Solutions for exercises marked with a '⋆' will be made available online, usually in the following week. Only if needed will these exercises be discussed during the tutorial. All other exercises are to be prepared at home and presented by the participants during the tutorial session.
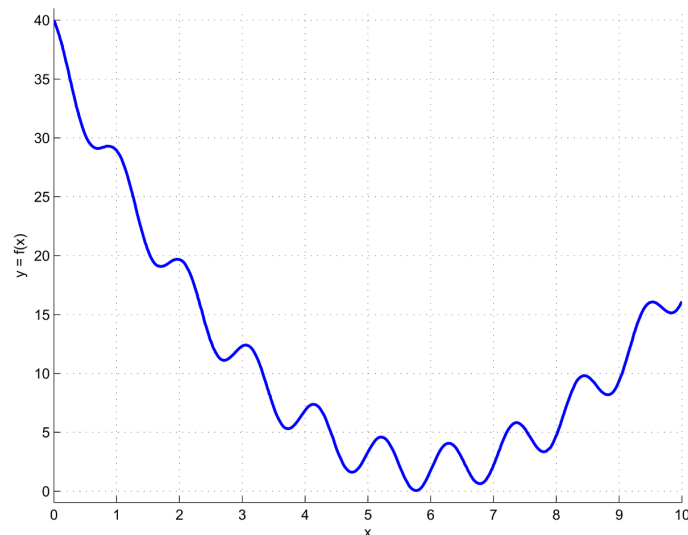
Exercise 8

8.1 Downhill Simplex

Consider the function

$$f(x) = 2\cos(6\,x) + (x-6)^2 + 2$$

for $x = [0,\,10]$. Its graph is shown below.

Use Scikit-learn's function fmin() to find the minimum of $f(x)$ via the Downhill-Simplex algorithm. The initial value is $x_0 = 1$.

Write a Python script to minimize $f(x)$ using the downhill simplex algorithm and perform appropriate plotting.

8.2 Gradient Descent⋆

Recall that the gradient $g(x)$ points in the direction of *steepest ascent* at point x. In order to minimize a function, we take a step into the *negative* gradient direction with some fixed step-size parameter (here: 0.005). Let the initial value be $x_0 = 1$ with initial gradient $\frac{d}{dx}f(x_0) = -6.65$.

Write your own implementation of the gradient descent algorithm with the step-size as a parameter. Use this implementation in a Python script to minimize $f(x)$ using different step-size values. Demonstrate your obervations by performing appropriate plotting.