Graphical Probability Models for Inference and Decision Making

Unit 7: Knowledge Engineering

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Learning Objectives

- Describe knowledge elicitation as a problem in system lifecycle engineering
  - Describe the stages in building a Bayesian network and/or decision graph model
  - Describe the activities that occur at each stage
  - Describe the products produced at each stage
- Describe how the KE process is managed
- Be prepared to carry out the process of developing, implementing and testing a Bayesian network or decision graph model for a problem of interest to you

Unit 7 Outline

- The Knowledge Acquisition Lifecycle
- Building the Model
- Managing and Evaluating the Model

Importance of Structured KE Process

- Graphical models have become well established tools for representing and reasoning under uncertainty
- Applications are growing more complex
- A formal, repeatable process for knowledge engineering is becoming more important
  - Early work on elicitation of probability models (1970's) focused on eliciting single probabilities or univariate probability distributions
  - Early work in graphical models tended to assume that structure elicitation was relatively straightforward
  - As models become more complex the KE process must be managed
- Knowledge elicitation for large Bayesian networks is a problem in systems engineering

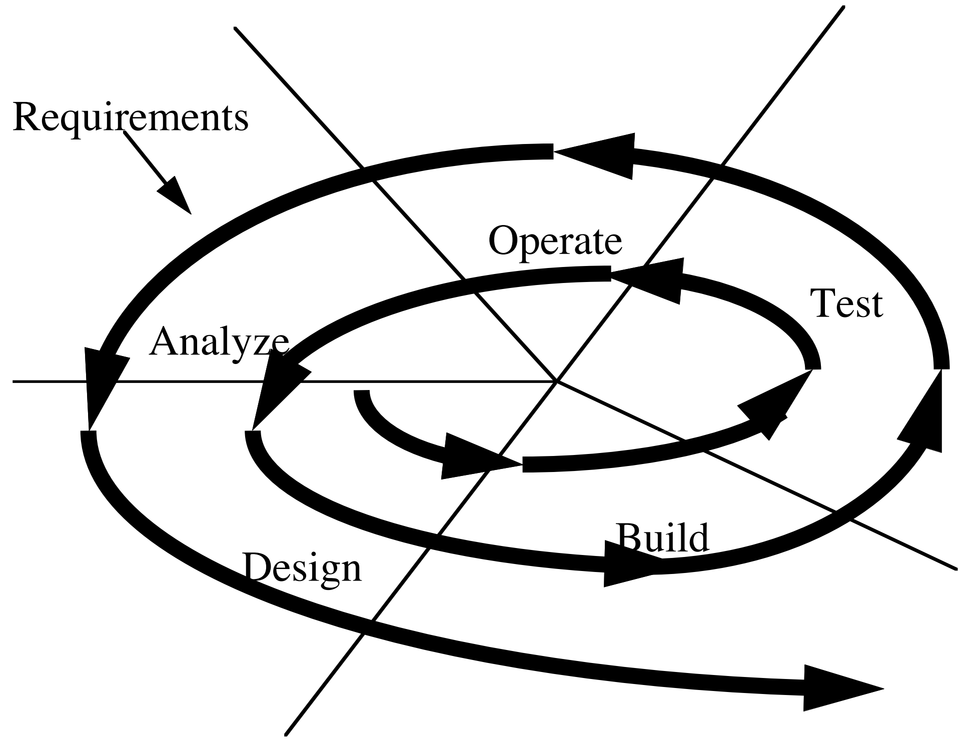
What is Knowledge Acquisition?

- Objective:
  - Construct a model to perform defined task
  - Develop knowledge base for use in solving problems in defined class
    - » Modularity
    - » Modifiability and reusability
- Participants: Collaboration between problem expert(s) and modeling expert(s)
- Process: Iterate until done
  - Define task objective
  - Construct model
  - Evaluate model

Systems Engineering

- System
  - A set of interacting components organized to serve a specified objective
- Systems engineering
  - The technical and managerial process by which a user need is translated into an operational system
- System life cycle
  - Systems evolve through predictable phases
    - » Design
    - » Development
    - » Operation
    - » Retirement
  - Systems engineering is organized around life cycle
    - » Support current phase
    - » Anticipate and plan for next phase

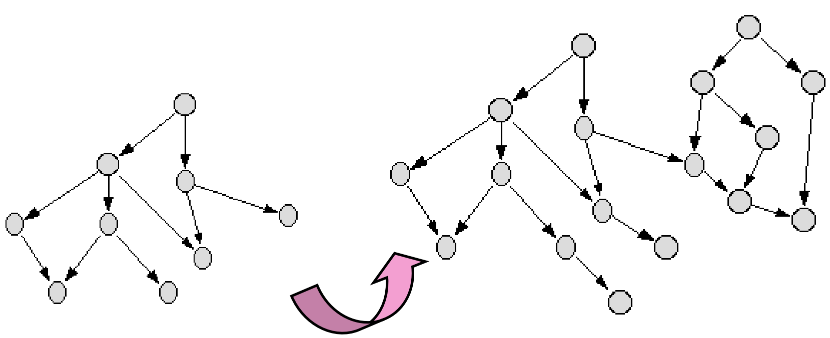
Spiral Model of Lifecycle Engineering

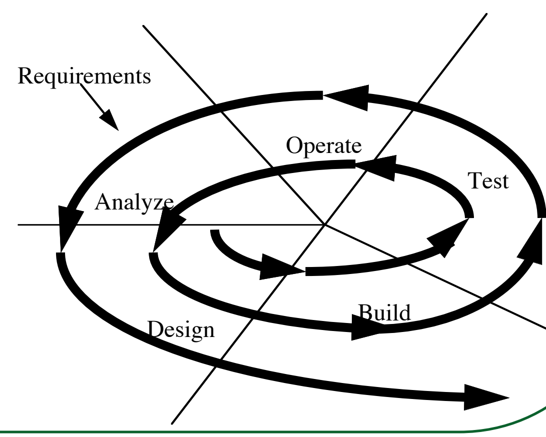


- System development viewed as repeating cycles of design, implementation, operation, evaluation
- Evaluation used to plan next cycle
- Early phases develop prototype for planning and risk mitigation
- Later phases develop operational versions

Spiral Model of Graphical Model Engineering

- Goal of knowledge engineering
  - Discovery and construction of appropriate model
  - Not extraction of pre-existing model
- Spiral model is necessary for systems in which requirements are discovered as development progresses
- Spiral KE
  - Constructs series of prototype models
  - Explores behavior of prototype model on sample problems
  - Evaluates prototypes and restructures as necessary
- KE changes both expert and elicitor
  - Understanding of expert and elicitor deepen as KE proceeds
  - Improves communication between elicitor and expert





Applying Spiral Knowledge Engineering

- Begin with a small subproblem
  - Self-contained
  - Can be completed in short time
  - Interesting in its own right
  - Reasonably representative of global problem
- Build and test model for subproblem
  - Look for common structures and processes that will recur
  - Think about more efficient ways to structure KE
  - Develop and document conventions ("style guide") to be followed as models are expanded
- Iteratively expand to more complex problems

Selecting a Subproblem

- Initial model or expansion of existing model
- Characteristics
  - Manageable size
  - Interesting in its own right
  - Path to expansion
  - Risk mitigation
- How to restrict
  - Focus or target variables - variables of direct interest to client
    - Restrict to subset of variables of interest
    - Restrict to subset of values
  - Evidence variables - variables for which information will be available; used to draw inferences about the focus variables
    - Restrict to subset of evidence sources
  - Context variables - variables that will be assumed known and will be set to definite values
    - Restrict to subset of contextual conditions (sensing conditions, background casual conditions; assignment of objects to sensors; number of objects)

The Participants

- Naive view
  - Put problem experts and modeling experts in a room together and magic will happen”
- Realistic view
  - Pure “problem experts” and pure “modeling experts” will talk past each other
  - Modeling experts must learn about the problem and problem experts must learn what models can do
  - This process can be time consuming and frustrating
  - Team will be more productive if both sides expect and tolerate this process
- Training
  - The most productive way of training modelers and problem experts is to construct very simple models of stylized domain problems
  - Goal is understanding and NOT realism or accuracy!
  - Beware: the training phase can seem pointless and frustrating
  - It is important to get expert buy-in

The Domain and the Expert

- Domains well suited to reliably and measurably good performance
  - Tasks are repeatable
  - Outcome feedback is available
  - Problems are decomposable
  - Phenomena are inherently predictable
  - Human behavior/"gaming" not involved
- Characteristics to look for in an expert
  - Expertise acknowledged by peers
  - Articulate
  - Interest and ability to reason about reasoning process
  - Tolerant of messy model-building process

Unit 7 Outline

- The Knowledge Acquisition Lifecycle
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- Managing and Evaluating the Model

Model Components

- What are the variables?
  - Random variables
  - Action and utility nodes
- What are their states?
- What is the graph structure?
  - Is there repeated structure?
- What is the structure of the local distributions?
- What are the parameters?
  - Probability distributions
  - Utility functions

The Clarity Test

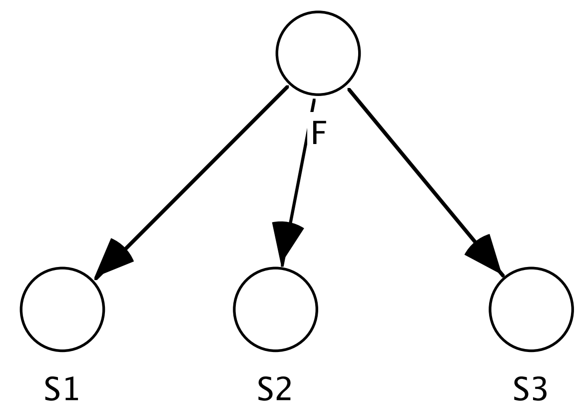
- Usually begin with loose structure to develop understanding of problem
- Final model should have clear operational meaning for all components
- Clarity test:
  - Could a clairvoyant unambiguously specify value of all nodes and states?
  - "Fever is high" does not pass clarity test
  - "Fever ≥ 103° F" passes clarity test

Defining the Variables

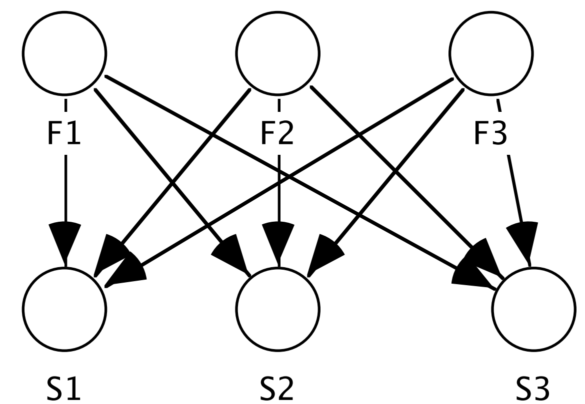
- Begin with “focus variable” and spread out to related variables
- Ask about causally related variables
  - Variables that could cause a state to be true
  - Variables that could prevent a state from being true
- Ask about enabling variables
  - Conditions that permit, enhance or inhibit operation of a cause
- Ask about effects of a variable
- Ask about associated variables
  - Knowing value provides information about another variable
- Ask about observables
  - What evidence could be observed that would enable you to infer state of a variable

Target or Focus Variable in Diagnosis

- Diagnosis problem: goal is to infer "fault," "disease," "problem" from a set of "findings," "symptoms" or "indicators"
  - Direction of inference is usually from effect to cause
- Modeling issue: single or multiple fault?
- Single fault
  - Collect all faults as states of a single node
  - Modeling simplicity and inference tractability
- Applicable domains:
  - Pathology- one disease/slide
  - Pediatrics- acute diseases
  - Highly maintained mechanical systems
- Modified single-disease hypothesis:
  - Include common combinations as explicit hypotheses



Single Fault Model



Multiple Fault Model

Target or Focus Variable in Prediction

- Prediction:
  - Objective is to predict a variable that has not yet occurred or is not known
  - Direction of inference is usually from cause to effect
- Applications:
  - Planning
  - Intelligence analysis
  - Policy modeling
  - Strategic decision making

States of Variables

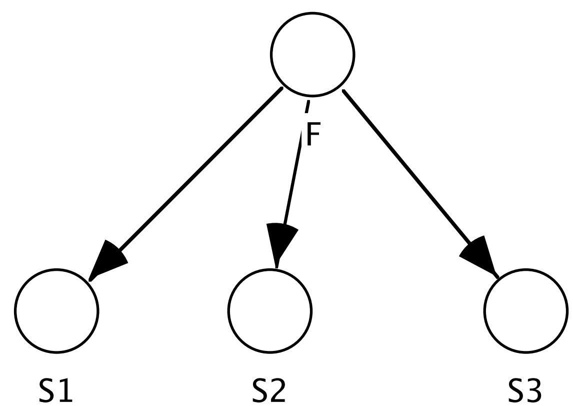
- States must be exclusive and exhaustive
  – Naive modelers sometimes create separate variables for different states of the same variable
- Types of variable
  – Binary (2-valued)
  – Qualitative
  – Numeric discrete
  – Numeric continuous
- Dealing with infinite and continuous state sets
  – Standard Bayesian network software requires finitely many states per random variable
  – Continuous random variables must be grouped into bins
  – Bin boundaries should represent meaningful differences in effect on related variables
  – Different resolutions may be appropriate for different purposes
  – Research on discretization is needed

Graph Structure

- Goal: develop model that is good enough for task
- Criteria to consider
  - Parameter parsimony
    - » Fewer nodes, fewer arcs, smaller state spaces, coarser partitions simplifies elicitation makes learning more efficient (fewer observations required)
  - Fidelity of model to problem
    - » Greater fidelity often requires more nodes, arcs, states, finer partitions
    - » Balance benefit against cost of additional modeling
    - » Too much detail can decrease accuracy
  - Maximize expert comfort with probability assessments
- Direction of arcs
  - Causal direction can increase:
    - » Conditional independence
    - » Ease of probability elicitation
    - » Efficiency of learning
  - Causal direction is required if modeling effects of interventions (planning)
  - It may be helpful to show user a graph with arcs in inferential direction even if BN has causal arcs

Naïve Bayes

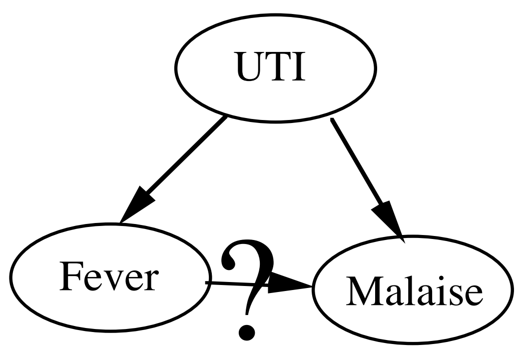
- Commonly applied in diagnosis problems
  - Simplifies elicitation
  - Simplifies inference
  - Simplifies learning
- Single parent node and multiple leaf nodes that are conditionally independent given parent
  - Also known as "idiot Bayes"
  - Simplifies knowledge engineering and speeds up computation
  - Often OK at least approximately



Naïve Bayes Model

Handling Dependency: Adding States to Parent Variable

- Problem: Symptoms not independent given fault
- Solution: Redefine parent variable to create model with independent symptoms
  - Incorporate into states of parent variable conditions that modify relationship between symptoms
- Example
  - P(Malaise|UTI, fever) > P(Malaise | UTI)
  - Redefine UTI states {absent, mild, moderate, severe}
  - P(Malaise | severe UTI, fever) ≈ P(Malaise | Severe UTI)

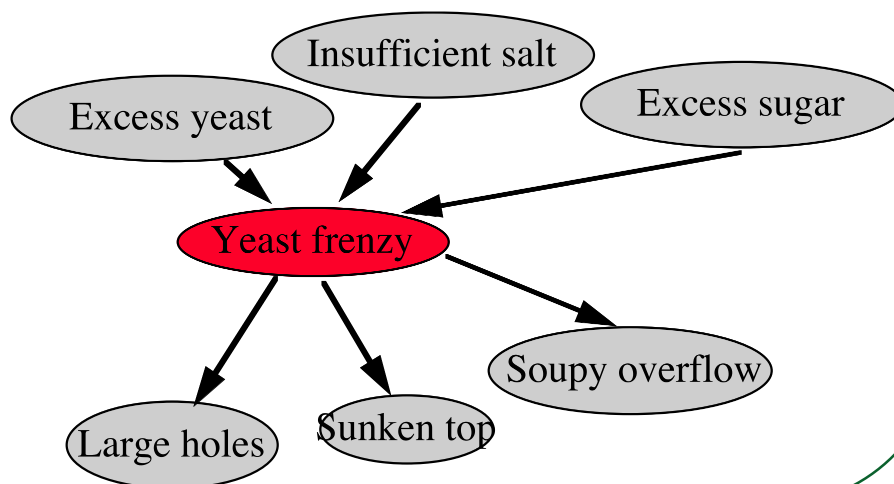


Example courtesy of Mike Shwe

Handling Dependency: Adding Intermediate Variables

- Intermediate variable is used to model dependency of children given parent
  - Symptoms are independent of fault given intermediate variable
  - Children are dependent given original parents
  - More parsimonious than drawing arcs between symptoms
- Examples:
  - "True state" variable creates conditional independence of sensor reports
  - Intermediate mechanism creates independence among a set of related findings

Using partitions or ICI can simplify specification of distribution of intermediate random variable



Example courtesy of Mike Shwe

Local Distribution Structure

- Local distributions:
  - One distribution for each combination of values of parent variables
  - Assessment is exponential in number of parent variables
  - Assessment can be reduced by exploiting structure
- Examples of local distribution structure
  - Elicitation by partition
  - Independence of causal influence
  - Divorcing

Elicitation by Partition

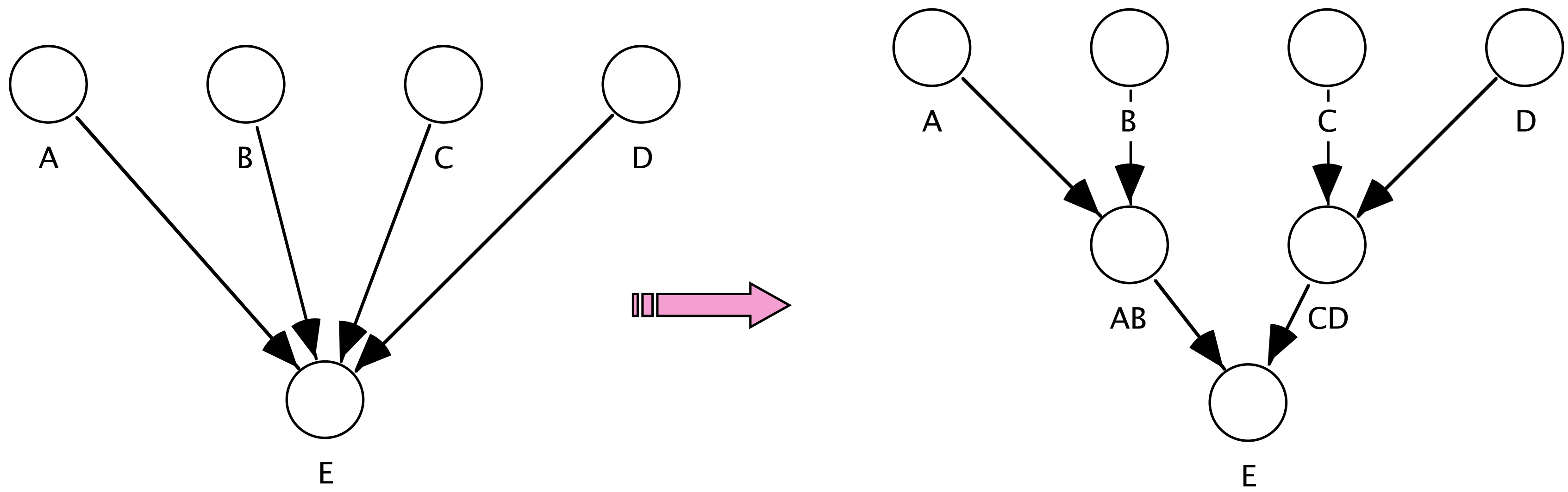
- Partition state set of parents into subsets
  - Set of subsets is called a partition
  - Each subset is a partition element
- Elicit one probability distribution per partition element
- Child is independent of parent given partition element
- Examples:
  - P(reported location | location, sensor-type, weather) independent of sensor type given weather=sunny
  - P(fever=high | disease) is the same for disease ∈ {flu, measles}
- Knowledge elicitation using partitions
  - For each child variable ask expert to group states of parent variable having same probability distributions
  - Sometimes several children induce same partition
  - Partitions may form basis of type hierarchy
    - » When many indicators induce a common partition element we may name that partition element as a subtype of the parent variable
    - » e.g., acute infectious disease "isa" infectious disease "isa" disease

Independence of Causal Influence

- Assumption: causal influences operate independently of each other in producing effect
  - Probability that C1 causes effect does not depend on whether C2 is operating
  - Excludes synergy or inhibition
- Examples
  - Noisy logic gates (Noisy-OR, Noisy-AND, Noisy-XOR)
  - Noisy adder
  - Noisy MAX
  - General noisy deterministic function
- Elicitation:
  - ICI structure:
    - » "Does the presence of C2, C3, … increase or decrease [strengthen or weaken] the impact of C1 on E?"
    - » "Does the presence of C2, C3, … increase or decrease the probability C1 will cause E to occur?"
  - Parameters:
    - » ICI structure allows calculation of entire probability table from single-cause distributions

Divorcing

- Divorcing generalizes partitions and ICI
- An intermediate variable summarizes the effect of a subset of parents on the child



Exploiting Context-Specific Indepdendence

- Context-specific independence can simplify elicitation
- Example:
  - Government supporters and apolitical people rarely criticize the government. Dissidents often do, as do government agents (because they are trying to lure Rechtian agents into thinking they are dissidents)
  - Need to specify only one distribution given {Supporter, Apolitical} and another probability given {Agent, Dissident}
- Eliciting partitions from expert:
  - Ask expert: "Which variables help to distinguish between supporters and agents?"
  - Expert answers: "Criticizing the government"
  - Ask: "Does this variable give any information to help distinguish between agents and dissidents? Between supporters and apolitical people?"
  - Expert answers: "No"
  - Result: Distribution for criticism is same for agents and dissidents, and same for supporters and apolitical people

Specialists and Generalists

- In many disciplines experts tend to partition problems into sub-categories exhibiting asymmetric independence.
- If we know which sub-category to focus on, we can ignore cues relevant for other categories.
  - Specialists focus on difficult cases in one sub-category
  - Generalists focus on
    - Solving easy cases in any category
    - Diagnosing when to call in a specialist and which specialist to call
- Asymmetric independence justifies and supports this strategy
  - Some variables are most useful for sorting cases into sub-categories, and may be (approximately) independent of which hypothesis is correct within sub-category
  - Other cues may be useful for discriminating within sub-categories
- Partitions can represent this type of reasoning

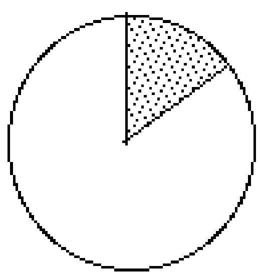
Assessing Probability Distributions

- Theory
  - Parametric expression may be suggested on theoretical grounds
    - » P(Node|Parents) = f(parents,parameter)
  - Elicit parameter from expert or estimate from observations
  - Test theory against data
- Statistical estimation (when data are available)
  - Frequencies (problem with zero probabilities)
  - Posterior distribution given Dirichlet prior distribution
    - » Uniform prior
    - » Elicit prior frequencies and virtual prior sample size from expert
  - Regression model (discrete or continuous)
- Direct elicitation
  - Probabilities
  - Odds (better for extreme probabilities)
  - Continuous distributions: percentiles, density function, parameters

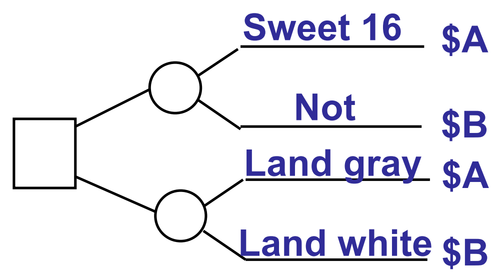
Direct Elicitation

Example: Will George Mason University make Sweet 16 in 2013?

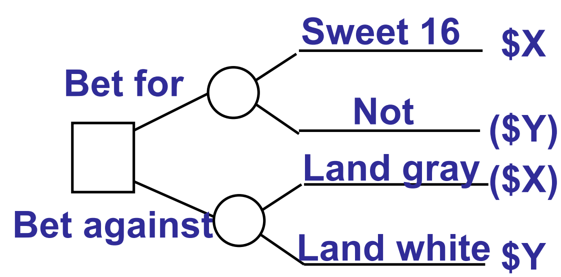
- Direct question: "What is the probability GMU will make Sweet 16 in 2009?"
- Visual representation
  - Adjust the size of the gray area so the probability that we will make Sweet 16 is the same as the probability that a spinner will land in the gray area



- Ask for probability or area on wheel so that decision maker is indifferent between 2 lotteries:



- Ask decision maker about betting odds and solve for probability



  - For small prizes, p=Y/(X+Y)

Comments

- Many decision makers are uncomfortable with numerical probabilities
  - Decision makers may prefer qualitative terms such as “fairly likely” or “improbable”
  - These phrases have ambiguous meaning and can cause miscommunication unless they are calibrated to agreed-upon numerical values
  - Sometimes visual devices are a good compromise. If manipulated on a computer screen they can be translated directly into numerical probabilities
  - Betting odds are often less useful in practice than asking directly about probabilities - betting odds come from probabilities and not vice versa
  - People find frequencies (3 out of every 100) easier than probabilities (0.03)
  - For very small probabilities orders of magnitude must be used
    - “State a1 is 100 times more likely than State a2”
    - “We will see 100 cases of State a1 for every case of State a2”
  - Assessing very small probabilities is difficult
- Probability assessors should be aware of systematic distortions of probability judgment
  - Treating low-probability events as impossible
  - Overconfidence and other anchoring effects
  - Neglect of base rates
  - Overweighting salient events
- If time permits it is good to phrase questions in multiple ways and to feed back consequences of judgments to decision maker

Assessing Odds

- Direct assessment of probabilities can yield very poor results on extreme probabilities
  - Direct assessment focuses on absolute magnitude
  - 0.01 seems "not much different" from 0.001
  - Orders of magnitude are important in Bayes Rule

$$\frac{P(H_1|E)}{P(H_2|E)} = \frac{P(E|H_1)P(H_1)}{P(E|H_2)P(H_2)}$$

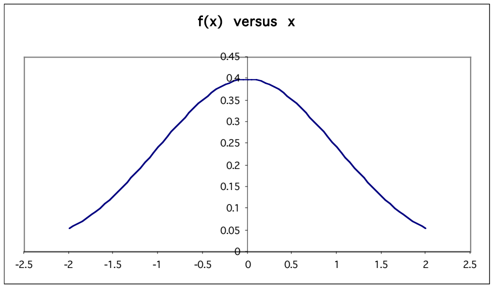
- Assessing by odds
  - "State a1 of variable A is 3 times more likely than state a2"
  - Yields equation P(a1) = 3P(a2)
  - Solve for P(a1) and P(a2)

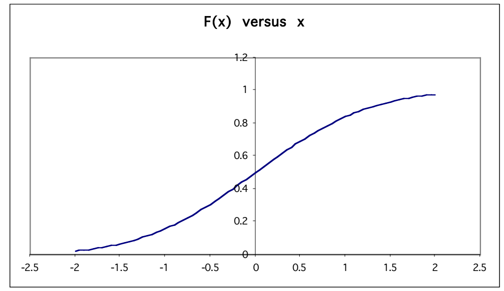
Continuous Distributions

- Continuous random variables can take on values on a continuum
  - Parametric models (e.g., Normal, Gamma, Chi-square)
  - Nonparametric models
  - "Semi-parametric" models (kernel density functions)
- The cumulative distribution function (cdf)
  - $F(x) = P(X \le x)$
  - Value of cdf at x is the probability that the random variable is less than or equal to the number x
  - F(x) is a step function for discrete variables, and a smoothly increasing function for continuous variables
- Probability density function (pdf)
  - The pdf measures the relative probability of different values of the continuous random variable. The value $f(x)\Delta x$ is approximately equal to the probability that X lies in the small interval $[x-\Delta x, x+\Delta x]$
  - The pdf is the derivative of the cdf:
    - » $f(x) = \frac{dF(x)}{dx}$
  - The cdf is the integral of the pdf:
    - » $F(x) = \int_{x' \le x} f(x')dx'$
  - The probability that X lies in the interval [a, b] is equal to the area under the curve defined by the cdf:
  - $P(a \le X \le b) = F(b) - F(a) = \int_a^b f(x)dx$

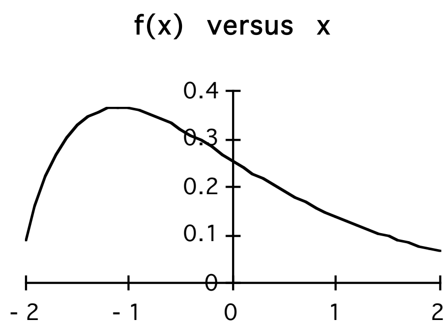
Example Types of Density Function

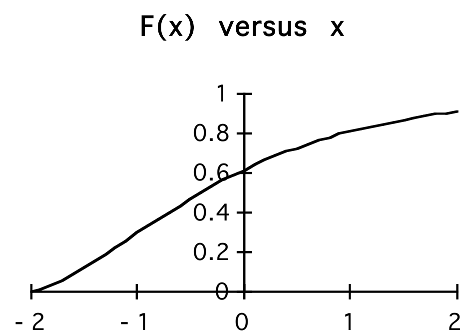
- Symmetric and unimodal



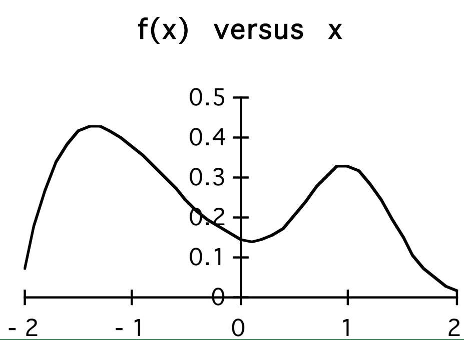


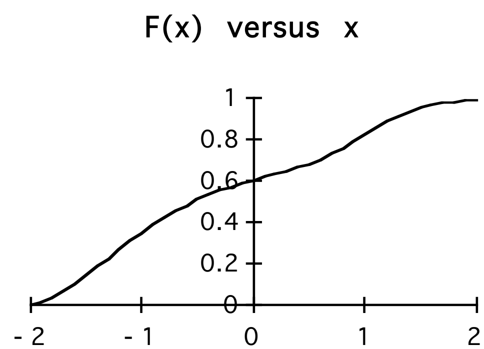
- Symmetric and skewed distribution





- Bimodal





We can show the expert different shapes and ask which best fits judgments

It may help to show density function and report percentiles ("With this density function, 23% of the cases will have value less than 1.5.")

Bimodal distribution may mean a missing parent

Assessing Continuous Distributions

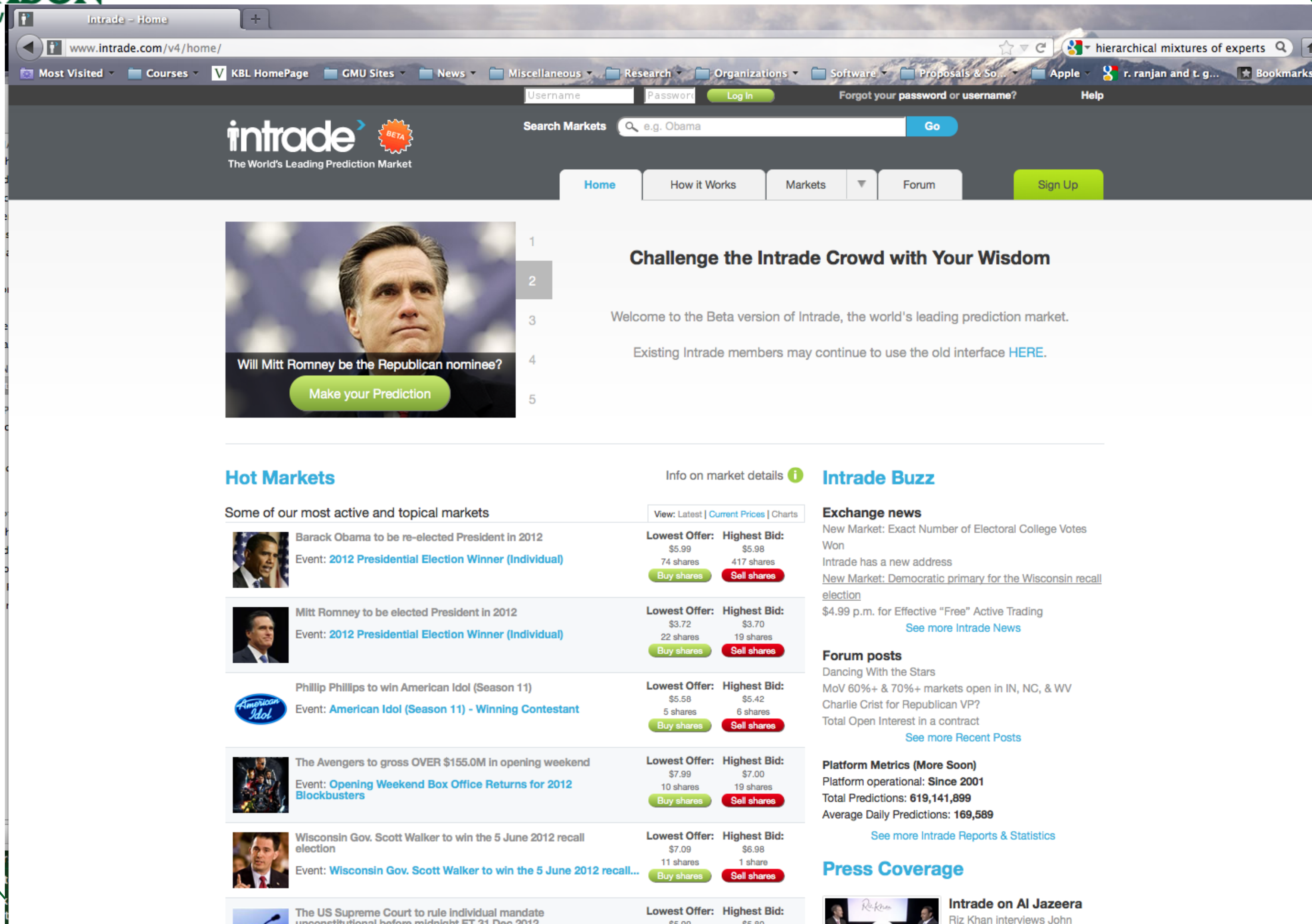
- Continuous distributions are often assessed by asking about the cdf
  - Pick values and assess probability (this method can also be used for discrete random variables) :
    - » P(sales ≤ $10,000) = ?
    - » P(sales ≤ $20,000) = ?
    - » P(sales ≤ $40,000) = ?
    - » P(sales ≤ $65,000) = ?
    - » P(sales ≤ $100,000) = ?
  - Pick probabilities and assess values:
    - » P(sales ≤ ?) = .10
    - » P(sales ≤ ?) = .30
    - » P(sales ≤ ?) = .50
    - » P(sales ≤ ?) = .70
    - » P(sales ≤ ?) = .90
- Depending on how the judgments will be used, we may interpolate between these points or we may fit a parameterized probability distribution to the expert's judgments
- This method may be problematic for parameters whose meaning is not straightforward to the decision maker
  - e.g., What is your cdf for the mean number of transmission errors per hour?
- Another method is to ask about shape of density function
- These methods can yield different results
  - Suggestion: use both and resolve inconsistencies

Parameterized Continuous Distributions

- Use of standard parameterized distributions may greatly economize on elicitation, e.g.:
  - Normal
  - Log-normal
  - Gamma
  - Exponential
- Assess several percentiles and select parameters to fit the percentiles (this may take some work)
- Check that shape of density function is acceptable
- Ask about "sufficient statistics"
  - What do you think is the average number of transmission errors per 8-hour period when averaged over many periods?
- When data are available they can be used to augment expert judgment
  - Use expert judgment to specify a prior distribution
  - Learn posterior distribution for parameters
  - Use structure learning method to check structural assumptions
    - » Partitions, presence/absence of arcs, functional form of distribution

Combining Models from Multiple Sources

- Problem decomposition
  - Some elements learned from data; others elicited from experts
  - Different experts specify different parts of model
- Aggregating different inputs on same model component
  - There is a large literature on combining probability estimates
  - We can combine estimates from multiple experts, multiple models, or both
  - Typically the combination rule is some kind of average
  - Many weighting schemes have been proposed; it has proven surprisingly difficult to beat simple averaging
- Prediction markets
  - Useful for forecasting well-defined events on which outcome feedback will become available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Help
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The World's Leading Prediction Market
Search Markets
e.g. Obama
Go
Home
How it Works
Markets
Forum
Sign Up
Will Mitt Romney be the Republican nominee?
Make your Prediction
1
2
3
4
5
Challenge the Intrade Crowd with Your Wisdom
Welcome to the Beta version of Intrade, the world's leading prediction market.
Existing Intrade members may continue to use the old interface HERE.
Hot Markets
Info on market details
Some of our most active and topical markets
View: Latest | Current Prices | Charts
Barack Obama to be re-elected President in 2012
Event: 2012 Presidential Election Winner (Individual)
Lowest Offer: $5.99 74 shares Buy shares
Highest Bid: $5.98 417 shares Sell shares
Mitt Romney to be elected President in 2012
Event: 2012 Presidential Election Winner (Individual)
Lowest Offer: $3.72 22 shares Buy shares
Highest Bid: $3.70 19 shares Sell shares
Phillip Phillips to win American Idol (Season 11)
Event: American Idol (Season 11) - Winning Contestant
Lowest Offer: $5.58 5 shares Buy shares
Highest Bid: $5.42 6 shares Sell shares
The Avengers to gross OVER $155.0M in opening weekend
Event: Opening Weekend Box Office Returns for 2012 Blockbusters
Lowest Offer: $7.99 10 shares Buy shares
Highest Bid: $7.00 19 shares Sell shares
Wisconsin Gov. Scott Walker to win the 5 June 2012 recall election
Event: Wisconsin Gov. Scott Walker to win the 5 June 2012 recall...
Lowest Offer: $7.09 11 shares Buy shares
Highest Bid: $6.98 1 share Sell shares
The US Supreme Court to rule individual mandate unconstitutional before midnight ET 31 Dec 2012
Lowest Offer: $5.99
Highest Bid: $5.80
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Platform operational: Since 2001
Total Predictions: 619,141,899
Average Daily Predictions: 169,589
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Riz Khan interviews John

Relational Knowledge Engineering

- Relational representations (e.g., MEBN, PRM, OOBN) require knowledge about:
  - Entity types (e.g., patients, diseases, tests)
  - Attributes of entities (e.g., gender of patient, sensitivity of test)
  - Relationships among entities (e.g., patients have diseases; patients take tests)
- Entity-relationship model specification is needed for database schema design; object-oriented software design; ontological engineering
- Literature in these areas usually does not treat uncertainty
- Methodology not very well developed
  - Software support is not widely available
  - Experience base is small of people knowledgeable in specifying relational models
  - This is a growth area in probabilistic modeling methodology

Unit 7 Outline

- The Knowledge Acquisition Lifecycle
- Building the Model
- Managing and Evaluating the Model

Managing Knowledge Acquisition

- Record rationale for modeling decisions
- Develop “style guide” to maintain consistency across multiple subproblems
  - Naming conventions
  - Variable definitions
  - Modeling conventions
- Enforce configuration management
  - History of model versions
  - Protocols for making and logging changes to current model
  - Rationale for changes
- Develop protocol for testing models
  - Record of test results traced to model changes and rationale

Configuration Management

- Formal process is required for managing evolution of complex models
- Configuration management includes:
  - Archiving history of evolving versions
  - Protocols for making and logging changes to current knowledge base
  - Protocols for documenting changes and rationale
  - Automated comparison of similarities and differences between different versions of knowledge base

Model Agility

- Requirement: rapid adaptation of model to a new situation
- Support for model agility
  - Libraries of reusable model fragments
  - Documentation of stable and changeable aspects of model
  - Development of data sources for inputs to changeable model components
    - Protocols for data collection and maintenance
    - Protocols for importing data into knowledge base
  - Automated support for propagating impact of changes

Model Evaluation

- Model walk-through
  - Present completed model to "fresh" experts and/or modelers
  - Evaluate all components of model
- Sensitivity analysis
  - Measures effect of one variable on another
  - Compare with expert intuition to evaluate model
  - Evaluate whether additional modeling is needed
- Case-based evaluation
  - Run model on set of test cases
    - » Cases to check local model fragments (component testing)
    - » Cases to test behavior of global model (whole-model testing)
  - Compare with expert judgment or “ground truth”
  - Important issue: selection of test cases

Summary and Synthesis

- BN model development is a problem in system lifecycle engineering
  - Begin with a small subproblem
    - » Self-contained
    - » Can be completed in short time
    - » Interesting in its own right
    - » Reasonably representative of global problem
  - Iteratively expand and refine
  - Test and evaluate at each stage
    - » Elicitation review
    - » Sensitivity analysis – quantitative and qualitative
    - » Case-based testing – local and global
    - » Evaluation against empirical data
- Effective management of knowledge engineering process improves
  - Communication between domain experts and knowledge engineers
  - Quality of model
  - °Reusability of results
- Software supports for BN lifecycle engineering are needed

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