Higher-Order DGFEM Transport Calculations on Polytope Meshes for Massively-Parallel Architectures

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The Continuous-Energy Transport Equation

$$\left[\mathbf{\Omega}\cdot\nabla + \sigma_t(\mathbf{r},E)\right]\psi(\mathbf{r},E,\mathbf{\Omega}) = \int_{4\pi}\int_0^{\infty}\sigma_s(\mathbf{r},E',E,\mathbf{\Omega}',\mathbf{\Omega})\psi(\mathbf{r},E',\mathbf{\Omega}')dE'd\Omega' + Q(\mathbf{r},E,\mathbf{\Omega})$$

$\mathbf{r}$ - neutron position (cm)
E - neutron energy (eV)
$\mathbf{\Omega}$ - neutron solid angle (steradians)
$\psi(\mathbf{r},E,\mathbf{\Omega})$ - angular flux
$Q(\mathbf{r},E,\mathbf{\Omega})$ - distributed neutron source
$\sigma_t(\mathbf{r},E)$ - total macroscopic cross section (1/cm)
$\sigma_s(\mathbf{r},E',E,\mathbf{\Omega}',\mathbf{\Omega})$ - total

Polytope Grid Motivation

- Other physics communities are now employing polytope grids due to decreased cell/face counts (CFD in particular)
- They allow for transition elements between different domain regions
- Hanging nodes from non-conforming meshes are not necessary
- Independently-generated simplicial grids (*i.e.* created in parallel) can be stitched together with polytopes without communicating the whole mesh across processors

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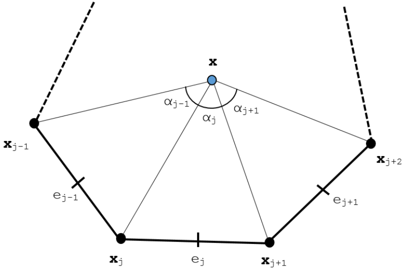
Linear Basis Functions on 2D Polygons



Wachspress Rational Functions

Piecewise Linear Functions

Mean Value Coordinates

Maximum Entropy Coordinates

Quadratic Serendipity Basis Functions on 2D Polygons

Linear Basis Functions on 3D Polyhedra

Theory

Theory

The Diffusion Equation and Boundary Conditions

The Diffusion Equation

$$-\nabla \cdot D \nabla \Phi(\mathbf{r}) + \sigma \Phi(\mathbf{r}) = q(\mathbf{r}), \qquad \mathbf{r} \in \mathcal{D}$$

Boundary Conditions

$$\begin{aligned}
\Phi(\mathbf{r}) &= \Phi_0(\mathbf{r}), && \mathbf{r} \in \partial \mathcal{D}^d \\
-D \partial_n \Phi(\mathbf{r}) &= J_0(\mathbf{r}), && \mathbf{r} \in \partial \mathcal{D}^n \\
\frac{1}{4}\Phi(\mathbf{r}) + \frac{1}{2} D \partial_n \Phi(\mathbf{r}) &= J^{inc}(\mathbf{r}), && \mathbf{r} \in \partial \mathcal{D}^r
\end{aligned}$$

Symmetric Interior Penalty (SIP) Form

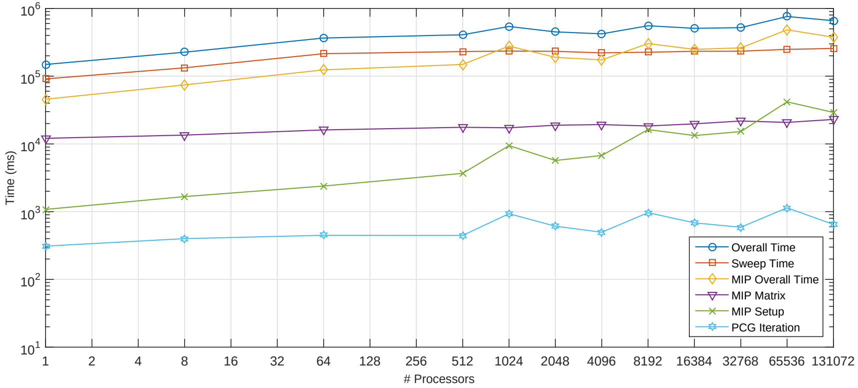
Bilinear Form

$$
\begin{aligned}
a(\Phi, b) = \left\langle D\vec{\nabla}\Phi, \vec{\nabla} b \right\rangle_{\mathcal{D}} + \left\langle \sigma\Phi, b \right\rangle_{\mathcal{D}} \\
+\left\{ \kappa_e^{SIP} [\![\Phi]\!], [\![b]\!] \right\}_{E_h^i} - \left\{ [\![\Phi]\!], \{\{D\partial_n b\}\} \right\}_{E_h^i} - \left\{ \{\{D\partial_n \Phi\}\}, [\![b]\!] \right\}_{E_h^i} \\
+\left\{ \kappa_e^{SIP} \Phi, b \right\}_{\partial\mathcal{D}^d} - \left\{ \Phi, D\partial_n b \right\}_{\partial\mathcal{D}^d} - \left\{ D\partial_n \Phi, b \right\}_{\partial\mathcal{D}^d} + \frac{1}{2}\left\{ \Phi, b \right\}_{\partial\mathcal{D}^r}
\end{aligned}
$$

Linear Form

$$
\ell(b) = \left\langle q, b \right\rangle_{\mathcal{D}} - \left\{ J_0, b \right\}_{\partial\mathcal{D}^n} + 2\left\{ J_{inc}, b \right\}_{\partial\mathcal{D}^r}
$$

$$
+\left\{ \kappa_e^{SIP} \Phi_0, b \right\} - \left\{ \Phi_0, D\partial_n b \right\}
$$


Time (ms)
10^6
10^5
10^4
10^3
10^2
10^1
1 2 4 8 16 32 64 128 256 512 1024 2048 4096 8192 16384 32768 65536 131072
Processors
Overall Time
Sweep Time
MIP Overall Time
MIP Matrix
MIP Setup
PCG Iteration