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Exercise 2

Solutions for exercises marked with a '$\star$' will be made available online, usually in the following week. Only if needed will these exercises be discussed during the tutorial. All other exercises are to be prepared at home and presented by the participants during the tutorial session.

2.1 Gauss-Jordan Elimination

Solving the linear system $A\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$ using Gauss-Jordan elimination: The first step of the algorithm was demonstrated during our first tutorial introducing the Python programming language when we created the augmented matrix $\mathbf{A}_{\text{aug}} = [\mathbf{A}, \mathbf{b}, \mathbf{I}_n]$. The next step is applying several operations to $\mathbf{A}_{\text{aug}}$ in order to arrive at the desired structure $[\mathbf{I}_n, \mathbf{x}, \mathbf{A}^{-1}]$ from which we can read off the solution vector $\mathbf{x}$ as well as the inverse of $\mathbf{A}$. Consult the lecture slides to review which operations are needed/allowed to implement this algorithm in the following cases and implement them.

- Gauss-Jordan Elimination without pivoting
- Gauss-Jordan Elimination with row pivoting
- Then try out your implementation on the following two problems. What do you observe?

$$A_1 = \begin{pmatrix} 2 & 3 & 2 \\ -1 & -1 & -3 \\ 3 & 5 & 5 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}.$$

$$A_2 = \begin{pmatrix} 1 & 2 & 5 & -3 \\ 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 1 \\ 3 & -3 & 2 & -1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} -2 \\ 6 \\ 5 \\ 1 \end{pmatrix}.$$