Michael W. Hackemack
Knolls Atomic Power Laboratory
P.O. Box 1072
Schenectady, NY 12301-1072, USA
Phone: (518) 395-7293
Email: michael.hackemack@unnpp.gov

<Date>

Professor Rémi Abgrall, Editor
Journal of Computational Physics

Dear Professor Abgrall,

Please find attached a copy of our manuscript titled "Quadratic Serendipity Discontinuous Finite Element Discretization for S_N Transport on Arbitrary Polygonal Grids" for submission to the *Journal of Computational Physics*.

In this paper, we analyze various basis functions for higher-order discontinuous finite element (DFEM) discretization of the S_N transport equation on arbitrary polygonal grids and propose a new quadratic serendipity trial space on polygonal grids. This work advances the current state-of-the-art of DFEM discretizations of the transport equation and extends the previous works of Teresa Bailey (LLNL), Marvin Adams (Texas A&M University), Jean Ragusa (Texas A&M University), Yaqi Wang (INL), and Todd Palmer (Oregon State). Our methodology closely follows previous works of the mathematics community by Alexander Rand (CD-adapco).

Quadratic serendipity functions are constructed from products of linear Generalized Barycentric Coordinates (GBC) that are compatible with polygonal mesh elements. They span the $\{ 1, x, y, x^2, xy, y^2 \}$ space of functions and they grow by $2n$ on a mesh element, where n is the number of the polygon's vertices. The mathematics community has provided a framework for this construction process, and we test with different GBCs. Inspired by their construction techniques, we further apply this methodology to the Piecewise Linear Discontinuous (PWLD) functions to convert them into Piecewise Quadratic Discontinuous (PWQD) functions. The PWLD functions have been extensively studied for S_N transport calculations and are fully compatible with arbitrary polygonal grids and the transport diffusion limit. Numerical tests confirm that these quadratic serendipity functions capture exactly quadratic solutions and appropriate convergence rates are observed, including a test case involving spatial adaptive mesh refinement. Finally, the functions are shown to retain full resolution in the thick diffusion limit.

FEM spatial discretizations of the transport equation have been mostly limited to first-order functions on simplical and tensor grids. This was for robustness and computational performance reasons. However, there has been a growing interest in higher-order functions for the DFEM transport equation on more arbitrary geometries. This would allow for more local computational work performed before parallel communication, along with greater flexibility in meshing strategies. These functions can allow for the use of higher-order functions for the DFEM transport equation on completely arbitrary 2D grids.

Thank you for considering this manuscript for publication in JCP.

Best Regards,
Michael W. Hackemack and Jean C. Ragusa