Categories in Proofgold

Blake Keiller

Draft of July 11, 2021

Contents

Chapter 1

Introduction

Proofgold is a peer to peer network for publishing formalized mathematics. The corresponding cryptocurrency is Proofgold bars. For the first half year (5000 blocks) half of each block reward (25 bars) was placed as a bounty on a randomly generated proposition. The idea is that users who join the network later can obtain this reward by proving the proposition or its negation. Most of these 125000 bars are still bounties on the generated propositions.

There were various problems with the random generation of propositions. One problem is simply that they are generally mathematically uninteresting.

In a hard fork in December 2020 the system was changed to place the 25 bars in a reward bounty fund. These could then be manually placed on more interesting propositions. The contribution to the reward bounty fund will continue until block 15000 (which should be created early in 2022).

The first roughly 25000 bars from the reward bounty fund were placed as bounties on some meaningful mathematical propositions, e.g., various instances of Ramsey's Theorem. However, it was suggested that an alternative would be to choose propositions that are part of a larger formalization program.

Proofgold is a fork of an earlier network called Dalilcoin. In Dalilcoin a large number of bounties were placed on Category Theory propositions as described in the documentation available at

```
https://github.com/aliibrahim80/
         dalilcoin/blob/master/doc/publishingformalmathematics.md
```

Later a user identified a number of bugs in the formulation of some of the "categories," along with providing a more readable formulation of the definitions and propositions. These were available on the dalilcoin.com forum, which is apparently defunct. The relevant post is available on archive.org:

```
https://web.archive.org/web/20200815133648/
     https://dalilcoin.com/forum/viewtopic.php?pid=598#p598
```

In any case, the formulations in Dalilcoin cannot directly be used in Proofgold since they make make use of polymorphism. To make Proofgold versions, the polymorphism has been eliminated by essentially assigning every type variable to the base type of sets. As a consequence, each object and each arrow of a (meta) category must now be represented by a set. The polymorphism was used to encode various structures to form categories (though this is where the most important bugs were identified).[1] Instead of relying polymorphism we instead encode structures as sets using the tools provided by Megalodon (see Section 2.15).

The relevant Category Theory definitions and results already published into the Proofgold blockchain are outlined in Chapter 2. The remaining chapters mostly consist of conjectured propositions on which bounties from the reward bounty fund have been placed. After each conjecture the address of the conjecture and the approximate amount of the bounty are given. (Due to transaction fees, the real bounty is slightly less than the approximation given.)

Megalodon and Proofgold files corresponding to the information in this report are available from here:

`https://proofgold.org/catfilesJuly2021.tgz`

[1] During the process of translating from Dalilcoin to Proofgold, more bugs in some of the definitions were found. For example, there was a missing condition in the definition of equalizers. One good reason to use these Category Theory propositions for bounties is simply that they have been around in one form or another for years and have been looked at by different people. Hopefully there are relatively few bugs left, if any.

Chapter 2

Basics

We will not give a full introduction to Category Theory here. The reader can find many accessible introductions freely available if one is needed. Our purpose is to relate the formal Proofgold/Megalodon definitions and previously proven propositions to the usual informal presentations.

A category is specified by giving four mathematical objects:

- $Obj : \iota \to o$ – a predicate carving out the objects of the category. Each object is represented by a set (i.e., something of type ι). Note that this may correspond to a proper class. $Obj\ X$ is true iff X is an object of the category.

- $Hom : \iota \to \iota \to \iota \to o$ – a predicate recognizing the arrows from one object to another. Again, each arrow is represented by a set. $Hom\ X\ Y\ f$ is true if f is an arrow from X to Y. We will sometimes abbreviate this proposition by $f : X \to Y$.

- $id : \iota \to \iota$ – a function taking an object X to the identity arrow $id\ X : X \to X$.

- $comp : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$ – a function taking three objects X, Y and Z, and two arrows $g : Y \to Z$ and $f : X \to Y$ and returns a composed arrow $comp\ X\ Y\ Z\ g\ f : X \to Z$. We will often write $comp\ X\ Y\ Z\ g\ f$ simply as $g \circ f$.

  We first define two propositions giving "typing" information for the arrows returned by *id* and *comp*.

  Definition 1. *We define* `idT` *to be*

  $$\forall X : \iota . Obj\ X \to Hom\ X\ X\ (id\ X)$$

  of type o.

  Definition 2. *We define* `compT` *to be*

  $$\forall X, Y, Z : \iota . \forall f, g : \iota . Obj\ X \to Obj\ Y \to Obj\ Z \to Hom\ X\ Y\ f \to Hom\ Y\ Z\ g \\ \to Hom\ X\ Z\ (comp\ X\ Y\ Z\ g\ f)$$

  of type o.

We next define three propositions giving the equations that should be satisfied by arrows in a category. In summary, identity arrows should be two-sided identities relative to composition and composition should be associative.

Definition 3. *We define* `idL` *to be*

$$\forall X, Y : \iota. \forall f : \iota. Obj\ X \to Obj\ Y \to Hom\ X\ Y\ f \to comp\ X\ X\ Y\ f\ (id\ X) = f$$

of type o.

Definition 4. *We define* `idR` *to be*

$$\forall X, Y : \iota. \forall f : \iota. Obj\ X \to Obj\ Y \to Hom\ X\ Y\ f \to comp\ X\ Y\ Y\ (id\ Y)\ f = f$$

of type o.

Definition 5. *We define* `compAssoc` *to be*

$$\begin{gathered}\forall X, Y, Z, W : \iota. \forall f, g, h : \iota. Obj\ X \to Obj\ Y \to Obj\ Z \to Obj\ W \\ \to Hom\ X\ Y\ f \to Hom\ Y\ Z\ g \to Hom\ Z\ W\ h \\ \to comp\ X\ Y\ W\ (comp\ Y\ Z\ W\ h\ g)\ f = comp\ X\ Z\ W\ h\ (comp\ X\ Y\ Z\ g\ f)\end{gathered}$$

of type o.

We now define a *metacategory* (leaving "category" for later to refer to sets that encode small metacategories) to be a relation on the four objects. It is defined to hold if all of the five properties defined above hold.

Definition 6. *We define* `MetaCat` *to be*

$$(\mathtt{idT} \wedge \mathtt{compT}) \wedge (\mathtt{idL} \wedge \mathtt{idR}) \wedge \mathtt{compAssoc}$$

of type o.

Working with multiple conjunctions is sometimes tedious, so we prove an introduction and elimination principle for `MetaCat`.

Theorem 1. *[*`MetaCat_I`*]*

$$\begin{gathered}\mathtt{idT} \to \mathtt{compT} \\ \to (\forall X, Y : \iota. \forall f : \iota. Obj\ X \to Obj\ Y \to Hom\ X\ Y\ f \to comp\ X\ X\ Y\ f\ (id\ X) = f) \\ \to (\forall X, Y : \iota. \forall f : \iota. Obj\ X \to Obj\ Y \to Hom\ X\ Y\ f \to comp\ X\ Y\ Y\ (id\ Y)\ f = f) \\ \to (\forall X, Y, Z, W : \iota. \forall f, g, h : \iota. Obj\ X \to Obj\ Y \to Obj\ Z \to Obj\ W \\ \to Hom\ X\ Y\ f \to Hom\ Y\ Z\ g \to Hom\ Z\ W\ h \\ \to comp\ X\ Y\ W\ (comp\ Y\ Z\ W\ h\ g)\ f = comp\ X\ Z\ W\ h\ (comp\ X\ Y\ Z\ g\ f)) \\ \to \mathtt{MetaCat}.\end{gathered}$$

Proof. The formal proof proceeds by assuming the 5 properties, then using variants of conjunction elimination (`andI` and `and3I`) to prove the conjunction. The definition of `MetaCat` is expanded by writing the expanded statement using the `prove` tactic. Here is the Megalodon proof:

```
assume H1 H2 H3 H4 H5.
prove (idT /\ compT) /\ (idL /\ idR) /\ compAssoc.
apply and3I.
- apply andI.
  + exact H1.
  + exact H2.
- apply andI.
  + exact H3.
  + exact H4.
- exact H5.
```

□

The elimination principle states that if we know `MetaCat` holds (for four objects we are currently leaving implicit), then we can prove any proposition p if we can prove p under the five extra assumptions given in the definition of `MetaCat`.

Theorem 2. *[*`MetaCat_E`*]*

$$\begin{gathered}\texttt{MetaCat}{\to}\forall p:o.\\ (\texttt{idT}{\to}\texttt{compT}\\ {\to}(\forall X,Y:\iota.\forall f:\iota.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f{\to}comp\ X\ X\ Y\ f\ (id\ X){=}f)\\ {\to}(\forall X,Y:\iota.\forall f:\iota.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f{\to}comp\ X\ Y\ Y\ (id\ Y)\ f{=}f)\\ {\to}(\forall X,Y,Z,W:\iota.\forall f,g,h:\iota.Obj\ X{\to}Obj\ Y{\to}Obj\ Z{\to}Obj\ W\\ {\to}Hom\ X\ Y\ f{\to}Hom\ Y\ Z\ g{\to}Hom\ Z\ W\ h\\ {\to}comp\ X\ Y\ W\ (comp\ Y\ Z\ W\ h\ g)\ f{=}comp\ X\ Z\ W\ h\ (comp\ X\ Y\ Z\ g\ f))\\ {\to}p)\\ {\to}p.\end{gathered}$$

Proof. In this case the definition of conjunction $A \wedge B$ as

$$\forall p : prop.(A \to B \to p) \to P$$

allows us to prove this by repeatedly applying assumed conjunctions. to obtain the conjuncts as extra assumptions. We start by assuming `MetaCat`, letting the proposition p be given and assuming we can prove p from the five assumptions. We then use apply on the assumed components of the definition of `MetaCat` until we have all five parts as assumptions and can complete the proof. Here is the Megalodon proof.

```
assume HC. let p. assume Hp.
apply HC. assume H14 H5. apply H14. assume H12 H34.
apply H12. assume H1 H2.
apply H34. assume H3 H4.
exact Hp H1 H2 H3 H4 H5.
```

□

2.1 Opposite Metacategory

Given a metacategory, we also have the opposite metacategory given by reversing the arrows

Theorem 3. *[*`MetaCatOp`*]*

$$\texttt{MetaCat}\ Obj\ Hom\ id\ comp$$
$$\to\texttt{MetaCat}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f).$$

Proof. Use Theorems 2 to split the assumption into the five properties. Apply Theorems 1 to split the conclusion into five subgoals. Each of these five subgoals is easy to prove from one of the five assumptions. □

2.2 Monics

A *monic* is an arrow $f : X \to Y$ such that $f \circ g = f \circ h$ implies $g = h$ for all appropriate arrows $g, h : Z \to X$.

Definition 7. *We define* `monic` *to be*

$$\lambda X, Y, f.Obj\ X \wedge Obj\ Y \wedge Hom\ X\ Y\ f$$
$$\wedge \forall Z : \iota.Obj\ Z \to \forall g, h : \iota.Hom\ Z\ X\ g \to Hom\ Z\ X\ h$$
$$\to comp\ Z\ X\ Y\ f\ g = comp\ Z\ X\ Y\ f\ h \to g = h$$

of type $\iota\to\iota\to\iota\to o$.

2.3 Limits and Colimits

An object Y is *terminal* if for every object X there is a unique arrow $h : X \to Y$.

Definition 8. *We define* `terminal_p` *to be*

$$\lambda Y, h.Obj\ Y \wedge \forall X : \iota.Obj\ X \to Hom\ X\ Y\ (h\ X) \wedge \forall h' : \iota.Hom\ X\ Y\ h' \to h' = h\ X$$

of type $\iota\to(\iota\to\iota)\to o$.

An object Y is *initial* if for every object X there is a unique arrow $h : Y \to X$.

Definition 9. *We define* `initial_p` *to be*

$$\lambda Y, h.Obj\ Y \wedge \forall X : \iota.Obj\ X \to Hom\ Y\ X\ (h\ X) \wedge \forall h' : \iota.Hom\ Y\ X\ h' \to h' = h\ X$$

of type $\iota\to(\iota\to\iota)\to o$.

Given two objects X and Y, a *product* of X and Y is given by four mathematical objects

– $Z : \iota$

- $\pi_0 : \iota$
- $\pi_1 : \iota$
- $pair : \iota \to \iota \to \iota \to \iota$

such that Z is an object, $\pi_0 : Z \to X$, $\pi_1 : Z \to Y$ and for all appropriate $h : W \to X$ and $k : W \to Y$ $pair\ W\ h\ k$ is the unique arrow $W \to Z$ such that $\pi_0 \circ pair = h$ and $\pi_1 \circ pair = k$.

Definition 10. *We define* `product_p` *to be*

$$\begin{gathered}\lambda X, Y, Z, \pi_0, \pi_1, pair. Obj\ X {\wedge} Obj\ Y {\wedge} Obj\ Z {\wedge} Hom\ Z\ X\ \pi_0 {\wedge} Hom\ Z\ Y\ \pi_1 \\ \wedge \forall W : \iota. Obj\ W {\to} \forall h, k : \iota. Hom\ W\ X\ h {\to} Hom\ W\ Y\ k \\ \to Hom\ W\ Z\ (pair\ W\ h\ k) {\wedge} comp\ W\ Z\ X\ \pi_0\ (pair\ W\ h\ k){=}h {\wedge} comp\ W\ Z\ Y\ \pi_1\ (pair\ W\ h\ k){=}k \\ \wedge \forall u : \iota. Hom\ W\ Z\ u {\to} comp\ W\ Z\ X\ \pi_0\ u{=}h {\to} comp\ W\ Z\ Y\ \pi_1\ u{=}k {\to} u{=}pair\ W\ h\ k\end{gathered}$$

of type $\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}(\iota{\to}\iota{\to}\iota{\to}\iota){\to}o$*.*

A *product constructor* is given by four mathematical objects

- $prod : \iota \to \iota \to \iota$
- $\pi_0 : \iota \to \iota \to \iota$
- $\pi_1 : \iota \to \iota \to \iota$
- $pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that $prod\ X\ Y$, $\pi_0\ X\ Y$, $\pi_1\ X\ Y$ and $pair\ X\ Y$ give a product of X and Y for all objects X and Y.

Definition 11. *We define* `product_constr_p` *to be*

$$\begin{gathered}\lambda prod, \pi_0, \pi_1, pair. \forall X, Y : \iota. Obj\ X {\to} Obj\ Y \\ \to \texttt{product\_p}\ X\ Y\ (prod\ X\ Y)\ (\pi_0\ X\ Y)\ (\pi_1\ X\ Y)\ (pair\ X\ Y)\end{gathered}$$

of type $(\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}o$*.*

Given two objects X and Y, a *coproduct* of X and Y is given by four mathematical objects

- $Z : \iota$
- $i_0 : \iota$
- $i_1 : \iota$
- $comb : \iota \to \iota \to \iota \to \iota$

such that Z is an object, $i_0 : X \to Z$, $i_1 : Y \to Z$ and for all appropriate $h : X \to W$ and $k : Y \to W$ $comb\ W\ h\ k$ is the unique arrow $Z \to W$ such that $comb \circ i_0 = h$ and $comb \circ i_1 = k$.

Definition 12. *We define* `coproduct_p` *to be*

$$\begin{gathered}\lambda X, Y, Z, i0, i1, comb. Obj\ X {\wedge} Obj\ Y {\wedge} Obj\ Z {\wedge} Hom\ X\ Z\ i0 {\wedge} Hom\ Y\ Z\ i1 \\ \wedge \forall W : \iota. Obj\ W {\to} \forall h, k : \iota. Hom\ X\ W\ h {\to} Hom\ Y\ W\ k \\ \to Hom\ Z\ W\ (comb\ W\ h\ k) {\wedge} comp\ X\ Z\ W\ (comb\ W\ h\ k)\ i0{=}h {\wedge} comp\ Y\ Z\ W\ (comb\ W\ h\ k)\ i1{=}k \\ \wedge \forall hk : \iota. Hom\ Z\ W\ hk {\to} comp\ X\ Z\ W\ hk\ i0{=}h {\to} comp\ Y\ Z\ W\ hk\ i1{=}k {\to} hk{=}comb\ W\ h\ k\end{gathered}$$

of type $\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}(\iota{\to}\iota{\to}\iota{\to}\iota){\to}o$*.*

A *coproduct constructor* is given by four mathematical objects

- $coprod : \iota \to \iota \to \iota$
- $i_0 : \iota \to \iota \to \iota$
- $i_1 : \iota \to \iota \to \iota$
- $comb : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that $coprod\ X\ Y$, $i_0\ X\ Y$, $i_1\ X\ Y$ and $comb\ X\ Y$ give a product of X and Y for all objects X and Y.

Definition 13. *We define* `coproduct_constr_p` *to be*

$$\lambda coprod, i0, i1, copair. \forall X, Y : \iota. Obj\ X \to Obj\ Y \\ \to \texttt{coproduct\_p}\ X\ Y\ (coprod\ X\ Y)\ (i0\ X\ Y)\ (i1\ X\ Y)\ (copair\ X\ Y)$$

of type $(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to o$.

Given arrows $f, g : X \to Y$ an *equalizer* is given by three mathematical objects

- $Q : \iota$
- $q : \iota$
- $fac : \iota \to \iota \to \iota$

such that Q is an object, q is an arrow $q : Q \to X$, $f \circ q = g \circ q$ and for arrows $h : W \to X$ where $f \circ h = g \circ h$, $fac\ W\ h$ is the unique arrow such that $q \circ fac\ W\ h = h$.

Definition 14. *We define* `equalizer_p` *to be*

$$\lambda X, Y, f, g, Q, q, fac. Obj\ X \wedge Obj\ Y \wedge Hom\ X\ Y\ f \wedge Hom\ X\ Y\ g \wedge Obj\ Q \wedge Hom\ Q\ X\ q \\ \wedge comp\ Q\ X\ Y\ f\ q{=}comp\ Q\ X\ Y\ g\ q \\ \wedge \forall W : \iota. Obj\ W \to \forall h : \iota. Hom\ W\ X\ h \to comp\ W\ X\ Y\ f\ h{=}comp\ W\ X\ Y\ g\ h \\ \to Hom\ W\ Q\ (fac\ W\ h) \wedge comp\ W\ Q\ X\ q\ (fac\ W\ h){=}h \\ \wedge \forall u : \iota. Hom\ W\ Q\ u \to comp\ W\ Q\ X\ q\ u{=}h \to u{=}fac\ W\ h$$

of type $\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to(\iota\to\iota\to\iota)\to o$.

A *equalizer constructor* is specified by three mathematical objects

- $quot : \iota \to \iota \to \iota \to \iota \to \iota$,
- $canonmap : \iota \to \iota \to \iota \to \iota \to \iota$ and
- $fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that for all objects X and Y and arrows $f, g : X \to Y$, $quot\ X\ Y\ f\ g$, $canonmap\ X\ Y\ f\ g$ and $fac\ X\ Y\ f\ g$ give an equalizer.

Definition 15. *We define* `equalizer_constr_p` *to be*

$$\lambda quot, canonmap, fac. \forall X, Y : \iota. Obj\ X \to Obj\ Y \to \forall f, g : \iota. Hom\ X\ Y\ f \to Hom\ X\ Y\ g \\ \to \texttt{equalizer\_p}\ X\ Y\ f\ g\ (quot\ X\ Y\ f\ g)\ (canonmap\ X\ Y\ f\ g)\ (fac\ X\ Y\ f\ g)$$

of type $(\iota\to\iota\to\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to o$.

Given arrows $f, g : X \to Y$ a *coequalizer* is given by three mathematical objects

- $Q : \iota$
- $q : \iota$
- $fac : \iota \to \iota \to \iota$

such that Q is an object, q is an arrow $q : X \to Q$, $q \circ f = q \circ g$ and for arrows $h : X \to W$ where $h \circ f = h \circ g$, $fac\ W\ h$ is the unique arrow such that $fac\ W\ h \circ q = h$.

Definition 16. *We define* `coequalizer_p` *to be*

$$\begin{gathered}\lambda X, Y, f, g, Q, q, fac.Obj\ X{\wedge}Obj\ Y{\wedge}Hom\ X\ Y\ f{\wedge}Hom\ X\ Y\ g{\wedge}Obj\ Q{\wedge}Hom\ Y\ Q\ q\\ {\wedge}comp\ X\ Y\ Q\ q\ f{=}comp\ X\ Y\ Q\ q\ g\\ {\wedge}\forall W : \iota.Obj\ W{\to}\forall h : \iota.Hom\ Y\ W\ h{\to}comp\ X\ Y\ W\ h\ f{=}comp\ X\ Y\ W\ h\ g\\ {\to}Hom\ Q\ W\ (fac\ W\ h){\wedge}comp\ Y\ Q\ W\ (fac\ W\ h)\ q{=}h\\ {\wedge}\forall u : \iota.Hom\ Q\ W\ u{\to}comp\ Y\ Q\ W\ u\ q{=}h{\to}u{=}fac\ W\ h\end{gathered}$$

of type $\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}(\iota{\to}\iota{\to}\iota){\to}o$.

A *coequalizer constructor* is specified by three mathematical objects

- $quot : \iota \to \iota \to \iota \to \iota \to \iota$,
- $canonmap : \iota \to \iota \to \iota \to \iota \to \iota$ and
- $fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that for all objects X and Y and arrows $f, g : X \to Y$, $quot\ X\ Y\ f\ g$, $canonmap\ X\ Y\ f\ g$ and $fac\ X\ Y\ f\ g$ give an coequalizer.

Definition 17. *We define* `coequalizer_constr_p` *to be*

$$\begin{gathered}\lambda quot, canonmap, fac.\forall X, Y : \iota.Obj\ X{\to}Obj\ Y{\to}\forall f, g : \iota.Hom\ X\ Y\ f{\to}Hom\ X\ Y\ g\\ {\to}\texttt{coequalizer\_p}\ X\ Y\ f\ g\ (quot\ X\ Y\ f\ g)\ (canonmap\ X\ Y\ f\ g)\ (fac\ X\ Y\ f\ g)\end{gathered}$$

of type $(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}o$.

Given arrows $f : X \to Z$ and $g : Y \to Z$ a *pullback* is given by four mathematical objects

- $P : \iota$,
- $\pi_0 : \iota$,
- $\pi_1 : \iota$ and
- $pair : \iota \to \iota \to \iota \to \iota$

such that P is an object, π_0 is an arrow $\pi_0 : P \to X$, π_1 is an arrow $\pi_1 : P \to Y$, $f \circ \pi_0 = g \circ \pi_1$ and forall objects W and arrows $h : W \to X$ and $k : W \to Y$ if $f \circ h = g \circ k$, then $pair\ W\ h\ k$ is the unique arrow from W to P such that $\pi_0 \circ pair\ W\ h\ k = h$ and $\pi_1 \circ pair\ W\ h\ k = k$.

Definition 18. *We define* `pullback_p` *to be*

$$\begin{gathered}\lambda X,Y,Z,f,g,P,\pi_0,\pi_1,pair.Obj\ X\wedge Obj\ Y\wedge Obj\ Z\wedge Hom\ X\ Z\ f\wedge Hom\ Y\ Z\ g\\ \wedge Obj\ P\wedge Hom\ P\ X\ \pi_0\wedge Hom\ P\ Y\ \pi_1\\ \wedge comp\ P\ X\ Z\ f\ \pi_0{=}comp\ P\ Y\ Z\ g\ \pi_1\\ \wedge\forall W:\iota.Obj\ W{\to}\forall h:\iota.Hom\ W\ X\ h{\to}\forall k:\iota.Hom\ W\ Y\ k\\ {\to}comp\ W\ X\ Z\ f\ h{=}comp\ W\ Y\ Z\ g\ k\\ {\to}Hom\ W\ P\ (pair\ W\ h\ k)\\ \wedge comp\ W\ P\ X\ \pi_0\ (pair\ W\ h\ k){=}h\wedge comp\ W\ P\ Y\ \pi_1\ (pair\ W\ h\ k){=}k\\ \wedge\forall u:\iota.Hom\ W\ P\ u{\to}comp\ W\ P\ X\ \pi_0\ u{=}h{\to}comp\ W\ P\ Y\ \pi_1\ u{=}k\\ {\to}u{=}pair\ W\ h\ k\end{gathered}$$

of type $\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}(\iota{\to}\iota{\to}\iota{\to}\iota){\to}o$.

A *pullback constructor* is given by four mathematical objects

- $pb : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$
- $\pi_0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$,
- $\pi_1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$ and
- $pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that for all objects X, Y, Z and arrows $f : X \to Z$ and $g : Y \to Z$ such that $pb\ X\ Y\ Z\ f\ g$, $\pi_0\ X\ Y\ Z\ f\ g$, $\pi_1\ X\ Y\ Z\ f\ g$ and $pair\ X\ Y\ Z\ f\ g$ give a pullback.

Definition 19. *We define* `pullback_constr_p` *to be*

$$\begin{gathered}\lambda pb,\pi_0,\pi_1,pair.\forall X,Y,Z:\iota.Obj\ X{\to}Obj\ Y{\to}Obj\ Z\\ {\to}\forall f,g:\iota.Hom\ X\ Z\ f{\to}Hom\ Y\ Z\ g\\ {\to}\texttt{pullback\_p}\ X\ Y\ Z\ f\ g\ (pb\ X\ Y\ Z\ f\ g)\\ (\pi_0\ X\ Y\ Z\ f\ g)\ (\pi_1\ X\ Y\ Z\ f\ g)\ (pair\ X\ Y\ Z\ f\ g)\end{gathered}$$

of type

$$\begin{gathered}(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}\\ (\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}(\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota){\to}o.\end{gathered}$$

Given arrows $f : X \to Z$ and $g : Y \to Z$ a *pushout* is given by four mathematical objects

- $P : \iota$,
- $i_0 : \iota$,
- $i_1 : \iota$ and
- $copair : \iota \to \iota \to \iota \to \iota$

such that P is an object, i_0 is an arrow $i_0 : X \to P$, i_1 is an arrow $i_1 : Y \to P$, $i_0 \circ f = i_1 \circ g$ and forall objects W and arrows $h : X \to W$ and $k : Y \to W$ if $h \circ f = k \circ g$, then $copair\ W\ h\ k$ is the unique arrow from P to W such that $copair\ W\ h\ k \circ i_0 = h$ and $copair\ W\ h\ k \circ i_1 = k$.

Definition 20. *We define* `pushout_p` *to be*

$$\begin{gathered}\lambda X,Y,Z,f,g,P,i0,i1,copair.Obj\ X\wedge Obj\ Y\wedge Obj\ Z\wedge Hom\ Z\ X\ f\wedge Hom\ Z\ Y\ g\\ \wedge Obj\ P\wedge Hom\ X\ P\ i0\wedge Hom\ Y\ P\ i1\\ \wedge comp\ Z\ X\ P\ i0\ f{=}comp\ Z\ Y\ P\ i1\ g\\ \wedge\forall W:\iota.Obj\ W\to\forall h:\iota.Hom\ X\ W\ h\to\forall k:\iota.Hom\ Y\ W\ k\\ \to comp\ Z\ X\ W\ h\ f{=}comp\ Z\ Y\ W\ k\ g\\ \to Hom\ P\ W\ (copair\ W\ h\ k)\\ \wedge comp\ X\ P\ W\ (copair\ W\ h\ k)\ i0{=}h\wedge comp\ Y\ P\ W\ (copair\ W\ h\ k)\ i1{=}k\\ \wedge\forall u:\iota.Hom\ P\ W\ u\to comp\ X\ P\ W\ u\ i0{=}h\to comp\ Y\ P\ W\ u\ i1{=}k\\ \to u{=}copair\ W\ h\ k\end{gathered}$$

of type $\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to(\iota\to\iota\to\iota\to\iota)\to o$.

A *pushout constructor* is given by four mathematical objects

- $po : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$
- $i_0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$,
- $i_1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$ and
- $copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that for all objects X, Y, Z and arrows $f : Z \to X$ and $g : Z \to Y$ such that $po\ X\ Y\ Z\ f\ g$, $i_0\ X\ Y\ Z\ f\ g$, $i_1\ X\ Y\ Z\ f\ g$ and $copair\ X\ Y\ Z\ f\ g$ give a pushout.

Definition 21. *We define* `pushout_constr_p` *to be*

$$\begin{gathered}\lambda po,i0,i1,copair.\forall X,Y,Z:\iota.Obj\ X\to Obj\ Y\to Obj\ Z\to\\ \forall f,g:\iota.Hom\ Z\ X\ f\to Hom\ Z\ Y\ g\\ \to\texttt{pushout\_p}\ X\ Y\ Z\ f\ g\ (po\ X\ Y\ Z\ f\ g)\\ (i0\ X\ Y\ Z\ f\ g)\ (i1\ X\ Y\ Z\ f\ g)\ (copair\ X\ Y\ Z\ f\ g)\end{gathered}$$

of type

$$\begin{gathered}(\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to\\ (\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to o.\end{gathered}$$

2.4 Exponents

Let Obj, Hom, id and $comp$ of appropriate types forming a metacategory be given. Assume $prod$, π_0, π_1 and $pair$ give a product constructor for the metacategory. Let X and Y be objects. An *exponent for* X *and* Y is given by three mathematical objects

- $Z : \iota$,
- $a : \iota$ and
- $lm : \iota \to \iota \to \iota$

such that Z is an object, $a : prod\ Z\ X \to Y$ is an arrow and for all objects W and arrows $f : prod\ W\ X \to Y$ $lm\ W\ f$ is the unique arrow such that

$$pair \cdots (lm\ W\ f \circ \pi_1\ W\ X)\ (\pi_2\ W\ X) = f.$$

Definition 22. *We define* `exponent_p` *to be*

$$\begin{gathered}\lambda prod,\pi_0,\pi_1,pair,X,Y,Z,a,lm.Obj\ X\wedge Obj\ Y\wedge Obj\ Z\wedge Hom\ (prod\ Z\ X)\ Y\ a\\ \wedge\forall W:\iota.Obj\ W\to\forall f:\iota.Hom\ (prod\ W\ X)\ Y\ f\\ \to Hom\ W\ Z\ (lm\ W\ f)\\ \wedge comp\ (prod\ W\ X)\ (prod\ Z\ X)\ Y\ a\\ (pair\ Z\ X\ (prod\ W\ X)\ (comp\ (prod\ W\ X)\ W\ Z\ (lm\ W\ f)\ (\pi_0\ W\ X))\ (\pi_1\ W\ X)){=}f\\ \wedge\forall g:\iota.Hom\ W\ Z\ g\\ \to comp\ (prod\ W\ X)\ (prod\ Z\ X)\ Y\ a\\ (pair\ Z\ X\ (prod\ W\ X)\ (comp\ (prod\ W\ X)\ W\ Z\ g\ (\pi_0\ W\ X))\ (\pi_1\ W\ X)){=}f\\ \to g{=}lm\ W\ f\end{gathered}$$

of type $(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to\iota\to\iota\to\iota\to\iota\to(\iota\to\iota\to\iota)\to o.$

An *product-exponent constructor* is given by seven mathematical objects

- $prod : \iota \to \iota \to \iota$
- $\pi_0 : \iota \to \iota \to \iota$
- $\pi_1 : \iota \to \iota \to \iota$
- $pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota$
- $exp : \iota \to \iota \to \iota$,
- $a : \iota \to \iota \to \iota$ and
- $lm : \iota \to \iota \to \iota \to \iota \to \iota$

such that

- $prod$, π_1, π_2 and $pair$ give a product constructor and
- for all objects X and Y, $exp\ X\ Y$, $a\ X\ Y$ and $lm\ X\ Y$ give an exponent for X and Y.

Definition 23. *We define* `product_exponent_constr_p` *to be*

$$\begin{gathered}\lambda prod,\pi_0,\pi_1,pair,exp,a,lm.\texttt{product\_constr\_p}\ prod\ \pi_0\ \pi_1\ pair\\ \wedge\forall X,Y:\iota.Obj\ X\to Obj\ Y\\ \to\texttt{exponent\_p}\ prod\ \pi_0\ \pi_1\ pair\ X\ Y\ (exp\ X\ Y)\ (a\ X\ Y)\ (lm\ X\ Y)\end{gathered}$$

of type $(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to$

2.5 Subobject Classifiers

Let Obj, Hom, id and $comp$ of appropriate types forming a metacategory be given. A *subobject classifier* is given by six mathematical objects

- $one : \iota$
- $uniqa : \iota \to \iota$
- $Omega : \iota$
- $tru : \iota$
- $ch : \iota \to \iota \to \iota \to \iota$

– $constr\_p : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota$

such that

– one and $uniqa$ give a terminal object,

– $Omega$ is an object,

– tru is an arrow $tru : one \to Omega$ and

– for all monics $m : X \to Y$ $ch\ X\ Y\ m$ is an arrow from Y to $Omega$ and X, $uniqa\ X$, m and $constr\_p\ X\ Y\ m$ give a pullback for for one, Y, $Omega$, tru and $ch\ X\ Y\ m$.

Definition 24. *We define* `subobject_classifier_p` *to be*

$$\begin{gathered}\lambda one, uniqa, Omega, tru, ch, constr\_p.\texttt{terminal\_p}\ one\ uniqa\\ \wedge Obj\ Omega\\ \wedge Hom\ one\ Omega\ tru\\ \wedge\forall X, Y : \iota.\forall m : \iota.\texttt{monic}\ X\ Y\ m\\ \to Hom\ Y\ Omega\ (ch\ X\ Y\ m)\\ \wedge\texttt{pullback\_p}\ one\ Y\ Omega\ tru\ (ch\ X\ Y\ m)\ X\ (uniqa\ X)\ m\ (constr\_p\ X\ Y\ m)\end{gathered}$$

of type $\iota\to(\iota\to\iota)\to\iota\to\iota\to(\iota\to\iota\to\iota\to\iota)\to(\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota)\to o$.

2.6 Natural Number Objects

Let Obj, Hom, id and $comp$ of appropriate types forming a metacategory be given. A natural numbers object is given by five mathematical objects

– $one : \iota$,

– $uniqa : \iota \to \iota$,

– $N : \iota$,

– $zer : \iota$,

– $suc : \iota$ and

– $rec : \iota \to \iota \to \iota \to \iota$

such that

– one and $uniqa$ give a terminal object,

– N is an object,

– zer is an arrow $zer : one \to N$,

– suc is an arrow $suc : N \to N$ and

– for all objects X and arrows $x : one \to X$ and $f : X \to X$ $rec\ X\ x\ f$ is the arrow $rec\ X\ x\ f : N \to X$ such that $rec\ X\ x\ f \circ zer = x$ and $rec\ X\ x\ f \circ suc = f \circ rec\ X\ x\ f$.

Definition 25. *We define* `nno_p` *to be*

$$\begin{gathered}\lambda one, uniqa, N, zer, suc, rec.\texttt{terminal\_p}\ one\ uniqa \wedge Obj\ N\\ \wedge Hom\ one\ N\ zer\\ \wedge Hom\ N\ N\ suc\\ \wedge\forall X:\iota.\forall x:\iota.\forall f:\iota.Obj\ X\to Hom\ one\ X\ x\to Hom\ X\ X\ f\\ \to Hom\ N\ X\ (rec\ X\ x\ f)\wedge comp\ one\ N\ X\ (rec\ X\ x\ f)\ zer{=}x\\ \wedge comp\ N\ N\ X\ (rec\ X\ x\ f)\ suc{=}comp\ N\ X\ X\ f\ (rec\ X\ x\ f)\\ \wedge\forall u:\iota.Hom\ N\ X\ u\\ \to comp\ one\ N\ X\ u\ zer{=}x\\ \to comp\ N\ N\ X\ u\ suc{=}comp\ N\ X\ X\ f\ u\\ \to u{=}rec\ X\ x\ f\end{gathered}$$

of type $\iota\to(\iota\to\iota)\to\iota\to\iota\to\iota\to(\iota\to\iota\to\iota\to\iota)\to o$.

2.7 Relationships between Limits and Colimits

We next describe a number of proven relationships between limits and colimits in metacategories and their opposites. Let *Obj*, *Hom*, *id* and *comp* of appropriate types be fixed.

A product in a metacategory gives a coproduct in its opposite.

Theorem 4. *[*`product_coproduct_Op`*]*

$$\begin{gathered}\forall X,Y,Z:\iota.\forall\pi_0,\pi_1:\iota.\forall pair:\iota\to\iota\to\iota\to\iota.\\ \texttt{product\_p}\ Obj\ Hom\ id\ comp\ X\ Y\ Z\ \pi_0\ \pi_1\ pair\\ \to\texttt{coproduct\_p}\ Obj\ (\lambda X,Y.Hom\ Y\ X)\ id\\ (\lambda X,Y,Z,f,g.comp\ Z\ Y\ X\ g\ f)\ X\ Y\ Z\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proof. This is trivial since once the definitions are expanded the assumption converts to the conclusion. Here is the Megalodon proof:

```
let X Y Z pi0 pi1 pair.
assume H1. exact H1.
```

□

A product constructor in a metacategory gives a coproduct constructor in its opposite.

Theorem 5. *[*`product_coproduct_constr_Op`*]*

$$\begin{gathered}\forall prod:\iota\to\iota\to\iota.\forall\pi_0,\pi_1:\iota\to\iota\to\iota.\forall pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{product\_constr\_p}\ Obj\ Hom\ id\ comp\ prod\ \pi_0\ \pi_1\ pair\\ \to\texttt{coproduct\_constr\_p}\ Obj\ (\lambda X,Y.Hom\ Y\ X)\ id\\ (\lambda X,Y,Z,f,g.comp\ Z\ Y\ X\ g\ f)\ prod\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

A coproduct in a metacategory gives a product in its opposite.

Theorem 6. *[*`coproduct_product_Op`*]*

$$\begin{gathered}\forall X, Y, Z : \iota.\forall i0, i1 : \iota.\forall copair : \iota{\to}\iota{\to}\iota{\to}\iota.\\ \texttt{coproduct\_p}\ Obj\ Hom\ id\ comp\ X\ Y\ Z\ i0\ i1\ copair\\ {\to}\texttt{product\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ X\ Y\ Z\ i0\ i1\ copair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

A coproduct constructor in a metacategory gives a product constructor in its opposite.

Theorem 7. *[*`coproduct_product_constr_Op`*]*

$$\begin{gathered}\forall coprod : \iota{\to}\iota{\to}\iota.\forall i0, i1 : \iota{\to}\iota{\to}\iota.\forall copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \texttt{coproduct\_constr\_p}\ Obj\ Hom\ id\ comp\ coprod\ i0\ i1\ copair\\ {\to}\texttt{product\_constr\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ coprod\ i0\ i1\ copair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

An equalizer in a metacategory gives a coequalizer in its opposite.

Theorem 8. *[*`equalizer_coequalizer_Op`*]*

$$\begin{gathered}\forall X, Y : \iota.\forall f, g : \iota.\forall Q : \iota.\forall q : \iota.\forall fac : \iota{\to}\iota{\to}\iota.\\ \texttt{equalizer\_p}\ Obj\ Hom\ id\ comp\ X\ Y\ f\ g\ Q\ q\ fac\\ {\to}\texttt{coequalizer\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ Y\ X\ f\ g\ Q\ q\ fac.\end{gathered}$$

Proof. Straightforward. The conjunction $Obj\ X \wedge Obj\ Y$ needs to be swapped to be $Obj\ Y \wedge Obj\ X$. □

An equalizer constructor in a metacategory gives a coequalizer constructor in its opposite.

Theorem 9. *[*`equalizer_coequalizer_constr_Op`*]*

$$\begin{gathered}\forall quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\forall canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\forall fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \texttt{equalizer\_constr\_p}\ Obj\ Hom\ id\ comp\ quot\ canonmap\ fac\\ {\to}\texttt{coequalizer\_constr\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ (\lambda X, Y, f, g.quot\ Y\ X\ f\ g)\\ (\lambda X, Y, f, g.canonmap\ Y\ X\ f\ g)\ (\lambda X, Y, f, g.fac\ Y\ X\ f\ g).\end{gathered}$$

Proof. Use Theorem 8. □

A coequalizer in a metacategory gives an equalizer in its opposite.

Theorem 10. *[*`coequalizer_equalizer_Op`*]*

$$\begin{gathered}\forall X, Y : \iota.\forall f, g : \iota.\forall Q : \iota.\forall q : \iota.\forall fac : \iota{\rightarrow}\iota{\rightarrow}\iota.\\ \texttt{coequalizer\_p}\ Obj\ Hom\ id\ comp\ X\ Y\ f\ g\ Q\ q\ fac\\ \rightarrow\texttt{equalizer\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ Y\ X\ f\ g\ Q\ q\ fac.\end{gathered}$$

Proof. Straightforward. □

A coequalizer constructor in a metacategory gives an equalizer constructor in its opposite.

Theorem 11. *[*`coequalizer_equalizer_constr_Op`*]*

$$\begin{gathered}\forall quot : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\forall canonmap : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\forall fac : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\\ \texttt{coequalizer\_constr\_p}\ Obj\ Hom\ id\ comp\ quot\ canonmap\ fac\\ \rightarrow\texttt{equalizer\_constr\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ (\lambda X, Y, f, g.quot\ Y\ X\ f\ g)\\ (\lambda X, Y, f, g.canonmap\ Y\ X\ f\ g)\ (\lambda X, Y, f, g.fac\ Y\ X\ f\ g).\end{gathered}$$

Proof. Use Theorem 10. □

A pullback in a metacategory gives a pushout in its opposite.

Theorem 12. *[*`pullback_pushout_Op`*]*

$$\begin{gathered}\forall X, Y, Z : \iota.\forall f, g : \iota.\forall P : \iota.\forall \pi_0, \pi_1 : \iota.\forall pair : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\\ \texttt{pullback\_p}\ Obj\ Hom\ id\ comp\ X\ Y\ Z\ f\ g\ P\ \pi_0\ \pi_1\ pair\\ \rightarrow\texttt{pushout\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ X\ Y\ Z\ f\ g\ P\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

A pullback constructor in a metacategory gives a pushout constructor in its opposite.

Theorem 13. *[*`pullback_pushout_constr_Op`*]*

$$\begin{gathered}\forall pb : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\forall \pi_0, \pi_1 : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\\ \forall pair : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\\ \texttt{pullback\_constr\_p}\ Obj\ Hom\ id\ comp\ pb\ \pi_0\ \pi_1\ pair\\ \rightarrow\texttt{pushout\_constr\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ pb\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

A pushout in a metacategory gives a pullback in its opposite.

Theorem 14. *[*`pushout_pullback_Op`*]*

$$\begin{gathered}\forall X, Y, Z : \iota.\forall f, g : \iota.\forall P : \iota.\forall i0, i1 : \iota.\forall copair : \iota{\rightarrow}\iota{\rightarrow}\iota{\rightarrow}\iota.\\ \texttt{pushout\_p}\ Obj\ Hom\ id\ comp\ X\ Y\ Z\ f\ g\ P\ i0\ i1\ copair\\ \rightarrow\texttt{pullback\_p}\ Obj\ (\lambda X, Y.Hom\ Y\ X)\ id\\ (\lambda X, Y, Z, f, g.comp\ Z\ Y\ X\ g\ f)\ X\ Y\ Z\ f\ g\ P\ i0\ i1\ copair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

A pushout constructor in a metacategory gives a pullback constructor in its opposite.

Theorem 15. *[*`pushout_pullback_constr_Op`*]*

$$\begin{gathered}\forall po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \forall i0, i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \forall copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \texttt{pushout\_constr\_p}\ Obj\ Hom\ id\ comp\ po\ i0\ i1\ copair \\ \to \texttt{pullback\_constr\_p}\ Obj\ (\lambda X, Y. Hom\ Y\ X)\ id \\ (\lambda X, Y, Z, f, g. comp\ Z\ Y\ X\ g\ f)\ po\ i0\ i1\ copair.\end{gathered}$$

Proof. Trivial: assumption converts to conclusion. □

A product constructor and an equalizer constructor can be combined to give a pullback constructor.

Theorem 16. *[*`product_equalizer_pullback_constr`*]*

$$\begin{gathered}\texttt{MetaCat}\ Obj\ Hom\ id\ comp \\ \to \forall quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \forall canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \forall fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \texttt{equalizer\_constr\_p}\ Obj\ Hom\ id\ comp\ quot\ canonmap\ fac \\ \to \forall prod : \iota{\to}\iota{\to}\iota. \forall \pi_0 : \iota{\to}\iota{\to}\iota. \forall \pi_1 : \iota{\to}\iota{\to}\iota. \\ \forall pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \texttt{product\_constr\_p}\ Obj\ Hom\ id\ comp\ prod\ \pi_0\ \pi_1\ pair \\ \to \texttt{pullback\_constr\_p}\ Obj\ Hom\ id\ comp \\ (\lambda X, Y, Z, f, g. quot\ (prod\ X\ Y)\ Z\ (comp\ (prod\ X\ Y)\ X\ Z\ f\ (\pi_0\ X\ Y)) \\ (comp\ (prod\ X\ Y)\ Y\ Z\ g\ (\pi_1\ X\ Y))) \\ (\lambda X, Y, Z, f, g. comp\ (quot\ (prod\ X\ Y)\ Z\ (comp\ (prod\ X\ Y)\ X\ Z\ f\ (\pi_0\ X\ Y)) \\ (comp\ (prod\ X\ Y)\ Y\ Z\ g\ (\pi_1\ X\ Y)))\ (prod\ X\ Y)\ X\ (\pi_0\ X\ Y) \\ (canonmap\ (prod\ X\ Y)\ Z\ (comp\ (prod\ X\ Y)\ X\ Z\ f\ (\pi_0\ X\ Y)) \\ (comp\ (prod\ X\ Y)\ Y\ Z\ g\ (\pi_1\ X\ Y)))) \\ (\lambda X, Y, Z, f, g. comp\ (quot\ (prod\ X\ Y)\ Z\ (comp\ (prod\ X\ Y)\ X\ Z\ f\ (\pi_0\ X\ Y)) \\ (comp\ (prod\ X\ Y)\ Y\ Z\ g\ (\pi_1\ X\ Y)))\ (prod\ X\ Y)\ Y\ (\pi_1\ X\ Y) \\ (canonmap\ (prod\ X\ Y)\ Z\ (comp\ (prod\ X\ Y)\ X\ Z\ f\ (\pi_0\ X\ Y)) \\ (comp\ (prod\ X\ Y)\ Y\ Z\ g\ (\pi_1\ X\ Y)))) \\ (\lambda X, Y, Z, f, g, W, h, k. fac\ (prod\ X\ Y)\ Z\ (comp\ (prod\ X\ Y)\ X\ Z\ f\ (\pi_0\ X\ Y)) \\ (comp\ (prod\ X\ Y)\ Y\ Z\ g\ (\pi_1\ X\ Y)) W\ (pair\ X\ Y\ W\ h\ k)).\end{gathered}$$

Proof. Straightforward. □

An existential version of the previous theorem (abstracting away the construction).

Theorem 17. *[*`product_equalizer_pullback_constr_ex`*]*

$$\begin{array}{c}\texttt{MetaCat}\ Obj\ Hom\ id\ comp\\ \to(\exists quot:\iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap:\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{equalizer\_constr\_p}\ Obj\ Hom\ id\ comp\ quot\ canonmap\ fac)\\ \to(\exists prod:\iota\to\iota\to\iota.\exists\pi_0:\iota\to\iota\to\iota.\exists\pi_1:\iota\to\iota\to\iota.\\ \exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{product\_constr\_p}\ Obj\ Hom\ id\ comp\ prod\ \pi_0\ \pi_1\ pair)\\ \to\exists pb:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{pullback\_constr\_p}\ Obj\ Hom\ id\ comp\ pb\ \pi_0\ \pi_1\ pair.\end{array}$$

Proof. Use Theorem 16. □

A coproduct constructor and a coequalizer constructor can be combined to give a pushout constructor.

Theorem 18. *[*`coproduct_coequalizer_pushout_constr_ex`*]*

$$\begin{array}{c}\texttt{MetaCat}\ Obj\ Hom\ id\ comp\\ \to(\exists quot:\iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap:\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{coequalizer\_constr\_p}\ Obj\ Hom\ id\ comp\ quot\ canonmap\ fac)\\ \to(\exists coprod:\iota\to\iota\to\iota.\exists i0:\iota\to\iota\to\iota.\exists i1:\iota\to\iota\to\iota.\\ \exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{coproduct\_constr\_p}\ Obj\ Hom\ id\ comp\ coprod\ i0\ i1\ copair)\\ \to\exists po:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{pushout\_constr\_p}\ Obj\ Hom\ id\ comp\ po\ i0\ i1\ copair.\end{array}$$

Proof. Use Theorem 3 to prove the opposite category is a metacategory. Then use Theorem 17 with the opposite category. Finally use Theorem 11. (One could also use Theorems 7 and 13, but these are not necessary since their proofs are trivial and can be repeated in place.) □

2.8 Categores of Sets I

We define `SetHom` so that `SetHom` X Y is the predicate true on precisely the set theoretic functions from X to Y. Note that Y^X is notation for `setexp` X Y, the set containing the set theoretic functions as members. Likewise, `setexp` X Y is defined as `Pi` X $(\lambda x.Y)$ where `Pi` A B is the set of set theoretic functions f with domain A such that for all $a \in A$, applying f to a gives a value in the set B a. We use the binder notation $\Pi x \in A.C$ as syntactic sugar for the term `Pi` A $(\lambda x.C)$.[1]

[1] In the default Egal based HOTG theory of Megalodon/Proofgold, set theoretic functions are represented using the Aczel trace representation instead of the more traditional graph based representation, but this is unlikely to be relevant.

Definition 26. *We define* `SetHom` *to be*

$$\lambda X, Y, f.f \in Y^X$$

of type $\iota{\to}\iota{\to}\iota{\to}o$.

A metacategory of sets is given by a predicate *Obj* that determines which sets are included. The value of *Hom* will always be given by `SetHom`. Likewise, the value of *id* and *comp* will be given by set theoretic identity mapping and composition of mappings. Since Megalodon/Proofgold includes set level binders and allows one to write fx for set level application (where $f, x : \iota$), we can write the value of *id* as $(\lambda X : \iota.\lambda x \in X.x)$ and the value of *comp* as $(\lambda X, Y, Z, f, g : \iota.\lambda x \in X.f(gx))$. Without this syntactic sugar, the terms would be written as

$$(\lambda X : \iota.\mathtt{lam}\, X\, (\lambda x : \iota.x)$$

and

$$(\lambda X, Y, Z, f, g : \iota.\mathtt{lam}\, X\, (\lambda x : \iota.\mathtt{ap}\ f\ (\mathtt{ap}\ g\ x)))$$

where $\mathtt{lam} : \iota \to (\iota \to \iota) \to \iota$ and $\mathtt{ap} : \iota \to \iota \to \iota$. Here are some important (previously proven) properties of `lam` and `ap`:

- `ap_Pi`:

  $$\forall X : \iota.\forall Y : \iota \to \iota.\forall f x : \iota.f \in (\Pi x \in X.Y\ x) \to x \in X \to f\ x \in Y\ x.$$

  This can be used to reduce proving $f\ x$ (the result of applying `ap` to f and x) is a member of $Y\ x$ by proving $f \in \Pi x \in X.Y\ x$ and $x \in X$.

- `lam_Pi`:

  $$\forall X : \iota.\forall Y F : \iota \to \iota.(\forall x \in X.F\ x \in Y\ x) \to (\lambda x \in X.F\ x) \in Pix \in X.Y\ x.$$

  This can be used to reduce proving $(\lambda x \in X.t)$ is in Y^X to proving $\forall x \in X.t \in Y$.

- `beta`:

  $$\forall X : \iota.\forall F : \iota \to \iota.\forall x \in X.(\lambda x \in X.F\ x)\ x = F\ x.$$

  This can be used to β reduce a set level λ-abstraction applied to an argument, assuming one can prove the argument is in the domain.

- `lam_ext`:

  $$\forall X : \iota.\forall F G : \iota \to \iota.(\forall x \in X.F\ x = G\ x) \to (\lambda x \in X.F\ x) = (\lambda x \in X.G\ x).$$

  This can be used to prove two set level λ-abstractions are equal by proving they have the same domain and give the same values on that domain.

- `Pi_eta`:

  $$\forall X : \iota.\forall Y : \iota \to \iota.\forall f \in (\Pi x \in X.Y\ x) \to (\lambda x \in X.f\ x) = f.$$

  This can be used to η-reduce or η-expand sets in $\Pi x \in X.Y\ x$.

We also have the following previously published definitions:

```
Definition lam_id : set -> set
   := fun A => fun x :e A => x.
Definition lam_comp : set -> set -> set -> set
   := fun A f g => fun x :e A => f (g x).
```

Using these we can more concisely say `lam_id` and `lam_comp` specify the identity arrows and compositions for metacategories of sets. We will sometimes use `lam_id` and `lam_comp` and sometimes use the expanded versions.

Three specific metacategories of sets we will consider are:

- the metacategory of all sets where *Obj* is $\lambda X : \iota.\top$,
- the metacategory of hereditarily finite sets where *Obj* is $\lambda X : \iota.X \in$ `UnivOf` 0[2]
- and the metacategory of small sets where *Obj* is $\lambda X : \iota.X \in$ `UnivOf` (`UnivOf` 0)

We will not prove these are metacategories until Section 2.14 after we have general results about concrete categories.

We now prove some basic conditions on a predicate *Obj* that will guarantee that certain constructions exist in metacategories of sets where *Obj* recognizes the sets included in the metacategory. Each metacategory of sets will

If 0 (the empty set) is an object, then there is an initial object.

Theorem 19. *[*`MetaCatSet_initial_gen`*]*

$$\begin{gathered}\forall Obj : \iota{\to}o.Obj\ 0 \\ \to\exists Y : \iota.\exists uniqa : \iota{\to}\iota. \\ \texttt{initial\_p}\ Obj\ \texttt{SetHom} \\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ \ (g\ x)))\ Y\ uniqa.\end{gathered}$$

Proof. Use 0 as the witness. Given an object X we can prove $\lambda x \in 0.0$ is the unique arrow in `SetHom` $0\ X$. Here is the Megalodon proof:

```
let Obj. assume H0.
witness 0. witness (fun X => (fun x :e 0 => 0)).
prove Obj 0
   /\ forall X:set, Obj X
       -> SetHom 0 X (fun x :e 0 => 0)
       /\ forall h':set, SetHom 0 X h'
             -> h' = (fun x :e 0 => 0).
apply andI.
- exact H0.
- let X. assume HX. apply andI.
  + prove SetHom 0 X (fun x :e 0 => 0).
    prove (fun x :e 0 => 0) :e Pi_ x :e 0, X.
```

[2]The Grothendieck universe operator `UnivOf` is part of the HOTG set theory and gives the least transitive set closed under power sets, unions and replacement containing its argument as a member.

```
    apply lam_Pi. let x. assume Hx: x :e 0.
    prove False. exact EmptyE x Hx.
  + let h. assume Hh: h :e Pi_ x :e 0, X.
    prove h = (fun x :e 0 => 0).
    transitivity (fun x :e 0 => h x).
    * symmetry. exact Pi_eta 0 (fun _ => X) h Hh.
    * apply lam_ext. let x. assume Hx: x :e 0.
      prove False. exact EmptyE x Hx.
```

□

The metacategory of all sets (which we do not yet know is a metacategory) will obviously have an initial object.

Theorem 20. [MetaCatSet_initial]

$$\begin{gathered}\exists Y:\iota.\exists uniqa:\iota{\to}\iota.\\ \texttt{initial\_p}\ (\lambda\_.True)\ \texttt{SetHom}\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ \ (g\ \ x)))\ Y\ uniqa.\end{gathered}$$

Proof. Apply Theorem 19 to TrueI (a proof of $\top$). □

If the ordinal 1 (provably equal to $\{0\}$) is an object, then the metacategory will have a terminal object.

Theorem 21. [MetaCatSet_terminal_gen]

$$\begin{gathered}\forall Obj:\iota{\to}o.Obj\ 1\\ {\to}\exists Y:\iota.\exists uniqa:\iota{\to}\iota.\\ \texttt{terminal\_p}\ Obj\ \texttt{SetHom}\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ \ (g\ \ x)))\ Y\ uniqa.\end{gathered}$$

Proof. Use 1 as the witness and $\lambda x\in X.0$ as the unique arrow in SetHom X 1. Here is the Megalodon proof:

```
let Obj. assume HO.
witness 1. witness (fun X => (fun x :e X => 0)).
prove Obj 1
   /\ forall X:set, Obj X
       -> SetHom X 1 (fun x :e X => 0)
      /\ forall h':set, SetHom X 1 h'
            -> h' = (fun x :e X => 0).
apply andI.
- exact HO.
- let X. assume HX. apply andI.
  + prove SetHom X 1 (fun x :e X => 0).
    prove (fun x :e X => 0) :e Pi_ x :e X, 1.
    apply lam_Pi. let x. assume Hx: x :e X.
    exact In_0_1.
  + let h. assume Hh: h :e Pi_ x :e X, 1.
    prove h = (fun x :e X => 0).
```

```
transitivity (fun x :e X => h x).
* symmetry. exact Pi_eta X (fun _ => 1) h Hh.
* apply lam_ext. let x. assume Hx: x :e X.
  prove h x = 0.
  apply SingE 0 (h x).
  prove h x :e {0}.
  rewrite <- eq_1_Sing0.
  prove h x :e 1.
  exact ap_Pi X (fun _ => 1) h x Hh Hx.
```

□

The metacategory of all sets will have a terminal object.

Theorem 22. [MetaCatSet_terminal]

$$\begin{gathered}\exists Y:\iota.\exists uniqa:\iota\to\iota.\\ \texttt{terminal\_p}\ (\lambda\_.True)\ \texttt{SetHom}\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\ Y\ uniqa.\end{gathered}$$

Proof. Apply Theorem 21 to TrueI (a proof of $\top$). □

For sets X and Y, the set setsum X Y is a representation of the disjoint union of X and Y. There are particular functions $\mathrm{Inj0}, \mathrm{Inj1} : \iota \to \iota$ such that the elements of setsum X Y are precisely those of the form Inj0 x for $x \in X$ and Inj1 y for $y \in Y$. We sometimes write $X \oplus Y$ for setsum X Y. If Obj is closed under setsum then the metacategory will have a coproduct constructor.

Theorem 23. [MetaCatSet_coproduct_gen]

$$\begin{gathered}\forall Obj:\iota\to o.(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (\text{setsum}\ X\ Y))\\ \to\exists coprod:\iota\to\iota\to\iota.\exists i1,i2:\iota\to\iota\to\iota.\\ \exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{coproduct\_constr\_p}\ Obj\ \texttt{SetHom}\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\ coprod\ i1\ i2\ copair.\end{gathered}$$

Proof. This is relatively straightforward using previously published definitions and theorems. The reader can study the Megalodon proof and relevant parts of the preamble file for details. □

The metacategory of all sets will have coproducts.

Theorem 24. [MetaCatSet_coproduct]

$$\begin{gathered}\exists coprod:\iota\to\iota\to\iota.\exists i1,i2:\iota\to\iota\to\iota.\\ \exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \texttt{coproduct\_constr\_p}\ (\lambda\_.True)\ \texttt{SetHom}\\ (\lambda X.\lambda x\in X.x)\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\ coprod\ i1\ i2\ copair.\end{gathered}$$

Proof. Trivial. □

Let X and Y be sets. setprod X Y is the set of all pairs (x, y) where $x \in X$ and $y \in Y$.[3] Set level application to 0 and 1 provide projection operations. We sometimes write $X \times Y$ for setprod X Y.

Technically setprod X Y is defined to be Sigma X $(\lambda x.Y)$. Here Sigma A B is the set of pairs (a, b) where $a \in A$ and $b \in B$ a. We sometimes write $\Sigma x \in A.C$ for Sigma A $(\lambda x.C)$.

If Obj is clused under setprod, then the metacategory will have products.

Theorem 25. *[*`MetaCatSet_product_gen`*]*

$$\begin{gathered}\forall Obj : \iota{\to}o.(\forall X.Obj\ X{\to}\forall Y.Obj\ Y{\to}Obj\ (\text{setprod}\ X\ Y))\\ {\to}\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \texttt{product\_constr\_p}\ Obj\ \texttt{SetHom}\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\ prod\ \pi_1\ \pi_2\ pair.\end{gathered}$$

Proof. We use *setprod*, $(\lambda X, Y.(\lambda z \in X{\times}Y.z\ 0))$ and $(\lambda X, Y.(\lambda z \in X{\times}Y.z\ 1))$ to witness the relevant existential quantifiers. We use $(\lambda X, Y, W, h, k.(\lambda w \in W.(h\ w, k\ w)))$ to witness the existential quantifier for the unique arrow for each W, $h : W \to X$ and $k : W \to Y$. □

The metacategory of all sets will have products.

Theorem 26. *[*`MetaCatSet_product`*]*

$$\begin{gathered}\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \texttt{product\_constr\_p}\ (\lambda\_.True)\ \texttt{SetHom}\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\ prod\ \pi_1\ \pi_2\ pair.\end{gathered}$$

Proof. We prove the intermediate claim $L1$: $\forall X.True{\to}\forall Y.True{\to}True$. Exact $(\lambda\_, \_, \_, \_.TrueI)$. Exact Theorem 25 $(\lambda\_.True)$ $L1$. □

In order to apply the results above the metacategories of hereditarily finite sets and small sets (both of which we have yet to prove are metacategories), we need some closure properties of Grothendieck universes. The proofs are straightforward and omitted. Note that ZF_closed U means U is closed under power sets, unions and replacement.

Theorem 27. *[*`UnivOf_Subq_closed`*]* $\forall N.\forall X \in \text{UnivOf}\ N.\forall Q \subseteq X.Q \in \text{UnivOf}\ N$.

Definition 27. *We define* `famunion_closed` *to be*

$$\lambda U : \iota.\forall X \in U.\forall F : \iota{\to}\iota.(\forall x \in X.F\ x \in U){\to}famunion\ X\ F \in U$$

of type $\iota{\to}o$.

Theorem 28. *[*`Union_Repl_famunion_closed`*]*

$$\forall U : \iota.Union\_closed\ U{\to}Repl\_closed\ U{\to}\texttt{famunion\_closed}\ U.$$

[3] In the default Egal based HOTG theory of Megalodon/Proofgold, pairs are represented by functions with the ordinal 2 as domain (provably equivalent to $x \oplus y$) instead of the more traditional Kuratowski pairs, but this is unlikely to be relevant.

Theorem 29. *[*`ZF_closed_0`*]* $\forall U, X.TransSet\ U \to ZF\_closed\ U \to X \in U \to 0 \in U.$

Theorem 30. *[*`ZF_Inj1_closed`*]* $\forall U.TransSet\ U \to ZF\_closed\ U \to \forall X \in U.Inj1\ X \in U.$

Theorem 31. *[*`ZF_Inj0_closed`*]* $\forall U.TransSet\ U \to ZF\_closed\ U \to \forall X \in U.Inj0\ X \in U.$

Theorem 32. *[*`ZF_setsum_closed`*]* $\forall U.TransSet\ U \to ZF\_closed\ U \to \forall X, Y \in U.(X \oplus Y) \in U.$

Theorem 33. *[*`ZF_Sigma_closed`*]*

$$\forall U.TransSet\ U \to ZF\_closed\ U$$
$$\to \forall X \in U.\forall Y : \iota \to \iota.(\forall x \in X.Y\ x \in U) \to (\Sigma x \in X.Y\ x) \in U.$$

Theorem 34. *[*`ZF_setprod_closed`*]* $\forall U.TransSet\ U \to ZF\_closed\ U \to \forall X, Y \in U.(X \times Y) \in U.$

Theorem 35. *[*`ZF_Pi_closed`*]*

$$\forall U.TransSet\ U \to ZF\_closed\ U$$
$$\to \forall X \in U.\forall Y : \iota \to \iota.(\forall x \in X.Y\ x \in U) \to (\Pi x \in X.Y\ x) \in U.$$

Theorem 36. *[*`ZF_setexp_closed`*]* $\forall U.TransSet\ U \to ZF\_closed\ U \to \forall X, Y \in U.(Y^X) \in U.$

The metacategory of hereditarily finite sets will have an initial object.

Theorem 37. *[*`MetaCatHFSet_initial`*]*

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$\texttt{initial\_p}\ (\lambda X.X \in \mathrm{UnivOf}\ Empty)\ \texttt{SetHom}$$
$$(\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\ Y\ uniqa.$$

Proof. Use Theorem 19 and the fact that $0 \in \mathrm{UnivOf}\ 0$. □

The metacategory of small sets will have an initial object.

Theorem 38. *[*`MetaCatSmallSet_initial`*]*

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$\texttt{initial\_p}\ (\lambda X.X \in \mathrm{UnivOf}\ (\mathrm{UnivOf}\ Empty))\ \texttt{SetHom}$$
$$(\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\ Y\ uniqa.$$

Proof. Use Theorem 29 to prove $0 \in \mathrm{UnivOf}\ (\mathrm{UnivOf}\ 0)$ and then Theorem 19. □

The metacategory of hereditarily finite sets will have an initial object.

Theorem 39. *[*`MetaCatHFSet_terminal`*]*

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$\texttt{terminal\_p}\ (\lambda X.X \in \mathrm{UnivOf}\ Empty)\ \texttt{SetHom}$$
$$(\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\ Y\ uniqa.$$

Proof. Prove $1 \in \mathrm{UnivOf}\ 0$ using $mathrmZF\_ordsucc\_closed$ (from previously proven results). Then use Theorem 29. □

The metacategory of small sets will have an initial object.

Theorem 40. *[*`MetaCatSmallSet_terminal`*]*

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$\texttt{terminal\_p}\ (\lambda X . X \in \mathrm{UnivOf}\ (\mathrm{UnivOf}\ \mathit{Empty}))\ \texttt{SetHom}$$
$$(\lambda X . (\lambda x \in X . x))\ (\lambda X, Y, Z, f, g . (\lambda x \in X . f\ \ (g\ \ x)))\ Y\ \ uniqa.$$

Proof. Prove $1 \in \mathrm{UnivOf}\ (\mathrm{UnivOf}\ \mathit{Empty})$ using $mathrmZF\_ordsucc\_closed$ and Theorem 29. Then use Theorem 21. □

The category of hereditarily finite sets will have a coproduct constructor.

Theorem 41. *[*`MetaCatHFSet_coproduct`*]*

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota .$$
$$\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\texttt{coproduct\_constr\_p}\ (\lambda X . X \in \mathrm{UnivOf}\ \ \mathit{Empty})\ \texttt{SetHom}$$
$$(\lambda X . \lambda x \in X . x)\ (\lambda X, Y, Z, f, g . (\lambda x \in X . f\ \ (g\ \ x)))\ coprod\ \ i1\ \ i2\ \ copair.$$

Proof. Use Theorems 32 and 23. □

The category of small sets will have a coproduct constructor.

Theorem 42. *[*`MetaCatSmallSet_coproduct`*]*

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota .$$
$$\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\texttt{coproduct\_constr\_p}\ (\lambda X . X \in \mathrm{UnivOf}\ (\mathrm{UnivOf}\ \mathit{Empty}))\ \texttt{SetHom}$$
$$(\lambda X . (\lambda x \in X . x))\ (\lambda X, Y, Z, f, g . (\lambda x \in X . f\ \ (g\ \ x)))\ coprod\ \ i1\ \ i2\ \ copair.$$

Proof. Use Theorems 32 and 23. □

The category of hereditarily finite sets will have a product constructor.

Theorem 43. *[*`MetaCatHFSet_product`*]*

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota .$$
$$\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\texttt{product\_constr\_p}\ (\lambda X . X \in \mathrm{UnivOf}\ \ \mathit{Empty})\ \texttt{SetHom}$$
$$(\lambda X . (\lambda x \in X . x))\ (\lambda X, Y, Z, f, g . (\lambda x \in X . f\ \ (g\ \ x)))\ prod\ \ \pi_1\ \ \pi_2\ \ pair.$$

Proof. Use Theorem 34 and 25. □

The category of small sets will have a product constructor.

Theorem 44. *[*`MetaCatSmallSet_product`*]*

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota .$$
$$\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\texttt{product\_constr\_p}\ (\lambda X . X \in \mathrm{UnivOf}\ (\mathrm{UnivOf}\ \mathit{Empty}))\ \texttt{SetHom}$$
$$(\lambda X . (\lambda x \in X . x))\ (\lambda X, Y, Z, f, g . (\lambda x \in X . f\ \ (g\ \ x)))\ prod\ \ \pi_1\ \ \pi_2\ \ pair.$$

Proof. Use Theorems 34 and 25. □

2.9 Functors

Let Obj, Hom, id, $comp$, Obj', Hom', id' and $comp'$ for two metacategories be given. A *metafunctor* from the first metacategory to the second is given by two mathematical objects:

- $F0 : \iota{\to}\iota$ – a mapping from objects of the first category to the second.
- $F1 : \iota{\to}\iota{\to}\iota{\to}\iota$ – a mapping from arrows of the first category to the second, parameterized by two objects.

We will define `MetaFunctor` to hold (relative to all the objects mentioned above) when the following conditions hold:

- If X is an object of the first metacategory, then $F0\ X$ is an object of the second.
- If $f : X \to Y$ is an arrow of the first category, then $F1\ X\ Y\ f : F0\ X \to F0\ Y$ is an arrow of the second.
- $F1$ sends identity arrows to identity arrows.
- $F1$ respects composition.

Definition 28. *We define* `MetaFunctor` *to be*

$$\begin{gathered}(\forall X.Obj\ X{\to}Obj'\ (F0\ X))\\ \wedge(\forall X,Y,f.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f{\to}Hom'\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\\ \wedge(\forall X.Obj\ X{\to}F1\ X\ X\ (id\ X){=}id'\ (F0\ X))\\ \wedge(\forall X,Y,Z,f,g.Obj\ X{\to}Obj\ Y{\to}Obj\ Z{\to}Hom\ X\ Y\ f{\to}Hom\ Y\ Z\ g\\ {\to}F1\ X\ Z\ (comp\ X\ Y\ Z\ g\ f){=}comp'\ (F0\ X)\ (F0\ Y)\ (F0\ Z)\ (F1\ Y\ Z\ g)\ (F1\ X\ Y\ f))\end{gathered}$$

of type o.

To make it easier to reason about metafunctors, we prove theorems giving introduction and elimination principles. The proofs are omitted and involve basic manipulations of conjunctions.

Theorem 45. *[*`MetaFunctorI`*]*

$$\begin{gathered}(\forall X.Obj\ X{\to}Obj'\ (F0\ X))\\ {\to}(\forall X,Y,f.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f{\to}Hom'\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\\ {\to}(\forall X.Obj\ X{\to}F1\ X\ X\ (id\ X){=}id'\ (F0\ X))\\ {\to}(\forall X,Y,Z,f,g.Obj\ X{\to}Obj\ Y{\to}Obj\ Z{\to}Hom\ X\ Y\ f{\to}Hom\ Y\ Z\ g\\ {\to}F1\ X\ Z\ (comp\ X\ Y\ Z\ g\ f){=}comp'\ (F0\ X)\ (F0\ Y)\ (F0\ Z)\ (F1\ Y\ Z\ g)\ (F1\ X\ Y\ f))\\ {\to}\texttt{MetaFunctor}.\end{gathered}$$

Theorem 46. *[*`MetaFunctorE`*]*

$$\begin{gathered}\texttt{MetaFunctor}{\to}\forall p : o.\\ ((\forall X.Obj\ X{\to}Obj'\ (F0\ X))\\ {\to}(\forall X,Y,f.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f{\to}Hom'\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\\ {\to}(\forall X.Obj\ X{\to}F1\ X\ X\ (id\ X){=}id'\ (F0\ X))\\ {\to}(\forall X,Y,Z,f,g.Obj\ X{\to}Obj\ Y{\to}Obj\ Z{\to}Hom\ X\ Y\ f{\to}Hom\ Y\ Z\ g\\ {\to}F1\ X\ Z\ (comp\ X\ Y\ Z\ g\ f){=}comp'\ (F0\ X)\ (F0\ Y)\ (F0\ Z)\ (F1\ Y\ Z\ g)\ (F1\ X\ Y\ f))\\ {\to}p)\\ {\to}p.\end{gathered}$$

We also define a strict version of metafunctor. The strict version also requires Obj, Hom, id, $comp$ to give a metacategory and Obj', Hom', id', $comp'$ to give a metacategory. We also prove introduction and elimination principles, omitting the straightforward proofs here.

Definition 29. *We define* `MetaFunctor_strict` *to be*

$$\texttt{MetaCat}\ Obj\ Hom\ id\ comp \wedge \texttt{MetaCat}\ Obj'\ Hom'\ id'\ comp' \wedge \texttt{MetaFunctor}$$

of type o.

Theorem 47. *[*`MetaFunctor_strict_I`*]*

$$\texttt{MetaCat}\ Obj\ Hom\ id\ comp \to \texttt{MetaCat}\ Obj'\ Hom'\ id'\ comp' \to \texttt{MetaFunctor} \to \texttt{MetaFunctor\_strict}.$$

Theorem 48. *[*`MetaFunctor_strict_E`*]*

$$\begin{gathered}\texttt{MetaFunctor\_strict} \to \forall p : o.\\ (\texttt{MetaCat}\ Obj\ Hom\ id\ comp \to \texttt{MetaCat}\ Obj'\ Hom'\ id'\ comp' \to \texttt{MetaFunctor} \to p)\\ \to p.\end{gathered}$$

There is an identity functor given by taking $F0$ to be $\lambda X.X$ and $F1$ to be $\lambda X, Y, f.f$.

Theorem 49. *[*`MetaCat_IdFunctor`*]*

$$\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp\ (\lambda X.X)\ (\lambda X, Y, f.f).$$

Proof. Use Theorem 45 to reduce the goal to proving the four properties. All four properties are trivial. □

If Obj, Hom, id, $comp$ is a metacategory, then the identitfy functor is a strict metafunctor.

Theorem 50. *[*`MetaCat_IdFunctor_strict`*]*

$$\begin{gathered}\texttt{MetaCat}\ Obj\ Hom\ id\ comp\\ \to \texttt{MetaFunctor\_strict}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp\ (\lambda X.X)\ (\lambda X, Y, f.f).\end{gathered}$$

Proof. Use Theorems 47 and 49. □

We can also compose metafunctors in the obvious ways. Here assume we have three metacategories specified by Obj, Hom, id, $comp$, Obj', Hom', id', $comp'$, Obj'', Hom'', id'' and $comp''$. Let $F0$ and $F1$ specify a metafunctor from the first to the second and $G0$ and $G1$ specify a metafunctor from the second to the third. We can prove the composition gives a metafunctor.

Theorem 51. *[*`MetaCat_CompFunctors`*]*

$$\begin{gathered}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ \to \texttt{MetaFunctor}\ Obj'\ Hom'\ id'\ comp'\ Obj''\ Hom''\ id''\ comp''\ G0\ G1\\ \to \texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj''\ Hom''\ id''\ comp''\\ (\lambda X.G0\ (F0\ X))\\ (\lambda X, Y, f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f)).\end{gathered}$$

Proof. Use Theorems 46, 46 and 45. □

The composition is also strict under the appropriate assumptions.

Theorem 52. [`MetaCat_CompFunctors_strict`]

$$\begin{array}{c}\texttt{MetaFunctor\_strict}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ \to\texttt{MetaFunctor\_strict}\ Obj'\ Hom'\ id'\ comp'\ Obj''\ Hom''\ id''\ comp''\ G0\ G1\\ \to\texttt{MetaFunctor\_strict}\ Obj\ Hom\ id\ comp\ Obj''\ Hom''\ id''\ comp''\\ (\lambda X.G0\ (F0\ X))\\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f)).\end{array}$$

Proof. Use Theorems 48, 48, 47 and 51. □

2.10 Natural Transformations

Assume we have two metacategories specified by Obj, Hom, id, $comp$, Obj', Hom', id' and $comp'$ and two metafunctors F and G from the first to the second specified by $F0$, $F1$, $G0$ and $G1$. A *meta natural transformation* (from F to G) is specifed by $\eta : \iota{\to}\iota$ satisfying the following properties:

– If X is an object of the first metacategory, then $\eta\ X : F0\ X \to G0\ X$ is an arrow of the second metacategory.

– If $f : X \to Y$ is an arrow of the first metacategory, then $G1\ X\ Y\ f \circ \eta\ X = \eta\ X \circ F1\ X\ Y\ f$.

We define this formally as `MetaNatTrans`.

Definition 30. *We define* `MetaNatTrans` *to be*

$$\begin{array}{c}(\forall X.Obj\ X{\to}Hom'\ (F0\ X)\ (G0\ X)\ (\eta\ X))\\ \wedge(\forall X,Y,f.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f{\to}\\ comp'\ (F0\ X)\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ f)\ (\eta\ X)\\ =comp'\ (F0\ X)\ (F0\ Y)\ (G0\ Y)\ (\eta\ Y)\ (F1\ X\ Y\ f))\end{array}$$

of type o.

To make it easier to reason about meta natural transformations, we prove theorems giving introduction and elimination principles. The proofs are trivial since there is only one conjunction.

Theorem 53. [`MetaNatTransI`]

$$\begin{array}{c}(\forall X.Obj\ X{\to}Hom'\ (F0\ X)\ (G0\ X)\ (\eta\ X))\\ \to(\forall X,Y,f.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f\\ \to comp'\ (F0\ X)\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ f)\ (\eta\ X)\\ =comp'\ (F0\ X)\ (F0\ Y)\ (G0\ Y)\ (\eta\ Y)\ (F1\ X\ Y\ f))\\ \to\texttt{MetaNatTrans}.\end{array}$$

Proof. The proof is simply the conjunction introduction principle with the two conjuncts as arguments. Here is the Megalodon proof:

```
exact andI (forall X, Obj X -> Hom' (F0 X) (G0 X) (eta X))
           (forall X Y f, Obj X -> Obj Y -> Hom X Y f
             -> comp' (F0 X) (G0 X) (G0 Y) (G1 X Y f) (eta X)
              = comp' (F0 X) (F0 Y) (G0 Y) (eta Y) (F1 X Y f)).
```

□

Theorem 54. *[*`MetaNatTransE`*]*

$$\begin{gathered}\texttt{MetaNatTrans}{\to}\forall p:o.\\ ((\forall X.Obj\ X{\to}Hom'\ (F0\ X)\ (G0\ X)\ (\eta\ X))\\ {\to}(\forall X,Y,f.Obj\ X{\to}Obj\ Y{\to}Hom\ X\ Y\ f\\ {\to}comp'\ (F0\ X)\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ f)\ (\eta\ X)\\ {=}comp'\ (F0\ X)\ (F0\ Y)\ (G0\ Y)\ (\eta\ Y)\ (F1\ X\ Y\ f))\\ {\to}p)\\ {\to}p.\end{gathered}$$

Proof. This is particular trivial since the formulation precisely matches the definition of conjunction. Here is the Megalodon proof.

```
assume H. exact H.
```

□

We also define a strict version ensuring the appropriate mathematical objects give metacategories and metafunctors.

Definition 31. *We define* `MetaNatTrans_strict` *to be*

$$\begin{gathered}\texttt{MetaCat}\ Obj\ Hom\ id\ comp\\ {\wedge}\texttt{MetaCat}\ Obj'\ Hom'\ id'\ comp'\\ {\wedge}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ {\wedge}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ G0\ G1\\ {\wedge}\texttt{MetaNatTrans}\end{gathered}$$

of type o.

We again prove introduction and elimination principles, omitting the straightforward proofs.

Theorem 55. *[*`MetaNatTrans_strict_I`*]*

$$\begin{gathered}\texttt{MetaCat}\ Obj\ Hom\ id\ comp{\to}\texttt{MetaCat}\ Obj'\ Hom'\ id'\ comp'\\ {\to}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ {\to}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ G0\ G1\\ {\to}\texttt{MetaNatTrans}{\to}\texttt{MetaNatTrans\_strict}.\end{gathered}$$

Theorem 56. *[*`MetaNatTrans_strict_E`*]*

$$\begin{gathered}\texttt{MetaNatTrans\_strict}{\to}\forall p:o.\\ (\texttt{MetaCat}\ Obj\ Hom\ id\ comp{\to}\texttt{MetaCat}\ Obj'\ Hom'\ id'\ comp'\\ {\to}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ {\to}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ G0\ G1\\ {\to}\texttt{MetaNatTrans}{\to}p)\\ {\to}p.\end{gathered}$$

We can compose meta natural transformations and metafunctors to obtain new meta natural transformations. Assume we have three metacategories specified by *Obj*, *Hom*, *id*, *comp*, *Obj'*, *Hom'*, *id'*, *comp'*, *Obj''*, *Hom''*, *id''* and *comp''*, two metafunctors F and G from the first to then second specified by $F0$, $F1$, $G0$ and $G1$ and a metafunctor H from the second to the third specified by $H0$ and $H1$. Finally assume we have a meta natural transformation η from F to G. The following theorem states that composing H with η (say, $H \circ \eta$) gives a meta natural transformation from $H \circ F$ to $H \circ G$ The proof is tedious but not difficult. The interested reader can study the Megalodon proof.

Theorem 57. [`MetaCat_CompFunctorNatTrans`]

$$\begin{array}{c}
\texttt{MetaFunctor}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}'\ \mathit{Hom}'\ \mathit{id}'\ \mathit{comp}'\ F0\ F1 \\
\to\texttt{MetaFunctor}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}'\ \mathit{Hom}'\ \mathit{id}'\ \mathit{comp}'\ G0\ G1 \\
\to\texttt{MetaNatTrans}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}'\ \mathit{Hom}'\ \mathit{id}'\ \mathit{comp}'\ F0\ F1\ G0\ G1\ \eta \\
\to\texttt{MetaFunctor}\ \mathit{Obj}'\ \mathit{Hom}'\ \mathit{id}'\ \mathit{comp}'\ \mathit{Obj}''\ \mathit{Hom}''\ \mathit{id}''\ \mathit{comp}''\ H0\ H1 \\
\to\texttt{MetaNatTrans}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}''\ \mathit{Hom}''\ \mathit{id}''\ \mathit{comp}'' \\
(\lambda X.H0\ (F0\ X)) \\
(\lambda X,Y,f.H1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f)) \\
(\lambda X.H0\ (G0\ X)) \\
(\lambda X,Y,f.H1\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ f)) \\
(\lambda X.H1\ (F0\ X)\ (G0\ X)\ (\eta\ X)).
\end{array}$$

Proof. Use Theorems 46 and 54. □

Now assume we have three metacategories specified by *Obj*, *Hom*, *id*, *comp*, *Obj'*, *Hom'*, *id'*, *comp'*, *Obj''*, *Hom''*, *id''* and *comp''*, two metafunctors F and G from the second to then third specified by $F0$, $F1$, $G0$ and $G1$, a metafunctor H from the first to the second specified by $H0$ and $H1$ and a meta natural transformation η from F to G. The next theorem proves the composition of η and H (say, $\eta \circ H$) is a meta natural transformation from $F \circ H$ to $G \circ H$. Again, we leave the reader to study the Megalodon proof if interested.

Theorem 58. [`MetaCat_CompNatTransFunctor`]

$$\begin{array}{c}
\texttt{MetaNatTrans}\ \mathit{Obj}'\ \mathit{Hom}'\ \mathit{id}'\ \mathit{comp}'\ \mathit{Obj}''\ \mathit{Hom}''\ \mathit{id}''\ \mathit{comp}''\ F0\ F1\ G0\ G1\ \eta \\
\to\texttt{MetaFunctor}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}'\ \mathit{Hom}'\ \mathit{id}'\ \mathit{comp}'\ H0\ H1 \\
\to\texttt{MetaNatTrans}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}''\ \mathit{Hom}''\ \mathit{id}''\ \mathit{comp}'' \\
(\lambda X.F0\ (H0\ X)) \\
(\lambda X,Y,f.F1\ (H0\ X)\ (H0\ Y)\ (H1\ X\ Y\ f)) \\
(\lambda X.G0\ (H0\ X)) \\
(\lambda X,Y,f.G1\ (H0\ X)\ (H0\ Y)\ (H1\ X\ Y\ f)) \\
(\lambda X.\eta\ (H0\ X)).
\end{array}$$

Proof. Use Theorems 54 and 46. □

2.11 Monads

Let Obj, Hom, id and $comp$ of appropriate types specifying a metacategory be fixed. A *monad* is specified by four objects:

- $T0 : \iota \to \iota$ – the object part of a metafunctor T from the metacategory to itself.
- $T1 : \iota \to \iota \to \iota \to \iota$ – the arrow part of a metafunctor T from the metacategory to itself.
- $\eta : \iota \to \iota$ – a meta natural transformation from the identity functor to T.
- $\mu : \iota \to \iota$ – a meta natural transformation from $T \circ T$ to T.

To be a metamonad it must also satisfy the following for objects X:

- $\mu\ X \circ T1 \cdots (\mu\ X) = \mu\ X \circ \mu\ (T0\ X)$,
- $\mu\ X \circ \eta\ (T0\ X) = id\ (T0\ X)$ and
- $\mu\ X \circ T1 \cdots (\eta\ X) = id\ (T0\ X)$.

We define `MetaMonad` and a strict version `MetaMonad_strict`.

Definition 32. *We define* `MetaMonad` *to be*

$$\begin{gathered}(\forall X. Obj\ X \to comp\ (T0\ (T0\ (T0\ X)))\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (T1\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)) \\ = comp\ (T0\ (T0\ (T0\ X)))\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (\mu\ (T0\ X))) \\ \wedge (\forall X. Obj\ X \to comp\ (T0\ X)\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (\eta\ (T0\ X)) = id\ (T0\ X)) \\ \wedge (\forall X. Obj\ X \to comp\ (T0\ X)\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (T1\ X\ (T0\ X)\ (\eta\ X)) = id\ (T0\ X))\end{gathered}$$

of type o.

Definition 33. *We define* `MetaMonad_strict` *to be*

$$\begin{gathered}\texttt{MetaNatTrans\_strict}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp \\ (\lambda X.X)\ (\lambda X, Y, f.f)\ T0\ T1\ \eta \\ \wedge \texttt{MetaNatTrans\_strict}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp \\ (\lambda X.T0\ (T0\ X)) \\ (\lambda X, Y, f.T1\ (T0\ X)\ (T0\ Y)\ (T1\ X\ Y\ f))\ T0\ T1\ \mu \\ \wedge \texttt{MetaMonad}\end{gathered}$$

of type o.

Introduction and elimination principles are easy to prove.

Theorem 59. *[*`MetaMonadI`*]*

$$\begin{gathered}(\forall X. Obj\ X \to comp\ (T0\ (T0\ (T0\ X)))\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (T1\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)) \\ = comp\ (T0\ (T0\ (T0\ X)))\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (\mu\ (T0\ X))) \\ \to (\forall X. Obj\ X \to comp\ (T0\ X)\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (\eta\ (T0\ X)) = id\ (T0\ X)) \\ \to (\forall X. Obj\ X \to comp\ (T0\ X)\ (T0\ (T0\ X))\ (T0\ X)\ (\mu\ X)\ (T1\ X\ (T0\ X)\ (\eta\ X)) = id\ (T0\ X)) \\ \to \texttt{MetaMonad}.\end{gathered}$$

Theorem 60. *[*`MetaMonad_strict_I`*]*

$$\begin{array}{c}\texttt{MetaNatTrans\_strict}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\\ (\lambda X.X)\ (\lambda X,Y,f.f)\ T0\ T1\ \eta\\ \to\texttt{MetaNatTrans\_strict}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\\ (\lambda X.T0\ (T0\ X))\\ (\lambda X,Y,f.T1\ (T0\ X)\ (T0\ Y)\ (T1\ X\ Y\ f))\\ T0\ T1\ \mu\\ \to\texttt{MetaMonad}\to\texttt{MetaMonad\_strict}.\end{array}$$

Theorem 61. *[*`MetaMonad_strict_E`*]*

$$\begin{array}{c}\texttt{MetaMonad\_strict}\to\forall p:o.\\ (\texttt{MetaNatTrans\_strict}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\\ (\lambda X.X)\ (\lambda X,Y,f.f)\ T0\ T1\ \eta\\ \to\texttt{MetaNatTrans\_strict}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\ \mathit{Obj}\ \mathit{Hom}\ \mathit{id}\ \mathit{comp}\\ (\lambda X.T0\ (T0\ X))\\ (\lambda X,Y,f.T1\ (T0\ X)\ (T0\ Y)\ (T1\ X\ Y\ f))\\ T0\ T1\ \mu\\ \to\texttt{MetaMonad}\\ \to p)\\ \to p.\end{array}$$

2.12 Adjunctions

Assume we have two metacategories specified by *Obj*, *Hom*, *id*, *comp*, Obj', Hom', id' and $comp'$. A *metaadjunction* is specified by six mathematical objects.

- $F0 : \iota\to\iota$ – the object part of a metafunctor F from the first to the second
- $F1 : \iota\to\iota\to\iota\to\iota$ – the arrow part of a metafunctor F from the first to the second
- $G0 : \iota\to\iota$ – the object part of a metafunctor G from the second to the first
- $G1 : \iota\to\iota\to\iota\to\iota$ – the arrow part of a metafunctor G from the second to the first
- $\eta : \iota \to \iota$ – a meta natural transformation from the identity metafunctor (on the first metacategory) to $G \circ F$.
- $\varepsilon : \iota \to \iota$ – a meta natural transformation from $F \circ G$ to the identity metafunctor (on the second metacategory).

To be a metaadjunction it must satisfy the following:

- For each object X of the first metacategory, $\varepsilon\ (F0\ X) \circ F1 \cdots (\eta\ X) = id'\ (F0\ X)$.
- For each object Y of the second metacategory, $G1 \cdots (\varepsilon\ Y) \circ \eta\ (G0\ Y) = id\ (G0\ Y)$.

We formally define this as `MetaAdjunction` and a strict version as `MetaAdjunction_strict`. Introduction and elimination principles are easy to prove.

Definition 34. *We define* `MetaAdjunction` *to be*

$$\begin{gathered}(\forall X.Obj\ X \rightarrow \\ comp'\ (F0\ X)\ (F0\ (G0\ (F0\ X)))\ (F0\ X)\ (\varepsilon\ (F0\ X))\ (F1\ X\ (G0\ (F0\ X))\ (\eta\ X)) \\ = id'\ (F0\ X)) \\ \wedge(\forall Y.Obj'\ Y \rightarrow \\ comp\ (G0\ Y)\ (G0\ (F0\ (G0\ Y)))\ (G0\ Y)\ (G1\ (F0\ (G0\ Y))\ Y\ (\varepsilon\ Y))\ (\eta\ (G0\ Y)) \\ = id\ (G0\ Y))\end{gathered}$$

of type o.

Definition 35. *We define* `MetaAdjunction_strict` *to be*

$$\begin{gathered}\texttt{MetaFunctor\_strict}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1 \\ \wedge\texttt{MetaFunctor}\ Obj'\ Hom'\ id'\ comp'\ Obj\ Hom\ id\ comp\ G0\ G1 \\ \wedge\texttt{MetaNatTrans}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp \\ (\lambda X.X)\ (\lambda X,Y,f.f) \\ (\lambda X.G0\ (F0\ X))\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\ \eta \\ \wedge\texttt{MetaNatTrans}\ Obj'\ Hom'\ id'\ comp'\ Obj'\ Hom'\ id'\ comp' \\ (\lambda Y.F0\ (G0\ Y))\ (\lambda X,Y,g.F1\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ g)) \\ (\lambda Y.Y)\ (\lambda X,Y,g.g)\ \varepsilon \\ \wedge\texttt{MetaAdjunction}\end{gathered}$$

of type o.

Theorem 62. *[*`MetaAdjunctionI`*]*

$$\begin{gathered}(\forall X.Obj\ X \rightarrow \\ comp'\ (F0\ X)\ (F0\ (G0\ (F0\ X)))\ (F0\ X)\ (\varepsilon\ (F0\ X))\ (F1\ X\ (G0\ (F0\ X))\ (\eta\ X)) \\ = id'\ (F0\ X)) \\ \rightarrow(\forall Y.Obj'\ Y \rightarrow \\ comp\ (G0\ Y)\ (G0\ (F0\ (G0\ Y)))\ (G0\ Y)\ (G1\ (F0\ (G0\ Y))\ Y\ (\varepsilon\ Y))\ (\eta\ (G0\ Y)) \\ = id\ (G0\ Y)) \\ \rightarrow\texttt{MetaAdjunction}.\end{gathered}$$

Theorem 63. *[*`MetaAdjunctionE`*]*

$$\begin{gathered}\texttt{MetaAdjunction}\rightarrow\forall p : o. \\ ((\forall X.Obj\ X \rightarrow \\ comp'\ (F0\ X)\ (F0\ (G0\ (F0\ X)))\ (F0\ X)\ (\varepsilon\ (F0\ X))\ (F1\ X\ (G0\ (F0\ X))\ (\eta\ X)) \\ = id'\ (F0\ X)) \\ \rightarrow(\forall Y.Obj'\ Y \rightarrow \\ comp\ (G0\ Y)\ (G0\ (F0\ (G0\ Y)))\ (G0\ Y)\ (G1\ (F0\ (G0\ Y))\ Y\ (\varepsilon\ Y))\ (\eta\ (G0\ Y)) \\ = id\ (G0\ Y)) \\ \rightarrow p) \\ \rightarrow p.\end{gathered}$$

Theorem 64. [MetaAdjunction_strict_I]

$$\begin{gathered}\texttt{MetaFunctor\_strict}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ \to\texttt{MetaFunctor}\ Obj'\ Hom'\ id'\ comp'\ Obj\ Hom\ id\ comp\ G0\ G1\\ \to\texttt{MetaNatTrans}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp\\ (\lambda X.X)\ (\lambda X,Y,f.f)\\ (\lambda X.G0\ (F0\ X))\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\ \eta\\ \to\texttt{MetaNatTrans}\ Obj'\ Hom'\ id'\ comp'\ Obj'\ Hom'\ id'\ comp'\\ (\lambda Y.F0\ (G0\ Y))\ (\lambda X,Y,g.F1\ (G0\ X)\\ (G0\ Y)\ (G1\ X\ Y\ g))\ (\lambda Y.Y)\ (\lambda X,Y,g.g)\ \varepsilon\\ \to\texttt{MetaAdjunction}\to\texttt{MetaAdjunction\_strict}.\end{gathered}$$

Theorem 65. [MetaAdjunction_strict_E]

$$\begin{gathered}\texttt{MetaAdjunction\_strict}\to\forall p:o.\\ (\texttt{MetaFunctor\_strict}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ \to\texttt{MetaFunctor}\ Obj'\ Hom'\ id'\ comp'\ Obj\ Hom\ id\ comp\ G0\ G1\\ \to\texttt{MetaNatTrans}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp\\ (\lambda X.X)\ (\lambda X,Y,f.f)\\ (\lambda X.G0\ (F0\ X))\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\ \eta\\ \to\texttt{MetaNatTrans}\ Obj'\ Hom'\ id'\ comp'\ Obj'\ Hom'\ id'\ comp'\\ (\lambda Y.F0\ (G0\ Y))\ (\lambda X,Y,g.F1\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ g))\\ (\lambda Y.Y)\ (\lambda X,Y,g.g)\ \varepsilon\\ \to\texttt{MetaAdjunction}\\ \to p)\\ \to p.\end{gathered}$$

A metaadjunction can be used to construct a metamonad. This is tedious to prove formally. The interested reader can study the Megalodon proof.

Theorem 66. [MetaAdjunctionMonad]

$$\begin{gathered}\texttt{MetaFunctor}\ Obj\ Hom\ id\ comp\ Obj'\ Hom'\ id'\ comp'\ F0\ F1\\ \to\texttt{MetaFunctor}\ Obj'\ Hom'\ id'\ comp'\ Obj\ Hom\ id\ comp\ G0\ G1\\ \to\texttt{MetaNatTrans}\ Obj\ Hom\ id\ comp\ Obj\ Hom\ id\ comp\\ (\lambda X.X)\ (\lambda X,Y,f.f)\\ (\lambda X.G0\ (F0\ X))\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\ \eta\\ \to\texttt{MetaNatTrans}\ Obj'\ Hom'\ id'\ comp'\ Obj'\ Hom'\ id'\ comp'\\ (\lambda Y.F0\ (G0\ Y))\ (\lambda X,Y,g.F1\ (G0\ X)\ (G0\ Y)\ (G1\ X\ Y\ g))\\ (\lambda Y.Y)\ (\lambda X,Y,g.g)\ \varepsilon\\ \to\texttt{MetaAdjunction}\\ \to\texttt{MetaMonad}\ Obj\ Hom\ id\ comp\\ (\lambda X.G0\ (F0\ X))\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\\ \eta\ (\lambda X.G1\ (F0\ (G0\ (F0\ X)))\ (F0\ X)\ (\varepsilon\ (F0\ X))).\end{gathered}$$

We can also prove a strict version of the result.

Theorem 67. [MetaAdjunctionMonad_strict]

$$\begin{gathered}\texttt{MetaAdjunction\_strict}\\ \to\texttt{MetaMonad\_strict}\ Obj\ Hom\ id\ comp\\ (\lambda X.G0\ (F0\ X))\ (\lambda X,Y,f.G1\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f))\\ \eta\ (\lambda X.G1\ (F0\ (G0\ (F0\ X)))\ (F0\ X)\ (\varepsilon\ (F0\ X))).\end{gathered}$$

Proof. Use Theorems 65, 48, 51, 58, 57, 60, 55, 49 and 66. □

2.13 Concrete Categories

A *concrete metacategory* is specified by three mathematical objects

- $Obj : \iota \to o$ – giving the objects,
- $U : \iota \to \iota$ – mapping the objects to sets and
- $Hom : \iota \to \iota \to \iota \to o$ giving the arrows

satisfying the following:

- If $f : X \to Y$, then $f \in U\ Y^{U\ X}$. That is, arrows are always set theoretic functions on the sets given by U.
- The identity function $U\ X$ is an arrow for objects X.
- Arrows are closed under composition of functions.

We will not explicitly define concrete metacategories formally, but prove that the conditions are sufficient to give a metacategory. Furthermore, U will give a metafunctor to the metacategory of all sets (which we still have not proven is a metacategory).

Theorem 68. *[*`MetaCatConcrete`*]*

$$\begin{gathered}(\forall X, Y, f. Obj\ X \to Obj\ Y \to Hom\ X\ Y\ f \to f \in U\ Y^{U\ X})\\ \to (\forall X. Obj\ X \to Hom\ X\ X\ (lam\_id\ (U\ X)))\\ \to (\forall X, Y, Z, f, g. Obj\ X \to Obj\ Y \to Obj\ Z \to Hom\ X\ Y\ f \to Hom\ Y\ Z\ g\\ \to Hom\ X\ Z\ (lam\_comp\ (U\ X)\ g\ f))\\ \to \texttt{MetaCat}\ Obj\ Hom\ (\lambda X. lam\_id\ (U\ X))\ (\lambda X, Y, Z, g, f. lam\_comp\ (U\ X)\ g\ f).\end{gathered}$$

Proof. Use Theorem 1 and check the conditions. □

Theorem 69. *[*`MetaCatConcreteForgetful`*]*

$$\begin{gathered}(\forall X, Y, f. Obj\ X \to Obj\ Y \to Hom\ X\ Y\ f \to f \in U\ Y^{U\ X})\\ \to \texttt{MetaFunctor}\ Obj\ Hom\\ (\lambda X. lam\_id\ (U\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ (U\ X)\ f\ g))\ (\lambda \_. True)\\ \texttt{SetHom}\ (\lambda X. lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ U\ (\lambda X, Y, f. f).\end{gathered}$$

Proof. Use Theorem 45 and check the conditions. □

2.14 Categories of Sets II

We can now finally conclude that the metacategories of all sets, hereditarily finite sets and small sets are metacategories. They are each concrete metacategories taking U to be $\lambda X.X$.

Theorem 70. *[*`MetaCatSet`*]*

$$\texttt{MetaCat}\ (\lambda \_. True)\ \texttt{SetHom}\ (\lambda X. lam\_id\ X)\ (\lambda X, Y, Z, g, f. lam\_comp\ X\ g\ f).$$

Proof. Use Theorem 68 with $\lambda X.X$ for U. □

Theorem 71. *[*`MetaCatHFSet`*]*

$$\texttt{MetaCat}\ (\lambda X.X \in \text{UnivOf}\ \mathit{Empty})\ \texttt{SetHom}\ (\lambda X.\mathit{lam\_id}\ X)\ (\lambda X, Y, Z, f, g.(\mathit{lam\_comp}\ X\ f\ g)).$$

Proof. Use Theorem 68 with $\lambda X.X$ for U. □

Theorem 72. *[*`MetaCatSmallSet`*]*

$$\texttt{MetaCat}\ (\lambda X.X \in \text{UnivOf}\ (\text{UnivOf}\ \mathit{Empty}))\ \texttt{SetHom}\ (\lambda X.\mathit{lam\_id}\ X)\ (\lambda X, Y, Z, f, g.(\mathit{lam\_comp}\ X\ f$$

Proof. Use Theorem 68 with $\lambda X.X$ for U. □

We can now prove the strict version of Theorem 69.

Theorem 73. *[*`MetaCatConcreteForgetful_strict`*]*

$$\begin{gathered}
\forall \mathit{Obj} : \iota{\to}o.\forall U : \iota{\to}\iota.\forall \mathit{Hom} : \iota{\to}\iota{\to}\iota{\to}o. \\
(\forall X, Y, f.\mathit{Obj}\ X{\to}\mathit{Obj}\ Y{\to}\mathit{Hom}\ X\ Y\ f{\to}f \in U\ Y^{U\ X}) \\
{\to}(\forall X.\mathit{Obj}\ X{\to}\mathit{Hom}\ X\ X\ (\mathit{lam_id}\ (U\ X))) \\
{\to}(\forall X, Y, Z, f, g.\mathit{Obj}\ X{\to}\mathit{Obj}\ Y{\to}\mathit{Obj}\ Z{\to}\mathit{Hom}\ X\ Y\ f{\to}\mathit{Hom}\ Y\ Z\ g \\
{\to}\mathit{Hom}\ X\ Z\ (\mathit{lam_comp}\ (U\ X)\ g\ f)) \\
{\to}\texttt{MetaFunctor_strict}\ \mathit{Obj}\ \mathit{Hom} \\
(\lambda X.\mathit{lam_id}\ (U\ X))\ (\lambda X, Y, Z, f, g.(\mathit{lam_comp}\ (U\ X)\ f\ g)) \\
(\lambda_.\mathit{True})\ \texttt{SetHom} \\
(\lambda X.\mathit{lam_id}\ X)\ (\lambda X, Y, Z, f, g.(\mathit{lam_comp}\ X\ f\ g)) \\
U\ (\lambda X, Y, f.f).
\end{gathered}$$

Proof. Use Theorems 47, 68, 70 and 69. □

2.15 Categories of Structures

We now turn to the kinds of concrete metacategories that will supply the bulk of our examples. The objects will be given by a "structure" given by an $n+1$-tuple $(A, c_1, \ldots, c_n)$ where A is the carrier set and each c_i encodes either an element of A, a unary or binary predicate on A, a unary or binary operation on A, or a collection of subsets of A. We adopt the following conventions: `e` means element, `p` means (unary) predicate, `r` means (binary) relation, `u` means unary operation, `b` means binary operation and `c` means collection of subsets of A. In general we will write suffixes with these codes to indicate the signature of the structure. For example, `_b_b_e` means the structure is a 4-tuple (A, f, g, c) where f and g encode binary operations on A and c is an element of A. The basic infrastructure of structures of various signatures have already been formalized in Megalodon and published into the Proofgold blockchain. We can simply make use of them here. In particular, for a given signature with suffix s we use we assume we already have the following:

- `struct_`s : $\iota \to o$ – giving a predicate that is true on such structures.
- `pack_`s : $\alpha_1 \to \ldots \to \alpha_n \to \iota$ – a function to construct a structure given the information. The value of n and the types α_i depend on the signature.
- `unpack_`s`i` : $\iota \to (\alpha_1 \to \ldots \to \alpha_n \to \iota) \to \iota$ – a matching construct useful for defining sets depending on structures.

- `unpack_so` : $\iota \to (\alpha_1 \to \ldots \to \alpha_n \to \iota) \to o$ – a matching construct useful for defining propositions depending on structures.

A number of results have also been previously proven an published. We will freely use these below when necessary.

The signatures we will consider are indicated by the following suffixes:

- `e` : an element of the carrier (giving pointed sets).
- `p` : a unary predicate on the carrier.
- `r` : a binary relation on the carrier.
- `u` : a unary function on the carrier.
- `b` : a binary operation on the carrier.
- `c` : a collection of subsets of the carrier.
- `b_b_e` : two binary operations on the carrier and an element of the carrier.
- `b_b_e_e` : two binary operations on the carrier and two elements of the carrier.
- `b_b_r_e_e` : two binary operations on the carrier, a binary relation on the carrier and two elements of the carrier.

For each of these signatures we define `Hom_struct_`s of type $\iota \to \iota \to \iota \to o$ that will give homorphisms as functions preserving the structure. These will provide the Hom value for the concrete metacategories.

Definition 36. *We define* `Hom_struct_e` *to be*

$$\lambda X, Y, f.unpack\_e\_o\ X\ (\lambda X', eX.unpack\_e\_o\ Y(\lambda Y', eY.\\ f \in {Y'}^{X'} \wedge f\ eX{=}eY))$$

of type $\iota{\to}\iota{\to}\iota{\to}o$*.*

Definition 37. *We define* `Hom_struct_u` *to be*

$$\lambda X, Y, f.unpack\_u\_o\ X\ (\lambda X', uX.unpack\_u\_o\ Y\ (\lambda Y', uY.\\ f \in {Y'}^{X'} \wedge \forall x \in X'.f\ (uX\ x){=}uY\ (f\ x)))$$

of type $\iota{\to}\iota{\to}\iota{\to}o$*.*

Definition 38. *We define* `Hom_struct_b` *to be*

$$\lambda X, Y, f.unpack\_b\_o\ X\ (\lambda X', opX.unpack\_b\_o\ Y\ (\lambda Y', opY.\\ f \in {Y'}^{X'} \wedge \forall x, y \in X'.f\ (opX\ x\ y){=}opY\ (f\ x)\ (f\ y)))$$

of type $\iota{\to}\iota{\to}\iota{\to}o$*.*

Definition 39. *We define* `Hom_struct_p` *to be*

$$\lambda X, Y, f.unpack\_p\_o\ X\ (\lambda X', pX.unpack\_p\_o\ Y\ (\lambda Y', pY.\\ f \in {Y'}^{X'} \wedge \forall x \in X'.pX\ x{\to}pY\ (f\ x)))$$

of type $\iota{\to}\iota{\to}\iota{\to}o$*.*

Definition 40. *We define* `Hom_struct_r` *to be*

$$\lambda X, Y, f.unpack\_r\_o\ X\ (\lambda X', rX.unpack\_r\_o\ Y\ (\lambda Y', rY.\\ f \in {Y'}^{X'} \wedge \forall x, y \in X'.rX\ x\ y \to rY\ (f\ x)\ (f\ y)))$$

of type $\iota \to \iota \to \iota \to o$.

Definition 41. *We define* `Hom_struct_c` *to be*

$$\lambda X, Y, f.unpack\_c\_o\ X\ (\lambda X', CX.unpack\_c\_o\ Y\ (\lambda Y', CY.\\ f \in {Y'}^{X'} \wedge \forall U : \iota \to o.(\forall y.U\ y \to y \in Y') \to CY\ U \to CX\ (\lambda x.x \in X' \wedge U\ (f\ x))))$$

of type $\iota \to \iota \to \iota \to o$.

Definition 42. *We define* `Hom_struct_b_b_e` *to be*

$$\lambda X, Y, f.unpack\_b\_b\_e\_o\ X\ (\lambda X', opX, op2X, eX.\\ unpack\_b\_b\_e\_o\ Y\ (\lambda Y', opY, op2Y, eY.\\ f \in {Y'}^{X'}\\ \wedge(\forall x, y \in X'.f\ (opX\ x\ y) = opY\ (f\ x)\ (f\ y))\\ \wedge(\forall x, y \in X'.f\ (op2X\ x\ y) = op2Y\ (f\ x)\ (f\ y))\\ \wedge f\ eX = eY))$$

of type $\iota \to \iota \to \iota \to o$.

Definition 43. *We define* `Hom_struct_b_b_e_e` *to be*

$$\lambda X, Y, f.unpack\_b\_b\_e\_e\_o\ X\ (\lambda X', opX, op2X, eX, e2X.\\ unpack\_b\_b\_e\_e\_o\ Y\ (\lambda Y', opY, op2Y, eY, e2Y.\\ f \in {Y'}^{X'}\\ \wedge(\forall x, y \in X'.f\ (opX\ x\ y) = opY\ (f\ x)\ (f\ y))\\ \wedge(\forall x, y \in X'.f\ (op2X\ x\ y) = op2Y\ (f\ x)\ (f\ y))\\ \wedge f\ eX = eY \wedge f\ e2X = e2Y))$$

of type $\iota \to \iota \to \iota \to o$.

Definition 44. *We define* `Hom_struct_b_b_r_e_e` *to be*

$$\lambda X, Y, f.unpack\_b\_b\_r\_e\_e\_o\ X\ (\lambda X', opX, op2X, rX, eX, e2X.\\ unpack\_b\_b\_r\_e\_e\_o\ Y\ (\lambda Y', opY, op2Y, rY, eY, e2Y.\\ f \in {Y'}^{X'}\\ \wedge(\forall x, y \in X'.f\ (opX\ x\ y) = opY\ (f\ x)\ (f\ y))\\ \wedge(\forall x, y \in X'.f\ (op2X\ x\ y) = op2Y\ (f\ x)\ (f\ y))\\ \wedge(\forall x, y \in X'.rX\ x\ y \to rY\ (f\ x)\ (f\ y))\\ \wedge f\ eX = eY\\ \wedge f\ e2X = e2Y))$$

of type $\iota \to \iota \to \iota \to o$.

We next prove the definitions above behave as we wish when applied structures explicitly given using `pack_`s. The proofs are not difficult, but rely on previously proven relationships between `pack_`s and `unpack_`s`o`. We omit the details here.

Theorem 74. [`Hom_struct_e_pack`]

$$\begin{gathered}\forall X, Y, eX, eY, f.\\ (\texttt{Hom\_struct\_e}\ (pack\_e\ X\ eX)\ (pack\_e\ Y\ eY)\ f)\\ =(f \in Y^X \wedge f\ eX = eY).\end{gathered}$$

Theorem 75. [`Hom_struct_u_pack`]

$$\begin{gathered}\forall X, Y. \forall opX, opY : \iota \to \iota. \forall f.\\ (\texttt{Hom\_struct\_u}\ (pack\_u\ X\ opX)\ (pack\_u\ Y\ opY)\ f)\\ =(f \in Y^X \wedge (\forall x \in X. f\ (opX\ x) = opY\ (f\ x))).\end{gathered}$$

Theorem 76. [`Hom_struct_b_pack`]

$$\begin{gathered}\forall X, Y. \forall opX, opY : \iota \to \iota \to \iota. \forall f.\\ (\texttt{Hom\_struct\_b}\ (pack\_b\ X\ opX)\ (pack\_b\ Y\ opY)\ f)\\ =(f \in Y^X \wedge (\forall x, y \in X. f\ (opX\ x\ y) = opY\ (f\ x)\ (f\ y))).\end{gathered}$$

Theorem 77. [`Hom_struct_p_pack`]

$$\begin{gathered}\forall X, Y. \forall pX, pY : \iota \to o. \forall f.\\ (\texttt{Hom\_struct\_p}\ (pack\_p\ X\ pX)\ (pack\_p\ Y\ pY)\ f)\\ =(f \in Y^X \wedge (\forall x \in X. pX\ x \to pY\ (f\ x))).\end{gathered}$$

Theorem 78. [`Hom_struct_r_pack`]

$$\begin{gathered}\forall X, Y. \forall rX, rY : \iota \to \iota \to o. \forall f.\\ (\texttt{Hom\_struct\_r}\ (pack\_r\ X\ rX)\ (pack\_r\ Y\ rY)\ f)\\ =(f \in Y^X \wedge (\forall x, y \in X. rX\ x\ y \to rY\ (f\ x)\ (f\ y))).\end{gathered}$$

Theorem 79. [`Hom_struct_c_pack`]

$$\begin{gathered}\forall X, Y. \forall CX, CY : (\iota \to o) \to o. \forall f.\\ (\texttt{Hom\_struct\_c}\ (pack\_c\ X\ CX)\ (pack\_c\ Y\ CY)\ f)\\ =(f \in Y^X \wedge (\forall U : \iota \to o. (\forall y. U\ y \to y \in Y) \to CY\ U \to CX\ (\lambda x. x \in X \wedge U\ (f\ x)))).\end{gathered}$$

Theorem 80. [`Hom_struct_b_b_e_pack`]

$$\begin{gathered}\forall X, Y. \forall opX, op2X, opY, op2Y : \iota \to \iota \to \iota. \forall eX, eY, f.\\ (\texttt{Hom\_struct\_b\_b\_e}\ (pack\_b\_b\_e\ X\ opX\ op2X\ eX)\ (pack\_b\_b\_e\ Y\ opY\ op2Y\ eY)\ f)\\ =(f \in Y^X\\ \wedge (\forall x, y \in X. f\ (opX\ x\ y) = opY\ (f\ x)\ (f\ y))\\ \wedge (\forall x, y \in X. f\ (op2X\ x\ y) = op2Y\ (f\ x)\ (f\ y))\\ \wedge f\ eX = eY).\end{gathered}$$

Theorem 81. [`Hom_struct_b_b_e_e_pack`]

$$\begin{gathered}\forall X, Y. \forall opX, op2X, opY, op2Y : \iota \to \iota \to \iota. \forall eX, e2X, eY, e2Y, f.\\ (\texttt{Hom\_struct\_b\_b\_e\_e}\ (pack\_b\_b\_e\_e\ X\ opX\ op2X\ eX\ e2X)\\ (pack\_b\_b\_e\_e\ Y\ opY\ op2Y\ eY\ e2Y)\ f)\\ =(f \in Y^X\\ \wedge (\forall x, y \in X. f\ (opX\ x\ y) = opY\ (f\ x)\ (f\ y))\\ \wedge (\forall x, y \in X. f\ (op2X\ x\ y) = op2Y\ (f\ x)\ (f\ y))\\ \wedge f\ eX = eY \wedge f\ e2X = e2Y).\end{gathered}$$

Theorem 82. [Hom_struct_b_b_r_e_e_pack]

$$\begin{array}{c}\forall X,Y.\forall opX,op2X,opY,op2Y:\iota\to\iota\to\iota.\forall rX,rY:\iota\to\iota\to o.\forall eX,e2X,eY,e2Y,f.\\ (\mathtt{Hom\_struct\_b\_b\_r\_e\_e}\ (pack\_b\_b\_r\_e\_e\ X\ opX\ op2X\ rX\ eX\ e2X)\\ (pack\_b\_b\_r\_e\_e\ Y\ opY\ op2Y\ rY\ eY\ e2Y)\ f)\\ =(f\in Y^X\\ \wedge(\forall x,y\in X.f\ (opX\ x\ y)=opY\ (f\ x)\ (f\ y))\\ \wedge(\forall x,y\in X.f\ (op2X\ x\ y)=op2Y\ (f\ x)\ (f\ y))\\ \wedge(\forall x,y\in X.rX\ x\ y\to rY\ (f\ x)\ (f\ y))\\ \wedge f\ eX=eY\wedge f\ e2X=e2Y).\end{array}$$

Next for each signature we prove a generic theorem stating that we obtain a (concrete) metacategory if we select some of the structures to be the objects. The U in each case will be the function taking the structure X to its carrier set $X\ 0$. (Recall that X is an $n+1$-tuple so that applying X to 0 will give the zeroth element of the $n+1$-tuple, which is always the carrier set.) We also prove this U gives a (forgetful) metafunctor.

In each case below assume we have some arbitrary $Obj:\iota\to o$.

Theorem 83. [MetaCat_struct_e_gen]

$$\begin{array}{c}(\forall X.Obj\ X\to struct\_e\ X)\\ \to\mathtt{MetaCat}\ Obj\ \mathtt{Hom\_struct\_e}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).\end{array}$$

Proof. Use Theorems 68 and 74. ☐

Theorem 84. [MetaCat_struct_e_Forgetful_gen]

$$\begin{array}{c}(\forall X.Obj\ X\to struct\_e\ X)\\ \to\mathtt{MetaFunctor}\ Obj\ \mathtt{Hom\_struct\_e}\\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)\\ (\lambda\_.True)\ \mathtt{SetHom}\\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).\end{array}$$

Proof. Use Theorems 74 and 69. ☐

Theorem 85. [MetaCat_struct_p_gen]

$$\begin{array}{c}(\forall X.Obj\ X\to struct\_p\ X)\\ \to\mathtt{MetaCat}\ Obj\ \mathtt{Hom\_struct\_p}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).\end{array}$$

Proof. Use Theorems 68 and 77. ☐

Theorem 86. [MetaCat_struct_p_Forgetful_gen]

$$\begin{array}{c}(\forall X.Obj\ X\to struct\_p\ X)\\ \to\mathtt{MetaFunctor}\ Obj\ \mathtt{Hom\_struct\_p}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)\\ (\lambda\_.True)\ \mathtt{SetHom}\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).\end{array}$$

Proof. Use Theorems 77 and 69. ☐

Theorem 87. *[*`MetaCat_struct_r_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_r\ X)$$
$${\rightarrow}\texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_r}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 78. □

Theorem 88. *[*`MetaCat_struct_r_Forgetful_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_r\ X)$$
$${\rightarrow}\texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_r}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 78 and 69. □

Theorem 89. *[*`MetaCat_struct_u_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_u\ X)$$
$${\rightarrow}\texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_u}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 75. □

Theorem 90. *[*`MetaCat_struct_u_Forgetful_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_u\ X)$$
$${\rightarrow}\texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_u}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 75 and 69. □

Theorem 91. *[*`MetaCat_struct_b_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_b\ X)$$
$${\rightarrow}\texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_b}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 76. □

Theorem 92. *[*`MetaCat_struct_b_Forgetful_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_b\ X)$$
$${\rightarrow}\texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_b}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 76 and 69. □

Theorem 93. *[*`MetaCat_struct_c_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_c\ X)$$
$${\rightarrow}\texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_c}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 79. □

Theorem 94. *[*`MetaCat_struct_c_Forgetful_gen`*]*

$$(\forall X.Obj\ X{\rightarrow}struct\_c\ X)$$
$${\rightarrow}\texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_c}\ (\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 79 and 69. □

Theorem 95. [`MetaCat_struct_b_b_e_gen`]

$$(\forall X.Obj\ X \to struct\_b\_b\_e\ X)$$
$$\to \texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_b\_b\_e}$$
$$(\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 80. □

Theorem 96. [`MetaCat_struct_b_b_e_Forgetful_gen`]

$$(\forall X.Obj\ X \to struct\_b\_b\_e\ X)$$
$$\to \texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_b\_b\_e}$$
$$(\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}\ (\lambda X.lam\_id\ X)$$
$$(\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 80 and 69. □

Theorem 97. [`MetaCat_struct_b_b_e_e_gen`]

$$(\forall X.Obj\ X \to struct\_b\_b\_e\_e\ X)$$
$$\to \texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_b\_b\_e\_e}$$
$$(\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 81. □

Theorem 98. [`MetaCat_struct_b_b_e_e_Forgetful_gen`]

$$(\forall X.Obj\ X \to struct\_b\_b\_e\_e\ X)$$
$$\to \texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_b\_b\_e\_e}$$
$$(\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}\ (\lambda X.lam\_id\ X)$$
$$(\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 81 and 69. □

Theorem 99. [`MetaCat_struct_b_b_r_e_e_gen`]

$$(\forall X.Obj\ X \to struct\_b\_b\_r\_e\_e\ X)$$
$$\to \texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_b\_b\_r\_e\_e}$$
$$(\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f).$$

Proof. Use Theorems 68 and 82. □

Theorem 100. [`MetaCat_struct_b_b_r_e_e_Forgetful_gen`]

$$(\forall X.Obj\ X \to struct\_b\_b\_r\_e\_e\ X)$$
$$\to \texttt{MetaFunctor}\ Obj\ \texttt{Hom\_struct\_b\_b\_r\_e\_e}$$
$$(\lambda X.lam\_id\ (X\ 0))\ (\lambda X,Y,Z,g,f.lam\_comp\ (X\ 0)\ g\ f)$$
$$(\lambda\_.True)\ \texttt{SetHom}(\lambda X.lam\_id\ X)$$
$$(\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. Use Theorems 82 and 69. □

To make the presentation of metacategories of structures more concise we make two more definitions, given here as Megalodon definitions:

```
Definition struct_id : set -> set
   := fun X => lam_id (X 0).
Definition struct_comp : set -> set -> set -> set -> set -> set
   := fun X Y Z f g => lam_comp (X 0) f g.
```

Using these definitions, Theorem 83 converts to

$$(\forall X. Obj\ X \rightarrow struct\_e\ X)$$
$$\rightarrow \texttt{MetaCat}\ Obj\ \texttt{Hom\_struct\_e}\ \texttt{struct\_id}\ \texttt{struct\_comp}.$$

This is typically how propositions we will write propositions from now on.

Chapter 3

Categories of Sets, Small Sets and Hereditarily Finite Sets

For the metacategories of all sets, small sets and hereditarily finite sets, there are yet unproven conjectures corresponding to the remaining constructions of limits, colimits, exponents, a subobject classifier and a natural numbers object. The metacategory of hereditarily finite sets will not have a natural numbers object so resolving the conjecture involves proving the negation, i.e., proving false under the assumption there is a natural numbers object. The bounty can be claimed by proving either the conjecture or its negation.

Conjecture 1.

$$\begin{gathered}\forall Obj : \iota\to o.(\forall X.Obj\ X\to\forall Q \subseteq X.Obj\ Q)\\ \to\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ coequalizer\_constr\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMGzQMhpn8ck6dyJBxVratv6nYVQDcgUENU
Bounty amount: approximately 125 bars

Conjecture 2.

$$\begin{gathered}\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ coequalizer\_constr\_p\ (\lambda\_.True)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMNYeCbbgpFo4jw6wDNDmrQ2PJEQjvPwuCz
Bounty amount: approximately 125 bars

Conjecture 3.

$$\begin{gathered}\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ coequalizer\_constr\_p\ (\lambda X.X \in UnivOf\ Empty)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMJqmUUJbD9FqCAPKpe8oEBu8Fimq3zb8HL
Bounty amount: approximately 125 bars

Conjecture 4.

$$\begin{gathered}\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ coequalizer\_constr\_p\ (\lambda X.X \in UnivOf\ (UnivOf\ Empty))\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMLfDT23ATtV8dHsuxVzWCBMJFoyHqBhFbC
Bounty amount: approximately 125 bars

Conjecture 5.

$$\begin{gathered}\forall Obj : \iota\to o.(\forall X.Obj\ X\to\forall Q \subseteq X.Obj\ Q)\\ \to\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ equalizer\_constr\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMammcWFp9eXqrm4GXk1Cnt4jtXNAuJs6Xk
Bounty amount: approximately 125 bars

Conjecture 6.

$$\begin{gathered}\forall Obj : \iota\to o.(\forall X.Obj\ X\to\forall Q \subseteq X.Obj\ Q)\\ \to\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ equalizer\_constr\_p\ (\lambda X.X \in UnivOf\ Empty)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMWXsQaTKuXV5W3VA2ccPb9r2pXJTYonedD
Bounty amount: approximately 125 bars

Conjecture 7.

$$\begin{gathered}\forall Obj : \iota\to o.(\forall X.Obj\ X\to\forall Q \subseteq X.Obj\ Q)\\ \to\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ equalizer\_constr\_p\ (\lambda X.X \in UnivOf\ (UnivOf\ Empty))\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X, Y, Z, f, g.(\lambda x \in X.f\ (g\ x)))\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMQo9idLgVuBRTKPr3jSZ5giFAJvWqjUuBC
Bounty amount: approximately 125 bars

Conjecture 8.

$$\begin{gathered}\forall Obj : \iota\to o. MetaCat\ Obj\ SetHom\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ \to(\forall X.Obj\ X\to\forall Q \subseteq X.Obj\ Q)\to(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (setprod\ X\ Y))\\ \to\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pullback\_constr\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ pb\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proofgold proposition address: TMSDwc5BEADA1KmCWXqDmvNedvSTpMptbPB
Bounty amount: approximately 250 bars

Conjecture 9.

$$\begin{gathered}\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pullback\_constr\_p\ (\lambda\_.True)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ pb\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proofgold proposition address: TMJDadmQ65GBynJ58sqwoV69gWpCpQnKUhe
Bounty amount: approximately 250 bars

Conjecture 10.

$$\begin{gathered}\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pullback\_constr\_p\ (\lambda X.X \in UnivOf\ Empty)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ pb\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proofgold proposition address: TMTP7ZHoKpLHnvoTAGxBEndefpYosi1rBfv
Bounty amount: approximately 250 bars

Conjecture 11.

$$\begin{gathered}\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pullback\_constr\_p\ (\lambda X.X \in UnivOf\ (UnivOf\ Empty))\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ pb\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proofgold proposition address: TMQWGS2nY9ULiEBRV3aNKgGM4bxm9Unug6b
Bounty amount: approximately 250 bars

Conjecture 12.

$$\begin{gathered}\forall Obj : \iota\to o. MetaCat\ Obj\ SetHom\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ \to(\forall X.Obj\ X\to\forall Q \subseteq X.Obj\ Q)\to(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (setsum\ X\ Y))\\ \to\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pushout\_constr\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ po\ i0\ i1\ copair.\end{gathered}$$

Proofgold proposition address: TMV38ja91ikyogEtvZsXASzEe4V9WjckV63
Bounty amount: approximately 250 bars

Conjecture 13.

$$\begin{gathered}\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pushout\_constr\_p\ (\lambda\_.True)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ po\ i0\ i1\ copair.\end{gathered}$$

Proofgold proposition address: TMH4HNFv8xWohQG7sa2kjRXnmJLQ7vg6cyM
Bounty amount: approximately 250 bars

Conjecture 14.

$$\begin{gathered}\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pushout\_constr\_p\ (\lambda X.X \in UnivOf\ Empty)\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ po\ i0\ i1\ copair.\end{gathered}$$

Proofgold proposition address: TMR9h5uWj9NUTYyUy9WF7JykMqEevDe825j
Bounty amount: approximately 250 bars

Conjecture 15.

$$\begin{gathered}\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ pushout\_constr\_p\ (\lambda X.X \in UnivOf\ (UnivOf\ Empty))\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ po\ i0\ i1\ copair.\end{gathered}$$

Proofgold proposition address: TMPUcBde6Q2eqt26tX9hjAwY8giJuFiCtDG
Bounty amount: approximately 250 bars

Conjecture 16.

$$\begin{gathered}\forall Obj : \iota\to o.(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (setprod\ X\ Y))\\ \to(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (Y^X))\\ \to product\_exponent\_constr\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x \in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x \in X.f\ (g\ x)))\\ setprod\\ (\lambda X,Y.(\lambda z \in X\times Y.z\ 0))\ (\lambda X,Y.(\lambda z \in X\times Y.z\ 1))\\ (\lambda X,Y,W,h,k.(\lambda w \in W.(h\ w,k\ w)))\\ (\lambda X,Y.Y^X)\\ (\lambda X,Y.(\lambda fx \in (Y^X)\times X.fx\ 0\ (fx\ 1)))\\ (\lambda X,Y,W,f.(\lambda w \in W.\lambda x \in X.f\ (w,x))).\end{gathered}$$

Proofgold proposition address: TMThKdA3AsHcGrYMxsKbp2FfdhzjBiDY2F1
Bounty amount: approximately 250 bars

Conjecture 17.

$$\begin{gathered}\forall Obj:\iota\to o.(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (setprod\ X\ Y))\\ \to(\forall X.Obj\ X\to\forall Y.Obj\ Y\to Obj\ (Y^X))\\ \to\exists prod:\iota\to\iota\to\iota.\exists\pi_1,\pi_2:\iota\to\iota\to\iota.\\ \exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists exp:\iota\to\iota\to\iota.\exists a:\iota\to\iota\to\iota.\\ \exists lm:\iota\to\iota\to\iota\to\iota\to\iota.\\ product\_exponent\_constr\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{gathered}$$

Proofgold proposition address: TMah8YAC66gD3gAn7EUQm1XCQnkzMgFzGg5
Bounty amount: approximately 250 bars

Conjecture 18.

$$\begin{gathered}\exists prod:\iota\to\iota\to\iota.\exists\pi_1,\pi_2:\iota\to\iota\to\iota.\\ \exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists exp:\iota\to\iota\to\iota.\exists a:\iota\to\iota\to\iota.\\ \exists lm:\iota\to\iota\to\iota\to\iota\to\iota.\\ product\_exponent\_constr\_p\ (\lambda\_.True)\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{gathered}$$

Proofgold proposition address: TMdKbr822oovftV3M28uVDTtAspBzJDZeKh
Bounty amount: approximately 250 bars

Conjecture 19.

$$\begin{gathered}\exists prod:\iota\to\iota\to\iota.\exists\pi_1,\pi_2:\iota\to\iota\to\iota.\\ \exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists exp:\iota\to\iota\to\iota.\exists a:\iota\to\iota\to\iota.\\ \exists lm:\iota\to\iota\to\iota\to\iota\to\iota.\\ product\_exponent\_constr\_p\ (\lambda X.X\in UnivOf\ Empty)\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{gathered}$$

Proofgold proposition address: TMdmRsimtqAGYGn6aFtE8HegkRkKEmSMJZW
Bounty amount: approximately 250 bars

Conjecture 20.

$$\begin{gathered}\forall Obj:\iota\to o.\forall X,Y.\forall f:\iota.\\ monic\ Obj\ SetHom\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\ X\ Y\ f\\ \to\forall u,v\in X.f\ u=f\ v\to u=v.\end{gathered}$$

Proofgold proposition address: TMaV5KXL7pmZ9ioxeQni9ueRtjvhEmp6TAn
Bounty amount: approximately 125 bars

Conjecture 21.

$$\begin{gathered}\forall Obj:\iota\to o.Obj\ 1\to Obj\ 2\\ \to subobject\_classifier\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ 1\ (\lambda X:\iota.(\lambda x\in X.0))\ 2\ (\lambda\_\in 1.1)\\ (\lambda X,Y:\iota.\lambda m:\iota.(\lambda y\in Y.\mathbf{if}\ (\exists x\in X.m\ x=y)\ \mathbf{then}\ 1\ \mathbf{else}\ 0))\\ (\lambda X,Y:\iota.\lambda m:\iota.\lambda W:\iota.\lambda h,k:\iota.(\lambda w\in W.inv\ X\ (\lambda x.m\ x)\ (k\ w))).\end{gathered}$$

Proofgold proposition address: TMLy4ydWkhFQrhFEQ58sbgf43JCJZQGbScY
Bounty amount: approximately 250 bars

Conjecture 22.

$$\begin{array}{c}\forall Obj:\iota\to o.Obj\ 1\to Obj\ 2\\ \to\exists one:\iota.\exists uniqa:\iota\to\iota.\exists Omega:\iota.\exists tru:\iota.\exists ch:\iota\to\iota\to\iota\to\iota.\\ \exists constr:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{array}$$

Proofgold proposition address: TMP86ZJT6ySKNVBo34KDLLx4UTSXQGqEvGC
Bounty amount: approximately 250 bars

Conjecture 23.

$$\begin{array}{c}\exists one:\iota.\exists uniqa:\iota\to\iota.\exists Omega:\iota.\exists tru:\iota.\exists ch:\iota\to\iota\to\iota\to\iota.\\ \exists constr:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ (\lambda\_.True)\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{array}$$

Proofgold proposition address: TMSLscVf86HGZthk3uGdGSSgZsQGvc5WFD3
Bounty amount: approximately 250 bars

Conjecture 24.

$$\begin{array}{c}\exists one:\iota.\exists uniqa:\iota\to\iota.\exists Omega:\iota.\exists tru:\iota.\exists ch:\iota\to\iota\to\iota\to\iota.\\ \exists constr:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ (\lambda X.X\in UnivOf\ Empty)\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{array}$$

Proofgold proposition address: TMUBwRJV23Axv6tUgLuk3yUdbrrkUBnnpAE
Bounty amount: approximately 250 bars

Conjecture 25.

$$\begin{array}{c}\exists one:\iota.\exists uniqa:\iota\to\iota.\exists Omega:\iota.\exists tru:\iota.\exists ch:\iota\to\iota\to\iota\to\iota.\\ \exists constr:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ (\lambda X.X\in UnivOf\ (UnivOf\ Empty))\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{array}$$

Proofgold proposition address: TMbRRcrJNhrxLTCxqYxPoxJFYe5fG2nr4wr
Bounty amount: approximately 250 bars

Conjecture 26.

$$\begin{array}{c}\forall Obj:\iota\to o.Obj\ 1\to Obj\ omega\\ \to nno\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ 1\ (\lambda X:\iota.(\lambda x\in X.0))\ omega\ (\lambda\_\in 1.0)\ (\lambda n\in omega.ordsucc\ n)\\ (\lambda X:\iota.\lambda x,f:\iota.(\lambda n\in omega.nat\_primrec\ (x\ 0)\ (\lambda\_,v.f\ v)\ n)).\end{array}$$

Proofgold proposition address: TMaB8f73oDN7KoEhAyBEGuRhR7uutjLxyXg
Bounty amount: approximately 250 bars

Conjecture 27.

$$\begin{gathered}\forall Obj:\iota\to o.Obj\ 1\to Obj\ omega\\ \to\exists one:\iota.\exists uniqa:\iota\to\iota.\exists N:\iota.\exists zer,suc:\iota.\exists rec:\iota\to\iota\to\iota\to\iota.\\ nno\_p\ Obj\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ N\ zer\ suc\ rec.\end{gathered}$$

Proofgold proposition address: TMRDjRCdvyGMF98V7Fm1VeMRiJUJswMti9D
Bounty amount: approximately 250 bars

Conjecture 28.

$$\begin{gathered}\exists one:\iota.\exists uniqa:\iota\to\iota.\exists N:\iota.\exists zer,suc:\iota.\exists rec:\iota\to\iota\to\iota\to\iota.\\ nno\_p\ (\lambda\_.True)\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ N\ zer\ suc\ rec.\end{gathered}$$

Proofgold proposition address: TMHfQvvq4MrYBbqyb5GWgYeFEXGbyd51tQz
Bounty amount: approximately 250 bars

Conjecture 29.

$$\begin{gathered}\exists one:\iota.\exists uniqa:\iota\to\iota.\exists N:\iota.\exists zer,suc:\iota.\exists rec:\iota\to\iota\to\iota\to\iota.\\ nno\_p\ (\lambda X.X\in UnivOf\ (UnivOf\ Empty))\ SetHom\\ (\lambda X.(\lambda x\in X.x))\ (\lambda X,Y,Z,f,g.(\lambda x\in X.f\ (g\ x)))\\ one\ uniqa\ N\ zer\ suc\ rec.\end{gathered}$$

Proofgold proposition address: TMXouZ5TCuZXX4MJv6mbcsXKGfW9U4TMgEe
Bounty amount: approximately 250 bars

Chapter 4

Structures with an Element

In this chapter we consider the metacategory of structures with an element of the carrier (pointed sets). We will follow a general pattern at this point of introducing a metacategory of structures, proving the proposed metacategory is a metacategory and that the forgetful metafunctor given by $\lambda X.X\ 0$ is a metafunctor to the metacategory of all sets. The remaining propositions will correspond to constructions of limits, colimits, etc., with the exception of what is usually the last proposition. The last proposition will typically asssert the existence of a left adjoint to the forgetful metafunctor. The construction of such an adjoint is arguably the most interest proposition mathematically and so when this is left as a conjecture the bounty is higher.

Using Theorems 83 and 84 we can easily prove we have a metacategory and forgetful metafunctor.

Theorem 101. *[*`MetaCat_struct_e`*]* $MetaCat\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp.$

Proof. Exact $MetaCat\_struct\_e\_gen\ struct\_e\ (\lambda X, H.H)$. □

Theorem 102. *[*`MetaCat_struct_e_Forgetful`*]*

$$\begin{gathered} MetaFunctor\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_e\_Forgetful\_gen\ struct\_e\ (\lambda X, H.H)$. □

In the remaining presentations of metacategories we will begin to assert conjectures (with bounties) at this point. In this case we include a proof that this metacategory has an initial object. The initial object is given by the pointed set with carrier 1 and element $0 \in 1$.

Theorem 103. *[*`MetaCat_struct_e_initial`*]*

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota{\to}\iota. \\ initial\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proof. We use $pack\_e\ 1\ 0$ to witness the existential quantifier for the object. We also need a function giving a unique arrow to other objects. Other objects X

will be structures with carrier set $X\ 0$ and selected element $X\ 1$. The obvious choice to take is the (set-level) function taking the only element of 1 to the selected element $X\ 1$. Thus we witness the second existential quantifier by the term

$$(\lambda X.\lambda x \in 1.X\ 1).$$

Using the fact `pack_struct_e_I` from the library we immediatly know struct_e (pack_e 1 0) holds so that we have an object of the metacategory. Next let X be an object, i.e., a set satisfying struct_e X. By the definition of struct_e we know X is (X', e) for some set X' and some $e \in X'$. In particular $X\ 0 = X'$ and $X\ 1 = e$. In order to prove

$$\text{Hom\_struct\_e (pack\_e 1 0) (pack\_e } X'\ e)\ (\lambda x \in 1.e)$$

we can use Theorem 74 to reduce to proving $(\lambda x \in 1.e)$ is in X'^1 (which is easy since $e \in X'$) and $(\lambda x \in 1.e)0 = e$ (which is easy using the `beta` property and $0 \in 1$). We finally need to prove uniqueness of the arrow. Suppose we have $h' \in X'^1$ such that $h'0 = e$. By the `Pi_eta` property $h' = (\lambda x \in 1.h'x)$. In order to prove $h' = (\lambda x \in 1.e)$ we can use the `lam_ext` property to reduce the problem to prove $\forall x \in 1.h'x = e$. Since 0 is the only member of 1 and $h'0 = e$ we are done.

Here is the corresponding Megalodon proof:

```
witness pack_e 1 0.
witness (fun X => fun x :e 1 => X 1).
prove struct_e (pack_e 1 0)
   /\ forall X:set, struct_e X
         -> Hom_struct_e (pack_e 1 0) X (fun x :e 1 => X 1)
         /\ forall h':set, Hom_struct_e (pack_e 1 0) X h'
                 -> h' = (fun x :e 1 => X 1).
apply andI.
- apply pack_struct_e_I. prove 0 :e 1. exact In_0_1.
- let X. assume HX: struct_e X.
  apply HX (fun u => Hom_struct_e (pack_e 1 0) u (fun x :e 1 => u 1)
                 /\ forall h':set, Hom_struct_e (pack_e 1 0) u h'
                         -> h' = (fun x :e 1 => u 1)).
  let X' e. assume He: e :e X'.
  prove Hom_struct_e (pack_e 1 0) (pack_e X' e) (fun x :e 1 => pack_e X' e 1)
     /\ forall h':set, Hom_struct_e (pack_e 1 0) (pack_e X' e) h'
           -> h' = (fun x :e 1 => pack_e X' e 1).
  rewrite <- pack_e_1_eq2.
  prove Hom_struct_e (pack_e 1 0) (pack_e X' e) (fun x :e 1 => e)
     /\ forall h':set, Hom_struct_e (pack_e 1 0) (pack_e X' e) h'
           -> h' = (fun x :e 1 => e).
  apply andI.
  - rewrite Hom_struct_e_pack.
    prove (fun x :e 1 => e) :e X' :^: 1
       /\ (fun x :e 1 => e) 0 = e.
    apply andI.
    + prove (fun x :e 1 => e) :e Pi_ x :e 1, X'.
      apply lam_Pi.
```

```
    let x. assume Hx. exact He.
  + exact beta 1 (fun x => e) 0 In_0_1.
- let h'. rewrite Hom_struct_e_pack.
  assume Hh'. apply Hh'.
  assume Hh'1: h' :e X' :^: 1.
  assume Hh'2: h' 0 = e.
  prove h' = (fun x :e 1 => e).
  transitivity (fun x :e 1 => h' x).
  + symmetry. exact Pi_eta 1 (fun _ => X') h' Hh'1.
  + apply lam_ext.
    let x. assume Hx: x :e 1.
    prove h' x = e.
    apply cases_1 x Hx (fun u => h' u = e).
    prove h' 0 = e.
    exact Hh'2.
```

□

Conjecture 30. [`MetaCat_struct_e_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$terminal\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMMSceAr6eTYJj8NKDk5kR3Ao4JFhyjzM4h
Bounty amount: approximately 25 bars

Conjecture 31. [`MetaCat_struct_e_coproduct_constr`]

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMTRDbdjbkAcgN13ZqPRwKtWNCCCBoZXXyC
Bounty amount: approximately 100 bars

Conjecture 32. [`MetaCat_struct_e_product_constr`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMXcjqWq1BzX7m3VJxwBWCgaGJ1UyF213X5
Bounty amount: approximately 100 bars

Conjecture 33. [`MetaCat_struct_e_coequalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMYdMP8kH73SzFDmuv485JUSKuu5Vo4c6cr
Bounty amount: approximately 125 bars

Conjecture 34. [`MetaCat_struct_e_equalizer_constr`]

$$\begin{gathered}\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ equalizer\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMTP64nfH97DVkUuyyS8qfe4bApRCv7RKha
Bounty amount: approximately 125 bars

Conjecture 35. [`MetaCat_struct_e_pushout_constr`]

$$\begin{gathered}\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ pushout\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ po\ i0\ i1\ copair.\end{gathered}$$

Proofgold proposition address: TMH8ViJad8xTFR9SRczU3RbDT57rtPomnqx
Bounty amount: approximately 250 bars

Conjecture 36. [`MetaCat_struct_e_pullback_constr`]

$$\begin{gathered}\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ pullback\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ pb\ \pi_0\ \pi_1\ pair.\end{gathered}$$

Proofgold proposition address: TMZdXpwg26tGHFLdYNNAoVhgF1HcCmt3J8H
Bounty amount: approximately 250 bars

Conjecture 37. [`MetaCat_struct_e_product_exponent`]

$$\begin{gathered}\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ product\_exponent\_constr\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{gathered}$$

Proofgold proposition address: TMF94AATqDPHiGkeqKfraGSc5bPFX7jcpDU
Bounty amount: approximately 250 bars

Conjecture 38. [`MetaCat_struct_e_subobject_classifier`]

$$\begin{gathered}\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ subobject\_classifier\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{gathered}$$

Proofgold proposition address: TMH7w6CANptWdfw1vEw1725r3vX1fXxDu5f
Bounty amount: approximately 250 bars

Conjecture 39. [`MetaCat_struct_e_nno`]

$$\begin{gathered}\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.\\ nno\_p\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ N\ zer\ suc\ rec.\end{gathered}$$

Proofgold proposition address: TMTUh8wspR34eLD49C5ciKgkRy1MjmaoXLD
Bounty amount: approximately 250 bars

The final proposition is given as a theorem, but will typically be left as the most important conjecture. Here we prove there is a left adjoint F to the forgetful functor. Such left adjoints often involve interesting constructions. In this case the left adjoint will take a set X to a set with an extra element (which we will point to in the structure). In terms of types, this corresponds to forming the "option" type of X where we have (copies of) all the previous members of X and a new "None" element. Mathematically we implement this "option type" as $1 \oplus X$. Here the copy of $x \in X$ is given by Inj1 $x \in 1 \oplus X$ and the "None" element is given by Inj0 $0 \in 1 \oplus X$. Since Inj0 0 is equal to 0, we can more simply say that we move the elements of X (injectively) away from the empty set and include the empty set as a new element.

We describe the proof briefly below, leaving the interested reader to look at the Megalodon proof for details.

Theorem 104. *[*`MetaCat_struct_e_left_adjoint_forgetful`*]*

$$\begin{gathered}\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ struct\_e\ Hom\_struct\_e\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.\end{gathered}$$

Proof. We specify the metafunctor by taking $F0$ to be the term

$$\lambda X.pack\_e\ (1{\oplus}X)\ 0$$

and $F1$ to be the term

$$\lambda X, Y, f.(\lambda x' \in 1{\oplus}X.combine\_funcs\ 1\ X\ (\lambda\_.0)\ (\lambda x.Inj1\ (f\ x))\ x')$$

We specify the meta natural transformations by taking *eta* to be the term

$$\lambda X.\lambda x \in X.Inj1\ x$$

and ε to be the term

$$\lambda X.(\lambda x' \in 1{\oplus}(X\ 0).combine\_funcs\ 1\ (X\ 0)\ (\lambda\_.X\ 1)\ (\lambda x.x)\ x').$$

We then prove the following properties.

1. $\forall X.struct\_e\ (F0\ X)$.
2. $\forall X.F0\ X\ 0{=}1{\oplus}X$.
3. $\forall X.F0\ X\ 1{=}0$.
4. $\forall X, Y, f.SetHom\ X\ Y\ f{\to}Hom\_struct\_e\ (F0\ X)\ (F0\ Y)\ (F1\ X\ Y\ f)$.
5. $\forall X, Y, f.\forall x \in 1.F1\ X\ Y\ f\ (Inj0\ x){=}0$.
6. $\forall X, Y, f.F1\ X\ Y\ f\ 0{=}0$.
7. $\forall X, Y, f.\forall x \in X.F1\ X\ Y\ f\ (Inj1\ x){=}Inj1\ (f\ x)$.

8. $\forall X, e.\forall x \in 1.\varepsilon\ (pack\_e\ X\ e)\ (Inj0\ x){=}e$.
9. $\forall X, e.\varepsilon\ (pack\_e\ X\ e)\ 0{=}e$.
10. $\forall X, e.\forall x \in X.\varepsilon\ (pack\_e\ X\ e)\ (Inj1\ x){=}x$.

We then apply Theorems 64 47 45 53 and 62 to reduce to the relevant subgoals and prove each subgoal using the facts above. $\square$

Chapter 5

Structures with a Predicate

Theorem 105. [`MetaCat_struct_p`] $MetaCat\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp.$

Proof. Exact $MetaCat\_struct\_p\_gen\ struct\_p\ (\lambda X, H.H)$. □

Theorem 106. [`MetaCat_struct_p_Forgetful`]

$$\begin{gathered} MetaFunctor\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_p\_Forgetful\_gen\ struct\_p\ (\lambda X, H.H)$. □

Conjecture 40. [`MetaCat_struct_p_initial`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota{\to}\iota. \\ initial\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMauEYXYbc7PfWVbFP1zhtmYfZVKmjUrnZZ
Bounty amount: approximately 25 bars

Conjecture 41. [`MetaCat_struct_p_terminal`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota{\to}\iota. \\ terminal\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMNohYHJY3rWwLGiicVQ7fP6a5t1XJwvnRb
Bounty amount: approximately 25 bars

Conjecture 42. [`MetaCat_struct_p_coproduct_constr`]

$$\begin{gathered} \exists coprod : \iota{\to}\iota{\to}\iota.\exists i1, i2 : \iota{\to}\iota{\to}\iota.\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ coproduct\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMXLQ7znFpZgjGPH7mmqwQ5uAJDTwjnM3tn
Bounty amount: approximately 100 bars

Conjecture 43. [`MetaCat_struct_p_product_constr`]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMPZNoADQCYtEerDGtRvs5QWAsBKqmLZx8Y
Bounty amount: approximately 100 bars

Conjecture 44. [`MetaCat_struct_p_coequalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMLKAuWugpebZnYypc4Z3u3h5TA1ULqm36Q
Bounty amount: approximately 125 bars

Conjecture 45. [`MetaCat_struct_p_equalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMXTP9Y1n13VDTdNbyyjkr5SzXLeNupQbNE
Bounty amount: approximately 125 bars

Conjecture 46. [`MetaCat_struct_p_pushout_constr`]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMWJsgie6PTdDFAW8cSQGEDaZhiTZpmZ8Pa
Bounty amount: approximately 250 bars

Conjecture 47. [`MetaCat_struct_p_pullback_constr`]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMSveu9pTvELKYdVExVpExwt4NamK9vA8CB
Bounty amount: approximately 250 bars

Conjecture 48. [`MetaCat_struct_p_product_exponent`]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMddGSdqvKEVriHp38eQwkK6F5UDLKZwDYC
Bounty amount: approximately 250 bars

Conjecture 49. [`MetaCat_struct_p_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMNsHWNYt5HVjVphPMJ9zraMa39RQ63wbCG
Bounty amount: approximately 250 bars

Conjecture 50. [`MetaCat_struct_p_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMdvAVYptTvnE1x4T2XUHNT3Xfpa11znUWb
Bounty amount: approximately 250 bars

Conjecture 51. [`MetaCat_struct_p_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$struct\_p\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMV4rXixLM1Rqb866KmMDhxKFqpo4f5ZnH4
Bounty amount: approximately 750 bars

5.1 Structures with a Nonempty Predicate

Definition 45. *We define* `struct_p_nonempty` *to be*

$$\lambda X.struct\_p\ X \wedge unpack\_p\_o\ X\ (\lambda X', p.\exists x \in X'.p\ x)$$

of type $\iota\to o$.

Theorem 107. [`MetaCat_struct_p_nonempty`]

$$MetaCat\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_p\_nonempty}\ X \to struct\_p\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_p\_gen$ `struct_p_nonempty` $L1$. ☐

Theorem 108. [`MetaCat_struct_p_nonempty_Forgetful`]

$$MetaFunctor\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_p_nonempty` $X \to struct\_p\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_p\_Forgetful\_gen$ `struct_p_nonempty` $L1$. □

Conjecture 52. *[*`MetaCat_struct_p_nonempty_initial`*]*

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$initial\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMKLpEpQciCLv742gBqVNG3WGWnRmxiV3Dt
Bounty amount: approximately 25 bars

Conjecture 53. *[*`MetaCat_struct_p_nonempty_terminal`*]*

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$terminal\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMVYvPPZj7pSddjoT43UWqWHC9ALzN8wgED
Bounty amount: approximately 25 bars

Conjecture 54. *[*`MetaCat_struct_p_nonempty_coproduct_constr`*]*

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMbFugwgA8GscX1NW9RMRJdwToE8s3coMPP
Bounty amount: approximately 100 bars

Conjecture 55. *[*`MetaCat_struct_p_nonempty_product_constr`*]*

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMHUnNdy5uCnoaqyHFgTEAVHzfBiygzpi51
Bounty amount: approximately 100 bars

Conjecture 56. *[*`MetaCat_struct_p_nonempty_coequalizer_constr`*]*

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMEmMz7a44GXrjggxdktQF99pNq8yHKnpbg
Bounty amount: approximately 125 bars

Conjecture 57. *[*`MetaCat_struct_p_nonempty_equalizer_constr`*]*

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMSZu6jeC5LUetUc1Q8g4hqbmZAB8ncyfzr
Bounty amount: approximately 125 bars

Conjecture 58. [`MetaCat_struct_p_nonempty_pushout_constr`]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMRf9DmGKSPuW1YmG9mfxkfmhtcAd5ikJsQ
Bounty amount: approximately 250 bars

Conjecture 59. [`MetaCat_struct_p_nonempty_pullback_constr`]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMKkmLLmsxgaNcRdwn6b3PQL9hPEvJSRDhZ
Bounty amount: approximately 250 bars

Conjecture 60. [`MetaCat_struct_p_nonempty_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMNJVnbMxHpxN16LDNZ4iShVPgUy83CXk87
Bounty amount: approximately 250 bars

Conjecture 61. [`MetaCat_struct_p_nonempty_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMPWwAAfn8sEat3XVf9qvVb9PCkAEc1tKK9
Bounty amount: approximately 250 bars

Conjecture 62. [`MetaCat_struct_p_nonempty_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMJA2BNfrHYi4AGmkYgP1ptFchCXdbtatYE
Bounty amount: approximately 250 bars

Conjecture 63. [`MetaCat_struct_p_nonempty_left_adjoint_forgetful`]

$$\begin{array}{c}\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ \mathtt{struct\_p\_nonempty}\ Hom\_struct\_p\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.\end{array}$$

Proofgold proposition address: TMWmTSe4y3TzSYMAZJF2h4oiXnWraWkbXyV
Bounty amount: approximately 750 bars
In this case we include an extra conjecture stating that we have a metafunctor from the metacategory of pointed sets into the metacategory with nonempty predicates given by taking the pointed element to its singleton.

Conjecture 64. [`MetaFunctor_struct_e_struct_p_nonempty`]

$$\begin{array}{c}MetaFunctor\ struct\_e\ Hom\_struct\_e\\ (\lambda X.(lam\_id\ (X\ 0)))\ (\lambda X, Y, Z, f, g.(lam\_comp\ (X\ 0)\ f\ g))\\ \mathtt{struct\_p\_nonempty}\ Hom\_struct\_p\ (\lambda X.(lam\_id\ (X\ 0)))\ (\lambda X, Y, Z, f, g.(lam\_comp\ (X\ 0)\ f\ g))\\ (\lambda X.unpack\_e\_i\ X\ (\lambda X', e.pack\_p\ X'\ (\lambda x.x{=}e)))\ (\lambda X, Y, f.f).\end{array}$$

Proofgold proposition address: TMdyw2byAdzW3zbcwPU86G7pmTtKKFjEQMN
Bounty amount: approximately 250 bars

Chapter 6

Structures with a Unary Function

Theorem 109. [`MetaCat_struct_u`] *MetaCat struct_u Hom_struct_u struct_id struct_comp.*

Proof. Exact $MetaCat\_struct\_u\_gen\ struct\_u\ (\lambda X, H.H)$. □

Theorem 110. [`MetaCat_struct_u_Forgetful`]

$$\begin{gathered} MetaFunctor\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_u\_Forgetful\_gen\ struct\_u\ (\lambda X, H.H)$. □

Conjecture 65. [`MetaCat_struct_u_initial`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMbFksCbpoZUF7W2znQPsnfAGSxii5TnpHu
Bounty amount: approximately 25 bars

Conjecture 66. [`MetaCat_struct_u_terminal`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMZ5bDjqkd66nob9dai7eiYLdhGprKgM3Qo
Bounty amount: approximately 25 bars

Conjecture 67. [`MetaCat_struct_u_coproduct_constr`]

$$\begin{gathered} \exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coproduct\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMXjV88ExHztv2rtPSKKSKyxsmnsU62f4UG
Bounty amount: approximately 100 bars

Conjecture 68. [`MetaCat_struct_u_product_constr`]

$$\exists prod : \iota{\to}\iota{\to}\iota. \exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota. \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMTNYUxT25C785x3QBSrmXaCcRcKxfZQEcr
Bounty amount: approximately 100 bars

Conjecture 69. [`MetaCat_struct_u_coequalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMac4F6NLrj5ywoVuhwykPqRcYQ3Cm5WPNm
Bounty amount: approximately 125 bars

Conjecture 70. [`MetaCat_struct_u_equalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMUx5hK79pD9gjJ9ZbWdtU1G5FdbquyqfCR
Bounty amount: approximately 125 bars

Conjecture 71. [`MetaCat_struct_u_pushout_constr`]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMctQs6MsMMwH3VVmPujkfnoTcrFi7w113Z
Bounty amount: approximately 250 bars

Conjecture 72. [`MetaCat_struct_u_pullback_constr`]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMHebk8iMAqRXyagjzjMPKPudkYwJh49fPg
Bounty amount: approximately 250 bars

Conjecture 73. [`MetaCat_struct_u_product_exponent`]

$$\exists prod : \iota{\to}\iota{\to}\iota. \exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota. \exists a : \iota{\to}\iota{\to}\iota. \exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMJQLtGzVXFzdBSBvLue6FXb5ZpygbpqCWM
Bounty amount: approximately 250 bars

Conjecture 74. [`MetaCat_struct_u_subobject_classifier`]

$$\exists one:\iota.\exists uniqa:\iota\to\iota.\exists Omega:\iota.\exists tru:\iota.\exists ch:\iota\to\iota\to\iota\to\iota.$$
$$\exists constr:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMMGrkhYLghEA6qAEfEHe6NbEEoEy5N7sMq
Bounty amount: approximately 250 bars

Conjecture 75. [`MetaCat_struct_u_nno`]

$$\exists one:\iota.\exists uniqa:\iota\to\iota.\exists N:\iota.\exists zer,suc:\iota.\exists rec:\iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMSz7UTm6S8ceqRQaqDV9jRsZFjsknEZGge
Bounty amount: approximately 250 bars

Conjecture 76. [`MetaCat_struct_u_left_adjoint_forgetful`]

$$\exists F0:\iota\to\iota.\exists F1:\iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon:\iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$struct\_u\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMMyzBYxovQXk75QbXJqjs32VsR3bZbjRuS
Bounty amount: approximately 750 bars

6.1 Injective Functions

Definition 46. *We define* `struct_u_inj` *to be*

$$\lambda X.struct\_u\ X\wedge unpack\_u\_o\ X\ (\lambda X',h.inj\ X'\ X'\ (\lambda x.h\ x))$$

of type $\iota\to o$.

Theorem 111. [`MetaCat_struct_u_inj`]

$$MetaCat\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_u\_inj}\ X\to struct\_u\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_u\_gen$ `struct_u_inj` $L1$. ☐

Theorem 112. [`MetaCat_struct_u_inj_Forgetful`]

$$MetaFunctor\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_u_inj` $X \rightarrow struct\_u\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_u\_Forgetful\_gen$ `struct_u_inj` $L1$. ☐

Conjecture 77. [`MetaCat_struct_u_inj_initial`]

$$\exists Y : \iota.\exists uniqa : \iota\rightarrow\iota.$$
$$initial\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMVoyFEkA6Uv6dT8Vk2MmZHJHS3VwTtWPT7
Bounty amount: approximately 25 bars

Conjecture 78. [`MetaCat_struct_u_inj_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota\rightarrow\iota.$$
$$terminal\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMaaRPZ2GwbG1yTU38kUmx5WFgBfH28qxVU
Bounty amount: approximately 25 bars

Conjecture 79. [`MetaCat_struct_u_inj_coproduct_constr`]

$$\exists coprod : \iota\rightarrow\iota\rightarrow\iota.\exists i1, i2 : \iota\rightarrow\iota\rightarrow\iota.\exists copair : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMMmdsxTYQx85smbyD9qqqqmZbfYbSznHK8
Bounty amount: approximately 100 bars

Conjecture 80. [`MetaCat_struct_u_inj_product_constr`]

$$\exists prod : \iota\rightarrow\iota\rightarrow\iota.\exists \pi_1, \pi_2 : \iota\rightarrow\iota\rightarrow\iota.\exists pair : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$product\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMcxh6n7Xm5swxDF6orWVoKUAVTJpHrXDqe
Bounty amount: approximately 100 bars

Conjecture 81. [`MetaCat_struct_u_inj_coequalizer_constr`]

$$\exists quot : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.\exists canonmap : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$\exists fac : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMUGbHzVmCrWQFSmdgSoSXoTsra6JBf5Ex2
Bounty amount: approximately 125 bars

Conjecture 82. [`MetaCat_struct_u_inj_equalizer_constr`]

$$\exists quot : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.\exists canonmap : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$\exists fac : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMRBKPjyZUdDnA3y24DN8HJqSigCiAwZbDu
Bounty amount: approximately 125 bars

Conjecture 83. [`MetaCat_struct_u_inj_pushout_constr`]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMasAmhxXr6HPrHN6v9GyDNByjXdWTk5fF4
Bounty amount: approximately 250 bars

Conjecture 84. [`MetaCat_struct_u_inj_pullback_constr`]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMQUm92ormpzHwQw8y5FSL2CqkgfsSqa9mL
Bounty amount: approximately 250 bars

Conjecture 85. [`MetaCat_struct_u_inj_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMSKibXKtFfzGqyeAHGrNr4PQN3Ue6iwcQV
Bounty amount: approximately 250 bars

Conjecture 86. [`MetaCat_struct_u_inj_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMNPr7hqjDxkta3E6A7CwqPisuZDkxf4SMD
Bounty amount: approximately 250 bars

Conjecture 87. [`MetaCat_struct_u_inj_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMXJ8kpdmFQkcnYTtsvRsGmM5kBnGuuaF12
Bounty amount: approximately 250 bars

Conjecture 88. *[*`MetaCat_struct_u_inj_left_adjoint_forgetful`*]*

$$\exists F0 : \iota \to \iota . \exists F1 : \iota \to \iota \to \iota \to \iota . \exists \eta, \varepsilon : \iota \to \iota .$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_u\_inj}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMQYMFMD8xzB54bdcP36XWZUroo7QU2WQTd
Bounty amount: approximately 750 bars

6.2 Bijective Functions

Definition 47. *We define* `struct_u_bij` *to be*

$$\lambda X.struct\_u\ X \wedge unpack\_u\_o\ X\ (\lambda X', h.bij\ X'\ X'\ (\lambda x.h\ x))$$

of type $\iota \to o$.

Theorem 113. *[*`MetaCat_struct_u_bij`*]*

$$MetaCat\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_u\_bij}\ X \to struct\_u\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_u\_gen$ `struct_u_bij` $L1$. □

Theorem 114. *[*`MetaCat_struct_u_bij_Forgetful`*]*

$$MetaFunctor\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_u\_bij}\ X \to struct\_u\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_u\_Forgetful\_gen$ `struct_u_bij` $L1$. □

Conjecture 89. *[*`MetaCat_struct_u_bij_initial`*]*

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$initial\_p\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMd93Hw4iWjf7wnjczgQ9bwjjBqYjad2mbi
Bounty amount: approximately 25 bars

Conjecture 90. *[*`MetaCat_struct_u_bij_terminal`*]*

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$terminal\_p\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMTwVPCiLLkNeDXxBdwKgLrVvBToi2dfXgS
Bounty amount: approximately 25 bars

Conjecture 91. [`MetaCat_struct_u_bij_coproduct_constr`]

$$\exists coprod : \iota{\to}\iota{\to}\iota.\exists i1, i2 : \iota{\to}\iota{\to}\iota.\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coproduct\_constr\_p \ \texttt{struct\_u\_bij} \ Hom\_struct\_u \ struct\_id \ struct\_comp$$
$$coprod \ i1 \ i2 \ copair.$$

Proofgold proposition address: TMSrZpp4YyvNup4E9GgtmyE5h14bWet2KNx
Bounty amount: approximately 100 bars

Conjecture 92. [`MetaCat_struct_u_bij_product_constr`]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p \ \texttt{struct\_u\_bij} \ Hom\_struct\_u \ struct\_id \ struct\_comp$$
$$prod \ \pi_1 \ \pi_2 \ pair.$$

Proofgold proposition address: TMSHoybVCBMUaF7LtAQBz1J9Rdd6S98EPEE
Bounty amount: approximately 100 bars

Conjecture 93. [`MetaCat_struct_u_bij_coequalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p \ \texttt{struct\_u\_bij} \ Hom\_struct\_u \ struct\_id \ struct\_comp$$
$$quot \ canonmap \ fac.$$

Proofgold proposition address: TMJ1QQ95wcwLjW5JvJw9a1MNLkHB2QSg8k4
Bounty amount: approximately 125 bars

Conjecture 94. [`MetaCat_struct_u_bij_equalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p \ \texttt{struct\_u\_bij} \ Hom\_struct\_u \ struct\_id \ struct\_comp$$
$$quot \ canonmap \ fac.$$

Proofgold proposition address: TMT2xzZjjmktic4gJTVVZX6FeZ1Uzm3KhEr
Bounty amount: approximately 125 bars

Conjecture 95. [`MetaCat_struct_u_bij_pushout_constr`]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p \ \texttt{struct\_u\_bij} \ Hom\_struct\_u \ struct\_id \ struct\_comp$$
$$po \ i0 \ i1 \ copair.$$

Proofgold proposition address: TMHBYot4zdGCjAbeT6fQMpxku5jTUHwupZy
Bounty amount: approximately 250 bars

Conjecture 96. [`MetaCat_struct_u_bij_pullback_constr`]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p \ \texttt{struct\_u\_bij} \ Hom\_struct\_u \ struct\_id \ struct\_comp$$
$$pb \ \pi_0 \ \pi_1 \ pair.$$

Proofgold proposition address: TMJMnxGpsKrZcQh98SkynbFbF5TR6JeZsLS
Bounty amount: approximately 250 bars

Conjecture 97. [`MetaCat_struct_u_bij_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMHPufVxHaoPgeWoxfqqBDG4jyZYkqrnWNW
Bounty amount: approximately 250 bars

Conjecture 98. [`MetaCat_struct_u_bij_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMQLgb1MFaJQo4fjAhdw9svqEp5JMQLzbyy
Bounty amount: approximately 250 bars

Conjecture 99. [`MetaCat_struct_u_bij_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMFdV6jYUVtg1RxuSvi4EuiMSTw9q3eH3vq
Bounty amount: approximately 250 bars

Conjecture 100. [`MetaCat_struct_u_bij_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists\eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_u\_bij}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMNXscwYJuVedfUGJ5UjMyVngF19B8ghnuy
Bounty amount: approximately 750 bars

6.3 Idempotent Functions

Definition 48. *We define* `struct_u_idem` *to be*

$$\lambda X.struct\_u\ X \wedge unpack\_u\_o\ X\ (\lambda X', h.\forall x \in X'.h\ (h\ x) = h\ x)$$

of type $\iota\to o$.

Theorem 115. [`MetaCat_struct_u_idem`]

$$MetaCat\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_u\_idem}\ X \rightarrow struct\_u\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_u\_gen$ $\texttt{struct\_u\_idem}$ $L1$. □

Theorem 116. [MetaCat_struct_u_idem_Forgetful]

$$\begin{array}{c} MetaFunctor\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{array}$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_u\_idem}\ X \rightarrow struct\_u\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_u\_Forgetful\_gen$ $\texttt{struct\_u\_idem}$ $L1$. □

Conjecture 101. [MetaCat_struct_u_idem_initial]

$$\begin{array}{c} \exists Y : \iota.\exists uniqa : \iota \rightarrow \iota. \\ initial\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa. \end{array}$$

Proofgold proposition address: TMdjwTKfquFSEs7LSWvytGh2BdE2HfhmZX3
Bounty amount: approximately 25 bars

Conjecture 102. [MetaCat_struct_u_idem_terminal]

$$\begin{array}{c} \exists Y : \iota.\exists uniqa : \iota \rightarrow \iota. \\ terminal\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\ Y\ uniqa. \end{array}$$

Proofgold proposition address: TMJ9wrNKF7QMgJssE2Rf19omvZuzBQtbCex
Bounty amount: approximately 25 bars

Conjecture 103. [MetaCat_struct_u_idem_coproduct_constr]

$$\begin{array}{c} \exists coprod : \iota \rightarrow \iota \rightarrow \iota.\exists i1, i2 : \iota \rightarrow \iota \rightarrow \iota.\exists copair : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota. \\ coproduct\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{array}$$

Proofgold proposition address: TMJTnqzQdtWwBRb8PDXvpF1qep2HpoPz2HR
Bounty amount: approximately 100 bars

Conjecture 104. [MetaCat_struct_u_idem_product_constr]

$$\begin{array}{c} \exists prod : \iota \rightarrow \iota \rightarrow \iota.\exists \pi_1, \pi_2 : \iota \rightarrow \iota \rightarrow \iota.\exists pair : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota. \\ product\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp \\ prod\ \pi_1\ \pi_2\ pair. \end{array}$$

Proofgold proposition address: TMFSFUYRQx5QdArjnYFzRsPAQD325CEiM6a
Bounty amount: approximately 100 bars

Conjecture 105. [MetaCat_struct_u_idem_coequalizer_constr]

$$\begin{array}{c} \exists quot : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota.\exists canonmap : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota. \\ \exists fac : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota. \\ coequalizer\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac. \end{array}$$

Proofgold proposition address: TMPeeaBtrURpe27Yd4yy6WMDQVMwq7grF7n
Bounty amount: approximately 125 bars

Conjecture 106. [MetaCat_struct_u_idem_equalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ equalizer\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMNYqnzWNJU4yygWQxcP7K7rY9XQbQ4dss7
Bounty amount: approximately 125 bars

Conjecture 107. [MetaCat_struct_u_idem_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ pushout\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\\ po\ i0\ i1\ copair.$$

Proofgold proposition address: TMXX7x468LhT2pCcLyxq9CBhDPuxYCvSYNK
Bounty amount: approximately 250 bars

Conjecture 108. [MetaCat_struct_u_idem_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ pullback\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\\ pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMWBn9gMpap5JBCSYSMBqbYbhxMLw35JCsn
Bounty amount: approximately 250 bars

Conjecture 109. [MetaCat_struct_u_idem_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ product\_exponent\_constr\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMYi8kKa6RGKUnVf7TZgMqnK8ZYaJKj16NW
Bounty amount: approximately 250 bars

Conjecture 110. [MetaCat_struct_u_idem_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.\\ \exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\\ subobject\_classifier\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp\\ one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMSEpe7rGVSCyWkowyS9UvrwpxFmntX8uc8
Bounty amount: approximately 250 bars

Conjecture 111. [MetaCat_struct_u_idem_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMT1hR8yBfJTnqt24ksUMQgCnngBtqkkjpw
Bounty amount: approximately 250 bars

Conjecture 112. [MetaCat_struct_u_idem_left_adjoint_forgetful]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_u\_idem}\ Hom\_struct\_u\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMXF89QccqpaLXPPrLD6rU88VB6fxNyFi94
Bounty amount: approximately 750 bars

Chapter 7

Structures with a Binary Relation

Theorem 117. [`MetaCat_struct_r`] $MetaCat\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp.$

Proof. Exact $MetaCat\_struct\_r\_gen\ struct\_r\ (\lambda X, H.H)$. □

Theorem 118. [`MetaCat_struct_r_Forgetful`]

$$\begin{gathered} MetaFunctor\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_r\_Forgetful\_gen\ struct\_r\ (\lambda X, H.H)$. □

Conjecture 113. [`MetaCat_struct_r_initial`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMQZQjRUMSAfQB6Z4i1p5EuWR38P6icgj58
Bounty amount: approximately 25 bars

Conjecture 114. [`MetaCat_struct_r_terminal`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMWpBLP5PyqZxGuKa6fzgfPRMgCto1dqvob
Bounty amount: approximately 25 bars

Conjecture 115. [`MetaCat_struct_r_coproduct_constr`]

$$\begin{gathered} \exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coproduct\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMZPieX1ezqmpRUVbG4ziLw45KAqGWqxWYs
Bounty amount: approximately 100 bars

Conjecture 116. [MetaCat_struct_r_product_constr]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMX55dEysJ7wVxrZwYEQsVe9RCxpEy1j3A9
Bounty amount: approximately 100 bars

Conjecture 117. [MetaCat_struct_r_coequalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMWLyEETzhMNiM4PTAKKW9JWhAotKLEGDS4
Bounty amount: approximately 125 bars

Conjecture 118. [MetaCat_struct_r_equalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMYWRjPZ1ij51VbRXP78mGeE7ySDe9EcbEA
Bounty amount: approximately 125 bars

Conjecture 119. [MetaCat_struct_r_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMJyAbKMNjdmqHHXoMqs2QGTZkryguW94LD
Bounty amount: approximately 250 bars

Conjecture 120. [MetaCat_struct_r_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMJSGf3tz71d4rfUJ8ECquXhwCYTfEaN9c5
Bounty amount: approximately 250 bars

Conjecture 121. [MetaCat_struct_r_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMa4jJpr37mTXdwsxuEAiVdfQTTmSkeWpNj
Bounty amount: approximately 250 bars

Conjecture 122. [`MetaCat_struct_r_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMQbgUFeaF55n81GanvbEoRYVhMWiSEBmv1
Bounty amount: approximately 250 bars

Conjecture 123. [`MetaCat_struct_r_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMFNmnC5NSVB2t2VnJCamvWSGtJWAgkTHXg
Bounty amount: approximately 250 bars

Conjecture 124. [`MetaCat_struct_r_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$struct\_r\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMLsWXe1LzSTu6fqKpPM1eMbXMhmKiTrFfx
Bounty amount: approximately 750 bars

7.1 Graphs

Definition 49. *We define* `struct_r_graph` *to be*

$$\lambda X.struct\_r\ X \wedge unpack\_r\_o\ X\ (\lambda X', r.$$
$$(\forall x \in X'.\neg r\ x\ x) \wedge (\forall x, y \in X'.r\ x\ y \to r\ y\ x))$$

of type $\iota\to o$.

Theorem 119. [`MetaCat_struct_r_graph`]

$$MetaCat\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_r_graph` $X \to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_gen$ `struct_r_graph` $L1$. □

Theorem 120. [`MetaCat_struct_r_graph_Forgetful`]

$$MetaFunctor\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$struct_r_graph $X \to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume H, _. Exact H. Exact $MetaCat\_struct\_r\_Forgetful\_gen$ struct_r_graph $L1$. □

Conjecture 125. [MetaCat_struct_r_graph_initial]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$initial\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMLJi4GJ16NuocmcLppd7i1tSQ4hJvDMQM9
Bounty amount: approximately 25 bars

Conjecture 126. [MetaCat_struct_r_graph_terminal]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$terminal\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMaUwRXma75kj6kS93jwVBpy8cv5wS8NG4W
Bounty amount: approximately 25 bars

Conjecture 127. [MetaCat_struct_r_graph_coproduct_constr]

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMTXeFNAtGNFCnB1RAtMwSHV1xvyogymJWC
Bounty amount: approximately 100 bars

Conjecture 128. [MetaCat_struct_r_graph_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMT4ACCFneUpzJdDvwwpSYWDCgZUUHY4th8
Bounty amount: approximately 100 bars

Conjecture 129. [MetaCat_struct_r_graph_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMLQwXkjXPLTa6hxNsjVPbEptXc4NUcTzXg
Bounty amount: approximately 125 bars

Conjecture 130. [MetaCat_struct_r_graph_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMXubAMeMsyo9qzbKG2poyhssLvr4nYpsCw
Bounty amount: approximately 125 bars

Conjecture 131. [MetaCat_struct_r_graph_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMGBNzdMMT2xEKQozUHQbJvhFg7fdB8AXM9
Bounty amount: approximately 250 bars

Conjecture 132. [MetaCat_struct_r_graph_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMHdpobu8o5AB7vFiHgj3nyyeVdVZB5T6GS
Bounty amount: approximately 250 bars

Conjecture 133. [MetaCat_struct_r_graph_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMJikvLxtBBXkjdWLLN7GrkPqcHqSecam5Y
Bounty amount: approximately 250 bars

Conjecture 134. [MetaCat_struct_r_graph_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMaBitiBZN7eyVHfwDQNe9UULiH9QJ57uFD
Bounty amount: approximately 250 bars

Conjecture 135. [MetaCat_struct_r_graph_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMZgBnFtbi8qBZdBcnRavDrtfL63u6Sds4H
Bounty amount: approximately 250 bars

Conjecture 136. [MetaCat_struct_r_graph_left_adjoint_forgetful]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon : \iota\to\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ \texttt{struct\_r\_graph}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMQvtNXfVrkGGh67ASGThM6oS6FcmhX5onR
Bounty amount: approximately 750 bars

7.2 Partial Equivalence Relations

Definition 50. *We define* struct_r_per *to be*

$$\lambda X.struct\_r\ X\wedge unpack\_r\_o\ X\ (\lambda X',r.\\ (\forall x,y\in X'.r\ x\ y\to r\ y\ x)\\ \wedge(\forall x,y,z\in X'.r\ x\ y\to r\ y\ z\to r\ x\ z))$$

of type $\iota\to o$.

Theorem 121. [MetaCat_struct_r_per]

$$MetaCat\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_per}\ X\to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_r\_gen$ struct_r_per $L1$. □

Theorem 122. [MetaCat_struct_r_per_Forgetful]

$$MetaFunctor\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_per}\ X\to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_r\_Forgetful\_gen$ struct_r_per $L1$. □

Conjecture 137. [MetaCat_struct_r_per_initial]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.\\ initial\_p\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMbkRHugarGwP9DbzKaMLMdTnzVpts6uimo
Bounty amount: approximately 25 bars

Conjecture 138. [MetaCat_struct_r_per_terminal]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.\\ terminal\_p\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMXimm6LfQ8hdrsSEzWgh2Jh1NcmhNLVnnZ
Bounty amount: approximately 25 bars

Conjecture 139. [`MetaCat_struct_r_per_coproduct_constr`]

$$\exists coprod : \iota{\to}\iota{\to}\iota. \exists i1, i2 : \iota{\to}\iota{\to}\iota. \exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ coproduct\_constr\_p \ \texttt{struct\_r\_per} \ Hom\_struct\_r \ struct\_id \ struct\_comp \\ coprod \ i1 \ i2 \ copair.$$

Proofgold proposition address: TMWp3phMDhecg9H7SXLc23PRtLPMwyeM9jG
Bounty amount: approximately 100 bars

Conjecture 140. [`MetaCat_struct_r_per_product_constr`]

$$\exists prod : \iota{\to}\iota{\to}\iota. \exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota. \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ product\_constr\_p \ \texttt{struct\_r\_per} \ Hom\_struct\_r \ struct\_id \ struct\_comp \\ prod \ \pi_1 \ \pi_2 \ pair.$$

Proofgold proposition address: TMWf3MJ89aRidif4WfeKaRhU3pBXEu4LphD
Bounty amount: approximately 100 bars

Conjecture 141. [`MetaCat_struct_r_per_coequalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ coequalizer\_constr\_p \ \texttt{struct\_r\_per} \ Hom\_struct\_r \ struct\_id \ struct\_comp \\ quot \ canonmap \ fac.$$

Proofgold proposition address: TMLq8s1qCCZAPV1wAwZbGuVR7RgBgHbcVSF
Bounty amount: approximately 125 bars

Conjecture 142. [`MetaCat_struct_r_per_equalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ equalizer\_constr\_p \ \texttt{struct\_r\_per} \ Hom\_struct\_r \ struct\_id \ struct\_comp \\ quot \ canonmap \ fac.$$

Proofgold proposition address: TMKZS5XNJCQp3BiJSptPVzxF9cZyV46t5vT
Bounty amount: approximately 125 bars

Conjecture 143. [`MetaCat_struct_r_per_pushout_constr`]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ pushout\_constr\_p \ \texttt{struct\_r\_per} \ Hom\_struct\_r \ struct\_id \ struct\_comp \\ po \ i0 \ i1 \ copair.$$

Proofgold proposition address: TMVMKgSYaiRC9zpEU66gEEHKX1QvUTbQwwt
Bounty amount: approximately 250 bars

Conjecture 144. [`MetaCat_struct_r_per_pullback_constr`]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ pullback\_constr\_p \ \texttt{struct\_r\_per} \ Hom\_struct\_r \ struct\_id \ struct\_comp \\ pb \ \pi_0 \ \pi_1 \ pair.$$

Proofgold proposition address: TMKsR8uNa1ta1n1Y7pa55FgToQ6565UPqgm
Bounty amount: approximately 250 bars

Conjecture 145. [`MetaCat_struct_r_per_product_exponent`]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMXndcqJHEJRG8kM1P5v6711shLxMtDtfTA
Bounty amount: approximately 250 bars

Conjecture 146. [`MetaCat_struct_r_per_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMFE6DNXhY7tKEwEgXm96UHVER2xXMMks9e
Bounty amount: approximately 250 bars

Conjecture 147. [`MetaCat_struct_r_per_nno`]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ \texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMHiNs3vVHfGzwoiX7cuxQxB8yEzfPxRTKd
Bounty amount: approximately 250 bars

Conjecture 148. [`MetaCat_struct_r_per_left_adjoint_forgetful`]

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_r\_per}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMJ3yPcVPJGMEFqxoBm8W4ZR5kGnH5iRvAZ
Bounty amount: approximately 750 bars

7.3 Equivalence Relations

Definition 51. *We define* `struct_r_equivreln` *to be*

$$\lambda X.struct\_r\ X{\wedge}unpack\_r\_o\ X\ (\lambda X', r.$$
$$(\forall x \in X'.r\ x\ x){\wedge}(\forall x, y \in X'.r\ x\ y{\to}r\ y\ x)$$
$${\wedge}(\forall x, y, z \in X'.r\ x\ y{\to}r\ y\ z{\to}r\ x\ z))$$

of type $\iota{\to}o$.

Theorem 123. [`MetaCat_struct_r_equivreln`]

$$MetaCat\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_equivreln}\ X \rightarrow struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_gen$ `struct_r_equivreln` $L1$. □

Theorem 124. [`MetaCat_struct_r_equivreln_Forgetful`]

$$\begin{gathered}MetaFunctor\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).\end{gathered}$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_equivreln}\ X \rightarrow struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_Forgetful\_gen$ `struct_r_equivreln` $L1$. □

Conjecture 149. [`MetaCat_struct_r_equivreln_initial`]

$$\begin{gathered}\exists Y : \iota.\exists uniqa : \iota \rightarrow \iota.\\ initial\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.\end{gathered}$$

Proofgold proposition address: TMJpsozkpCK8eBEsCzbzL6Xzeh9eLEuq94G
Bounty amount: approximately 25 bars

Conjecture 150. [`MetaCat_struct_r_equivreln_terminal`]

$$\begin{gathered}\exists Y : \iota.\exists uniqa : \iota \rightarrow \iota.\\ terminal\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.\end{gathered}$$

Proofgold proposition address: TMcUgfFJa1UTCBYxhK62HJctJPWQvjAm4UD
Bounty amount: approximately 25 bars

Conjecture 151. [`MetaCat_struct_r_equivreln_coproduct_constr`]

$$\begin{gathered}\exists coprod : \iota \rightarrow \iota \rightarrow \iota.\exists i1, i2 : \iota \rightarrow \iota \rightarrow \iota.\exists copair : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota.\\ coproduct\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ coprod\ i1\ i2\ copair.\end{gathered}$$

Proofgold proposition address: TMSyNsSbfgKg2Wy1qUEjjYZNBn1H182G7ku
Bounty amount: approximately 100 bars

Conjecture 152. [`MetaCat_struct_r_equivreln_product_constr`]

$$\begin{gathered}\exists prod : \iota \rightarrow \iota \rightarrow \iota.\exists \pi_1, \pi_2 : \iota \rightarrow \iota \rightarrow \iota.\exists pair : \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota \rightarrow \iota.\\ product\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair.\end{gathered}$$

Proofgold proposition address: TMU7nM3FWQJKwEHdisu6w6T84yUYNaWmCDT
Bounty amount: approximately 100 bars

Conjecture 153. [MetaCat_struct_r_equivreln_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMHqquXRBNaU7tB3guTKxefejxMGt9jWw96
Bounty amount: approximately 125 bars

Conjecture 154. [MetaCat_struct_r_equivreln_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMLUhjPhCbfgvNcX4nqP2hvsx946ixrYhyC
Bounty amount: approximately 125 bars

Conjecture 155. [MetaCat_struct_r_equivreln_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMHYcF7negKKvg5XE1DLmNng5e5FSUqFeWu
Bounty amount: approximately 250 bars

Conjecture 156. [MetaCat_struct_r_equivreln_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMLtXEMWbRSiMEhteYJsNf6rVUUyrN4SJwt
Bounty amount: approximately 250 bars

Conjecture 157. [MetaCat_struct_r_equivreln_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMLUGLQxEffYCLb2HDgxwmJB72raxzd6Rbo
Bounty amount: approximately 250 bars

Conjecture 158. [MetaCat_struct_r_equivreln_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMX5BadowpPCUhAjAEBbMPsg556dv2e8xQY
Bounty amount: approximately 250 bars

Conjecture 159. [`MetaCat_struct_r_equivreln_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.\\ nno\_p\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMcssuk9hjBTZYURay2LnPpxXKHTygdi4Nx
Bounty amount: approximately 250 bars

Conjecture 160. [`MetaCat_struct_r_equivreln_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ \texttt{struct\_r\_equivreln}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMKGkKqNZom9pJphT7fCnoueaEmLNRKsDs4
Bounty amount: approximately 750 bars

7.4 Partial Orderings

Definition 52. *We define* `struct_r_partialord` *to be*

$$\lambda X.struct\_r\ X \wedge unpack\_r\_o\ X\ (\lambda X', r.\\ (\forall x \in X'.\neg r\ x\ x)\\ \wedge(\forall x, y, z \in X'.r\ x\ y\to r\ y\ z\to r\ x\ z))$$

of type $\iota\to o$.

Theorem 125. [`MetaCat_struct_r_partialord`]

$$MetaCat\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_partialord}\ X\to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_gen$ `struct_r_partialord` $L1$. □

Theorem 126. [`MetaCat_struct_r_partialord_Forgetful`]

$$MetaFunctor\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_partialord}\ X\to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_Forgetful\_gen$ `struct_r_partialord` $L1$. □

Conjecture 161. [MetaCat_struct_r_partialord_initial]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMSTswt5nryoV4wpPtpQGAdL1YgQEHJFq6V
Bounty amount: approximately 25 bars

Conjecture 162. [MetaCat_struct_r_partialord_terminal]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMdRQ9R7JDvghbtJLBvLhbGeHZTx5YEswmp
Bounty amount: approximately 25 bars

Conjecture 163. [MetaCat_struct_r_partialord_coproduct_constr]

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coproduct\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMHY8777zzw6rMnrzwuynphSnQ1z69YzGCc
Bounty amount: approximately 100 bars

Conjecture 164. [MetaCat_struct_r_partialord_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ product\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMMxQrXXsTtGPzgapHptUbkWr6qMFxgQ5eD
Bounty amount: approximately 100 bars

Conjecture 165. [MetaCat_struct_r_partialord_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota. \\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coequalizer\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMEkYYYH5vWusdWRQ41DRgowsmTGGed6rE5
Bounty amount: approximately 125 bars

Conjecture 166. [MetaCat_struct_r_partialord_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota. \\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ equalizer\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMHhZpV5apR3HY7xPnPTXnLgfqNEuJEDwfd
Bounty amount: approximately 125 bars

Conjecture 167. [MetaCat_struct_r_partialord_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMYX1txrVaVs43pX9GadZzQHLJTbnwckBrQ
Bounty amount: approximately 250 bars

Conjecture 168. [MetaCat_struct_r_partialord_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMWuojAbRREboyDXAbh9MyBWmwDmHs9ewGo
Bounty amount: approximately 250 bars

Conjecture 169. [MetaCat_struct_r_partialord_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMbboU18NAv3j4ienUh28PfKiAxiDkCGdd3
Bounty amount: approximately 250 bars

Conjecture 170. [MetaCat_struct_r_partialord_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMamUUDKDjW2vuJCYcKukVokfHyH6FQAP2o
Bounty amount: approximately 250 bars

Conjecture 171. [MetaCat_struct_r_partialord_nno]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ \texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMXYGHhsWbJFz6sbPCMe8M2nKN8UKJaDsta
Bounty amount: approximately 250 bars

Conjecture 172. [MetaCat_struct_r_partialord_left_adjoint_forgetful]

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_r\_partialord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMUj1njojeoWdJUUaUuQf9m8crGntAWggwG
Bounty amount: approximately 750 bars

7.5 Orderings

Definition 53. *We define* `struct_r_ord` *to be*

$$\lambda X.struct\_r\ X \wedge unpack\_r\_o\ X\ (\lambda X', r.\ (\forall x \in X'.\neg r\ x\ x) \wedge (\forall x, y \in X'.r\ x\ y \vee r\ y\ x) \wedge (\forall x, y, z \in X'.r\ x\ y \to r\ y\ z \to r\ x\ z))$$

of type $\iota \to o$.

Theorem 127. [`MetaCat_struct_r_ord`]

$$MetaCat\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_r_ord` $X \to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_gen$ `struct_r_ord` $L1$. □

Theorem 128. [`MetaCat_struct_r_ord_Forgetful`]

$$MetaFunctor\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ (\lambda\_.True)\ SetHom\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_r_ord` $X \to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_Forgetful\_gen$ `struct_r_ord` $L1$. □

Conjecture 173. [`MetaCat_struct_r_ord_initial`]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.\ initial\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMcVRyBgo8d8GqD5ZSZ3YZxs8HKWadHZuzx
Bounty amount: approximately 25 bars

Conjecture 174. [`MetaCat_struct_r_ord_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.\ terminal\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMMV22MNB2AQoiRD9EyFZZC8Wt3ebKtiavB
Bounty amount: approximately 25 bars

Conjecture 175. [`MetaCat_struct_r_ord_coproduct_constr`]

$$\exists coprod : \iota \to \iota \to \iota.\exists i1, i2 : \iota \to \iota \to \iota.\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.\ coproduct\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMJAhZfChZP3DrqUzDHVTzXjy1TsYGsUxfF
Bounty amount: approximately 100 bars

Conjecture 176. [`MetaCat_struct_r_ord_product_constr`]

$$\exists prod : \iota{\to}\iota{\to}\iota. \exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota. \exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMQQd6VL8xWednPwMeTTyF4UFsdrAJ7SULo
Bounty amount: approximately 100 bars

Conjecture 177. [`MetaCat_struct_r_ord_coequalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMaZN2UTnTYkNxtPBvES5EVST6hcKv6BvC3
Bounty amount: approximately 125 bars

Conjecture 178. [`MetaCat_struct_r_ord_equalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMPbxpjyagvkZBvw5ag2A9WSRcUgAfRn7WR
Bounty amount: approximately 125 bars

Conjecture 179. [`MetaCat_struct_r_ord_pushout_constr`]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMV1M3RGku4UNnBiLPdrKHKULjQay1maMd4
Bounty amount: approximately 250 bars

Conjecture 180. [`MetaCat_struct_r_ord_pullback_constr`]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMX6gPmt5La4HYYbgVqdBKSW3kWrMAbcV2D
Bounty amount: approximately 250 bars

Conjecture 181. [`MetaCat_struct_r_ord_product_exponent`]

$$\exists prod : \iota{\to}\iota{\to}\iota. \exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota. \exists a : \iota{\to}\iota{\to}\iota. \exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMXGsgV54esncjBZiwby2ahakEB1fSrNNvp
Bounty amount: approximately 250 bars

Conjecture 182. *[*`MetaCat_struct_r_ord_subobject_classifier`*]*

$$\exists one:\iota.\exists uniqa:\iota\to\iota.\exists Omega:\iota.\exists tru:\iota.\exists ch:\iota\to\iota\to\iota\to\iota.\\ \exists constr:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMPfTapPD4mduwE94F4boysGDN8CYkh2zPw
Bounty amount: approximately 250 bars

Conjecture 183. *[*`MetaCat_struct_r_ord_nno`*]*

$$\exists one:\iota.\exists uniqa:\iota\to\iota.\exists N:\iota.\exists zer,suc:\iota.\exists rec:\iota\to\iota\to\iota\to\iota.\\ nno\_p\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMVDx46Kuuo9iixZmiY9yXUfNLfmMfDys23
Bounty amount: approximately 250 bars

Conjecture 184. *[*`MetaCat_struct_r_ord_left_adjoint_forgetful`*]*

$$\exists F0:\iota\to\iota.\exists F1:\iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon:\iota\to\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ \texttt{struct\_r\_ord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMNDjAqhTse4K8y2RxQPPYdyWzYSsqZgjeG
Bounty amount: approximately 750 bars

7.6 Well-Orderings

Definition 54. *We define* `struct_r_wellord` *to be*

$$\lambda X.struct\_r\ X\\ \wedge unpack\_r\_o\ X\ (\lambda X',r.\\ (\forall x\in X'.\neg r\ x\ x)\wedge(\forall x,y\in X'.r\ x\ y\vee r\ y\ x)\\ \wedge(\forall x,y,z\in X'.r\ x\ y\to r\ y\ z\to r\ x\ z)\\ \wedge(\forall p:\iota\to o.(\forall y\in X'.(\forall x\in X'.r\ x\ y\to p\ x)\to p\ y)\to\forall x\in X'.p\ x))$$

of type $\iota\to o$.

Theorem 129. *[*`MetaCat_struct_r_wellord`*]*

$$MetaCat\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_r_wellord` $X\to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_r\_gen$ `struct_r_wellord` $L1$. ☐

Theorem 130. [MetaCat_struct_r_wellord_Forgetful]

$$\begin{gathered}MetaFunctor\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).\end{gathered}$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_r\_wellord}\ X \to struct\_r\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_r\_Forgetful\_gen$ $\texttt{struct\_r\_wellord}$ $L1$. □

Conjecture 185. [MetaCat_struct_r_wellord_initial]

$$\begin{gathered}\exists Y:\iota.\exists uniqa:\iota\to\iota.\\ initial\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.\end{gathered}$$

Proofgold proposition address: TMFnSVcj5yYTAuDAuhdSGF8Mb9Hq5CbpMn6
Bounty amount: approximately 25 bars

Conjecture 186. [MetaCat_struct_r_wellord_terminal]

$$\begin{gathered}\exists Y:\iota.\exists uniqa:\iota\to\iota.\\ terminal\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\ Y\ uniqa.\end{gathered}$$

Proofgold proposition address: TMViFBt4uVpjLj3RbRU93JG3fmbtLRXGab7
Bounty amount: approximately 25 bars

Conjecture 187. [MetaCat_struct_r_wellord_coproduct_constr]

$$\begin{gathered}\exists coprod:\iota\to\iota\to\iota.\exists i1,i2:\iota\to\iota\to\iota.\exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ coproduct\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ coprod\ i1\ i2\ copair.\end{gathered}$$

Proofgold proposition address: TMNQogGtyCpYNkpjYCSiYjzTLDT7w9o2PGs
Bounty amount: approximately 100 bars

Conjecture 188. [MetaCat_struct_r_wellord_product_constr]

$$\begin{gathered}\exists prod:\iota\to\iota\to\iota.\exists\pi_1,\pi_2:\iota\to\iota\to\iota.\exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ product\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair.\end{gathered}$$

Proofgold proposition address: TMJs2V6hkZTMreFdyriTSfhhXrXZ1YTZjaN
Bounty amount: approximately 100 bars

Conjecture 189. [MetaCat_struct_r_wellord_coequalizer_constr]

$$\begin{gathered}\exists quot:\iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap:\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists fac:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ coequalizer\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp\\ quot\ canonmap\ fac.\end{gathered}$$

Proofgold proposition address: TMNVd3Nz9VwTPV7aEciSBmk7AyVJNNgUMeF
Bounty amount: approximately 125 bars

Conjecture 190. [MetaCat_struct_r_wellord_equalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMTdC3Zu5yUuSMHSfJHchojL5upcKG33z9p
Bounty amount: approximately 125 bars

Conjecture 191. [MetaCat_struct_r_wellord_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMZE2yxjuN7irUahppJaSQryiM8xLdXnhB3
Bounty amount: approximately 250 bars

Conjecture 192. [MetaCat_struct_r_wellord_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMYWeKbK8Ra32nESmKwZnEZfX4SBXMgyJCX
Bounty amount: approximately 250 bars

Conjecture 193. [MetaCat_struct_r_wellord_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMRkTqTWX5tUSQJmdmGTVdsoPhrAMWNGzYJ
Bounty amount: approximately 250 bars

Conjecture 194. [MetaCat_struct_r_wellord_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMGykHZW69KxyDvLoz4MrHV5LYHm8eeXYdT
Bounty amount: approximately 250 bars

Conjecture 195. [MetaCat_struct_r_wellord_nno]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMVofvi5bw3mhFCK18oVr7Py2uYUHfgFLx5
Bounty amount: approximately 250 bars

Conjecture 196. [MetaCat_struct_r_wellord_left_adjoint_forgetful]

$$\begin{array}{c}\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota. \\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom \\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ \texttt{struct\_r\_wellord}\ Hom\_struct\_r\ struct\_id\ struct\_comp \\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.\end{array}$$

Proofgold proposition address: TMV889FkbyiTfp7AS9XHjXuwXfeBkUXYZat
Bounty amount: approximately 750 bars

Chapter 8

Structures with a Binary Operation

Theorem 131. [`MetaCat_struct_b`] *MetaCat struct_b Hom_struct_b struct_id struct_comp.*

Proof. Exact $MetaCat\_struct\_b\_gen\ struct\_b\ (\lambda X, H.H)$. □

Theorem 132. [`MetaCat_struct_b_Forgetful`]

$$\begin{gathered} MetaFunctor\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_b\_Forgetful\_gen\ struct\_b\ (\lambda X, H.H)$. □

Conjecture 197. [`MetaCat_struct_b_initial`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMXrMH9HoshfS4bNWuA2HHch2CWr22pHZZQ
Bounty amount: approximately 25 bars

Conjecture 198. [`MetaCat_struct_b_terminal`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMRx87j3auhFihdZzyegZT7EqKFWv9P8adc
Bounty amount: approximately 25 bars

Conjecture 199. [`MetaCat_struct_b_coproduct_constr`]

$$\begin{gathered} \exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coproduct\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMa4YQxruup3mPJHHagFSE72cJiu1zErFhi
Bounty amount: approximately 100 bars

Conjecture 200. [MetaCat_struct_b_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMYvUnh6Ujwq5sipY861ebf5Ci2MWybXhgG
Bounty amount: approximately 100 bars

Conjecture 201. [MetaCat_struct_b_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMTq7w2tZHa3U5YB3dnKffDsAAekXJCEmB5
Bounty amount: approximately 125 bars

Conjecture 202. [MetaCat_struct_b_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMaF1A5Xeg2zjh3qNgckaUKdoyU7aLMVjaA
Bounty amount: approximately 125 bars

Conjecture 203. [MetaCat_struct_b_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMWt2rd3SBmFcMKpuU57cTTThppf1RjrRsF
Bounty amount: approximately 250 bars

Conjecture 204. [MetaCat_struct_b_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMMuJD83fMpyoxeG4d1f162KeMhQmU2hgQF
Bounty amount: approximately 250 bars

Conjecture 205. [MetaCat_struct_b_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMczYDEd1AuvAei71rzzziHGTBpYJ3vq53p
Bounty amount: approximately 250 bars

Conjecture 206. [MetaCat_struct_b_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMQ4nFrsVKcNv9AKbRWmghNfmVzVAhwWAUf
Bounty amount: approximately 250 bars

Conjecture 207. [MetaCat_struct_b_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMbxg4mDUSTPUz6MzQYtaSLGXh5ys8GunuC
Bounty amount: approximately 250 bars

Conjecture 208. [MetaCat_struct_b_left_adjoint_forgetful]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$struct\_b\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMFPpeHWnYp8jwkcXskQ5SvZ93APnZ3wRG3
Bounty amount: approximately 750 bars

8.1 Quasigroups

Definition 55. *We define* `struct_b_quasigroup` *to be*

$$\lambda X.struct\_b\ X \wedge unpack\_b\_o\ X\ (\lambda X', op.$$
$$(\forall a \in X'.bij\ X'\ X'\ (\lambda x.op\ a\ x))$$
$$\wedge(\forall a \in X'.bij\ X'\ X'\ (\lambda x.op\ x\ a)))$$

of type $\iota\to o$.

Theorem 133. [MetaCat_struct_b_quasigroup]

$$MetaCat\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_quasigroup}\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_gen$ `struct_b_quasigroup` $L1$. □

Theorem 134. [MetaCat_struct_b_quasigroup_Forgetful]

$$MetaFunctor\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_quasigroup` $X \rightarrow struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_Forgetful\_gen$ `struct_b_quasigroup` $L1$. □

Conjecture 209. [`MetaCat_struct_b_quasigroup_initial`]

$$\exists Y : \iota.\exists uniqa : \iota\rightarrow\iota.$$
$$initial\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMRiqnfhADT6LbWSudRJuynqX9mjo4kxMuh
Bounty amount: approximately 25 bars

Conjecture 210. [`MetaCat_struct_b_quasigroup_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota\rightarrow\iota.$$
$$terminal\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMVtPzfVbpQqaE54S3QYPFpSG4Lxi662VUc
Bounty amount: approximately 25 bars

Conjecture 211. [`MetaCat_struct_b_quasigroup_coproduct_constr`]

$$\exists coprod : \iota\rightarrow\iota\rightarrow\iota.\exists i1, i2 : \iota\rightarrow\iota\rightarrow\iota.\exists copair : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMbUoCtZaq6UBfPrSv2vkJJZNPnMSbN7Uug
Bounty amount: approximately 100 bars

Conjecture 212. [`MetaCat_struct_b_quasigroup_product_constr`]

$$\exists prod : \iota\rightarrow\iota\rightarrow\iota.\exists \pi_1, \pi_2 : \iota\rightarrow\iota\rightarrow\iota.\exists pair : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$product\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMVSnzTnMEaGNAtgTJTMYCSbuSCNS-buUUzm
Bounty amount: approximately 100 bars

Conjecture 213. [`MetaCat_struct_b_quasigroup_coequalizer_constr`]

$$\exists quot : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.\exists canonmap : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$\exists fac : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMZiW2nwJeRTg8xnUPGoJMh4Ssz9TQkyUsk
Bounty amount: approximately 125 bars

Conjecture 214. [`MetaCat_struct_b_quasigroup_equalizer_constr`]

$$\exists quot : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.\exists canonmap : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$\exists fac : \iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota\rightarrow\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMY3nMg1xsyNzNHMHDfC3UZXtUcxcJU5e1g
Bounty amount: approximately 125 bars

Conjecture 215. [MetaCat_struct_b_quasigroup_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMXWmX3QjrJ5r6VesucLbJfw5mSuXYG3d2G
Bounty amount: approximately 250 bars

Conjecture 216. [MetaCat_struct_b_quasigroup_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMYhkzUNLQaBWGJ8ZnG6UV6zs1PaQLuxdqN
Bounty amount: approximately 250 bars

Conjecture 217. [MetaCat_struct_b_quasigroup_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMdBgE64bDTCftQnN4oL8X38Q6bXuq6V1y2
Bounty amount: approximately 250 bars

Conjecture 218. [MetaCat_struct_b_quasigroup_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMXV8yPejUCG7A2XUfaRxngFiF6Kpipyazf
Bounty amount: approximately 250 bars

Conjecture 219. [MetaCat_struct_b_quasigroup_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMWuDYzLHGC1FJfqL9y2EAtY3g92ZBDL7bA
Bounty amount: approximately 250 bars

Conjecture 220. *[*`MetaCat_struct_b_quasigroup_left_adjoint_forgetful`*]*

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_quasigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMbeGBiLyZ6ZfkgoBc6y4d7CtbCyro7faFa
Bounty amount: approximately 750 bars

8.2 Loops

Definition 56. *We define* `struct_b_loop` *to be*

$$\lambda X.struct\_b\ X \wedge unpack\_b\_o\ X\ (\lambda X', op.$$
$$(\exists e \in X'.\forall x \in X'.op\ x\ e{=}x \wedge op\ e\ x{=}x)$$
$$\wedge(\forall a \in X'.bij\ X'\ X'\ (\lambda x.op\ a\ x))$$
$$\wedge(\forall a \in X'.bij\ X'\ X'\ (\lambda x.op\ x\ a)))$$

of type $\iota{\to}o$.

Theorem 135. *[*`MetaCat_struct_b_loop`*]*

$$MetaCat\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_loop` $X{\to}struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_gen$ `struct_b_loop` $L1$. ☐

Theorem 136. *[*`MetaCat_struct_b_loop_Forgetful`*]*

$$MetaFunctor\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_loop` $X{\to}struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_Forgetful\_gen$ `struct_b_loop` $L1$. ☐

Conjecture 221. *[*`MetaCat_struct_b_loop_initial`*]*

$$\exists Y : \iota.\exists uniqa : \iota{\to}\iota.$$
$$initial\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMdRm8CoSpPuQpjMtzFHgzX3g3qE1xWaRq4
Bounty amount: approximately 25 bars

Conjecture 222. *[*`MetaCat_struct_b_loop_terminal`*]*

$$\exists Y : \iota.\exists uniqa : \iota{\to}\iota.$$
$$terminal\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMXvG7zrdDR5mvBdxAV9w7fUni29uK81B4K
Bounty amount: approximately 25 bars

Conjecture 223. [MetaCat_struct_b_loop_coproduct_constr]

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota . \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ coproduct\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMM5KMzcoCRMi5VxwjdikuMuaFs1Ed8v8ae
Bounty amount: approximately 100 bars

Conjecture 224. [MetaCat_struct_b_loop_product_constr]

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota . \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ product\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMHNu2CcsyJMYCoppyVD7gCMxVVVJnWyQAp
Bounty amount: approximately 100 bars

Conjecture 225. [MetaCat_struct_b_loop_coequalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ coequalizer\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMZJfwv4vJEqBP9brw2hLxpqAifQGUNAkCf
Bounty amount: approximately 125 bars

Conjecture 226. [MetaCat_struct_b_loop_equalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ equalizer\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMP1r8ZMnYUsX3JLgqYwJFs7H2P9m7pFWof
Bounty amount: approximately 125 bars

Conjecture 227. [MetaCat_struct_b_loop_pushout_constr]

$$\exists po : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ pushout\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ po\ i0\ i1\ copair.$$

Proofgold proposition address: TMHDFJt2eu9tvwp9mCTxSzT9uH8MkeoZx9f
Bounty amount: approximately 250 bars

Conjecture 228. [MetaCat_struct_b_loop_pullback_constr]

$$\exists pb : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ pullback\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMayR9eP6qjmRSRN78SfxSjyg6RHfikRbPS
Bounty amount: approximately 250 bars

Conjecture 229. [MetaCat_struct_b_loop_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMUiKGMj13nouiUVbZHzcT32jd59BjY9vdT
Bounty amount: approximately 250 bars

Conjecture 230. [MetaCat_struct_b_loop_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMKpYXd9U9TQj61wv8ErZ6trgP17ojAr2Ad
Bounty amount: approximately 250 bars

Conjecture 231. [MetaCat_struct_b_loop_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMWsmBTTRQ9Zgb9DrQqT6swdo6HvEJYo3za
Bounty amount: approximately 250 bars

Conjecture 232. [MetaCat_struct_b_loop_left_adjoint_forgetful]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_loop}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMPMJzbjrCR3ayMY533QifQZMu1NsLdyadu
Bounty amount: approximately 750 bars

8.3 Semigroups

Definition 57. *We define* `struct_b_semigroup` *to be*

$$\lambda X.struct\_b\ X \wedge unpack\_b\_o\ X\ (\lambda X', op.$$
$$\forall x,y,z \in X'.op\ (op\ x\ y)\ z{=}op\ x\ (op\ y\ z))$$

of type $\iota\to o$.

Theorem 137. [MetaCat_struct_b_semigroup]

$$MetaCat\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_semigroup}\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_gen$ struct_b_semigroup $L1$. □

Theorem 138. [MetaCat_struct_b_semigroup_Forgetful]

$$MetaFunctor\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ (\lambda\_.True)\ SetHom\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_semigroup}\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_Forgetful\_gen$ struct_b_semigroup $L1$. □

Conjecture 233. [MetaCat_struct_b_semigroup_initial]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.\ initial\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMc8zPng7LjvD9mQ6AKsn9nBdTif7WgzGKH
Bounty amount: approximately 25 bars

Conjecture 234. [MetaCat_struct_b_semigroup_terminal]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.\ terminal\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMKFaMSXdWCZhXYKFPzAcg4XCC3gyPr7sXd
Bounty amount: approximately 25 bars

Conjecture 235. [MetaCat_struct_b_semigroup_coproduct_constr]

$$\exists coprod : \iota \to \iota \to \iota.\exists i1, i2 : \iota \to \iota \to \iota.\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.\ coproduct\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMJjD6iWbDhLMWXbDGV1E8QB3k6x58DpN1f
Bounty amount: approximately 100 bars

Conjecture 236. [MetaCat_struct_b_semigroup_product_constr]

$$\exists prod : \iota \to \iota \to \iota.\exists \pi_1, \pi_2 : \iota \to \iota \to \iota.\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.\ product\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMFizYqVDS5xLizVqzN2H4PPtv85VQEobUV
Bounty amount: approximately 100 bars

Conjecture 237. [MetaCat_struct_b_semigroup_coequalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMNNYRbUP2YUwWBJEHWMnK8u2sTAj6jrEqN
Bounty amount: approximately 125 bars

Conjecture 238. [MetaCat_struct_b_semigroup_equalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMQo5BvLn6Sz3dnQfPmmx9fgo6wSQsKt7xA
Bounty amount: approximately 125 bars

Conjecture 239. [MetaCat_struct_b_semigroup_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMcVZQMaudS2F7KCctMgZTH3vPfkKerfuCe
Bounty amount: approximately 250 bars

Conjecture 240. [MetaCat_struct_b_semigroup_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMRKZDED4egMepzUnbcZWsHyb3mdj5uvJ5q
Bounty amount: approximately 250 bars

Conjecture 241. [MetaCat_struct_b_semigroup_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMUCy5tAsP8CCiBYfo3jQZeQoSxjCTqyneP
Bounty amount: approximately 250 bars

Conjecture 242. [MetaCat_struct_b_semigroup_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMNdAw2Va5hwjpHzAbNrc1sCTXAtX56FjyM
Bounty amount: approximately 250 bars

Conjecture 243. [`MetaCat_struct_b_semigroup_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMcHYmX1Kd5Y7vUye3BJby5EWHGzu83vHB1
Bounty amount: approximately 250 bars

Conjecture 244. [`MetaCat_struct_b_semigroup_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_semigroup}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMa5NwztckyXUZtmMQqF88GifRt1pqzv88F
Bounty amount: approximately 750 bars

8.4 Monoids

Definition 58. *We define* `struct_b_monoid` *to be*

$$\lambda X.struct\_b\ X \wedge unpack\_b\_o\ X\ (\lambda X', op.$$
$$(\forall x, y, z \in X'.op\ (op\ x\ y)\ z{=}op\ x\ (op\ y\ z))$$
$$\wedge(\exists e \in X'.\forall x \in X'.op\ x\ e{=}x \wedge op\ e\ x{=}x))$$

of type $\iota\to o$.

Theorem 139. [`MetaCat_struct_b_monoid`]

$$MetaCat\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_monoid}\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_gen$ `struct_b_monoid` $L1$. □

Theorem 140. [`MetaCat_struct_b_monoid_Forgetful`]

$$MetaFunctor\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_monoid}\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_Forgetful\_gen$ `struct_b_monoid` $L1$. □

Conjecture 245. [`MetaCat_struct_b_monoid_initial`]

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$initial\_p \; \texttt{struct\_b\_monoid} \; Hom\_struct\_b \; struct\_id \; struct\_comp \; Y \; uniqa.$$

Proofgold proposition address: TMMJQomEgPJdPZXcAsPGuZRWyszbaxG-WfmN
Bounty amount: approximately 25 bars

Conjecture 246. [`MetaCat_struct_b_monoid_terminal`]

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$terminal\_p \; \texttt{struct\_b\_monoid} \; Hom\_struct\_b \; struct\_id \; struct\_comp \; Y \; uniqa.$$

Proofgold proposition address: TMEkk49Tn5thhqR1Mp3qYXxAoV5RWxWYWV3
Bounty amount: approximately 25 bars

Conjecture 247. [`MetaCat_struct_b_monoid_coproduct_constr`]

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota . \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coproduct\_constr\_p \; \texttt{struct\_b\_monoid} \; Hom\_struct\_b \; struct\_id \; struct\_comp$$
$$coprod \; i1 \; i2 \; copair.$$

Proofgold proposition address: TMMv76TaCxPX1Kgs9TSs9brnFEMk29LJvuZ
Bounty amount: approximately 100 bars

Conjecture 248. [`MetaCat_struct_b_monoid_product_constr`]

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota . \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$product\_constr\_p \; \texttt{struct\_b\_monoid} \; Hom\_struct\_b \; struct\_id \; struct\_comp$$
$$prod \; \pi_1 \; \pi_2 \; pair.$$

Proofgold proposition address: TMdMx6C4ZLPxdsErp6Mz5a4oHrVyrXYa6Ka
Bounty amount: approximately 100 bars

Conjecture 249. [`MetaCat_struct_b_monoid_coequalizer_constr`]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coequalizer\_constr\_p \; \texttt{struct\_b\_monoid} \; Hom\_struct\_b \; struct\_id \; struct\_comp$$
$$quot \; canonmap \; fac.$$

Proofgold proposition address: TMYxJoerhcREnxqQ8MWURXa5aSZoB9D8wPn
Bounty amount: approximately 125 bars

Conjecture 250. [`MetaCat_struct_b_monoid_equalizer_constr`]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$equalizer\_constr\_p \; \texttt{struct\_b\_monoid} \; Hom\_struct\_b \; struct\_id \; struct\_comp$$
$$quot \; canonmap \; fac.$$

Proofgold proposition address: TMVtNcNyyf9E87V4hCppb7MbtTppiLNyJaq
Bounty amount: approximately 125 bars

Conjecture 251. *[*`MetaCat_struct_b_monoid_pushout_constr`*]*

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMWYwRZkif5nUYBJgKBsioJtqp57TgtjUNp
Bounty amount: approximately 250 bars

Conjecture 252. *[*`MetaCat_struct_b_monoid_pullback_constr`*]*

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMYGx97qR7YKtNmHRFCZnM342Fp7iyC91p5
Bounty amount: approximately 250 bars

Conjecture 253. *[*`MetaCat_struct_b_monoid_product_exponent`*]*

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMKnezv5a1WeRW6fAdMbm3kKGwwhWWf4pWY
Bounty amount: approximately 250 bars

Conjecture 254. *[*`MetaCat_struct_b_monoid_subobject_classifier`*]*

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMRhf7JmQ13d8enS8dt61dFYjwhuXfcwiGP
Bounty amount: approximately 250 bars

Conjecture 255. *[*`MetaCat_struct_b_monoid_nno`*]*

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ \texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMF4c9kNhjHPcv3rbEpbQKYZ1FySc754Y6M
Bounty amount: approximately 250 bars

Conjecture 256. *[*`MetaCat_struct_b_monoid_left_adjoint_forgetful`*]*

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_monoid}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMShMMiRH5fhEi3vbKA5MPnTcEW4C4YHY2P
Bounty amount: approximately 750 bars

8.5 Groups

Theorem 141. [`MetaCat_struct_b_group`] *MetaCat Group Hom_struct_b struct_id struct_comp.*

Proof. We prove the intermediate claim $L1$: $\forall X.Group\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_gen\ Group\ L1$. ☐

Theorem 142. [`MetaCat_struct_b_group_Forgetful`]

$$MetaFunctor\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.Group\ X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_Forgetful\_gen\ Group\ L1$. ☐

Conjecture 257. [`MetaCat_struct_b_group_initial`]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$initial\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMTA9zccV8316TEve4g6B1WwK6RptHD1taL
Bounty amount: approximately 25 bars

Conjecture 258. [`MetaCat_struct_b_group_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$terminal\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMPHPjTFc7MSFep4dg6ooesJCjjWq41Rua8
Bounty amount: approximately 25 bars

Conjecture 259. [`MetaCat_struct_b_group_coproduct_constr`]

$$\exists coprod : \iota \to \iota \to \iota.\exists i1, i2 : \iota \to \iota \to \iota.\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.$$
$$coproduct\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMXnjHKBgJb5JtvYMwKoHfqv99h3prtv78u
Bounty amount: approximately 100 bars

Conjecture 260. [`MetaCat_struct_b_group_product_constr`]

$$\exists prod : \iota \to \iota \to \iota.\exists \pi_1, \pi_2 : \iota \to \iota \to \iota.\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.$$
$$product\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMHTnxyfUpFeWqo8YNxuKkRntBePvjUu127
Bounty amount: approximately 100 bars

Conjecture 261. [MetaCat_struct_b_group_coequalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMVA7fMLAS2n2HQ9khF3goh1J69U9CSmqNR
Bounty amount: approximately 125 bars

Conjecture 262. [MetaCat_struct_b_group_equalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMXADQbRGbbDVLgMzvFB2sRjaNYoEjBijXA
Bounty amount: approximately 125 bars

Conjecture 263. [MetaCat_struct_b_group_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMGi46emuXsEasSwRRTQApf6BtcjfR5Uha5
Bounty amount: approximately 250 bars

Conjecture 264. [MetaCat_struct_b_group_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMaF9ejunsYMdHfVGymxsrs1trPo2zdYs8N
Bounty amount: approximately 250 bars

Conjecture 265. [MetaCat_struct_b_group_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMJipFYs9FhBfWaFmwzhYuJbsZAVA3kT9dZ
Bounty amount: approximately 250 bars

Conjecture 266. [MetaCat_struct_b_group_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMXQHuS343NHGBhxsrMpRwcztMtxD3A9vbv
Bounty amount: approximately 250 bars

Conjecture 267. [MetaCat_struct_b_group_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota. \\ nno\_p\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMQ4QjmPyTgsiDq6acvZYkq747ByPDQFj4c
Bounty amount: approximately 250 bars

Conjecture 268. [MetaCat_struct_b_group_left_adjoint_forgetful]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota. \\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom \\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ Group\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMQvwY1m9iU5rev4qXQjWWGYTZDHwCseEMv
Bounty amount: approximately 750 bars

8.6 Abelian Groups

Definition 59. *We define* `struct_b_abelian_group` *to be*

$$\lambda X.struct\_b\ X \wedge unpack\_b\_o\ X \\ (\lambda X', op.explicit\_Group\ X'\ op \wedge explicit\_abelian\ X'\ op)$$

of type $\iota\to o$.

Theorem 143. [MetaCat_struct_b_abelian_group]

$$MetaCat\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_abelian_group` $X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_gen$ `struct_b_abelian_group` $L1$. □

Theorem 144. [MetaCat_struct_b_abelian_group_Forgetful]

$$MetaFunctor\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_abelian_group` $X \to struct\_b\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_Forgetful\_gen$ `struct_b_abelian_group` $L1$. □

Conjecture 269. [MetaCat_struct_b_abelian_group_initial]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$initial\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TManAuWHFZ4HQqSKX3DTSambvnnSSt9YK5g
Bounty amount: approximately 25 bars

Conjecture 270. [MetaCat_struct_b_abelian_group_terminal]

$$\exists Y : \iota.\exists uniqa : \iota \to \iota.$$
$$terminal\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMca1J4Srj3Ry7CL12zgowQd1eGtbVnPbGj
Bounty amount: approximately 25 bars

Conjecture 271. [MetaCat_struct_b_abelian_group_coproduct_constr]

$$\exists coprod : \iota \to \iota \to \iota.\exists i1, i2 : \iota \to \iota \to \iota.\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMPouPv8XsmUh5hKJ4xZYQmhnH74CgB6FGV
Bounty amount: approximately 100 bars

Conjecture 272. [MetaCat_struct_b_abelian_group_product_constr]

$$\exists prod : \iota \to \iota \to \iota.\exists \pi_1, \pi_2 : \iota \to \iota \to \iota.\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.$$
$$product\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMLcUixEBk1Z6FLwt7gMprESUZrYKf4LXZN
Bounty amount: approximately 100 bars

Conjecture 273. [MetaCat_struct_b_abelian_group_coequalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota.\exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota.$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMVYzbQkgnMHrJqfSWdrojaCFSDE5uhK9NZ
Bounty amount: approximately 125 bars

Conjecture 274. [MetaCat_struct_b_abelian_group_equalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota.\exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota.$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMSKhEBSQmgmxJN99m9Jtk5AXwgajFACmDA
Bounty amount: approximately 125 bars

Conjecture 275. [`MetaCat_struct_b_abelian_group_pushout_constr`]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMafZr6AFX98qRPtTHFhSX6mkZwVXwzC3ni
Bounty amount: approximately 250 bars

Conjecture 276. [`MetaCat_struct_b_abelian_group_pullback_constr`]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMMqAUg8SA4NyCrF4pPWgVhB3zvTSD7AoSn
Bounty amount: approximately 250 bars

Conjecture 277. [`MetaCat_struct_b_abelian_group_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMEwWxj3V9xGqEKVB4WF7KwvoQFpRdUirUp
Bounty amount: approximately 250 bars

Conjecture 278. [`MetaCat_struct_b_abelian_group_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMPTYrPmjjw3etCykop4YEsPS38RtTyeDzj
Bounty amount: approximately 250 bars

Conjecture 279. [`MetaCat_struct_b_abelian_group_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMSidMVZjKPKgYLPWNevu9kVUwWiSuro1vC
Bounty amount: approximately 250 bars

Conjecture 280. [`MetaCat_struct_b_abelian_group_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_abelian\_group}\ Hom\_struct\_b\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMPXaiBEbXVt48TjBUpGodNzx8iJ4vewxZS
Bounty amount: approximately 750 bars

Chapter 9

Structures with a Collection of Subsets

Theorem 145. [`MetaCat_struct_c`] *MetaCat struct_c Hom_struct_c struct_id struct_comp.*

Proof. Exact $MetaCat\_struct\_c\_gen\ struct\_c\ (\lambda X, H.H)$. □

Theorem 146. [`MetaCat_struct_c_Forgetful`]

$$\begin{gathered} MetaFunctor\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_c\_Forgetful\_gen\ struct\_c\ (\lambda X, H.H)$. □

Conjecture 281. [`MetaCat_struct_c_initial`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota{\to}\iota. \\ initial\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMdK7sVzofKwavCDvLY5KLDHcyDQdjRwEQY
Bounty amount: approximately 25 bars

Conjecture 282. [`MetaCat_struct_c_terminal`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota{\to}\iota. \\ terminal\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMcsAWgXh5P1HW4zbUWfUFeY8CM2yXLxydm
Bounty amount: approximately 25 bars

Conjecture 283. [`MetaCat_struct_c_coproduct_constr`]

$$\begin{gathered} \exists coprod : \iota{\to}\iota{\to}\iota.\exists i1, i2 : \iota{\to}\iota{\to}\iota.\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota. \\ coproduct\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMJTx8mFp3urcELt7mVPjakNFdxjN8EHh9t
Bounty amount: approximately 100 bars

Conjecture 284. [MetaCat_struct_c_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMHHipsCLgKdFYftzHR2iKYjKxMpokRZDkt
Bounty amount: approximately 100 bars

Conjecture 285. [MetaCat_struct_c_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMRbLHDHMEDsnwxAr4nXMqG1z4ohDNVtEiK
Bounty amount: approximately 125 bars

Conjecture 286. [MetaCat_struct_c_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMZWqV1RXK2uJjEvoLbA8yaKJ6fdhtvGL4Y
Bounty amount: approximately 125 bars

Conjecture 287. [MetaCat_struct_c_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMXvBTcutD6i6UCD82q1hu6fE5KxndKAPSk
Bounty amount: approximately 250 bars

Conjecture 288. [MetaCat_struct_c_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMQcZZhqK6dUsuBXm9ZzpPyQmpzDSWKDBba
Bounty amount: approximately 250 bars

Conjecture 289. [MetaCat_struct_c_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMbywqXGU4jCRdWr1HxXMjuMcp4smiCiVVZ
Bounty amount: approximately 250 bars

Conjecture 290. *[*`MetaCat_struct_c_subobject_classifier`*]*

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMXy3XCGuPnDyw7KZSGcaAuSySs6YYs1sat
Bounty amount: approximately 250 bars

Conjecture 291. *[*`MetaCat_struct_c_nno`*]*

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMbFHuQ3eiZEdq5ieweyC82CU4WdBy1uBHf
Bounty amount: approximately 250 bars

Conjecture 292. *[*`MetaCat_struct_c_left_adjoint_forgetful`*]*

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$struct\_c\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMYqjJoWCC5tZZGktEFyvXiTyzXRFwDJ9FR
Bounty amount: approximately 750 bars

9.1 Topologies

Definition 60. *We define* `struct_c_topology` *to be*

$$\lambda X.struct\_c\ X {\wedge} unpack\_c\_o\ X\ (\lambda X', Open.$$
$$Open\ (\lambda x.x \in X')$$
$$\wedge(\forall U, V : \iota{\to}o.Open\ U{\to}Open\ V{\to}Open\ (\lambda x.U\ x{\wedge}U\ x))$$
$$\wedge(\forall C : (\iota{\to}o){\to}o.(\forall U : \iota{\to}o.C\ U{\to}Open\ U)$$
$${\to}Open\ (\lambda x.\exists U : \iota{\to}o.C\ U{\wedge}U\ x)))$$

of type $\iota{\to}o$.

Theorem 147. *[*`MetaCat_struct_c_topology`*]*

$$MetaCat\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_c_topology` $X{\to}struct\_c\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_c\_gen$ `struct_c_topology` $L1$. □

Theorem 148. [`MetaCat_struct_c_topology_Forgetful`]

$$MetaFunctor\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ (\lambda\_.True)\ SetHom\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_c\_topology}\ X \to struct\_c\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume H, _. Exact H. Exact $MetaCat\_struct\_c\_Forgetful\_gen$ `struct_c_topology` $L1$. ☐

Conjecture 293. [`MetaCat_struct_c_topology_initial`]

$$\exists Y:\iota.\exists uniqa:\iota\to\iota.\ initial\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMWbwBtEspTGmhNnPj4zyVk61JXdLSXxMRA
Bounty amount: approximately 25 bars

Conjecture 294. [`MetaCat_struct_c_topology_terminal`]

$$\exists Y:\iota.\exists uniqa:\iota\to\iota.\ terminal\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMa5UUZmQb3EGuwW7cPg21uQqw6i4iyoQtK
Bounty amount: approximately 25 bars

Conjecture 295. [`MetaCat_struct_c_topology_coproduct_constr`]

$$\exists coprod:\iota\to\iota\to\iota.\exists i1,i2:\iota\to\iota\to\iota.\exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\ coproduct\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMaXyNP7wQNiDFEAThAQr1mHFwkb7uVHEKn
Bounty amount: approximately 100 bars

Conjecture 296. [`MetaCat_struct_c_topology_product_constr`]

$$\exists prod:\iota\to\iota\to\iota.\exists \pi_1,\pi_2:\iota\to\iota\to\iota.\exists pair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\ product\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMMvoLxK1LfSPVriYDLqkJLKWKcraT3snBJ
Bounty amount: approximately 100 bars

Conjecture 297. [`MetaCat_struct_c_topology_coequalizer_constr`]

$$\exists quot:\iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap:\iota\to\iota\to\iota\to\iota\to\iota.\ \exists fac:\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\ coequalizer\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMGucTrYbhAN8XPbeaNFBiE1hUjeQ3Mkzvc
Bounty amount: approximately 125 bars

Conjecture 298. [`MetaCat_struct_c_topology_equalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMF9MpMyw5e9z7RVnuuAzoGZwrRtoG69JVE
Bounty amount: approximately 125 bars

Conjecture 299. [`MetaCat_struct_c_topology_pushout_constr`]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMQFDCSUxe1i2PaGNmehE7DD8yYTTaBxicB
Bounty amount: approximately 250 bars

Conjecture 300. [`MetaCat_struct_c_topology_pullback_constr`]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMXWksqxmXnuvgV2BNBWK8kpUmRzsNT4Wjb
Bounty amount: approximately 250 bars

Conjecture 301. [`MetaCat_struct_c_topology_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMX3qgVtT7qWKdJC86diB88Nx47jGhWyMXn
Bounty amount: approximately 250 bars

Conjecture 302. [`MetaCat_struct_c_topology_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMcVXxGugpBcWjUv7m7e3WdLZkJfemGvHmY
Bounty amount: approximately 250 bars

Conjecture 303. [`MetaCat_struct_c_topology_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMUfewNq6NJ6vqveKXLvLAyzMFMV2WSBKpp
Bounty amount: approximately 250 bars

Conjecture 304. *[*`MetaCat_struct_c_topology_left_adjoint_forgetful`*]*

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_c\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMMXndYjxs5VdHdL4GvYjrR9xfNzAnXnQ2Q
Bounty amount: approximately 750 bars

9.2 T1 Topologies

Definition 61. *We define* `struct_c_T1_topology` *to be*

$$\lambda X.struct\_c\ X \wedge unpack\_c\_o\ X\ (\lambda X', Open.$$
$$Open\ (\lambda x.x \in X')$$
$$\wedge(\forall U, V : \iota\to o.Open\ U \to Open\ V \to Open\ (\lambda x.U\ x \wedge U\ x))$$
$$\wedge(\forall C : (\iota\to o)\to o.(\forall U : \iota\to o.C\ U \to Open\ U)$$
$$\to Open\ (\lambda x.\exists U : \iota\to o.C\ U \wedge U\ x))$$
$$\wedge(\forall a, b \in X'.a \neq b \to \exists U : \iota\to o.$$
$$Open\ U \wedge exactly1of2\ (U\ a)\ (U\ b)))$$

of type $\iota\to o$.

Theorem 149. *[*`MetaCat_struct_c_T1_topology`*]*

$$MetaCat\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_c\_T1\_topology}\ X \to struct\_c\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_c\_gen$ `struct_c_T1_topology` $L1$. □

Theorem 150. *[*`MetaCat_struct_c_T1_topology_Forgetful`*]*

$$MetaFunctor\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_c\_T1\_topology}\ X \to struct\_c\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_c\_Forgetful\_gen$ `struct_c_T1_topology` $L1$. □

Conjecture 305. *[*`MetaCat_struct_c_T1_topology_initial`*]*

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$initial\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMWEBNJtcXbF28cQX3acsBozNdxi3QLHnsH
Bounty amount: approximately 25 bars

Conjecture 306. [`MetaCat_struct_c_T1_topology_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota{\to}\iota.$$
$$terminal\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMSJvdfHaRB3gjMRpGDSwdYiLckUTbKadWT
Bounty amount: approximately 25 bars

Conjecture 307. [`MetaCat_struct_c_T1_topology_coproduct_constr`]

$$\exists coprod : \iota{\to}\iota{\to}\iota.\exists i1, i2 : \iota{\to}\iota{\to}\iota.\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMcJPbjcisP9VBF5U87xEGQxCFiaFPUw29n
Bounty amount: approximately 100 bars

Conjecture 308. [`MetaCat_struct_c_T1_topology_product_constr`]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMM4iedkF9uwttaDrHrcW5CCH8PyViT9JrB
Bounty amount: approximately 100 bars

Conjecture 309. [`MetaCat_struct_c_T1_topology_coequalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMKL8p3sEqwdEQCc7m8GpFymfCtpecaHDga
Bounty amount: approximately 125 bars

Conjecture 310. [`MetaCat_struct_c_T1_topology_equalizer_constr`]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMEkXuwWrQ6UK7VhiweagehjVzjmk9inRGc
Bounty amount: approximately 125 bars

Conjecture 311. [`MetaCat_struct_c_T1_topology_pushout_constr`]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMdPVveG7fE46aaP8vhFH5AMvWnnkuR8ccf
Bounty amount: approximately 250 bars

Conjecture 312. [MetaCat_struct_c_T1_topology_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMYx7h12kPi8izC7nHsYTVEh5VhDtNYQRXE
Bounty amount: approximately 250 bars

Conjecture 313. [MetaCat_struct_c_T1_topology_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMMVmX7Na2QbRLeinrJEnnrYM7dwB6yhKuZ
Bounty amount: approximately 250 bars

Conjecture 314. [MetaCat_struct_c_T1_topology_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMJ4j4Z5ePnYq8q6sGpx9kP19egqpMJp8BH
Bounty amount: approximately 250 bars

Conjecture 315. [MetaCat_struct_c_T1_topology_nno]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ \texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMcr86K2jkEXZWR64WhdMeKsNMyJkgsF9UE
Bounty amount: approximately 250 bars

Conjecture 316. [MetaCat_struct_c_T1_topology_left_adjoint_forgetful]

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists \eta, \varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_c\_T1\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMJ3xGXo7LhvVEydBdredwK4eJ7FUSDP7p4
Bounty amount: approximately 750 bars

9.3 Hausdorff Topologies

Definition 62. *We define* `struct_c_Hausdorff_topology` *to be*

$$\begin{array}{c}\lambda X.struct\_c\ X \wedge unpack\_c\_o\ X\ (\lambda X', Open.\\ Open\ (\lambda x.x \in X')\\ \wedge(\forall U, V : \iota \to o.Open\ U \to Open\ V \to Open\ (\lambda x.U\ x \wedge U\ x))\\ \wedge(\forall C : (\iota \to o) \to o.(\forall U : \iota \to o.C\ U \to Open\ U)\\ \to Open\ (\lambda x.\exists U : \iota \to o.C\ U \wedge U\ x))\\ \wedge(\forall a, b \in X'.a \neq b \to \exists U, V : \iota \to o.\\ Open\ U \wedge Open\ V \wedge U\ a \wedge V\ b \wedge (\forall x.U\ x \to \neg V\ x)))\end{array}$$

of type $\iota \to o$.

Theorem 151. *[*`MetaCat_struct_c_Hausdorff_topology`*]*

$MetaCat$ `struct_c_Hausdorff_topology` $Hom\_struct\_c\ struct\_id\ struct\_comp$.

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_c_Hausdorff_topology` $X \to struct\_c\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_c\_gen$ `struct_c_Hausdorff_topology` $L1$. □

Theorem 152. *[*`MetaCat_struct_c_Hausdorff_topology_Forgetful`*]*

$$\begin{array}{c}MetaFunctor\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).\end{array}$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_c_Hausdorff_topology` $X \to struct\_c\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_c\_Forgetful\_gen$ `struct_c_Hausdorff_topology` $L1$. □

Conjecture 317. *[*`MetaCat_struct_c_Hausdorff_topology_initial`*]*

$$\begin{array}{c}\exists Y : \iota.\exists uniqa : \iota \to \iota.\\ initial\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa.\end{array}$$

Proofgold proposition address: TMdriYzCfxvVkMW2fXdJrmYJWPMcCEZQdrt
Bounty amount: approximately 25 bars

Conjecture 318. *[*`MetaCat_struct_c_Hausdorff_topology_terminal`*]*

$$\begin{array}{c}\exists Y : \iota.\exists uniqa : \iota \to \iota.\\ terminal\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ Y\ uniqa.\end{array}$$

Proofgold proposition address: TMYMgEdgc1iwDfhj5oqfJpq8AANKuwHYP17
Bounty amount: approximately 25 bars

Conjecture 319. *[*`MetaCat_struct_c_Hausdorff_topology_coproduct_constr`*]*

$$\begin{array}{c}\exists coprod : \iota \to \iota \to \iota.\exists i1, i2 : \iota \to \iota \to \iota.\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.\\ coproduct\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\\ coprod\ i1\ i2\ copair.\end{array}$$

Proofgold proposition address: TMXcCpyuCHf3jkXvKSQPEqSjCgWVUDEMA2k
Bounty amount: approximately 100 bars

Conjecture 320. [MetaCat_struct_c_Hausdorff_topology_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMPt33qeKHq6pBM4DpRhZLe62bPBvXwtQ1B
Bounty amount: approximately 100 bars

Conjecture 321. [MetaCat_struct_c_Hausdorff_topology_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMSfc2bj7BawoS3sBBe1UNFKEwyVuJ6ptzf
Bounty amount: approximately 125 bars

Conjecture 322. [MetaCat_struct_c_Hausdorff_topology_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMNqFfcbwN5cr8qB7swNjQ5VDUYZaufYD4M
Bounty amount: approximately 125 bars

Conjecture 323. [MetaCat_struct_c_Hausdorff_topology_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ po\ i0\ i1\ copair.$$

Proofgold proposition address: TMZTGc5nktuopKJ1h4dDFvBusySQ1WxLNf3
Bounty amount: approximately 250 bars

Conjecture 324. [MetaCat_struct_c_Hausdorff_topology_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp\ pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMcZU5rz9rGGgYR3H19WqCZJLmmBeETHZ3X
Bounty amount: approximately 250 bars

Conjecture 325. [MetaCat_struct_c_Hausdorff_topology_product_exponent]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists\pi_1,\pi_2 : \iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists exp : \iota{\to}\iota{\to}\iota.\exists a : \iota{\to}\iota{\to}\iota.\exists lm : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMLzq3QjMszydz2TPLPV8HFfGh1pp7aRRtf
Bounty amount: approximately 250 bars

Conjecture 326. [MetaCat_struct_c_Hausdorff_topology_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists constr : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMTr5HC4ZfSZNXcsdgTRExPQLNH1hXxojvC
Bounty amount: approximately 250 bars

Conjecture 327. [MetaCat_struct_c_Hausdorff_topology_nno]

$$\exists one : \iota.\exists uniqa : \iota{\to}\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota{\to}\iota{\to}\iota{\to}\iota.$$
$$nno\_p\ \texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMNR6k1zFJgjfytL585hYt56VSTbrewtpjf
Bounty amount: approximately 250 bars

Conjecture 328. [MetaCat_struct_c_Hausdorff_topology_left_adjoint_forgetful]

$$\exists F0 : \iota{\to}\iota.\exists F1 : \iota{\to}\iota{\to}\iota{\to}\iota.\exists\eta,\varepsilon : \iota{\to}\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_c\_Hausdorff\_topology}\ Hom\_struct\_c\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMJK9bw7i9Yfw6Kf287tnycBja3rianqAgK
Bounty amount: approximately 750 bars

Chapter 10

Structures with Two Binary Operations and an Element

Theorem 153. [MetaCat_struct_b_b_e]

$$MetaCat\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp.$$

Proof. Exact $MetaCat\_struct\_b\_b\_e\_gen\ struct\_b\_b\_e\ (\lambda X, H.H)$. □

Theorem 154. [MetaCat_struct_b_b_e_Forgetful]

$$\begin{gathered} MetaFunctor\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_b\_b\_e\_Forgetful\_gen\ struct\_b\_b\_e\ (\lambda X, H.H)$. □

Conjecture 329. [MetaCat_struct_b_b_e_initial]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMQkNvS4supNuPqp7q6Xipu3E3sJ83fnGYo
Bounty amount: approximately 25 bars

Conjecture 330. [MetaCat_struct_b_b_e_terminal]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMYPemJSkmugHVQmCmFQxLATyiC79XuYpRY
Bounty amount: approximately 25 bars

Conjecture 331. [MetaCat_struct_b_b_e_coproduct_constr]

$$\begin{gathered} \exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coproduct\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMLd6AnkCVh62PB3qgAZo8mTkSaW6yqew1k
Bounty amount: approximately 100 bars

Conjecture 332. [MetaCat_struct_b_b_e_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMM186bUScgLfU9XJpgUwRdFRt7X8zwGxFX
Bounty amount: approximately 100 bars

Conjecture 333. [MetaCat_struct_b_b_e_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMHyGiW6DjAxttviFwRzh2UKUpAjRS5SAso
Bounty amount: approximately 125 bars

Conjecture 334. [MetaCat_struct_b_b_e_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMHnxcQJsTw15AH8KJ2XCjfbq1fKMXL1beP
Bounty amount: approximately 125 bars

Conjecture 335. [MetaCat_struct_b_b_e_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMRwATp4CthcGqCDVEJ3CuxxV8rhkNj3VM1
Bounty amount: approximately 250 bars

Conjecture 336. [MetaCat_struct_b_b_e_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMYw4VCTfno7KQBmboCFYWqpfuUeK4PJG8v
Bounty amount: approximately 250 bars

Conjecture 337. [`MetaCat_struct_b_b_e_product_exponent`]

$$\begin{gathered}\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.\\ \exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.\\ product\_exponent\_constr\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{gathered}$$

Proofgold proposition address: TMMLUno8tm1qVSZZwcGpe9Cb29YxHsnr14p
Bounty amount: approximately 250 bars

Conjecture 338. [`MetaCat_struct_b_b_e_subobject_classifier`]

$$\begin{gathered}\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.\\ \exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{gathered}$$

Proofgold proposition address: TMbrWpN9xugVi6wgYRiqJLYF3zzPhCdveX7
Bounty amount: approximately 250 bars

Conjecture 339. [`MetaCat_struct_b_b_e_nno`]

$$\begin{gathered}\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.\\ nno\_p\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ N\ zer\ suc\ rec.\end{gathered}$$

Proofgold proposition address: TMaCES9qhYUomwms1ArZFRFYvngWxAiEnLL
Bounty amount: approximately 250 bars

Conjecture 340. [`MetaCat_struct_b_b_e_left_adjoint_forgetful`]

$$\begin{gathered}\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon : \iota\to\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ struct\_b\_b\_e\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.\end{gathered}$$

Proofgold proposition address: TMWATC7Jrb9NxHqKvATYYb1QRdLQjPYHh5d
Bounty amount: approximately 750 bars

10.1 Rings without a Multiplicative Identity

Theorem 155. [`MetaCat_struct_b_b_e_rng`] $MetaCat\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp.$

Proof. We prove the intermediate claim $L1$: $\forall X.Rng\ X\to struct\_b\_b\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_gen\ Rng\ L1$. □

Theorem 156. [`MetaCat_struct_b_b_e_rng_Forgetful`]

$$\begin{gathered}MetaFunctor\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f).\end{gathered}$$

Proof. We prove the intermediate claim $L1$: $\forall X.Rng\ X \to struct\_b\_b\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_Forgetful\_gen\ Rng\ L1$. □

Conjecture 341. [`MetaCat_struct_b_b_e_rng_initial`]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$initial\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMSha4UxbHHjvywRCi6HndCHRqCtui9im2N
Bounty amount: approximately 25 bars

Conjecture 342. [`MetaCat_struct_b_b_e_rng_terminal`]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$terminal\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMTXtjKiEqEVton5pSC3pY54BZUp2L7Qc7J
Bounty amount: approximately 25 bars

Conjecture 343. [`MetaCat_struct_b_b_e_rng_coproduct_constr`]

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMNpny6Y4t79ydQgC3u5UWDSAXuxterMEbF
Bounty amount: approximately 100 bars

Conjecture 344. [`MetaCat_struct_b_b_e_rng_product_constr`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMPATwu9v38r76h2e7eA64fYRdLaGye7xf7
Bounty amount: approximately 100 bars

Conjecture 345. [`MetaCat_struct_b_b_e_rng_coequalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMPt2hrne9LvmBDXFtRRSBeL6o8FP47LcCx
Bounty amount: approximately 125 bars

Conjecture 346. [`MetaCat_struct_b_b_e_rng_equalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMQN6V2ijxPXb9ftL7gVrLEG6LimKvNMPRT
Bounty amount: approximately 125 bars

Conjecture 347. *[*`MetaCat_struct_b_b_e_rng_pushout_constr`*]*

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMNNNS9jg97naPVXASGBgfQQn4qwE2uQEUG
Bounty amount: approximately 250 bars

Conjecture 348. *[*`MetaCat_struct_b_b_e_rng_pullback_constr`*]*

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMKkpwcTpc2pNYFwazpb4UKS3QuzpBYs6b8
Bounty amount: approximately 250 bars

Conjecture 349. *[*`MetaCat_struct_b_b_e_rng_product_exponent`*]*

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMWwwhFgFhLz3MrddoyuSUipNyMQNkUeMcn
Bounty amount: approximately 250 bars

Conjecture 350. *[*`MetaCat_struct_b_b_e_rng_subobject_classifier`*]*

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMWx2xRTyyoht8rytcxbW1uMjJGzSGDjRUb
Bounty amount: approximately 250 bars

Conjecture 351. *[*`MetaCat_struct_b_b_e_rng_nno`*]*

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMFPePuuKEUxrPztiJvPqqaYfrrVkNRR-SAi
Bounty amount: approximately 250 bars

Conjecture 352. [MetaCat_struct_b_b_e_rng_left_adjoint_forgetful]

$$\exists F0 : \iota \to \iota . \exists F1 : \iota \to \iota \to \iota \to \iota . \exists \eta, \varepsilon : \iota \to \iota .$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$Rng\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMSTDTKY7nVkEVMnYWFSqk1tiBW7dVUvz3d
Bounty amount: approximately 750 bars

10.2 Commutative Rings without a Multiplicative Identity

Definition 63. *We define* `struct_b_b_e_crng` *to be*

$$\lambda R.struct\_b\_b\_e\ R \wedge unpack\_b\_b\_e\_o\ R\ (\lambda R, plus, mult, zero.$$
$$explicit\_Rng\ R\ zero\ plus\ mult$$
$$\wedge (\forall x, y \in R.mult\ x\ y = mult\ y\ x))$$

of type $\iota \to o$.

Theorem 157. [MetaCat_struct_b_b_e_crng]

$$MetaCat\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_b_e_crng` $X \to struct\_b\_b\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_gen$ `struct_b_b_e_crng` $L1$. □

Theorem 158. [MetaCat_struct_b_b_e_crng_Forgetful]

$$MetaFunctor\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_b_e_crng` $X \to struct\_b\_b\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_Forgetful\_gen$ `struct_b_b_e_crng` $L1$. □

Conjecture 353. [MetaCat_struct_b_b_e_crng_initial]

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$initial\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMS33Tk1ubr7irSNt6vYuD7Gv3ieNptC26W
Bounty amount: approximately 25 bars

Conjecture 354. [MetaCat_struct_b_b_e_crng_terminal]

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$terminal\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMHLKcACEsqirSrMBZ5jL3BZcCZUmz2YuA2
Bounty amount: approximately 25 bars

Conjecture 355. [MetaCat_struct_b_b_e_crng_coproduct_constr]

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMP41sKqC3WMSWFrfe7Ci95S8ftznHFLFnr
Bounty amount: approximately 100 bars

Conjecture 356. [MetaCat_struct_b_b_e_crng_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMc7XPrLHjh4KELGHgJbHCkjp8PEf3y5ASK
Bounty amount: approximately 100 bars

Conjecture 357. [MetaCat_struct_b_b_e_crng_coequalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMHvRPgmWFhw2GdhdGVEkVZogKaafxjqJ5U
Bounty amount: approximately 125 bars

Conjecture 358. [MetaCat_struct_b_b_e_crng_equalizer_constr]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMUL71PCgqk2LuZDU5yRY1aQYx5fYwJtLG8
Bounty amount: approximately 125 bars

Conjecture 359. [MetaCat_struct_b_b_e_crng_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMPucCpsKxkTkYLfPCQJKefkRgrEAvzP76y
Bounty amount: approximately 250 bars

Conjecture 360. [MetaCat_struct_b_b_e_crng_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMWUuLutJCDTpECRGxBw4roEXbPnvJ38Ns4
Bounty amount: approximately 250 bars

Conjecture 361. [`MetaCat_struct_b_b_e_crng_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMYz1ADCrv3XBaUn9i7m8Zm53iX9298xQhv
Bounty amount: approximately 250 bars

Conjecture 362. [`MetaCat_struct_b_b_e_crng_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMb7Qt1Lop9B4P6bPpBEN1caGBJkTgAT8Nt
Bounty amount: approximately 250 bars

Conjecture 363. [`MetaCat_struct_b_b_e_crng_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMPVvTp6mSZzzzzfC4N9s8y5fLp9FEFXH6v
Bounty amount: approximately 250 bars

Conjecture 364. [`MetaCat_struct_b_b_e_crng_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_b\_e\_crng}\ Hom\_struct\_b\_b\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMHo8UAybuPyQJtgRThf9aui3RfUkxN3ZZn
Bounty amount: approximately 750 bars

Chapter 11

Structures with Two Binary Operations and Two Elements

Theorem 159. [`MetaCat_struct_b_b_e_e`]

$$MetaCat\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp.$$

Proof. Exact $MetaCat\_struct\_b\_b\_e\_e\_gen\ struct\_b\_b\_e\_e\ (\lambda X, H.H)$. □

Theorem 160. [`MetaCat_struct_b_b_e_e_Forgetful`]

$$\begin{gathered} MetaFunctor\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{gathered}$$

Proof. Exact $MetaCat\_struct\_b\_b\_e\_e\_Forgetful\_gen\ struct\_b\_b\_e\_e\ (\lambda X, H.H)$. □

Conjecture 365. [`MetaCat_struct_b_b_e_e_initial`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota \to \iota. \\ initial\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMKr7hAeanCZy3UHSb6gNThorYzHfV3Aven
Bounty amount: approximately 25 bars

Conjecture 366. [`MetaCat_struct_b_b_e_e_terminal`]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota \to \iota. \\ terminal\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMKQtmUXDb8WJnxXVAW7k5qH59ofPo2s7Dv
Bounty amount: approximately 25 bars

Conjecture 367. [MetaCat_struct_b_b_e_e_coproduct_constr]

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota . \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coproduct\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMMskP97HNgM5sPcbo8aemVPgrNRch3p5jK
Bounty amount: approximately 100 bars

Conjecture 368. [MetaCat_struct_b_b_e_e_product_constr]

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota . \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$product\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMPVtGrkAwCWVCQGKHWXbeiLcN1g91wNwHU
Bounty amount: approximately 100 bars

Conjecture 369. [MetaCat_struct_b_b_e_e_coequalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coequalizer\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMHDRMwyaZuQxLBgY41ACq18utaw8nEpQi3
Bounty amount: approximately 125 bars

Conjecture 370. [MetaCat_struct_b_b_e_e_equalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$equalizer\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMLn8PX2C8WoacFQM7CjTcdVUcApck4YSS2
Bounty amount: approximately 125 bars

Conjecture 371. [MetaCat_struct_b_b_e_e_pushout_constr]

$$\exists po : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$pushout\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMKJeFwa74NGhESgrpdd6Lnnfi6hCRHYzYW
Bounty amount: approximately 250 bars

Conjecture 372. [MetaCat_struct_b_b_e_e_pullback_constr]

$$\exists pb : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$pullback\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMRUqttVyzvrsjuh3YQwPDpujFuRN4V1GHh
Bounty amount: approximately 250 bars

Conjecture 373. [MetaCat_struct_b_b_e_e_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMcgB5nZNBWpKpKXfLaxfpQauSt6Ntz6Gnp
Bounty amount: approximately 250 bars

Conjecture 374. [MetaCat_struct_b_b_e_e_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMWKuH95Ssz9BwSW5xLMHvvzKytWnnXfPPc
Bounty amount: approximately 250 bars

Conjecture 375. [MetaCat_struct_b_b_e_e_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMHKsKz9mATdYp6gASpUagsA7kVgPbadj6J
Bounty amount: approximately 250 bars

Conjecture 376. [MetaCat_struct_b_b_e_e_left_adjoint_forgetful]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$struct\_b\_b\_e\_e\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMZro9bQwEpBhfbCrSHxbYYjsQxkJXS28dL
Bounty amount: approximately 750 bars

11.1 Semirings

Definition 64. *We define* `struct_b_b_e_e_semiring` *to be*

$$\lambda R.struct\_b\_b\_e\_e\ R \wedge unpack\_b\_b\_e\_e\_o\ R\ (\lambda R, plus, mult, zero, one.$$
$$(\forall x, y, z \in R.plus\ (plus\ x\ y)\ z{=}plus\ x\ (plus\ y\ z))$$
$$\wedge(\forall x, y \in R.plus\ x\ y{=}plus\ y\ x)$$
$$\wedge(\forall x \in R.plus\ x\ zero{=}x)$$
$$\wedge(\forall x, y, z \in R.mult\ (mult\ x\ y)\ z{=}mult\ x\ (mult\ y\ z))$$
$$\wedge(\forall x \in R.mult\ x\ one{=}x \wedge mult\ one\ x{=}x)$$
$$\wedge(\forall x, y, z \in R.mult\ x\ (plus\ y\ z){=}plus\ (mult\ x\ y)\ (mult\ x\ z))$$
$$\wedge(\forall x, y, z \in R.mult\ (plus\ x\ y)\ z{=}plus\ (mult\ x\ z)\ (mult\ y\ z))$$
$$\wedge(\forall x \in R.mult\ x\ zero{=}zero)$$
$$\wedge(\forall x \in R.mult\ zero\ x{=}zero))$$

of type $\iota \to o$.

Theorem 161. *[*`MetaCat_struct_b_b_e_e_semiring`*]*

$MetaCat$ `struct_b_b_e_e_semiring` $Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$.

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_b_e_e_semiring` $X \to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_gen$ `struct_b_b_e_e_semiring` $L1$. ☐

Theorem 162. *[*`MetaCat_struct_b_b_e_e_semiring_Forgetful`*]*

$$MetaFunctor\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$(\lambda \_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.$`struct_b_b_e_e_semiring` $X \to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_Forgetful\_gen$ `struct_b_b_e_e_semiring` $L1$. ☐

Conjecture 377. *[*`MetaCat_struct_b_b_e_e_semiring_initial`*]*

$$\exists Y : \iota. \exists uniqa : \iota \to \iota.$$
$$initial\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMVTgRx5SiYPV9pxcGYkCPEWt5WFZHNwBvb
Bounty amount: approximately 25 bars

Conjecture 378. *[*`MetaCat_struct_b_b_e_e_semiring_terminal`*]*

$$\exists Y : \iota. \exists uniqa : \iota \to \iota.$$
$$terminal\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMckdj3FQHDFQNYJvNuptMhhhtzFM2Rfox7
Bounty amount: approximately 25 bars

Conjecture 379. [MetaCat_struct_b_b_e_e_semiring_coproduct_constr]

$$\exists coprod : \iota{\to}\iota{\to}\iota.\exists i1, i2 : \iota{\to}\iota{\to}\iota.\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMSRcLSwhp3Bo9GpEbfNvtwScu9FPAiQg5U
Bounty amount: approximately 100 bars

Conjecture 380. [MetaCat_struct_b_b_e_e_semiring_product_constr]

$$\exists prod : \iota{\to}\iota{\to}\iota.\exists \pi_1, \pi_2 : \iota{\to}\iota{\to}\iota.\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$product\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMHoPXQagBDYpAsUxMX1pcSSkjf2g7MjNKq
Bounty amount: approximately 100 bars

Conjecture 381. [MetaCat_struct_b_b_e_e_semiring_coequalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMR38pK1Uq8RAzRSfoNZdsGAijHCEdDhje2
Bounty amount: approximately 125 bars

Conjecture 382. [MetaCat_struct_b_b_e_e_semiring_equalizer_constr]

$$\exists quot : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists canonmap : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists fac : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMSm83X8KpPdtBFNVJydV4yCZjUG2E9g6Cn
Bounty amount: approximately 125 bars

Conjecture 383. [MetaCat_struct_b_b_e_e_semiring_pushout_constr]

$$\exists po : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists i1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists copair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMS8PU6Jbk43Li4ciEtMRTo3JizFMHXPKb6
Bounty amount: approximately 250 bars

Conjecture 384. [MetaCat_struct_b_b_e_e_semiring_pullback_constr]

$$\exists pb : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_0 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.\exists \pi_1 : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$\exists pair : \iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota{\to}\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMGSMaKp6YPbUUXyzoQ4vva2SJLZfVnELs8
Bounty amount: approximately 250 bars

Conjecture 385. [`MetaCat_struct_b_b_e_e_semiring_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMEr88Dp1QFes9zYAmJmt9NLdRgEj1ghSZr
Bounty amount: approximately 250 bars

Conjecture 386. [`MetaCat_struct_b_b_e_e_semiring_subobject_classifier`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMTFxNeNNRdfGLifPB6Jv2jJ3Li5cy3t9M5
Bounty amount: approximately 250 bars

Conjecture 387. [`MetaCat_struct_b_b_e_e_semiring_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMKpDiWFLUbSc4D1tFwn7q6U8asBKVys5CK
Bounty amount: approximately 250 bars

Conjecture 388. [`MetaCat_struct_b_b_e_e_semiring_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_b\_e\_e\_semiring}\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMNZHKsk8veqBkEzjS3n1ndQPpCGUfZr1Aj
Bounty amount: approximately 750 bars

11.2 Rings

Theorem 163. [`MetaCat_struct_b_b_e_e_ring`] $MetaCat\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struc$

Proof. We prove the intermediate claim $L1$: $\forall X.Ring\ X \to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_gen\ Ring\ L1$. □

Theorem 164. [MetaCat_struct_b_b_e_e_ring_Forgetful]

$$\begin{gathered} MetaFunctor\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f). \end{gathered}$$

Proof. We prove the intermediate claim $L1$: $\forall X.Ring\ X \to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_Forgetful\_gen\ Ring\ L1$. □

Conjecture 389. [MetaCat_struct_b_b_e_e_ring_initial]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMHDifwhhM4meCh4X71SYMfDp13MCnC6Ucb
Bounty amount: approximately 25 bars

Conjecture 390. [MetaCat_struct_b_b_e_e_ring_terminal]

$$\begin{gathered} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{gathered}$$

Proofgold proposition address: TMckNLm5HZV3y3W76c8QpqCuwRC8V4G9A7x
Bounty amount: approximately 25 bars

Conjecture 391. [MetaCat_struct_b_b_e_e_ring_coproduct_constr]

$$\begin{gathered} \exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coproduct\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ coprod\ i1\ i2\ copair. \end{gathered}$$

Proofgold proposition address: TMGSDWiPDrkqeUsc9phssciLeBwvD6b7ir2
Bounty amount: approximately 100 bars

Conjecture 392. [MetaCat_struct_b_b_e_e_ring_product_constr]

$$\begin{gathered} \exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ product\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ prod\ \pi_1\ \pi_2\ pair. \end{gathered}$$

Proofgold proposition address: TMVgv99A3a9nu7VBavJGerhu2KGpWkqdXGP
Bounty amount: approximately 100 bars

Conjecture 393. [MetaCat_struct_b_b_e_e_ring_coequalizer_constr]

$$\begin{gathered} \exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota. \\ \exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota. \\ coequalizer\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac. \end{gathered}$$

Proofgold proposition address: TMTr6ipjfBKGGP5rtg8iR1fEKaaSWCnv6N8
Bounty amount: approximately 125 bars

Conjecture 394. [MetaCat_struct_b_b_e_e_ring_equalizer_constr]

$$\begin{array}{c}\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ equalizer\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ quot\ canonmap\ fac.\end{array}$$

Proofgold proposition address: TMct8TCJvdci7KzFFzNdociCHqJFzrJAeMK
Bounty amount: approximately 125 bars

Conjecture 395. [MetaCat_struct_b_b_e_e_ring_pushout_constr]

$$\begin{array}{c}\exists po : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ pushout\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ po\ i0\ i1\ copair.\end{array}$$

Proofgold proposition address: TMJnyFBUU93HnXk2mL4hp9m8c5ntSNFacJx
Bounty amount: approximately 250 bars

Conjecture 396. [MetaCat_struct_b_b_e_e_ring_pullback_constr]

$$\begin{array}{c}\exists pb : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ pullback\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ pb\ \pi_0\ \pi_1\ pair.\end{array}$$

Proofgold proposition address: TMJTE2nR2NdBbTWd4NoVMWpsvM6KafXX8Ba
Bounty amount: approximately 250 bars

Conjecture 397. [MetaCat_struct_b_b_e_e_ring_product_exponent]

$$\begin{array}{c}\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota . \\ \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ \exists exp : \iota \to \iota \to \iota . \exists a : \iota \to \iota \to \iota . \exists lm : \iota \to \iota \to \iota \to \iota \to \iota . \\ product\_exponent\_constr\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{array}$$

Proofgold proposition address: TMXHC8DgcYsZxhqstnssCkd4WM5DF47am8S
Bounty amount: approximately 250 bars

Conjecture 398. [MetaCat_struct_b_b_e_e_ring_subobject_classifier]

$$\begin{array}{c}\exists one : \iota . \exists uniqa : \iota \to \iota . \exists Omega : \iota . \exists tru : \iota . \exists ch : \iota \to \iota \to \iota \to \iota . \\ \exists constr : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \\ subobject\_classifier\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{array}$$

Proofgold proposition address: TMZK7QasesUFK7J9oZxS2ZKYQkghTwxC19J
Bounty amount: approximately 250 bars

Conjecture 399. [MetaCat_struct_b_b_e_e_ring_nno]

$$\begin{array}{c}\exists one : \iota . \exists uniqa : \iota \to \iota . \exists N : \iota . \exists zer, suc : \iota . \exists rec : \iota \to \iota \to \iota \to \iota . \\ nno\_p\ Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp \\ one\ uniqa\ N\ zer\ suc\ rec.\end{array}$$

Proofgold proposition address: TMGUkRfLpvEGrsdL3mny39Zq45gxkpwhHAp
Bounty amount: approximately 250 bars

Conjecture 400. [`MetaCat_struct_b_b_e_e_ring_left_adjoint_forgetful`]

$$\exists F0:\iota\to\iota.\exists F1:\iota\to\iota\to\iota\to\iota.\exists\eta,\varepsilon:\iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$Ring\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X,Y,f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMUZj1zYJxGRQ64WTzQ2qaeMeeQXtaLqgVT
Bounty amount: approximately 750 bars

11.3 Commutative Rings

Theorem 165. [`MetaCat_struct_b_b_e_e_cring`] $MetaCat\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp.$

Proof. We prove the intermediate claim $L1$: $\forall X.CRing\ X\to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_gen\ CRing\ L1$. □

Theorem 166. [`MetaCat_struct_b_b_e_e_cring_Forgetful`]

$$MetaFunctor\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X,Y,Z,f,g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X,Y,f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.CRing\ X\to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H,\_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_Forgetful\_gen\ CRing\ L1$. □

Conjecture 401. [`MetaCat_struct_b_b_e_e_cring_initial`]

$$\exists Y:\iota.\exists uniqa:\iota\to\iota.$$
$$initial\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMQWkYk3yNbyVMFjw5bPeBh7FvmaPTYXWpH
Bounty amount: approximately 25 bars

Conjecture 402. [`MetaCat_struct_b_b_e_e_cring_terminal`]

$$\exists Y:\iota.\exists uniqa:\iota\to\iota.$$
$$terminal\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMKcisUhcVgLCZfhUgUEf5GrJ6D1kciJjrj
Bounty amount: approximately 25 bars

Conjecture 403. [`MetaCat_struct_b_b_e_e_cring_coproduct_constr`]

$$\exists coprod:\iota\to\iota\to\iota.\exists i1,i2:\iota\to\iota\to\iota.\exists copair:\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMHdmWEmUAgW4XfRRW1F8ozmFSQvpkRJtvZ
Bounty amount: approximately 100 bars

Conjecture 404. [`MetaCat_struct_b_b_e_e_cring_product_constr`]

$$\exists prod : \iota\to\iota\to\iota.\exists\pi_1,\pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMLjgxWgFWjRhhxmw2hHoNHHhiRWmGGTFhu
Bounty amount: approximately 100 bars

Conjecture 405. [`MetaCat_struct_b_b_e_e_cring_coequalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMGtVFEBHjBVEusTQtdCuzHKkFZt5sLgnAR
Bounty amount: approximately 125 bars

Conjecture 406. [`MetaCat_struct_b_b_e_e_cring_equalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota.\exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMR7ZnsJfsNGEhZwAybkkYXcKeKnMeJBPUE
Bounty amount: approximately 125 bars

Conjecture 407. [`MetaCat_struct_b_b_e_e_cring_pushout_constr`]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMPLdqcFhRzHLKir9xeoWoUXuK5QJRaUAdV
Bounty amount: approximately 250 bars

Conjecture 408. [`MetaCat_struct_b_b_e_e_cring_pullback_constr`]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists\pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMH9PFYAWoXKT5QZmULhR3tq4NBgKxgt24c
Bounty amount: approximately 250 bars

Conjecture 409. [MetaCat_struct_b_b_e_e_cring_product_exponent]

$$\begin{gathered}\exists prod : \iota \to \iota \to \iota. \exists \pi_1, \pi_2 : \iota \to \iota \to \iota.\\ \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota.\\ \exists exp : \iota \to \iota \to \iota. \exists a : \iota \to \iota \to \iota. \exists lm : \iota \to \iota \to \iota \to \iota \to \iota.\\ product\_exponent\_constr\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.\end{gathered}$$

Proofgold proposition address: TMG25Aq7HowKYhsJECPsRoNx5EWUoL8ekvf
Bounty amount: approximately 250 bars

Conjecture 410. [MetaCat_struct_b_b_e_e_cring_subobject_classifier]

$$\begin{gathered}\exists one : \iota. \exists uniqa : \iota \to \iota. \exists Omega : \iota. \exists tru : \iota. \exists ch : \iota \to \iota \to \iota \to \iota.\\ \exists constr : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota.\\ subobject\_classifier\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ Omega\ tru\ ch\ constr.\end{gathered}$$

Proofgold proposition address: TMQTSsci7h7GXbJ8Dw34gB6mLNnKzJhMF2e
Bounty amount: approximately 250 bars

Conjecture 411. [MetaCat_struct_b_b_e_e_cring_nno]

$$\begin{gathered}\exists one : \iota. \exists uniqa : \iota \to \iota. \exists N : \iota. \exists zer, suc : \iota. \exists rec : \iota \to \iota \to \iota \to \iota.\\ nno\_p\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ N\ zer\ suc\ rec.\end{gathered}$$

Proofgold proposition address: TMKnvM86hsJjMZGXk5bsCxnibbZTJTem44r
Bounty amount: approximately 250 bars

Conjecture 412. [MetaCat_struct_b_b_e_e_cring_left_adjoint_forgetful]

$$\begin{gathered}\exists F0 : \iota \to \iota. \exists F1 : \iota \to \iota \to \iota \to \iota. \exists \eta, \varepsilon : \iota \to \iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ CRing\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.\end{gathered}$$

Proofgold proposition address: TMSSGnfKWKvuBfRLiaVT845vintiiKKPySm
Bounty amount: approximately 750 bars

11.4 Fields

Theorem 167. [MetaCat_struct_b_b_e_e_field] $MetaCat\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp.$

Proof. We prove the intermediate claim $L1$: $\forall X.Field\ X \to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume $H, \_$. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_gen\ Field\ L1$. □

Theorem 168. [MetaCat_struct_b_b_e_e_field_Forgetful]

$$\begin{gathered}MetaFunctor\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\\ (\lambda\_.True)\ SetHom\\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f).\end{gathered}$$

Proof. We prove the intermediate claim $L1$: $\forall X.Field\ X \to struct\_b\_b\_e\_e\ X$. Let X be given. Assume HX. Apply HX to the current goal. Assume H, _. Exact H. Exact $MetaCat\_struct\_b\_b\_e\_e\_Forgetful\_gen\ Field\ L1$. ☐

Conjecture 413. [`MetaCat_struct_b_b_e_e_field_initial`]

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$initial\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMQxRHEzEV3MLxHVzdM85QZeiexqe2UAk8S
Bounty amount: approximately 25 bars

Conjecture 414. [`MetaCat_struct_b_b_e_e_field_terminal`]

$$\exists Y : \iota . \exists uniqa : \iota \to \iota .$$
$$terminal\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMYFYKJe2UqcbwvSFWcCinT7on9NeE1h6Nh
Bounty amount: approximately 25 bars

Conjecture 415. [`MetaCat_struct_b_b_e_e_field_coproduct_constr`]

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota . \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coproduct\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMGzhoCC4SEdXaC6RASPMpRrcwTpZqJEa7f
Bounty amount: approximately 100 bars

Conjecture 416. [`MetaCat_struct_b_b_e_e_field_product_constr`]

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota . \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$product\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMcGv62eaVeAcjo8LTEDSj3TSEfeeAwtgQE
Bounty amount: approximately 100 bars

Conjecture 417. [`MetaCat_struct_b_b_e_e_field_coequalizer_constr`]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coequalizer\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMHHMkUYjXzR35vPhsTzs9hzJ2bAhvgG6Z2
Bounty amount: approximately 125 bars

Conjecture 418. [`MetaCat_struct_b_b_e_e_field_equalizer_constr`]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$equalizer\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMdH5Az3tSoLoBfvJaT6Dw7QgqXng9nhZBA
Bounty amount: approximately 125 bars

Conjecture 419. [MetaCat_struct_b_b_e_e_field_pushout_constr]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMQPsyjczoqVyngxckDFs56v8rFUERDH5Lo
Bounty amount: approximately 250 bars

Conjecture 420. [MetaCat_struct_b_b_e_e_field_pullback_constr]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMTXtaXWNuXRjBAhAyA6CWrCviLAfQNGQXC
Bounty amount: approximately 250 bars

Conjecture 421. [MetaCat_struct_b_b_e_e_field_product_exponent]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMJGkS51snhjLxBcr8e4eJnfpNNPGK82LHK
Bounty amount: approximately 250 bars

Conjecture 422. [MetaCat_struct_b_b_e_e_field_subobject_classifier]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMaSfBGdB5npKNF6Qg3fards4Me9JxoBzUV
Bounty amount: approximately 250 bars

Conjecture 423. [MetaCat_struct_b_b_e_e_field_nno]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMcusRyChkCeAockgCrd5VQerXUD6LMR59X
Bounty amount: approximately 250 bars

Conjecture 424. *[*`MetaCat_struct_b_b_e_e_field_left_adjoint_forgetful`*]*

$$\exists F0 : \iota \to \iota . \exists F1 : \iota \to \iota \to \iota \to \iota . \exists \eta , \varepsilon : \iota \to \iota .$$
$$MetaAdjunction\_strict\ (\lambda \_ . True)\ SetHom$$
$$(\lambda X . (lam\_id\ X))\ (\lambda X , Y , Z , f , g . (lam\_comp\ X\ f\ g))$$
$$Field\ Hom\_struct\_b\_b\_e\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X . X\ 0)\ (\lambda X , Y , f . f)\ \eta\ \varepsilon .$$

Proofgold proposition address: TMZy5AcRroeMY2CNhBYYnxpdE48c4NGJ9Fh
Bounty amount: approximately 750 bars

Chapter 12

Structures with Two Binary Operations, a Binary Relation and Two Elements

Theorem 169. [MetaCat_struct_b_b_r_e_e]

$$MetaCat\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp.$$

Proof. Exact $MetaCat\_struct\_b\_b\_r\_e\_e\_gen\ struct\_b\_b\_r\_e\_e\ (\lambda X, H.H)$. ☐

Theorem 170. [MetaCat_struct_b_b_r_e_e_Forgetful]

$$\begin{array}{c} MetaFunctor\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp \\ (\lambda\_.True)\ SetHom \\ (\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g)) \\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f). \end{array}$$

Proof. Exact $MetaCat\_struct\_b\_b\_r\_e\_e\_Forgetful\_gen\ struct\_b\_b\_r\_e\_e\ (\lambda X, H.H)$. ☐

Conjecture 425. [MetaCat_struct_b_b_r_e_e_initial]

$$\begin{array}{c} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ initial\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{array}$$

Proofgold proposition address: TMVTnvsbKYzRQaDrabGEbGDWKY6MxYsEfUs
Bounty amount: approximately 25 bars

Conjecture 426. [MetaCat_struct_b_b_r_e_e_terminal]

$$\begin{array}{c} \exists Y : \iota.\exists uniqa : \iota\to\iota. \\ terminal\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa. \end{array}$$

Proofgold proposition address: TMXVttPanRBetH5SaRwiYWarUkRBxiv6S8M
Bounty amount: approximately 25 bars

Conjecture 427. [MetaCat_struct_b_b_r_e_e_coproduct_constr]

$$\exists coprod : \iota \to \iota \to \iota . \exists i1, i2 : \iota \to \iota \to \iota . \exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coproduct\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMYN3uNR5aeNpYu8PbWwzx8zFaoc6WXTBW1
Bounty amount: approximately 100 bars

Conjecture 428. [MetaCat_struct_b_b_r_e_e_product_constr]

$$\exists prod : \iota \to \iota \to \iota . \exists \pi_1, \pi_2 : \iota \to \iota \to \iota . \exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$product\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMHqJHEmaLADXQKCcdreupZK1KnPBLhwCyC
Bounty amount: approximately 100 bars

Conjecture 429. [MetaCat_struct_b_b_r_e_e_coequalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$coequalizer\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMdBjemNp3Ekpq5xhG9xND7uWgG7uVo6Ga3
Bounty amount: approximately 125 bars

Conjecture 430. [MetaCat_struct_b_b_r_e_e_equalizer_constr]

$$\exists quot : \iota \to \iota \to \iota \to \iota \to \iota . \exists canonmap : \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists fac : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$equalizer\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$quot\ canonmap\ fac.$$

Proofgold proposition address: TMRHWKG5xWFDZnGUZCb7Cw9xB3e1V2To9Zs
Bounty amount: approximately 125 bars

Conjecture 431. [MetaCat_struct_b_b_r_e_e_pushout_constr]

$$\exists po : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists i1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists copair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$pushout\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$po\ i0\ i1\ copair.$$

Proofgold proposition address: TMRgsdoYAxZUvrVg39NtVCFQEoCjGnKAsid
Bounty amount: approximately 250 bars

Conjecture 432. [MetaCat_struct_b_b_r_e_e_pullback_constr]

$$\exists pb : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_0 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota . \exists \pi_1 : \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$\exists pair : \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota \to \iota .$$
$$pullback\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMFdYJa32T7L7wm4TQ4Mrg9SARpTk8iShTZ
Bounty amount: approximately 250 bars

Conjecture 433. *[*`MetaCat_struct_b_b_r_e_e_product_exponent`*]*

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\\ \exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ \exists exp : \iota\to\iota\to\iota.\exists a : \iota\to\iota\to\iota.\exists lm : \iota\to\iota\to\iota\to\iota\to\iota.\\ product\_exponent\_constr\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMTdCBVHeQmjfaJDkg3D4yYcmNmZUoApGiL
Bounty amount: approximately 250 bars

Conjecture 434. *[*`MetaCat_struct_b_b_r_e_e_subobject_classifier`*]*

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists Omega : \iota.\exists tru : \iota.\exists ch : \iota\to\iota\to\iota\to\iota.\\ \exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.\\ subobject\_classifier\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMQA1myorbdMztpsTDhyKCVX9hbfS36xdbh
Bounty amount: approximately 250 bars

Conjecture 435. *[*`MetaCat_struct_b_b_r_e_e_nno`*]*

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.\\ nno\_p\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\\ one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMPZ5vVfJAgb638cTq9TuC264ki1gP1HyBe
Bounty amount: approximately 250 bars

Conjecture 436. *[*`MetaCat_struct_b_b_r_e_e_left_adjoint_forgetful`*]*

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.\\ MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom\\ (\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))\\ struct\_b\_b\_r\_e\_e\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\\ F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMcR6bujQrJ4kgCFUJCTYRvNdjYFykfW7A8
Bounty amount: approximately 750 bars

12.1 Ordered Fields

Definition 65. *We define* `struct_b_b_r_e_e_ordered_field` *to be*

$$\lambda R.struct\_b\_b\_r\_e\_e\ R \wedge unpack\_b\_b\_r\_e\_e\_o\ R\ (\lambda R, plus, mult, leq, zero, one.\\ explicit\_OrderedField\ R\ zero\ one\ plus\ mult\ leq)$$

of type $\iota\to o$.

Theorem 171. [MetaCat_struct_b_b_r_e_e_ordered_field]

$$MetaCat\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp.$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ X \to struct\_b\_b\_r\_e\_e$ Let X be given. Assume HX. Apply HX to the current goal. Assume H, _. Exact H. Exact $MetaCat\_struct\_b\_b\_r\_e\_e\_gen$ $\texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}$ $L1$. □

Theorem 172. [MetaCat_struct_b_b_r_e_e_ordered_field_Forgetful]

$$MetaFunctor\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$(\lambda\_.True)\ SetHom$$
$$(\lambda X.lam\_id\ X)\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$(\lambda X.X\ 0)\ (\lambda X, Y, f.f).$$

Proof. We prove the intermediate claim $L1$: $\forall X.\texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ X \to struct\_b\_b\_r\_e\_e$ Let X be given. Assume HX. Apply HX to the current goal. Assume H, _. Exact H. Exact $MetaCat\_struct\_b\_b\_r\_e\_e\_Forgetful\_gen$ $\texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}$ $L1$. □

Conjecture 437. [MetaCat_struct_b_b_r_e_e_ordered_field_initial]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$initial\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMWJiTaM7ZZjaBXMW69empfKAEB7JJBceyy
Bounty amount: approximately 25 bars

Conjecture 438. [MetaCat_struct_b_b_r_e_e_ordered_field_terminal]

$$\exists Y : \iota.\exists uniqa : \iota\to\iota.$$
$$terminal\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ Y\ uniqa.$$

Proofgold proposition address: TMVM17Kq27e54zMUvsPdS3hQjtDb9aT7Fyc
Bounty amount: approximately 25 bars

Conjecture 439. [MetaCat_struct_b_b_r_e_e_ordered_field_coproduct_constr]

$$\exists coprod : \iota\to\iota\to\iota.\exists i1, i2 : \iota\to\iota\to\iota.\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coproduct\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$coprod\ i1\ i2\ copair.$$

Proofgold proposition address: TMbLLzoP6mGuA333ypMrWuS3P9brjQ21fZ5
Bounty amount: approximately 100 bars

Conjecture 440. [MetaCat_struct_b_b_r_e_e_ordered_field_product_constr]

$$\exists prod : \iota\to\iota\to\iota.\exists \pi_1, \pi_2 : \iota\to\iota\to\iota.\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$prod\ \pi_1\ \pi_2\ pair.$$

Proofgold proposition address: TMcu6wdgPtM5De1zGPmij8fgEC69hnqCJLD
Bounty amount: approximately 100 bars

Conjecture 441. [`MetaCat_struct_b_b_r_e_e_ordered_field_coequalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota. \exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$coequalizer\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMT1LmJi4u6JtpeQorryfoJD9sMNJSQ7Vkx
Bounty amount: approximately 125 bars

Conjecture 442. [`MetaCat_struct_b_b_r_e_e_ordered_field_equalizer_constr`]

$$\exists quot : \iota\to\iota\to\iota\to\iota\to\iota. \exists canonmap : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists fac : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$equalizer\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ quot\ canonmap\ fac.$$

Proofgold proposition address: TMLYndc3qbe3LcXNJdLrzREkFwdmZMxB5Pu
Bounty amount: approximately 125 bars

Conjecture 443. [`MetaCat_struct_b_b_r_e_e_ordered_field_pushout_constr`]

$$\exists po : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \exists i0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \exists i1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists copair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pushout\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ po\ i0\ i1\ copair.$$

Proofgold proposition address: TMX33VSkKuqopsFrdbEmLS69cP8gMenyink
Bounty amount: approximately 250 bars

Conjecture 444. [`MetaCat_struct_b_b_r_e_e_ordered_field_pullback_constr`]

$$\exists pb : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \exists \pi_0 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota. \exists \pi_1 : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$pullback\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ pb\ \pi_0\ \pi_1\ pair.$$

Proofgold proposition address: TMTfqJUeJo5GAcfb6M8RZ57g9opYHXC16Qv
Bounty amount: approximately 250 bars

Conjecture 445. [`MetaCat_struct_b_b_r_e_e_ordered_field_product_exponent`]

$$\exists prod : \iota\to\iota\to\iota. \exists \pi_1, \pi_2 : \iota\to\iota\to\iota.$$
$$\exists pair : \iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$\exists exp : \iota\to\iota\to\iota. \exists a : \iota\to\iota\to\iota. \exists lm : \iota\to\iota\to\iota\to\iota\to\iota.$$
$$product\_exponent\_constr\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ prod\ \pi_1\ \pi_2\ pair\ exp\ a\ lm.$$

Proofgold proposition address: TMYyU8wZEmFV7Gom8EtYHNh6CY9gXA24yff
Bounty amount: approximately 250 bars

Conjecture 446. [`MetaCat_struct_b_b_r_e_e_ordered_field_subobject_classifier`]

$$\exists one : \iota. \exists uniqa : \iota\to\iota. \exists Omega : \iota. \exists tru : \iota. \exists ch : \iota\to\iota\to\iota\to\iota.$$
$$\exists constr : \iota\to\iota\to\iota\to\iota\to\iota\to\iota\to\iota.$$
$$subobject\_classifier\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp\ one\ uniqa\ Omega\ tru\ ch\ constr.$$

Proofgold proposition address: TMbrrLEyjSmLV6MtDC2cfMUKjEdqFgtJZaM
Bounty amount: approximately 250 bars

Conjecture 447. [`MetaCat_struct_b_b_r_e_e_ordered_field_nno`]

$$\exists one : \iota.\exists uniqa : \iota\to\iota.\exists N : \iota.\exists zer, suc : \iota.\exists rec : \iota\to\iota\to\iota\to\iota.$$
$$nno\_p\ \texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$one\ uniqa\ N\ zer\ suc\ rec.$$

Proofgold proposition address: TMTPcphxRqLytvp6zC6GRxsKkNs3XMsJUxc
Bounty amount: approximately 250 bars

Conjecture 448. [`MetaCat_struct_b_b_r_e_e_ordered_field_left_adjoint_forgetful`]

$$\exists F0 : \iota\to\iota.\exists F1 : \iota\to\iota\to\iota\to\iota.\exists \eta, \varepsilon : \iota\to\iota.$$
$$MetaAdjunction\_strict\ (\lambda\_.True)\ SetHom$$
$$(\lambda X.(lam\_id\ X))\ (\lambda X, Y, Z, f, g.(lam\_comp\ X\ f\ g))$$
$$\texttt{struct\_b\_b\_r\_e\_e\_ordered\_field}\ Hom\_struct\_b\_b\_r\_e\_e\ struct\_id\ struct\_comp$$
$$F0\ F1\ (\lambda X.X\ 0)\ (\lambda X, Y, f.f)\ \eta\ \varepsilon.$$

Proofgold proposition address: TMLstyvSrHJopdFttuhvwxLhkBmb1myWLWu
Bounty amount: approximately 750 bars