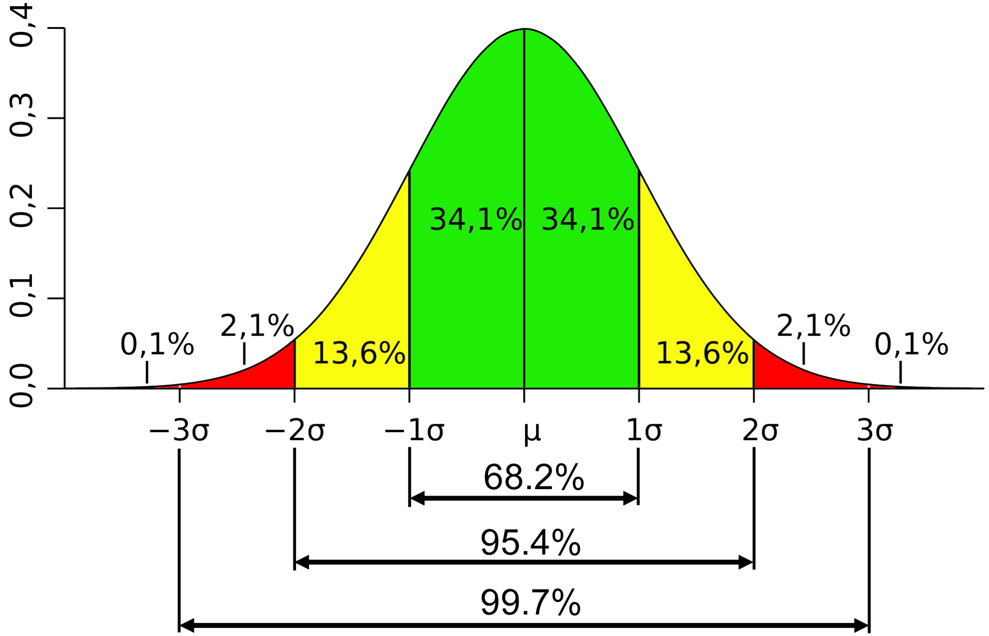
Preliminary Notions
Covariance

Take as random values the exam grades of all students from Control Engineering at math and English

$$\mathrm{Cov}(X,Y) = E[(\overline{x} - X)(\overline{y} - Y)] = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} (\overline{x}-x_i)(\overline{y}-y_i)}{N+M}$$

Express a linear dependence between two dimensions

Preliminary Notions
Gaussian Distribution



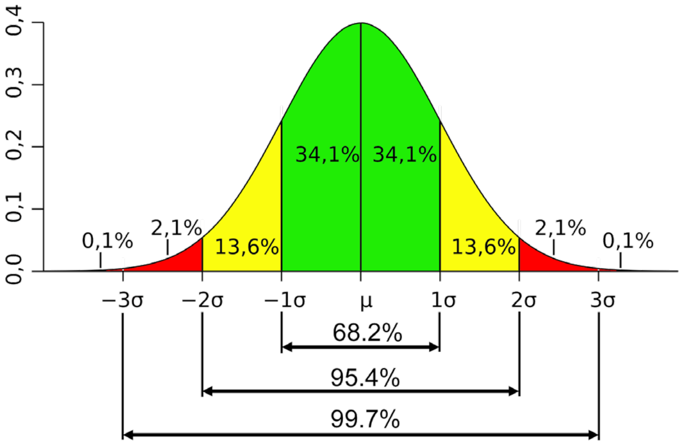
$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Probability density function

Take as random value the exam grade of all students from Control Engineering at math

How we interpret this graph ?

Preliminary Notions
Gaussian Distribution



$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X_f] = \mu$$

Grade of an average student

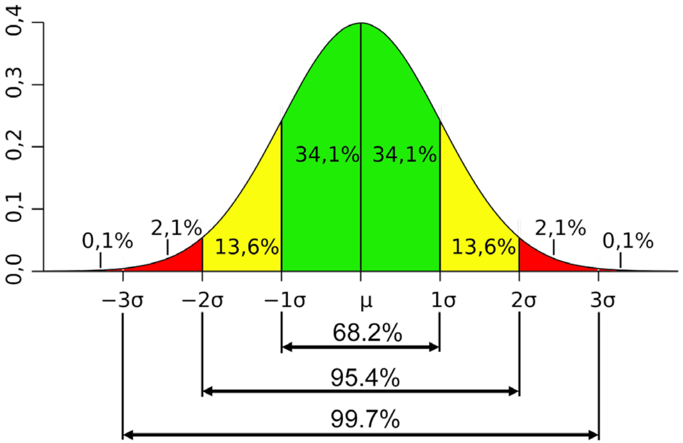
$$Var(X_f) = \sigma^2$$

How far away is the best and the worst student

Take as random value the exam grade of all students from Control Engineering at math

How we interpret this graph ?

Preliminary Notions
White Noise



$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X_f] = \mu = 0$$

We expect the noise to be zero

$$Var(X_f) = \sigma^2$$

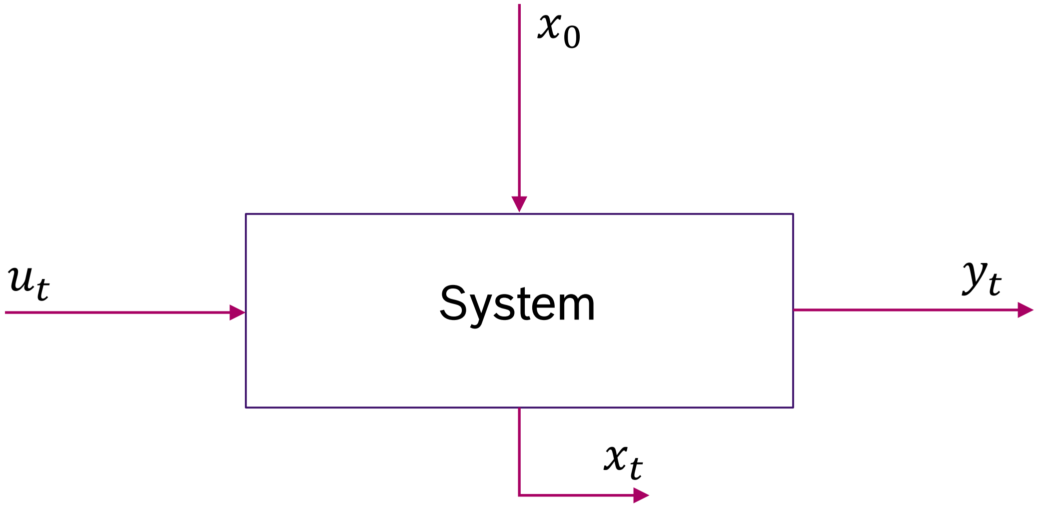
Depends on the signal source
⇔how distorted is the signal

A white noise is a deviation around the 0 which disturb the original signal

Example: Ground variation from an oscilloscope

SYSTEM MODEL

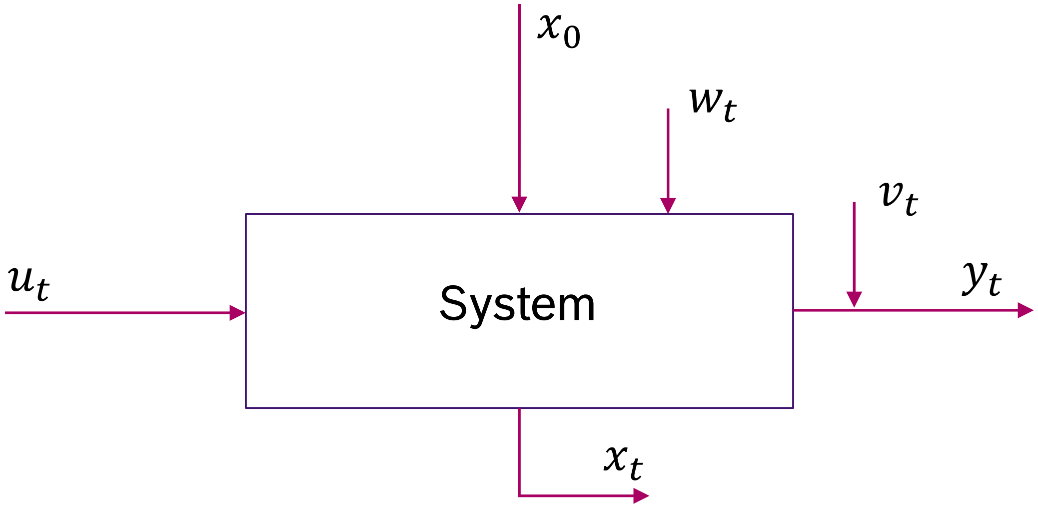
System Model
Introduction



$$x_{t+1} = A_t x_t + B_t u_t$$
$$y_t = H_t x_t$$

Image taken from www.quora.com

System Model
Noise on the System

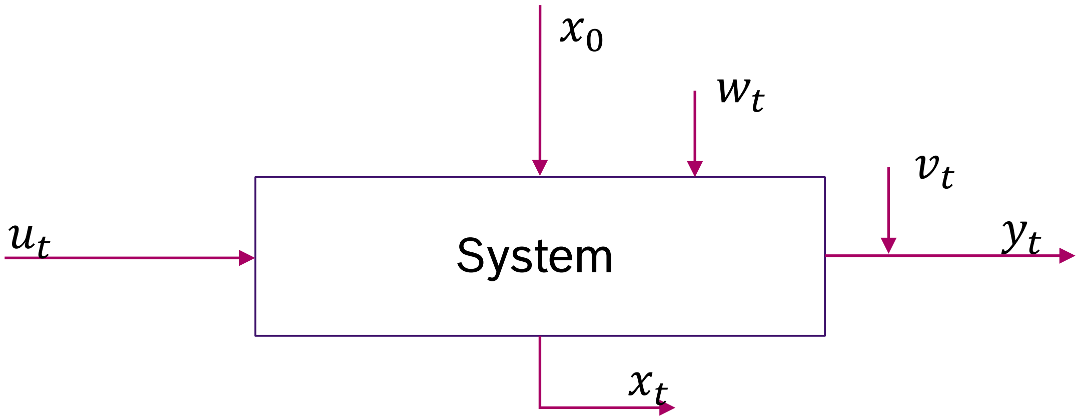


$$x_{t+1} = A_t x_t + B_t u_t + G w_t$$
$$y_t = H_t x_t + v_t$$

Image taken from www.quora.com

KALMAN FILTER

KALMAN FILTER
Initial asumptions



$v_t = measurement\ noise$
$w_t = process\ noise$

- v_t, w_t $- is\ assumed\ to\ be\ white\ noises$
- $v_t = N(0, R_t)$, R_t is covariance of measurement noise
- $w_t = N(0, Q_t), Q_t$ is covariance of process noise

Obs.: Process and measurement noise are uncorrelated

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KALMAN FILTER
Introduction

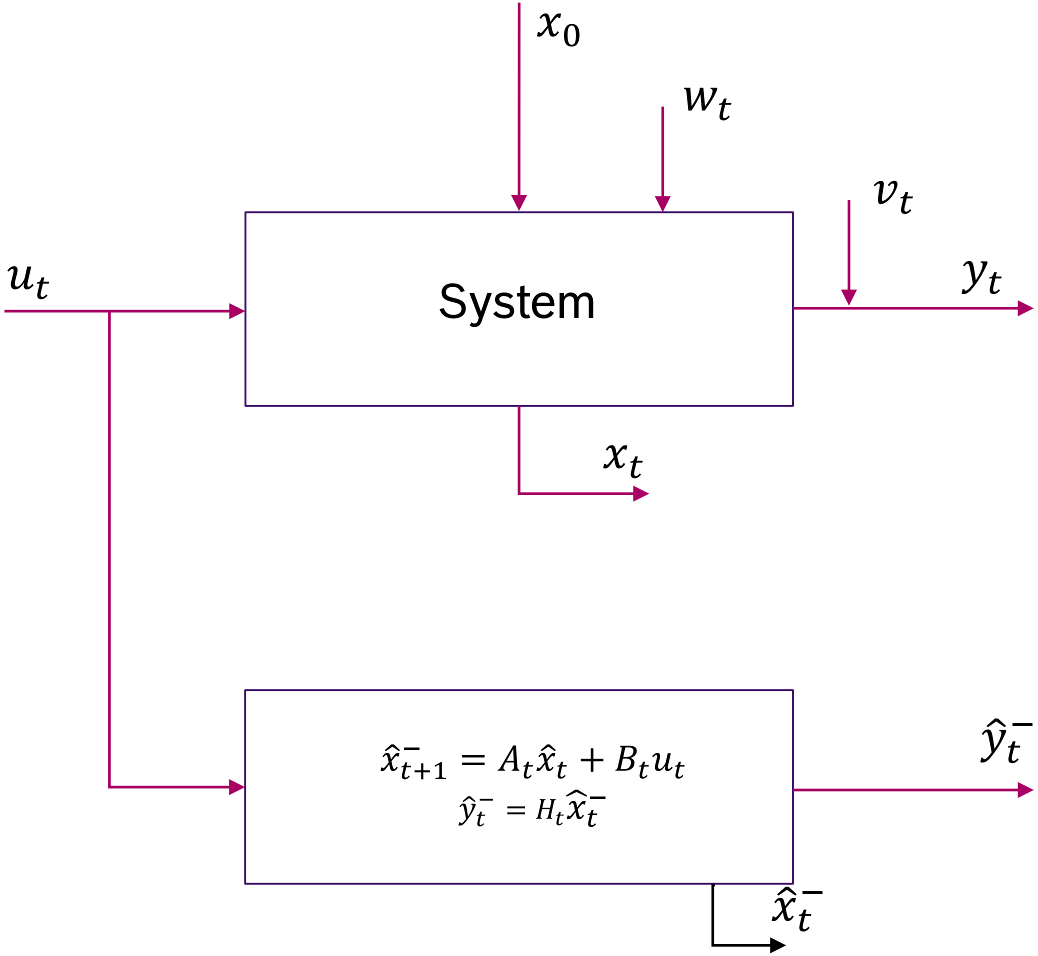
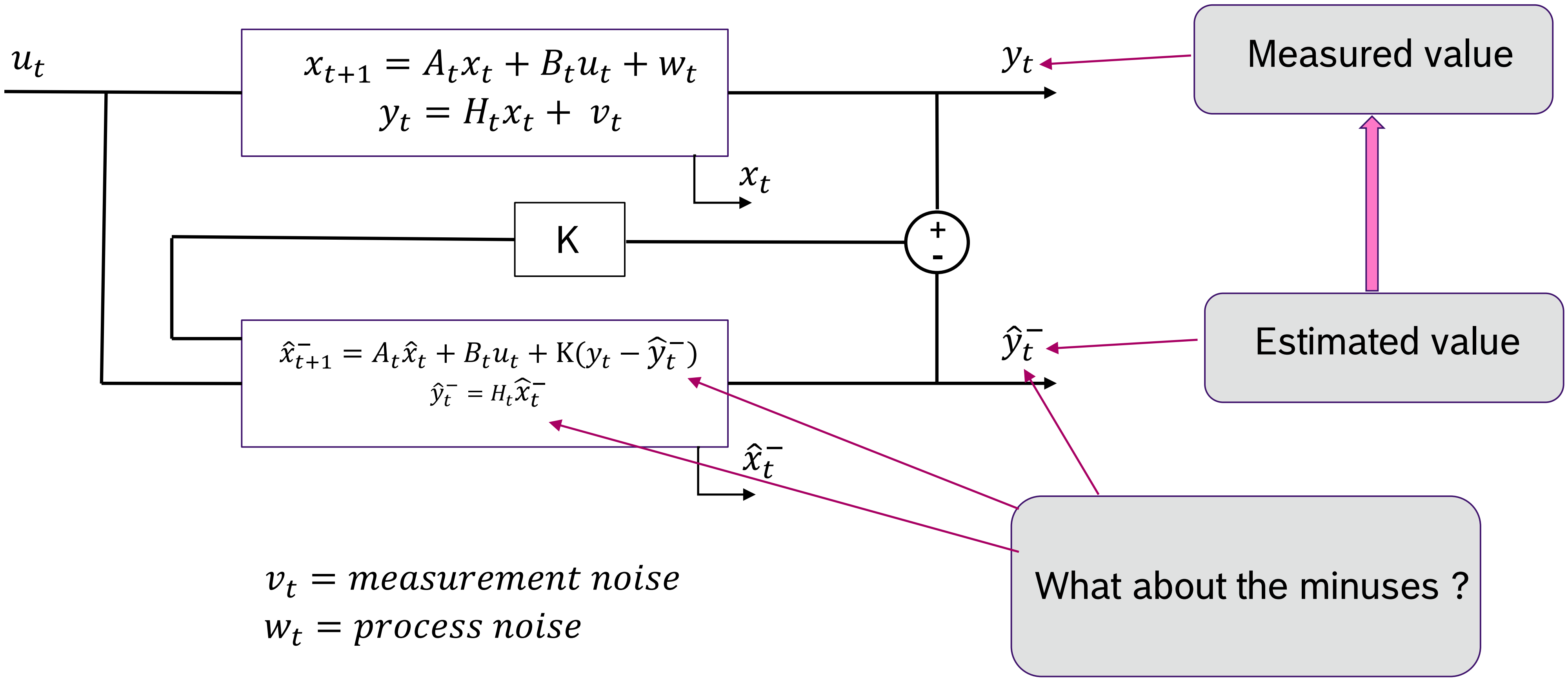


Image taken from www.quora.com

KALMAN FILTER



$v_t = measurement\ noise$

$w_t = process\ noise$

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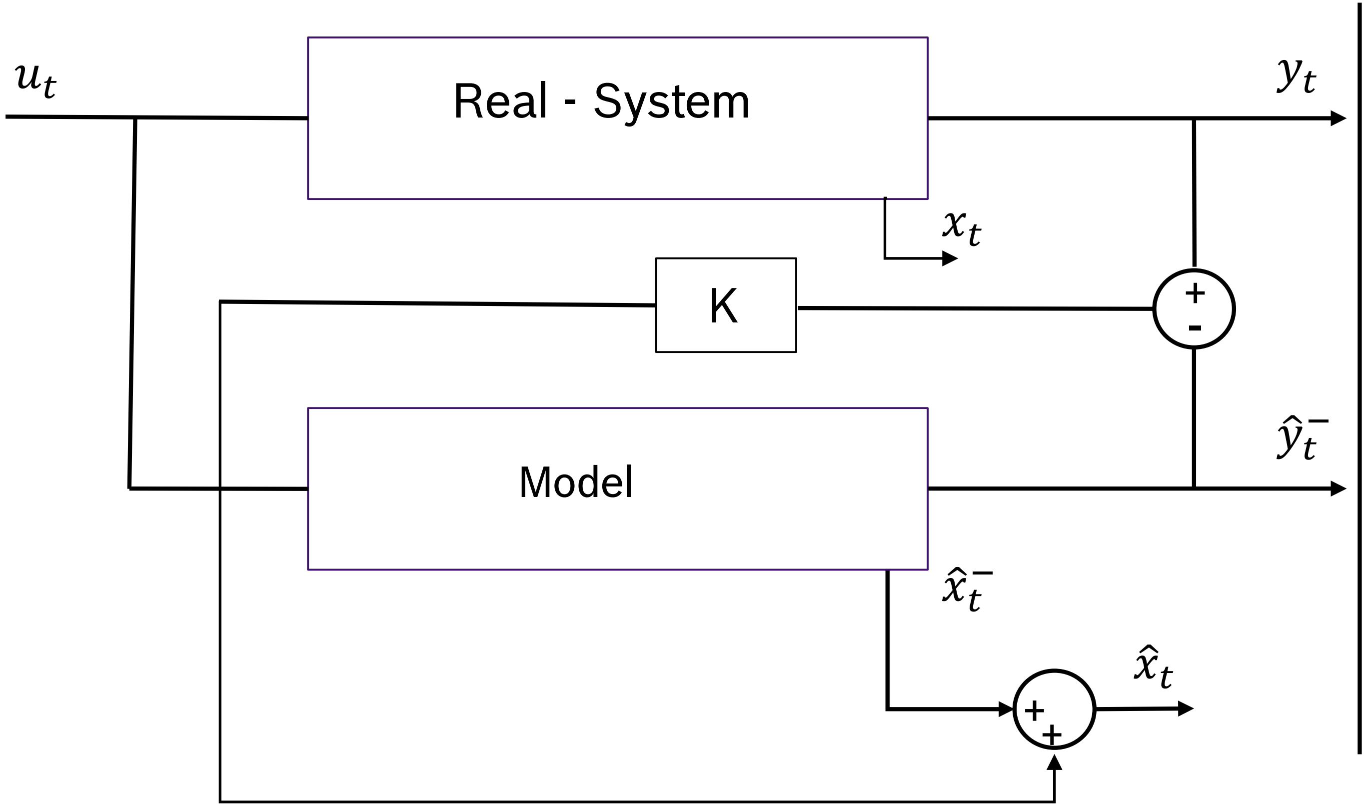
KALMAN FILTER



$$\hat{x}_t = \underbrace{A\hat{x}_{t-1} + Bu_t}_{\hat{x}_t^-} + K_t(y_t - H(\underbrace{A\hat{x}_{t-1} + Bu_t}_{\hat{x}_t^-}))$$

A priori Estimation

KALMAN FILTER

$$\hat{x}_t = A\hat{x}_{t-1} + Bu_t + K_t(y_t - H(A\hat{x}_{t-1} + Bu_t))$$

$$\hat{x}_t = \hat{x}_t^- + K_t(y_t - H\hat{x}_t^-)$$

A posteriori State Estimation

A priori State Estimation

Correction via Kalman Gain

KALMAN FILTER
Kalman Gain

- A posteriori estimation error: $\varepsilon = x_t - \widehat{x_t}$

Is a stochastic variable due to process noise w_t

- Take the expected value of a posteriori error as an estimation's measure

$$f(K) = E[\varepsilon^T \varepsilon]$$

KALMAN FILTER
Kalman Gain

- Take the expected value of a posteriori error as an estimation measure

$$f(K) = E[\varepsilon^T \varepsilon]$$

- Minimizing f with respect to K will give us the best K

$$\frac{df}{dK} = 0 \Longrightarrow K_t = \frac{P_t^- H^T}{H P_t^- H^T + R_t},$$ where P_t^- is a priori error covariance

$$P_t^- = \mathrm{E}[(x_t - \hat{x}_t^-)^T (x_t - \hat{x}_t^-)]$$

KALMAN FILTER
Updating the error covariance

- Starting from the definition of the a posteriori error covariance

$$P_t = E[\varepsilon^T \varepsilon]$$

- Expanding the above equation by using the model of a posteriori estimation we obtain that

$$P_t = (I - K_t H)P_t^-$$

KALMAN FILTER

Estimation of a priori error covariance

- As we estimate a priori the state of the system, we must estimate the a priori error covariance in order to compute the Kalman gain
- Starting from the definition of the a priori error covariance

$$P_t^- = \mathrm{E}[(x_t - \hat{x}_t^-)^T (x_t - \hat{x}_t^-)]$$

- By using the system transition model and considering the expected values of the independent signals to be 0 we obtain the a priori error covariance

$$P_t^- = \mathrm{A}P_{t-1}A^T + Q_t$$

KALMAN FILTER
Summary

Prediction

$$\hat{x}_t^- = A\hat{x}_{t-1} + Bu_{t-1}$$

$$P_t^- = AP_{t-1}A^T + Q_t$$

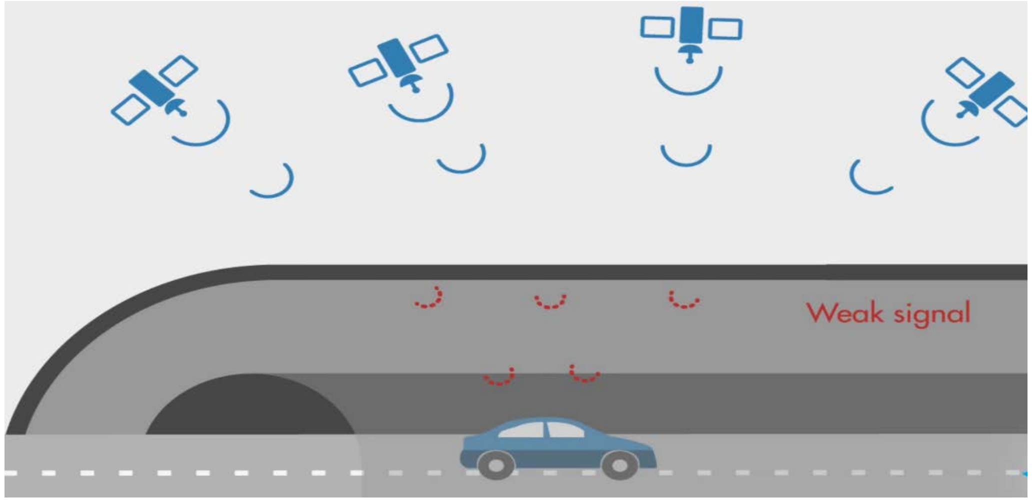
Update

$$\hat{x}_t = \hat{x}_t^- + K_t(y_t - H\hat{x}_t^-)$$

$$K_t = \frac{P_t^- H^T}{HP_t^- H^T + R_t}$$

$$P_t = (I - K_t H)P_t^-$$

KALMAN FILTER
Vehicle tracking



- Assume that the trajectory is a line
- You know the velocity of the car as input to the system
- You measure the position of the car via GPS system

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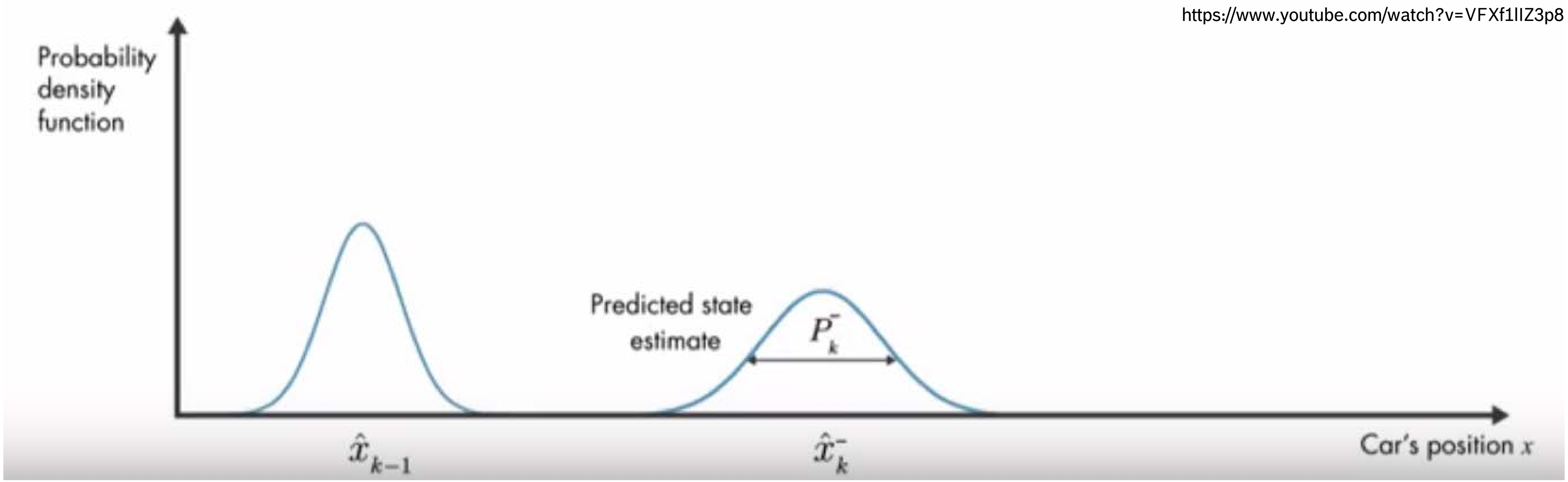
Vehicle tracking - solution

https://www.youtube.com/watch?v=VFXf1lIZ3p8



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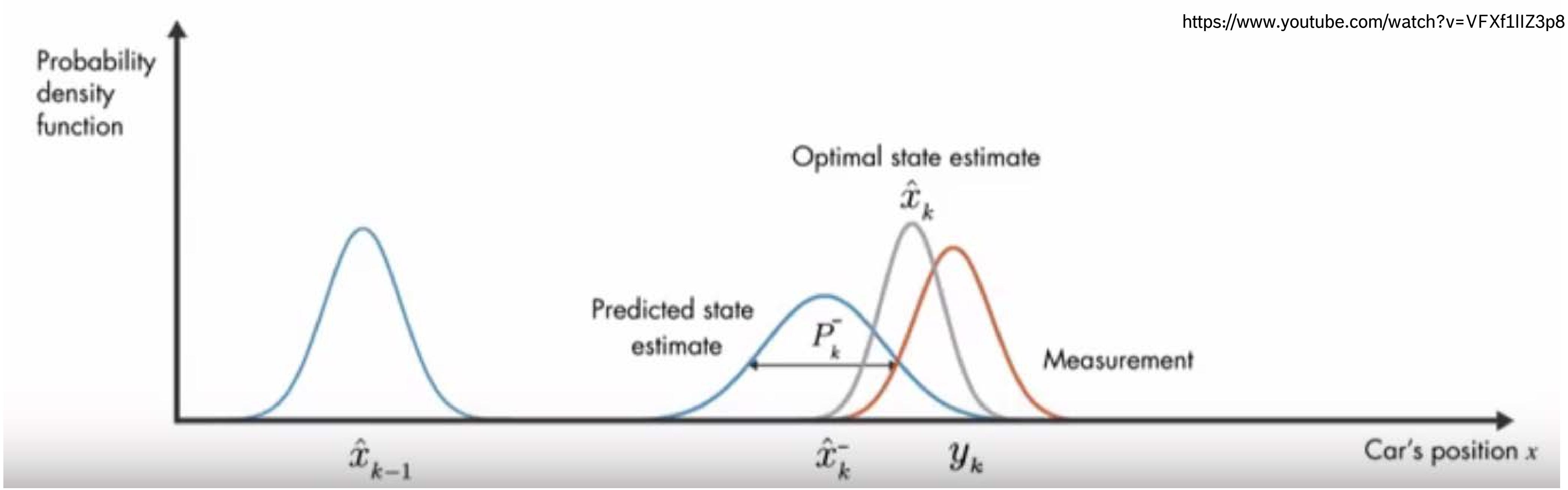

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KALMAN FILTER
Vehicle tracking - solution

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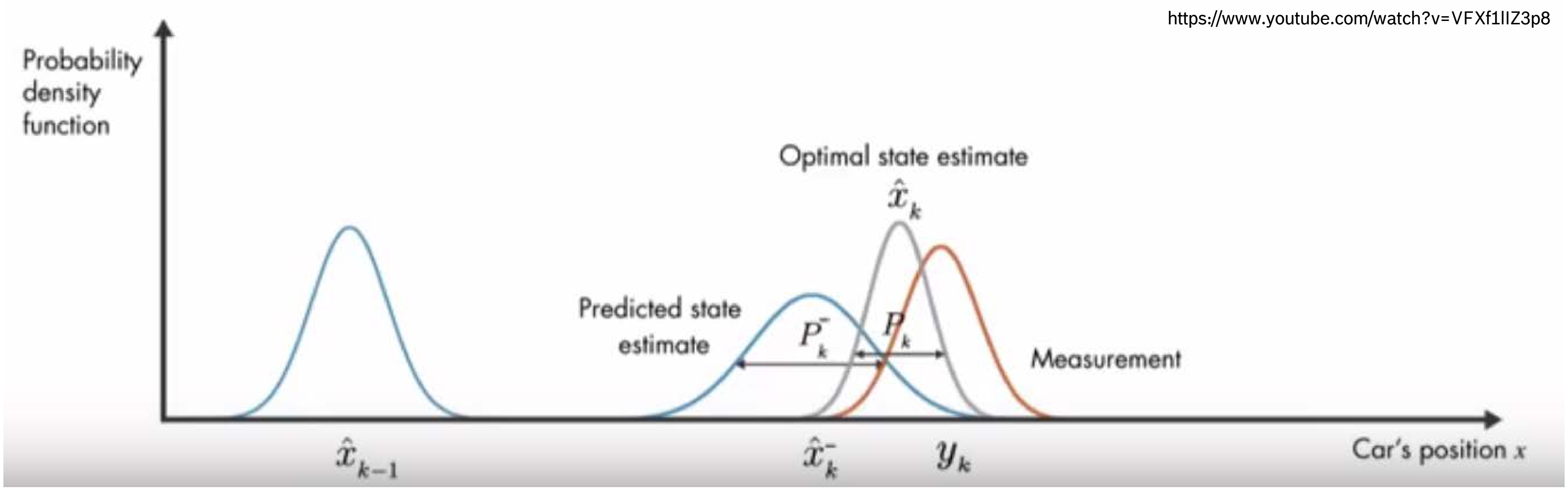

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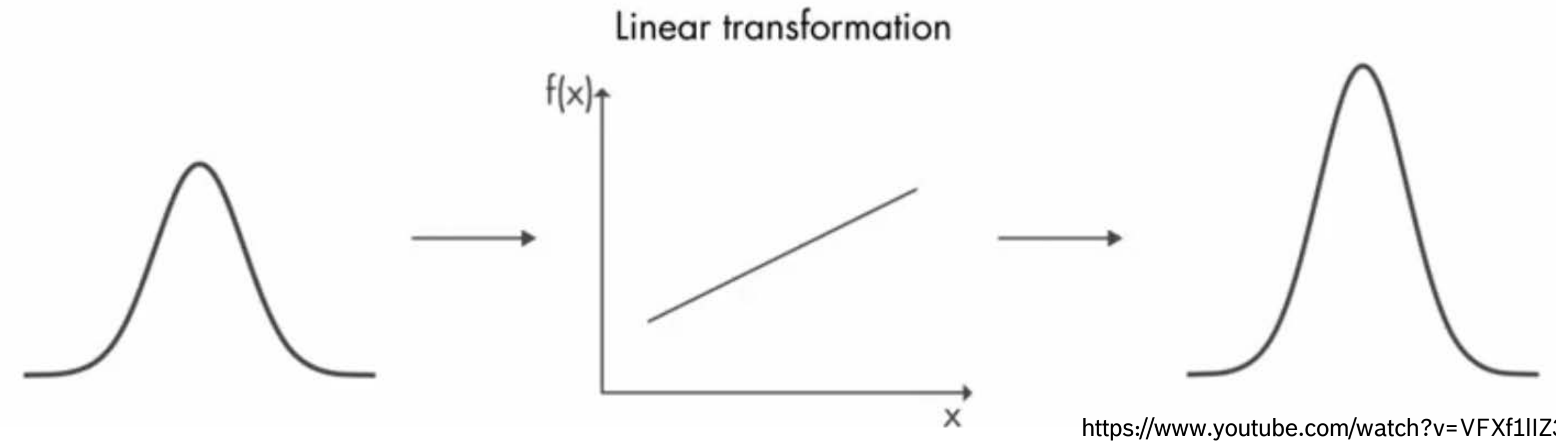
KALMAN FILTER
Principal drawback

- Kalman filter assume that the model of the system is linear

$$x_{t+1} = A_t x_t + B_t u_t$$
$$y_t = H_t x_t$$

- If we map an Gaussian distribution with a linear model we obtain another Gaussian distribution



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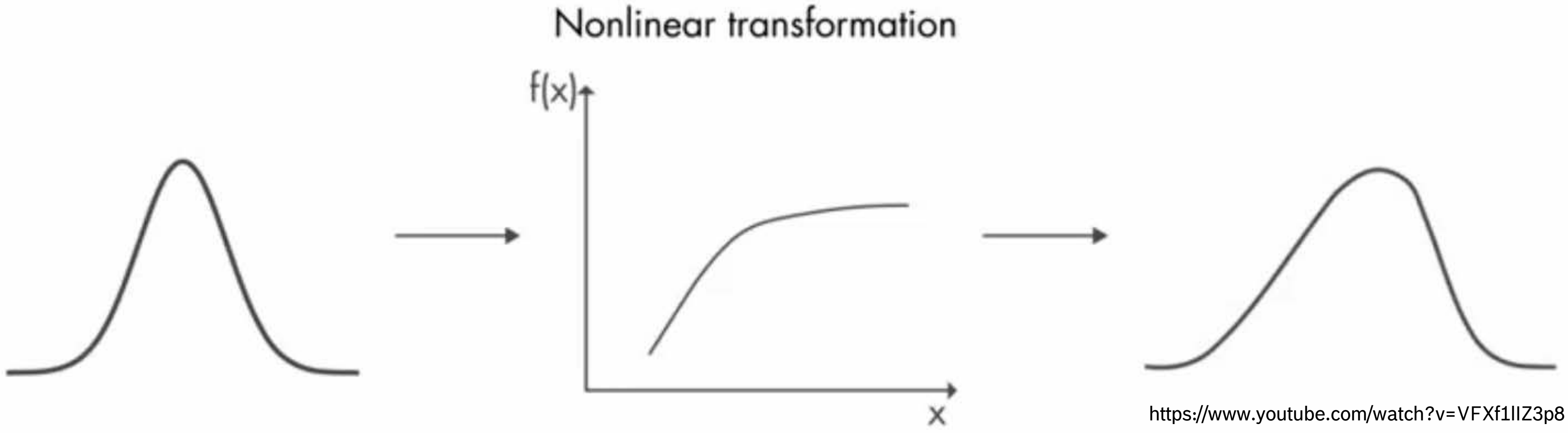
KALMAN FILTER
Principal drawback

- If model is nonlinear the Gaussian distribution is distorted

$$x_{t+1} = f(x_t, u_t)$$
$$y_t = g(x_t)$$



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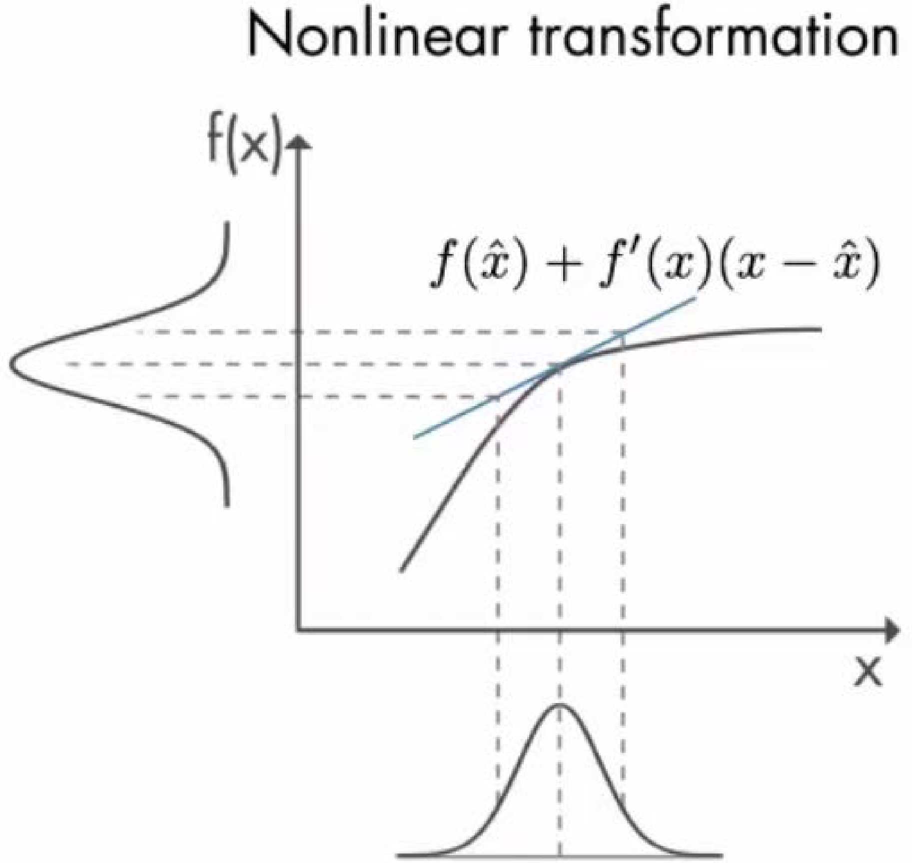
EXTENDED KALMAN FILTER

EXTENDED KALMAN FILTER
Model linearization



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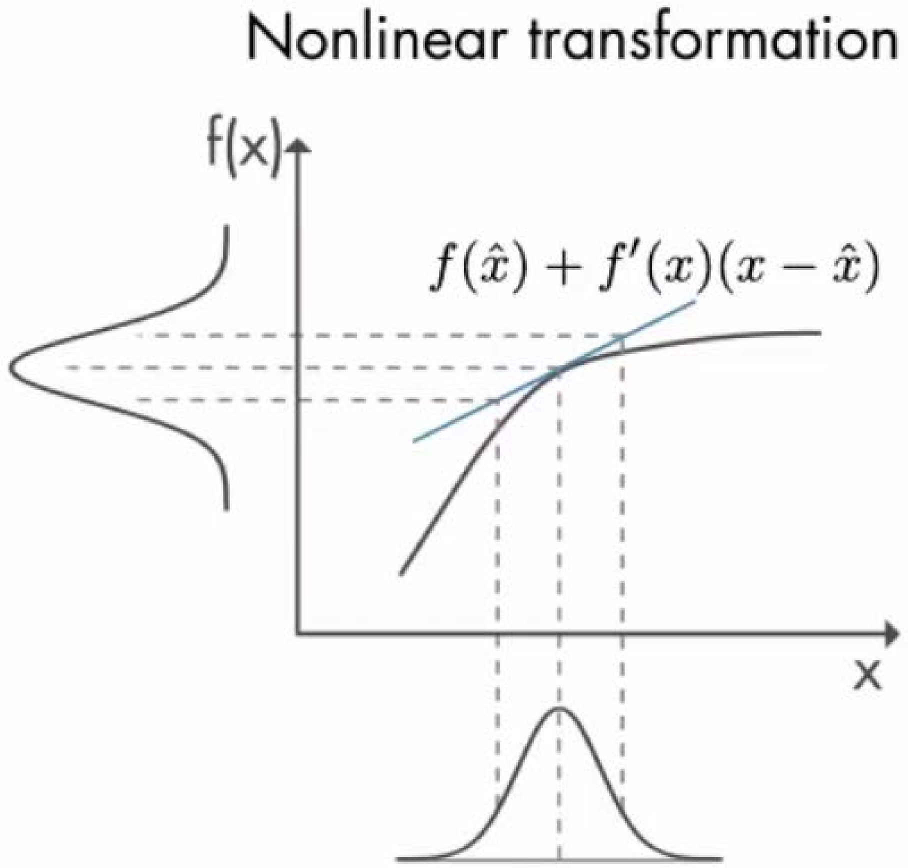
EXTENDED KALMAN FILTER
Model linearization



https://www.youtube.com/watch?v=VFXf1lIZ3p8

$$f(x) = f(x_k) + \frac{\partial f}{\partial x}(x_k - x)$$

For multidimensional function with multivariable input, here we have the Jacobian matrix

EXTENDED KALMAN FILTER
Model linearization

$$x_{t+1} = f(x_t, u_t)$$
$$y_t = g(x_t)$$

$$\Rightarrow \begin{cases} F_K = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{k-1},\hat{u}_k} \\ G_K = \frac{\partial g}{\partial x}\Big|_{\hat{x}_k} \end{cases}$$

Jacobians

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EXTENDED KALMAN FILTER
Jacobian Matrix

$$\begin{cases} F_K = \dfrac{\partial f}{\partial x}\Big|_{\hat{x}_{k-1},\hat{u}_k} \\ \\ G_K = \dfrac{\partial g}{\partial x}\Big|_{\hat{x}_k} \end{cases}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix}$$

EXTENDED KALMAN FILTER

Modified Kalman Algorithm

Prediction

$$\hat{x}_t^- = f(\hat{x}_{t-1}, u_{t-1})$$

$$P_t^- = F P_{t-1} F^T + Q_t$$

Update

$$\hat{x}_t = \hat{x}_t^- + K_t(y_t - h(\hat{x}_t^-))$$

$$K_t = \frac{P_t^- G^T}{G P_t^- G^T + R_t}$$

$$P_t = (I - K_t G) P_t^-$$

At each step, Jacobians must be recomputed !!

EXTENDED KALMAN FILTER
Modified Kalman Algorithm - drawbacks

- Jacobian matrix is difficult to compute analytically
- Numerical computation of Jacobian has a high computational cost
- If system has hard nonlinear parts the first order Taylor approximation fails
- If system has hard nonlinear parts, the kalman filter is not an optimal approach

References

- https://ocw.mit.edu/courses/mechanical-engineering/2-160-identification-estimation-and-learning-spring-2006/lecture-notes/lecture_5.pdf
- https://ocw.mit.edu/courses/mechanical-engineering/2-160-identification-estimation-and-learning-spring-2006/lecture-notes/lecture_6.pdf
- https://ocw.mit.edu/courses/mechanical-engineering/2-160-identification-estimation-and-learning-spring-2006/lecture-notes/lecture_8.pdf

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