In this work, we develop improvements to the discrete ordinates (Sn) neutron transport equation using a Discontinuous Galerkin Finite Element Method (DGFEM) spatial discretization on arbitrary polytope (polygonal and polyhedral) grids compatible for massively-parallel computer architectures. In particular, we focus on two topical areas of research. First, we discuss higher-order basis functions compatible to solve the DGFEM Sn transport equation on arbitrary polygonal meshes. Second, we assess Diffusion Synthetic Acceleration (DSA) schemes compatible with polytope grids for massively-parallel transport problems.

We first analyze basis functions compatible with arbitrary polygonal grids for the DGFEM transport equation. Four linearly-complete basis functions are utilized: the Wachspress rational functions, the PWL functions, the mean value coordinates, and the maximum entropy coordinates. We then describe the procedure to convert these polygonal linear basis functions into the quadratic serendipity space of functions. These functions preserve transport solutions in the thick diffusion limit. Maximum convergence rates of 2 and 3 are observed for regular transport solutions for the linear and quadratic basis functions, respectively.

DSA schemes are essential for transport problems with optically-thick configurations. The Modified Interior Penalty (MIP) method is used as the low-order DFEM diffusion form. MIP is Symmetric Positive Definite (SPD) and efficiently solved with Preconditioned Conjugate Gradient (PCG) with Algebraic MultiGrid (AMG) preconditioning. The method was implemented in the Parallel Deterministic Transport (PDT) code at Texas A\&M University. Good scalability was numerically verified out to around 131K processors. Finally, we have developed a novel methodology to accelerate transport problems dominated by thermal neutron upscattering. Compared to historical upscatter acceleration methods, our method is parallelizable and amenable to massively parallel transport calculations. Speedup factors of about 3-4 were observed with our new method.