

Searching, Sorting & Analysis

Unit 2

Chapter 8

CS 2308
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Definitions of Search and Sort

- Search: find a given item in an array, return the index of the item, or -1 if not found.
- Sort: rearrange the items in an array into some order (smallest to biggest, alphabetical order, etc.).
- There are various methods (algorithms) for carrying out these common tasks.
- Which ones are better? Why?

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Linear Search

- Very simple method.
- Compare first element to target value, if not found then compare second element to target value . . .
- Repeat until:
target value is found (return its index) or
we run out of items (return -1).

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Linear Search in C++

first attempt

```
int searchList (int list[], int size, int target) {  
    int position = -1;           //position of target  
    for (int i=0; i<size; i++)  
    {  
        if (list[i] == target) //found the target!  
            position = i;      //record which item  
    }  
    return position;  
}
```

Is this algorithm correct (does it calculate the right value)?

Is this algorithm efficient (does it do unnecessary work)?

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Linear Search in C++

second attempt

```
int searchList (int list[], int size, int target) {
    int position = -1;    //position of target
    bool found = false;  //flag, true when target is found

    for (int i=0; i < size && !found; i++)
    {
        if (list[i] == target) //found the target!
        {
            found = true;      //set the flag
            position = i;      //record which item
        }
    }
    return position;
}
```

Is this algorithm correct (does it calculate the right value)?

Is this algorithm efficient (does it do unnecessary work)?

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Program that uses linear search

```
#include <iostream>
using namespace std;

int searchList(int[], int, int);

int main() {
    const int SIZE=5;
    int idNums[SIZE] = {871, 750, 988, 100, 822};
    int results, id;

    cout << "Enter the employee ID to search for: ";
    cin >> id;

    results = searchList(idNums, SIZE, id);

    if (results == -1) {
        cout << "That id number is not registered\n";
    } else {
        cout << "That id number is found at location ";
        cout << results+1 << endl;
    }
}
```

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Evaluating the Algorithm

- Does it do any unnecessary work?
- Is it time efficient? How would we know?
- We measure time efficiency of algorithms in terms of number of main steps required to finish.
- For search algorithms, the main step is comparing an array element to the target value.
- Number of steps depends on:
 - size of input array
 - whether or not value is in array
 - where the value is in the array

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Efficiency of Linear Search

how many main steps (comparisons to target)?

N is the number of elements in the array

	N=50,000	In terms of N
Best Case:	1	1
Average Case:	25,000	N/2
Worst Case:	50,000	N

Note: if we search for many items that are not in the array, the average case will be greater than N/2.

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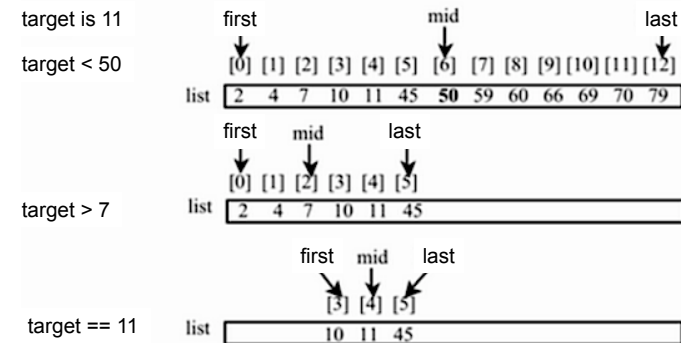
Binary Search

- Works only for SORTED arrays
- Divide and conquer style algorithm
- Compare target value to middle element in list.
 - if equal, then return its index
 - if less than middle element, repeat the search in the first half of list
 - if greater than middle element, repeat the search in last half of list
- If current search list is narrowed down to 0 elements, return -1

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Binary Search Algorithm example

We use first and last to indicate beginning and end of current search list



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Binary Search in C++

```
int binarySearch (int array[], int size, int target) {
    int first = 0,           //index of beginning of search list
        last = size - 1,    //index of end of search list
        middle,              //index of midpoint of search list
        position = -1;       //position of target value
    bool found = false;      //flag

    while (first <= last && !found) {
        middle = (first + last) / 2;    //calculate midpoint

        if (array[middle] == target) {
            found = true;
            position = middle;
        } else if (target < array[middle]) {
            last = middle - 1;          //search list = lower half
        } else {
            first = middle + 1;         //search list = upper half
        }
    }
    return position;
}
```

What if first + last is odd?
What if first==last?

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Program using Binary Search

```
#include <iostream>
using namespace std;

int binarySearch(int[], int, int);

int main() {
    const int SIZE=5;
    int idNums[SIZE] = {100, 750, 822, 871, 988};
    int results, id;

    cout << "Enter the employee ID to search for: ";
    cin >> id;

    results = binarySearch(idNums, SIZE, id);

    if (results == -1) {
        cout << "That id number is not registered\n";
    } else {
        cout << "That id number is found at location ";
        cout << results+1 << endl;
    }
}
```

How is this program different
from the one on slide 6?

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Efficiency of Binary Search

Calculate worst case (target not in list) for $N=1024$

# Items left to search	# Comparisons so far
1024	0
512	1
256	2
128	3
64	4
32	5
16	6
8	7
4	8
2	9
1	10

Goal: calculate this value from N

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$$1024 = 2^{10} \iff \log_2 1024 = 10$$

Efficiency of Binary Search

If N is the number of elements in the array, how many comparisons (steps)?

$$1024 = 2^{10} \iff \log_2 1024 = 10$$

$$N = 2^{\text{steps}} \iff \log_2 N = \text{steps}$$

To what power do I raise 2 to get N?

	$N=50,000$	In terms of N
Best Case:	1	1
Worst Case:	16	$\log_2 N$

Rounded up to next whole number

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Is $\log_2 N$ better than N ?

Is binary search better than linear search?

Is this really a fair comparison?

Compare values of $N/2$, N , and $\log_2 N$ as N increases:

N	$N/2$	$\log_2 N$
5	2.5	2.3
50	25	5.6
500	250	9
5,000	2,500	12.3
50,000	25,000	15.6

N and $N/2$ are growing much faster than $\log N$!

slower growing is more efficient (fewer steps).

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8.3 Sorting Algorithms

- Sort: rearrange the items in an array into ascending or descending order.
- Bubble Sort
- Selection Sort



55 112 78 14 20 179 42 67 190 7 101 1 122 170 8

unsorted

1 7 8 14 20 42 55 67 78 101 112 122 170 179 190

sorted

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The Bubble Sort

- On each pass:
 - Compare first two elements. If the first is bigger, they exchange places (swap).
 - Compare second and third elements. If second is bigger, exchange them.
 - Repeat until last two elements of the list are compared.
- Repeat this process (keep doing passes) until a pass completes with no exchanges

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Bubble sort

Example: first pass

- 7 2 3 8 9 1 7 > 2, swap
- 2 7 3 8 9 1 7 > 3, swap
- 2 3 7 8 9 1 !(7 > 8), no swap
- 2 3 7 8 9 1 !(8 > 9), no swap
- 2 3 7 8 9 1 9 > 1, swap
- 2 3 7 8 1 9 finished pass 1, did 3 swaps

Note: largest element is now in last position

Note: This is one complete pass!

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Bubble sort

Example: second and third pass

- 2 3 7 8 1 9 2 < 3 < 7 < 8, no swap, !(8 < 1), swap
- 2 3 7 1 8 9 (8 < 9) no swap
- finished pass 2, did one swap
- 2 3 7 1 8 9 2 < 3 < 7, no swap, !(7 < 1), swap
- 2 3 1 7 8 9 7 < 8 < 9, no swap
- finished pass 3, did one swap

2 largest elements
in last 2 positions

3 largest elements
in last 3 positions

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Bubble sort

Example: passes 4, 5, and 6

- 2 3 1 7 8 9 2 < 3, !(3 < 1) swap, 3 < 7 < 8 < 9
- 2 1 3 7 8 9
- finished pass 4, did one swap
- 2 1 3 7 8 9 !(2 < 1) swap, 2 < 3 < 7 < 8 < 9
- 1 2 3 7 8 9
- finished pass 5, did one swap
- 1 2 3 7 8 9 1 < 2 < 3 < 7 < 8 < 9, no swaps
- finished pass 6, no swaps, list is sorted!

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Bubble sort

how does it work?

- At the end of the first pass, the largest element is moved to the end (it's bigger than all its neighbors)
- At the end of the second pass, the second largest element is moved to just before the last element.
- The back end (tail) of the list remains sorted.
- Each pass increases the size of the sorted portion.
- No exchanges implies each element is smaller than its next neighbor (so the list is sorted).

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Bubble Sort in C++

```
void bubbleSort (int array[], int size) {  
    bool swap;  
    int temp;  
    do {  
        swap = false;  
        for (int i = 0; i < (size-1); i++) {  
            if (array [i] > array[i+1]) {  
                temp = array[i];  
                array[i] = array[i+1];  
                array[i+1] = temp;  
                swap = true;  
            }  
        }  
    } while (swap);  
}
```

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Program using bubble sort

```
#include <iostream>  
using namespace std;  
  
void bubbleSort(int [], int);  
void showArray(int [], int);  
  
int main() {  
    int values[6] = {7, 2, 3, 8, 9, 1};  
  
    cout << "The unsorted values are: \n";  
    showArray (values, 6);  
  
    bubbleSort (values, 6);  
  
    cout << "The sorted values are: \n";  
    showArray(values, 6);  
}  
  
void showArray (int array[], int size) {  
    for (int i=0; i<size; i++)  
        cout << array[i] << " " ;  
    cout << endl;  
}
```

Output:

```
The unsorted values are:  
7 2 3 8 9 1  
The sorted values are:  
1 2 3 7 8 9
```

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Selection Sort

- There is a pass for each position (0..size-1)
- On each pass, the smallest (minimum) element in the rest of the list is exchanged (**swapped**) with element at the current position.
- The first part of the list (the part that is already processed) is always sorted
- Each pass increases the size of the sorted portion.

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Selection sort

Example

- **7** 2 3 8 9 1 1 is the min a[5], swap with a[0]
- 1 **2** 3 8 9 7 2 is the min a[1], self-swap a[1]
- 1 2 **3** 8 9 7 3 is the min a[2], self-swap a[2]
- 1 2 3 **8** 9 7 7 is the min a[5], swap with a[3]
- 1 2 3 7 **9** 8 8 is the min a[5], swap with a[4]
- 1 2 3 7 8 9 sorted

Note: underlined portion of list is sorted.

Note: This is five passes

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Selection Sort in C++

My version

```
// Returns the index of the smallest element, starting at start
int findIndexOfMin (int array[], int size, int start) {
    int minIndex = start;
    for (int i = start+1; i < size; i++) {
        if (array[i] < array[minIndex]) {
            minIndex = i;
        }
    }
    return minIndex;
}
```

Note: saving the index

We need to find the index of the minimum value so that we can do the swap

```
// Sorts an array, using findIndexOfMin
void selectionSort (int array[], int size) {
    int temp;
    int minIndex;
    for (int index = 0; index < (size -1); index++) {
        minIndex = findIndexOfMin(array, size, index);
        //swap
        temp = array[minIndex];
        array[minIndex] = array[index];
        array[index] = temp;
    }
}
```

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Program using Selection Sort

```
#include <iostream>
using namespace std;

int findIndexOfMin (int [], int, int);
void selectionSort(int [], int);
void showArray(int [], int);

int main() {
    int values[6] = {7, 2, 3, 8, 9, 1};

    cout << "The unsorted values are: \n";
    showArray (values, 6);

    selectionSort (values, 6);

    cout << "The sorted values are: \n";
    showArray(values, 6);
}

void showArray (int array[], int size) {
    for (int i=0; i<size; i++)
        cout << array[i] << " ";
    cout << endl;
}
```

Output:

```
The unsorted values are:
7 2 3 8 9 1
The sorted values are:
1 2 3 7 8 9
```

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Analysis of Algorithms

using Big O notation

- Which algorithm is better, linear search or binary search?
- Which algorithm is better, bubble sort or selection sort?
- How can we answer these questions?
- **Analysis of algorithms** is the determination of the amount of resources (such as time and storage) necessary to execute them.

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Time Efficiency of Algorithms

- To classify the time efficiency of an algorithm:
 - Express “time” (using number of main steps), as a mathematical function of input size (or n below).
- Binary search: $f(n) = \log_2(n)$
- Need a way to be able to compare these math functions to determine which is better.
 - We are mostly concerned with which function has smaller values (# of steps) at very large data sizes.
 - We compare the growth rates of the functions and prefer the one that grows more slowly.

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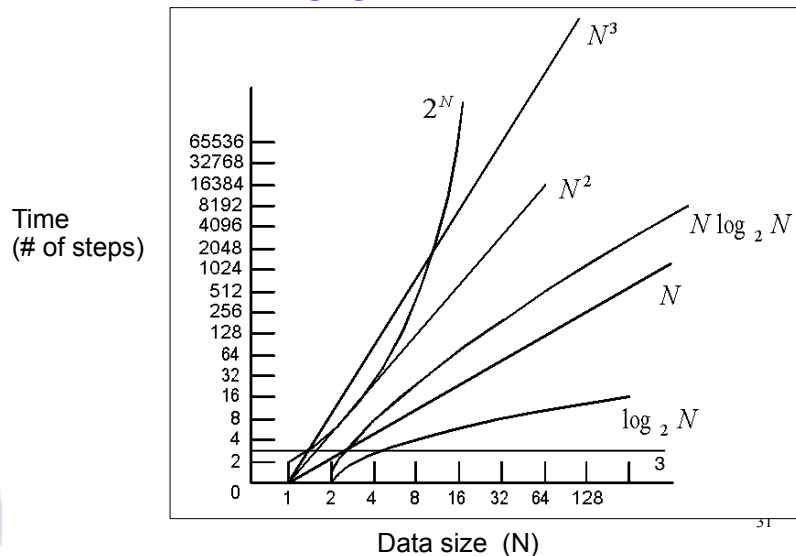
Classifications of (math) functions

Constant	$f(x)=b$	$O(1)$
Logarithmic	$f(x)=\log_b(x)$	$O(\log n)$
Linear	$f(x)=ax+b$	$O(n)$
Linearithmic	$f(x)=x \log_b(x)$	$O(n \log n)$
Quadratic	$f(x)=ax^2+bx+c$	$O(n^2)$
Exponential	$f(x)=2^x$	$O(2^n)$

- Last column is “big O notation”, used in CS.
- It ignores all but dominant term, constant factors

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Comparing growth of functions



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Time Efficiency of Algorithms

- To classify the time efficiency of an algorithm:
 - Express “time” (using number of main steps), as a mathematical function of input size.
 - Determine which classification the function fits into.
- Nearer to the top of the classification chart (on slide 30) is slower growth, and more efficient (constant is better than logarithmic, etc.)

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Efficiency of Searches

(Assuming the array is already sorted)

- Linear Search, worst case:

Linear search: $f(n) = n$

$O(N)$

- Binary Search, worst case:

Binary search: $f(n) = \log_2(n)$

$O(\log N)$

- Which is slower growing (and thus fewer steps at large input sizes)?

$O(\log N)$

- Which search algorithm is more time efficient?

Binary search

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Efficiency of Selection Sort

- N is the number of elements in the list
- Outer loop executes $N-1$ times
- Inner loop executes $N-1$, then $N-2$, then $N-3$, ... then once. One comparison per loop iteration.
- Total number of comparisons (in inner loop):

$f(N) = (N-1) + (N-2) + \dots + 2 + 1 = \text{sum of } 1 \text{ to } N-1$

sum of $1..N$: $N + (N-1) + (N-2) + \dots + 2 + 1 = N(N+1)/2$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Subtract N from each side:

$(N-1) + (N-2) + \dots + 2 + 1 = N(N+1)/2 - N$

$= (N^2+N)/2 - 2N/2$

$= (N^2+N-2N)/2$

$= N^2/2 - N/2$

$O(N^2)$

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Efficiency of Bubble Sort

- Each pass makes $N-1$ comparisons
- There will be (at most) N passes
- So worst case it's: $f(N) = (N-1)*N = N^2 - N$
- If you change the algorithm to look at only the **unsorted** part of the array in each pass, it's exactly like the selection sort:

$O(N^2)$

$(N-1) + (N-2) + \dots + 2 + 1 = N^2/2 - N/2$

still $O(N^2)$

- Neither algorithm is more efficient in the worst case.

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