Monetary Policy Transmission, Central Bank Digital Currency, and Bank Market Power*

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Abstract

Interest rates on new central bank digital currencies (CBDCs) can be expected to enter the monetary policy toolkit soon. Using an extended Sidrauski (1967) model featuring an oligopsonistic banking sector, we study the complex transmission of interest rates on CBDC, which generally involve both *direct* and *indirect* effects. This is because a CBDC rate cut does not only affect the rate on the CBDC itself, but also induces the non-competitive deposit providers to adjust their spreads, as the new substitute for their products becomes relatively less attractive. A calibration exercise suggests that the indirect effects depend strongly on the sources of deposit market power: If driven by high concentration, they substantially amplify the aggregate effects of the CBDC policy rate, both in response to transitory shocks as well as regarding its long-run welfare effects. This contrasts them with policies directed at the banking sector which are weakened by a less competitive deposit market.

Keywords: CBDC, Digital Currency, Bank Market Power, Monetary Transmission

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1 Introduction

Monetary authorities around the world are exploring the possibility of issuing a new digital payment instrument widely accessible to the public. As of May 2024, 134 countries and currency unions, which account for 98% of the global GDP, are considering a central bank digital currency (CBDC) (Atlantic Council, 2024). Motivations for such a new payment instrument include ensuring adequate public money, reducing systemic risk and improving financial stability, increasing competition in payments, and promoting financial inclusion (Engert and Fung, 2017).

One of the less-discussed aspects of CBDC is its potential to enable a direct implementation of monetary policy (e.g., Auer et al., 2022; Bank for International Settlement, 2020). Interest on CBDC could become a new policy instrument, providing policy-makers with greater flexibility to influence the real returns of money assets and sidestep financial intermediaries. However, banks will not idly stand by if the central bank makes it more attractive to hold an asset providing similar services as their deposits. In turn, the actual equilibrium impact of CBDC rates should depend both on households' liquidity preferences and the response of financial sector agents with substantial market power.

A key contribution of our work compared to the existing literature is that regarding the latter channel, we explicitly distinguish between different sources of bank market power, *market concentration* and *differentiation*: Current theories tend to focus either on one or the other (see below for further discussion), but clearly, both aspects are relevant in reality: The market for bank deposits does not only display substantial concentration (see e.g. Corbae and D'Erasmo, 2020) but the respective deposits also differ in practice due to regionally differing branch networks, bundling with different payment cards, etc. Indeed, we find the actual source of bank market power to have important implications for the effect of a CBDC rate as a policy tool, which suggests assumptions on the form of imperfect bank competition to be crucial for the outcomes of quantitatively modeling CBDC policy.

In particular, we conduct our analysis using an extended Sidrauski (1967) model building on the framework proposed by Niepelt (2024): In the model, households gain utility from holding different forms of liquid assets which allows us to capture various related aspects such as CBDC design and the private sector's need for deposits of different banks in a parsimonious way close to textbook theories. However, it should be noted that such a model has the implication that CBDC is not "special" compared to alternate forms of government-provided liquidity (in the sense of explicitly modeled design features). In turn, our theoretical analysis would equally apply to other liquidity types as long as central banks can affect their returns flexibly enough.

Crucially, in our framework, we allow for a banking sector in which bank market power is derived *both* from market concentration and households' imperfect ability to substitute between banks. We assume a common deposit market in which a set of non-competitive banks compete by offering differentiated deposits, but do not restrict it to be either monopsonistic (as e.g. in Niepelt, 2024) or monopsonistically competitive (as e.g. in Bacchetta and Perazzi, 2021). Rather, such settings are nested as limit cases, allowing us to vary the degree of deposit market concentration in order to demonstrate its importance for the transmission of

CBDC rates. We also do not restrict different banks' deposits to be fundamentally the same to households as in Chiu et al. (2023), so that oligopsonistic settings remain characterized by different sources of market power.

Additionally, we assume limited substitutability between CBDC and bank deposits, with perfect substitutability as a limit case. We believe there are good reasons to expect that in practice, CBDC would not be almost perfectly substitutable with bank deposits. For example, bank deposits are typically bundled with other financial services such as credit lines (e.g. overdraft facilities, credit cards), while CBDC may be perceived as offering more privacy and security. Additionally, other features such as the interoperability between CBDC and deposits and the ability to conduct international transactions might also limit practical substitutability (Bacchetta and Perazzi, 2021).

For the purpose of this paper, we consider the interest rates on CBDC as the main policy instrument of interest but also discuss implications for reserve rates. In our model, both can be shown to affect the real allocation through the *average cost of liquidity*, but the influence of the CBDC spread consists of both a *direct* and an *indirect* effect. Clearly, an increase in the CBDC spread (relative to a risk-free rate) directly increases the households' cost of liquidity. At the same time, the rising spread also enables banks to widen the spreads on the deposits they offer, as the alternative source of liquidity becomes comparatively less attractive. This introduces the indirect effect, reminiscent of the deposit channel of monetary policy proposed by Drechsler et al. (2017).

While the quantitative magnitude of the direct effect depends simply on the amount of CBDC households will choose to hold given its design, the strength of the indirect effect is more nuanced, depending crucially on the source of market power in the deposit market. Intuitively, if deposit market concentration is low, individual banks are small and cannot affect the amount of CBDC households will choose to hold. In turn, changes in the CBDC spread have little impact on the equilibrium deposit spread and the indirect effect is small. On the other hand, if the deposit market is highly concentrated, banks can practically compete with CBDC and adjust their spreads more, making the indirect effect relatively large. Our calibration exercises suggest the indirect effect to substantially amplify the aggregate response to CBDC rate changes in settings with high market concentration but less so if concentration is lower and *differentiation* constitutes a relatively more important source of deposit market power.

Such considerations do not only apply to temporary changes in interest rates, but also to how central bank policy can affect long-run welfare through inducing a more efficient liquidity mix. Indeed, we not only demonstrate that in a setting with more deposit market concentration, the effects of the CBDC rate on deposit spreads and welfare become more pronounced, but also through a decomposition that the former is indeed the cause of the latter. This also implies that the monetary policy chosen by a Ramsey planner with limited instruments will depend on deposit market concentration.

In contrast to CBDC policy, the impact of reserve rates, which we briefly analyze as stand-in for monetary policy instruments directed at the banking sector, *decreases* with higher deposit market concentration. Under the assumption that the shock makes it more expensive for banks to provide deposits, it causes banks to increase their deposit spreads and households' cost of liquidity. This effect is smaller if market concentration is high, as this makes the spreads charged by banks relatively more dependent on their demand schedule, which is otherwise not directly affected by the policy.

Our work relates to the growing and recent literature on CBDC, which has studied these potential new payment instruments from a variety of perspectives. For example, Agur et al. (2022) analyze the trade-offs associated with CBDC design given heterogeneous household preferences over payment instruments and network effects regarding their use. They conclude that central banks should indeed issue interest-bearing CBDCs and choose their rate so that other payment instruments remain in use. Similarly, Keister and Sanches (2023) highlight trade-offs associated with CBDC design choices. In particular, they argue that a CBDC with a deposit-like design would have positive effects by increasing payment- and exchange efficiency, but may also decrease investment by inducing higher funding costs for banks. Piazzesi and Schneider (2022), in turn, warn that CBDC crowding out bank deposits may decrease efficiency in financial intermediation due to a complementarity between offering both deposits and credit lines. Other work has studied the impact of CBDC adoption on financial stability with differing findings, i.e. that CBDC may either improve financial stability (Fernández-Villaverde et al., 2021) or encourage banking panics (Williamson, 2022).

Given that we study CBDC in a set-up with non-competitive banks, our work is particularly related to Andolfatto (2021), Bacchetta and Perazzi (2021) as well as Chiu et al. (2023), which all study the impact of CBDC introduction in the presence of a non-competitive banking sector. Andolfatto (2021) focuses on the impact of CBDC introduction on bank lending and economic activity and finds that a CBDC may not impede either. In fact, non-competitive banks forced to increase their deposit rates will be subject to an additional inflow of deposits due to the more attractive rates and, in turn, convert this additional funding into lending. Chiu et al. (2023) obtain similar results in a different set-up allowing for differing degrees of bank market power. In contrast to our work, these papers only consider *concentration* as a source of deposit bank market power and focus on the long-run effects of CBDC on bank lending and general economic activity and consider, while we also analyze the transmission of short-term shocks. While Bacchetta and Perazzi (2021) also share the long-run focus, they consider a monopolistic competitive deposit market on which deposit market power is only driven by *differentiation*.

Jiang and Zhu (2021) and Garratt et al. (2022) share our focus by studying monetary pass-through in settings with imperfectly competitive or heterogeneous banks, respectively. Jiang and Zhu (2021) study the pass-through of both reserve and CBDC rates in a framework similar to Chiu et al. (2023). In the presence of a non-competitive banking sector, the introduction of CBDC is shown to potentially weaken the reserve pass-through, as perfect substitutability forces banks to match the CBDC rate on the deposit market. CBDC can essentially "dictate" the economy. The CBDC rate, in turn, may have a particularly strong pass-through to deposit rates, while its effects on loan rates depend on the reserve rate in an ambiguous way. Major differences between the work of Jiang and Zhu (2021) and ours are that they also only consider one margin of banking competition, and, due to the assumption of perfect substitutability between bank deposits and CBDC, rule out the presence of the indirect effects discussed above, as the CBDC rate will either determine the deposit rate completely or not affect it all. Garratt et al. (2022) consider a framework with differing bank types ("large" and "small") competing for deposits from workers having heterogeneous preferences over the non-monetary benefits (e.g. extensive branch networks) they offer. They find that the pass-through of the CBDC rate to the deposit rate is stronger if the CBDC rate is high compared to the reserve rate, which, however, hurts the “small” bank. In contrast to our work, their focus is on bank heterogeneity, from which we abstract. Also, in their setup, no one actually ends up holding CBDC (the digital currency can again perfectly substitute for bank deposits and is out-competed by banks), so their model cannot provide for indirect effects of the CBDC rate on households’ liquidity costs either.

Furthermore, our research is related to several studies analyzing the interaction of bank market power and monetary policy transmission. In particular, Drechsler et al. (2017) propose a deposit channel of monetary policy. As interest rate increases raise the opportunity costs of holding cash, non-competitive banks are able to increase the deposit spread in response to tighter monetary policy, consequently reducing the overall amount of deposits. This, in turn, can affect both the liquidity premium and bank lending. Choi and Rocheteau (2023) study this channel theoretically in a search-theoretic model of deposit markets. Additionally, estimating a structural model of the banking sector, Wang et al. (2022) similarly find bank market power to have important effects on the transmission of rate changes to deposit rates. In addition to the Drechsler et al. (2017) mechanism, their model also explicitly considers an oligopolistic lending market, where banks additionally respond by adjusting their lending rate markups.

The rest of the paper is organized as follows: Section 2 describes the elements of the model economy and characterizes its equilibrium. Section 3 analyzes the transmission mechanisms of the interest rates on CBDC and reserves by qualitatively characterizing the channels through which the interest rates affect real allocation. Section 4 calibrates the model to conduct numerical exercises. Then, Section 5 quantitatively demonstrates the extent to which deposit market power affects policy transmissions under short-run monetary policy shocks. Section 6 investigates the implications of deposit market power for the efficacy of CBDC policy in the long run. Next, we conduct robustness tests in Section 7. Finally, Section 8 summarizes the results and concludes.

2 Model

We study an extended Sidrauski (1967) model, building on Niepelt (2024), in which both the government and banks provide liquidity to households that gain utility from holding it. Households substitute imperfectly between a government-issued form of liquid asset that we interpret as CBDC and commercial bank deposits. Banks fund themselves by borrowing deposits from the households and invest in capital and reserves which are used to “back up” deposit issuance. We follow Drechsler et al. (2017) and assume that banks are non-competitive in the deposit market. Banks have market power due to both market concentration and imperfect substitutability between banks’ deposit services. Neoclassical firms produce a common consumption good using capital and labor, and a consolidated government/central bank issues CBDC and reserves.

2.1 Households

We consider an economy consisting of many identical and infinitely-lived households, with the measure normalized to one. The representative household values consumption, c_t and liquidity services, z_{t+1}, according to a period utility function of the form

$$u(c_t, z_{t+1}) = \frac{\left((1-\upsilon)c_t^{1-\psi} + \upsilon z_{t+1}^{1-\psi}\right)^{\frac{1-\sigma}{1-\psi}}}{1-\sigma} \ ,$$

where $\upsilon \in (0,1)$ is the relative weight of liquidity services in utility, $\psi \in (0,1)$ is the inverse intratemporal elasticity of substitution between consumption and liquidity, and $\sigma > 0$ is the inverse intertemporal elasticity of substitution between consumption-liquidity bundles across time. We assume that CBDC and deposits are imperfect substitutes: Liquidity services are derived from real holdings of CBDC, m_{t+1}, and deposits, n_{t+1}, according to a Constant Elasticity of Substitution (CES) aggregator

$$z_{t+1} = \left((1-\gamma)m_{t+1}^{1-\epsilon} + \gamma n_{t+1}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \ ,$$

where $\gamma \in (0,1)$ is the relative liquidity weight of bank deposits, and $\epsilon \in (0,1)$ is the inverse elasticity of substitution between CBDC and deposits. The liquidity weight parameter, γ, captures how useful deposits are for the purpose of holding liquidity relative to the same quantity of CBDC. We follow Drechsler et al. (2017) in assuming that deposits are themselves a composite good issued by a set of N non-competitive banks. Each bank i has mass $1/N$ and produces deposits of a quantity n_{t+1}^i/N. The household values deposits at different banks such that

$$n_{t+1} = \left(\frac{1}{N}\sum_{i=1}^{N}\left(n_{t+1}^i\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \ , \tag{1}$$

where η denotes the inverse elasticity of substitution between banks. The representative household can be thought of as an aggregation of many individual households who may have diverse preferences for holding deposits at different banks. Therefore, the representative household substitutes deposits imperfectly across banks, which implies that $0 < \eta < 1$.

In our framework, in addition to deposits and CBDC, households can invest directly in capital. This is necessary for the model to feature realistic amounts of capital and liquid assets, as the aggregate amount of the former typically far exceeds the amount of the latter in modern economies.[1] Since we found it to be not crucial for our results, we abstract from a labor supply choice to simplify the analysis and instead assume the representative agent to inelastically supply a constant amount of labor $\bar{l}$. The household's budget constraint is then given by

$$c_t + k_{t+1}^h + m_{t+1} + \frac{1}{N}\sum_{i=1}^{N} n_{t+1}^i + \tau_t = w_t\bar{l} + \pi_t + k_t^h R_t^k + m_t R_t^m + \frac{1}{N}\sum_{i=1}^{N} n_t^i R_t^{n,i}, \tag{2}$$

[1] Note that our assumption is isomorphic to alternatively assuming that households do not hold capital directly but also provide funding to banks through a competitive asset market not providing liquidity services.

where k^h_{t+1} are direct holdings of capital, τ_t is the lump-sum tax net of government transfer, w_t is the wage rate, π_t is the dividends from firms and banks, R^k_t is the return on capital, R^m_{t+1} is the real gross interest rate on CBDC, and $R^{n,i}_{t+1}$ is the real gross interest rate on deposits at bank i. We assume that the returns on CBDC and deposits are risk-free, i.e. R^m_{t+1} and $R^{n,i}_{t+1}$ are known at time t. The household, taking prices, profits, and taxes as given, solves

$$\max_{\{c_t, k^h_{t+1}, m_{t+1}, n^i_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, z_{t+1})$$

$$\text{s.t.} \quad c_t + k^h_{t+1} + m_{t+1} + \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1} + \tau_t = w_t \bar{l} + \pi_t + k^h_t R^k_t + m_t R^m_t + \frac{1}{N}\sum_{i=1}^{N} n^i_t R^{n,i}_t,$$

$$k^h_{t+1}, m_{t+1}, n^i_{t+1} \geq 0.$$

We now turn to the first-order optimality conditions of the household program. Detailed derivations are provided in the Appendix A.1. First, the household optimally allocates resources between deposits at individual banks according to

$$n^i_{t+1} = n_{t+1} \left(\frac{\chi^{n,i}_{t+1}}{\chi^n_{t+1}} \right)^{-\frac{1}{\eta}}, \tag{3}$$

which closely resembles demand equations for differentiated consumption goods commonly derived in New Keynesian models. The relative share of deposits at bank i, n^i_{t+1}/n_{t+1}, must relate negatively to its corresponding relative cost, $\chi^{n,i}_{t+1}/\chi^n_{t+1}$. Here, $\chi^{n,i}_{t+1}$ is the interest-rate differential between the risk-free rate, R^f_{t+1}, and the deposit rate offered by bank i

$$\chi^{n,i}_{t+1} = 1 - \frac{R^{n,i}_{t+1}}{R^f_{t+1}},$$

which represents the opportunity cost of holding deposits at bank i and which we hereafter refer to as *deposit spread*. The risk-free rate is defined in the standard way as the inverse of the expected value of the household's stochastic discount factor, Λ_{t+1},

$$R^f_{t+1} = \frac{1}{\mathbb{E}_t[\Lambda_{t+1}]}, \tag{4}$$

where the stochastic discount factor is defined as follows:

$$\Lambda_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}}.$$

Given that the household can freely invest in capital, the risk-free rate will equal the expected return on capital.

We further define χ^n_{t+1} to represent the index that can be shown to capture the deposit spread associated with one unit of the aggregate deposit bundle n_{t+1} given demand schedule

If banks were not only the only agents able to hold capital and only obtain funding through deposits, the

(3):

$$\chi^n_{t+1} = \left(\frac{1}{N} \sum_{i=1}^{N} \left(\chi^{n,i}_{t+1} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{5}$$

This quantity can be interpreted as an aggregate price of deposits.

Next, optimization requires that the marginal rate of substitution between consumption and each of the liquid assets equals their respective costs. These conditions can be combined to derive an expression for the velocity of consumption

$$\frac{c_t}{z_{t+1}} = \left(\frac{1-\upsilon}{\upsilon} \chi^z_{t+1} \right)^{\frac{1}{\psi}}, \tag{6}$$

where χ^z_{t+1} is a weighted average of the spreads on deposits and CBDC

$$\chi^z_{t+1} = \frac{\chi^m_{t+1} \chi^n_{t+1}}{\left((1-\gamma)^{\frac{1}{\epsilon}} \left(\chi^n_{t+1} \right)^{\frac{1-\epsilon}{\epsilon}} + \gamma^{\frac{1}{\epsilon}} \left(\chi^m_{t+1} \right)^{\frac{1-\epsilon}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}}}. \tag{7}$$

Here, the *CBDC spread*, χ^m_{t+1}, is defined similarly to the deposit spread, as the interest-rate differential between the risk-free rate and the CBDC rate,

$$\chi^m_{t+1} = 1 - \frac{R^m_{t+1}}{R^f_{t+1}} \tag{8}$$

and denotes the opportunity cost of holding CBDC. Thus, we interpret χ^z_{t+1} as being the household's *average cost of liquidity*. Note that consumption velocity is increasing in this cost. As liquidity becomes more expensive, the household would want to economize on its liquidity holdings, and therefore, velocity increases. In the limiting case where the relative utility weight of liquidity goes to zero, i.e. $\upsilon \to 0$, consumption velocity goes to infinity and the model economy converges to the standard "cashless limit" case. Moreover, the household's demand for CBDC and deposits can be expressed as

$$\frac{m_{t+1}}{z_{t+1}} = (1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^m_{t+1}}{\chi^z_{t+1}} \right)^{-\frac{1}{\epsilon}}, \tag{9}$$

$$\frac{n_{t+1}}{z_{t+1}} = \gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^n_{t+1}}{\chi^z_{t+1}} \right)^{-\frac{1}{\epsilon}}. \tag{10}$$

We see that the relative demand for each liquid asset is increasing in their relative liquidity weights and decreasing in their costs relative to the average cost. In the special case where the liquidity weight of deposits goes to zero or the relative cost of deposits goes to infinity, CBDC becomes the household's only source of liquidity, i.e. $z_{t+1} = m_{t+1}$. The opposite occurs if the weight of deposits goes to one or the relative cost of CBDC goes to infinity.

Lastly, we derive the intertemporal Euler equation of the household, which has the form

$$c_t^{-\sigma} \Omega^c_t = \beta \mathbb{E}_t \left[R^k_{t+1} c^{-\sigma}_{t+1} \Omega^c_{t+1} \right], \tag{11}$$

with the standard interpretation of the representative agent equating the current-period marginal cost of savings, given by the marginal utility of consumption (left-hand side), with the next-period discounted expected return on savings (right-hand side). Notice that relative to a textbook real business cycle (RBC) model, the Euler equation (11) contains the term Ω_t^c

$$\Omega_t^c = (1-\upsilon)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\psi}} \left(\chi_{t+1}^z\right)^{1-\frac{1}{\psi}}\right)^{\frac{\psi-\sigma}{1-\psi}}, \tag{12}$$

which summarizes the impact of liquidity services on the marginal utility of consumption.

2.2 Banks

There is a set of N non-competitive banks that produce differentiated deposit services and invest in capital and reserves. The balance sheet of a typical bank is

$$k_{t+1}^i + r_{t+1}^i = n_{t+1}^i, \tag{13}$$

where k_{t+1}^i and r_{t+1}^i denote the bank's capital and reserve holdings, respectively. We follow Niepelt (2024) and assume that maturity transformation requires bank resources. Banks incur a cost per unit of deposit funding, and a role for reserves is introduced by assuming that larger reserve holdings relative to deposits reduce these costs. Unlike in Niepelt (2024), we do not assume positive externalities of reserve holdings as it does not substantially affect our results. A bank's per-unit cost of deposit, ν_t^i, is thus assumed to be a decreasing function of just its own reserves-to-deposits ratio, $\zeta_{t+1}^i = r_{t+1}^i / n_{t+1}^i$,

$$\nu_t^i\left(\zeta_{t+1}^i\right) = \omega + \phi\left(\zeta_{t+1}^i\right)^{1-\varphi},$$

where $\omega, \phi \geq 0$; $\varphi > 1$. We assume that all banks face the same cost function.

At time t, bank i decides on its reserve holdings and deposit rate, subject to its deposit demand schedule (3) and the balance sheet constraint (13). Returns on the bank's assets, net of interest payments, are realized in the subsequent period. The bank retains no earnings and distributes its entire profit to the household every period. The date-t program of a typical bank is

$$\begin{aligned} \max_{r_{t+1}^i, R_{t+1}^{n,i}} \quad & -n_{t+1}^i \nu_t^i + \mathbb{E}_t\left[\Lambda_{t+1}\left(k_{t+1}^i R_{t+1}^k + r_{t+1}^i R_{t+1}^r - n_{t+1}^i R_{t+1}^{n,i}\right)\right] \\ \text{s.t.} \quad & n_{t+1}^i = n_{t+1}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^n}\right)^{-\frac{1}{\eta}}, \\ & k_{t+1}^i = n_{t+1}^i - r_{t+1}^i, \end{aligned}$$

where R_{t+1}^r is the gross interest rate on reserve balances.

model would either feature way too much deposits, way too little capital, or way too low bank leverage ratios.

We focus on a symmetric industry equilibrium: Since all banks are identical, they will choose identical deposit rates (and thus identical deposit spreads) and levels of reserve holdings, i.e. $\chi^{n,i}_{t+1} = \chi^{n,j}_{t+1}$ and $r^i_{t+1} = r^j_{t+1}$ for all i and j. Identical deposit spread across banks, given banks' demand schedule (3), implies that the household's demand for each bank's deposits is also identical, i.e. $n^i_{t+1} = n^j_{t+1}$ for all i and j. Using equations (1) and (5), we can establish that $n_{t+1} = n^i_{t+1}$ and $\chi^n_{t+1} = \chi^{n,i}_{t+1}$. Moreover, identical levels of reserve holdings mean that the aggregate reserve holdings of the whole banking sector are $r_{t+1} = (1/N)\sum_{i=1}^N r^i_{t+1} = r^i_{t+1}$. Then, the reserves-to-deposits ratios have to be equal across banks too, i.e. $\zeta^i_{t+1} = \zeta_{t+1}$. As all banks are identical, we hereafter drop the individual superscript i.

We now turn to the first-order conditions of a bank's optimization problem. Detailed derivations are provided in the Appendix A.2. Firstly, the first-order condition with respect to reserves yields the bank's desired reserves-to-deposits ratio

$$\zeta_{t+1} = \left(\frac{\chi^r_{t+1}}{\phi(\varphi - 1)}\right)^{-\frac{1}{\varphi}}. \tag{14}$$

The ratio depends on the *reserve spread*, χ^r_{t+1}, which represents the opportunity cost of holding reserves

$$\chi^r_{t+1} = 1 - \frac{R^r_{t+1}}{R^f_{t+1}}. \tag{15}$$

As the reserve spread increases and reserves become more expensive, the bank's desired reserves-to-deposits ratio decreases, and its cost of deposit issuance increases. Equation (14) also shows that the bank's choice of reserves equalizes their marginal (opportunity) cost of holding reserves, χ^r_{t+1}, to the marginal gain stemming from a lower cost of deposit issuance, $-(1-\varphi)\phi\zeta^{-\varphi}_{t+1}$.

Secondly, the first-order condition with respect to deposit rate yields the condition that determines the equilibrium deposit spread

$$\chi^n_{t+1} + \chi^n_{t+1}\left(-\frac{1}{N}\left(\frac{1-s_t}{\psi} + \frac{s_t}{\epsilon}\right) - \left(1 - \frac{1}{N}\right)\frac{1}{\eta}\right)^{-1} = \omega + \varphi\phi\zeta^{1-\varphi}_{t+1}. \tag{16}$$

The right-hand side of equation (16) shows the marginal cost of deposit issuance. The marginal unit of deposit not only implies an extra cost of $\nu_t = \omega + \phi\zeta^{1-\varphi}_{t+1}$, but also increases the cost for inframarginal units, given by $-\frac{\partial \nu_t}{\partial \zeta_{t+1}}\zeta_{t+1}$. The two components add up to $\omega + \varphi\phi\zeta^{1-\varphi}_{t+1}$. The left-hand side of (16) shows the banks' marginal benefit of raising deposit funding. The first term on the left-hand side, χ^n_{t+1}, entails (if positive) a return in excess of the reference risk-free rate. That is, deposits are a cheap source of funding for the bank and the spread denotes a marginal gain from deposit issuance. However, recall that the deposit spread represents an (opportunity) cost for the household. The second term on the left-hand side shows the decrease in the spread the bank must make in order to incentivize the household to provide more deposits

$$\chi^n_{t+1}\left(-\frac{1}{N}\left(\frac{1-s_t}{\psi} + \frac{s_t}{\epsilon}\right) - \left(1 - \frac{1}{N}\right)\frac{1}{\eta}\right)^{-1} < 0. \tag{17}$$

Expression (17) can be thought of as a markup over the marginal cost that the non-competitive bank imposes on the household. The deposit spread markup depends negatively on the elasticity of demand that the bank faces, given by

$$-\frac{1}{N}\left(\frac{1-s_t}{\psi}+\frac{s_t}{\epsilon}\right)-\left(1-\frac{1}{N}\right)\frac{1}{\eta}, \tag{18}$$

where $s_t \in [0,1]$ is a relative weight

$$s_t = (1-\gamma)^{\frac{1}{\epsilon}}\left(\frac{\chi^z_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1-\epsilon}{\epsilon}}. \tag{19}$$

The elasticity of demand (18) shows that the changes in the demand for deposits are the sum of two effects. Firstly, suppose the bank decreases its deposit rate and thus widens its deposit spread. It raises the aggregate deposit spread index, χ^n_{t+1}, by the amount equal to its mass, $1/N$. This makes deposits more costly overall for the household and induces a substitution away from deposits at a rate $\left(-\frac{1-s_t}{\psi}-\frac{s_t}{\epsilon}\right)<0$. This aggregate effect is more pronounced in a more concentrated deposit market since the actions of each bank have a larger impact on the overall cost of deposits. Secondly, given a decrease in the deposit rate, its deposit spread increases by $1-1/N$ relative to the aggregate index. This induces an outflow of deposits from the bank at the rate of the elasticity of substitution between banks, $1/\eta$. This interbank effect is larger when the elasticity of substitution between banks is large or when market concentration is low.

The aggregate elasticity of demand for deposits, as represented by the first term in equation (18), is a weighted average of their elasticity of substitution to consumption, $1/\psi$, and CBDC, $1/\epsilon$. The weight s_t reflects the relative cost of CBDC m_{t+1} within the liquidity bundle z_{t+1}. When the interest rate on CBDC is high, the CBDC spread within the bundle is relatively low, leading to more outflow from deposits towards CBDC. This scenario makes the aggregate elasticity of demand close to $1/\epsilon$. Conversely, if CBDC becomes less attractive, consumption captures most of the substitution out of deposits.

We consider limit cases here. Suppose deposits at different banks are perfectly substitutable, i.e. $\eta \to 0$. Then, the elasticity of demand goes to infinity. The bank becomes competitive and sets the deposit spread equal to its marginal cost of deposit issuance

$$\chi^n_{t+1} = \omega + \varphi\phi\zeta^{1-\varphi}_{t+1}. \tag{20}$$

Alternatively, suppose the deposit market is perfectly dispersed, i.e. $1/N \to 0$. The bank becomes monopsonistically competitive and charges a constant multiplicative markdown $1/(1-\eta)$ over its marginal cost. Then, the deposit spread becomes

$$\chi^n_{t+1} = \frac{\omega + \varphi\phi\zeta^{1-\varphi}_{t+1}}{1-\eta}. \tag{21}$$

2.3 Firms

Competitive firms produce common consumption goods using capital and labor. The representative firm maximizes its profit by solving the following problem:

$$\begin{aligned} \max_{k_t, l_t} \quad & a_t f(k_t, l_t) - k_t \left(R_t^k - 1 + \delta\right) - w_t l_t \\ \text{s.t.} \quad & f(k_t, l_t) = k_t^{\alpha} l_t^{1-\alpha}, \end{aligned}$$

where α is the capital share of output, a_t is productivity, k_t and l_t are the firm's demand for capital and labor, and δ is the capital depreciation rate. The first-order conditions of the firm pin down the capital return and the wage rate, respectively,

$$R_t^k = 1 - \delta + a_t \alpha \left(\frac{k_t}{l_t}\right)^{\alpha - 1}, \tag{22}$$

$$w_t = a_t (1 - \alpha) \left(\frac{k_t}{l_t}\right)^{\alpha}. \tag{23}$$

Since firms operate in a perfectly competitive environment and the production function exhibits constant returns to scale, equilibrium profits are equal to zero.

2.4 Consolidated government

A consolidated government/central bank issues CBDC and reserves and invests in capital. The government incurs a per-unit cost, μ, when issuing (and managing) CBDC and a per-unit cost, ρ, when issuing (and managing) reserves. The budget constraint of the government reads

$$k_{t+1}^g - m_{t+1}(1 - \mu) - \frac{1}{N} \sum_{i=1}^{N} r_{t+1}^i (1 - \rho) = k_t^g R_t^k + \tau_t - m_t R_t^m - \frac{1}{N} \sum_{i=1}^{N} r_t^i R_t^r, \tag{24}$$

where k_{t+1}^g is the government's capital holdings. We assume that the government sets the interest rates on CBDC and reserves, and the level of lump-sum tax. The specific way in which the government sets these interest rates will be discussed in detail in the next sections.

2.5 Market clearing and aggregate resource constraint

Market clearing in the labor market requires that firms' labor demand equals the household's inelastic labor supply, i.e. $l_t = \bar{l}$. On the capital market, the firms' capital demand has to equal the sum of the capital holdings of the household, banks, and the government

$$k_t = k_t^h + \frac{1}{N} \sum_{i=1}^{N} k_t^i + k_t^g \; .$$

Lastly, total dividends distributed to the household must equal the sum of banks' and firms' profits

$$\pi_t = \frac{1}{N}\sum_{i=1}^{N}\left(-n^i_{t+1}\nu^i_t + k^i_t R^k_t + r^i_t R^r_t - n^i_t R^{n,i}_t\right) + a_t k^\alpha_t l^{1-\alpha}_t - k_t\left(R^k_t - 1 + \delta\right) - w_t l_t \ .$$

Aggregate resource constraint is derived by combining the budget constraints of the household and the government, market clearing conditions, and total dividends

$$c_t + k_{t+1} - k_t(1-\delta) = a_t k^\alpha_t \bar{l}^{1-\alpha} - Q_t \ ,$$

where

$$Q_t = m_{t+1}\mu + n_{t+1}\left(\nu_t + \zeta_{t+1}\rho\right) \ .$$

The resource constraint has the standard interpretation that available output in the economy is split between consumption, c_t, and investment, $k_{t+1}-k_t(1-\delta)$. However, there are resource costs associated with the provision of liquidity to the household, summarized by the term Q: μ per unit of CBDC and $\nu_t+\zeta_{t+1}\rho$ per unit of deposit. The resource cost of deposits has two terms because the banking sector incurs a cost of deposit issuance, ν_t, and the government incurs a cost of issuing reserves used by the banking sector, $\zeta_{t+1}\rho$, to "back up" deposit issuance. As the household demands liquidity services in proportion to consumption, we can combine the terms c_t and Q_t, and rewrite the resource constraint as

$$c_t\Omega^{rc}_t + k_{t+1} - k_t(1-\delta) = a_t k^\alpha_t \bar{l}^{1-\alpha}, \tag{25}$$

where $\Omega^{rc}_t \geq 1$ is given by

$$\Omega^{rc}_t = 1 + \left(\frac{\upsilon}{1-\upsilon}\frac{1}{\chi^z_{t+1}}\right)^{\frac{1}{\psi}}\left(\left((1-\gamma)\frac{\chi^z_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1}{\epsilon}}\mu + \left(\gamma\frac{\chi^z_{t+1}}{\chi^n_{t+1}}\right)^{\frac{1}{\epsilon}}\left(\omega + \phi\zeta^{1-\varphi}_{t+1} + \zeta_{t+1}\rho\right)\right). \tag{26}$$

2.6 Policy and equilibrium

The consolidated government sets the interest rates on CBDC and reserves and elastically supplies these assets to households and banks to meet demand. A policy consists of $\{R^m_{t+1}, R^r_{t+1}, \tau_t\}_{t\geq 0}$ and an equilibrium conditional on policy consist of

- a set of positive prices, $\{w_t, R^k_{t+1}, R^f_{t+1}, \chi^m_{t+1}, \chi^n_{t+1}, \chi^z_{t+1}, \chi^r_{t+1}\}_{t\geq 0}$;
- a positive allocation, $\{c_t, k_{t+1}\}_{t\geq 0}$;
- and positive CBDC, deposits and reserves holdings, $\{m_{t+1}, n_{t+1}, z_{t+1}, r_{t+1}\}_{t\geq 0}$,

such that (4), (6), (7), (8), (9), (10), (11), (14), (15), (16), (22), (23) and (25) are satisfied.

3 Monetary Policy Transmission

In this section, we elaborate on the transmission mechanisms of the interest rates on CBDC and analytically characterize the channels through which they affect the allocation. We also briefly discuss the transmission of reserve rates, given that we will contrast their effects with CBDC below. Overall, this analysis builds the foundation for the quantitative exercise in the next sections where we study how monetary policy affects the real economy.

3.1 Real effects of monetary policy

The two key conditions that characterize the equilibrium allocation, the Euler equation (11) and the resource constraint (25), all closely parallel the conditions of a textbook RBC model. The differences relative to an RBC model are the quantities Ω^c_{t+1} and Ω^{rc}_{t+1}. Importantly, the direct impact of liquidity on the household's consumption/savings decision, captured by Ω^c_{t+1}, depends solely on the average cost of liquidity, χ^z_{t+1}. So we will mostly focus on the effects of policy on χ^z_{t+1} when studying transmission below. For this purpose, it is instructive to first lay down how the average cost of liquidity works through our model economy.

The Euler equation (11) shows that the household's consumption/savings choice depends on liquidity through the marginal utility of consumption, which changes with the average cost of liquidity according to

$$\frac{\partial u_{c,t}}{\partial \chi^z_{t+1}} = c_t^{-\sigma} \frac{\partial \Omega^c_t}{\partial \chi^z_{t+1}}, \text{ where } \frac{\partial \Omega^c_t}{\partial \chi^z_{t+1}} \propto \frac{\sigma - \psi}{\psi}.$$

We see that the sign of the impact on the marginal utility of consumption depends on the relative magnitudes of ψ and σ. If the household's intratemporal elasticity of substitution between consumption and liquidity is smaller than the intertemporal elasticity of substitution, i.e. $\psi > \sigma$, an increase in the cost of liquidity leads to a decrease in the marginal utility of consumption. This is driven by the fact that an increase in the cost of liquidity, according to (6), reduces the household's demand for it. A decrease in the level of liquidity then decreases the marginal utility of consumption, and hence there is consumption-liquidity complementarity. On the other hand, when $\psi < \sigma$, an increase in the cost of liquidity leads to an increase in the marginal utility of consumption. In the case where $\psi = \sigma$, the household's utility is separable in consumption and liquidity and the cost of liquidity has no direct impact on consumption/savings choices.

Moreover, the spreads on CBDC and deposits also show up in the aggregate resource constraint (25) through the term Ω^{rc}_t. This reflects the resource costs associated with liquidity provision, incurred by the government and the banking sector.

In the special case where the household does not value liquidity services, i.e. $\upsilon \to 0$, both Ω^c_{t+1} and Ω^{rc}_{t+1} converge to one. At this "cashless limit", the cost of liquidity has no impact on the household's consumption/savings since no liquid assets are held. Therefore, there are also no resource costs associated with liquidity provision. Then, the model collapses into a standard RBC model.

To conclude, we have seen that the household's consumption/savings decision only depends on the average cost of liquidity, which in turn is a function of the spreads on CBDC and deposits. As these are also the sole endogenous determinants of the liquidity cost term Ω^{rc}_{t+1}, the government can affect the allocation only insofar as it affects these spreads. While the government controls the CBDC spread directly through the CBDC rate, the deposit spread is determined by the banking sector. But as we will see below, the government can influence its behavior through the interest rates on *both* reserves and CBDC.

3.2 Interest on CBDC

We now explain the channels through which the household's average cost of liquidity can be influenced by the CBDC rate. Suppose the government lowers the CBDC rate so that the CBDC spread widens.[2] Differentiating the average cost of liquidity, given by (7), with respect to the CBDC spread yields

$$\frac{\partial \chi^z_{t+1}}{\partial \chi^m_{t+1}} = \underbrace{(1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^m_{t+1}}{\chi^z_{t+1}}\right)^{-\frac{1}{\epsilon}}}_{\text{direct effect}} + \underbrace{\gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^n_{t+1}}{\chi^z_{t+1}}\right)^{-\frac{1}{\epsilon}} \frac{\partial \chi^n_{t+1}}{\partial \chi^m_{t+1}}}_{\text{indirect effect}}. \tag{27}$$

This expression shows that the CBDC spread works through two channels: Firstly, it directly increases the cost of liquidity by the first term. The strength of this direct effect is increasing in the relative liquidity weight of CBDC, $1-\gamma$, and decreasing in how costly CBDC is relative to the average cost of liquidity, $\chi^m_{t+1}/\chi^z_{t+1}$. Comparing the direct effect with the household's demand for CBDC (9), we see that it is just the share of CBDC in the total stock of liquidity, m_{t+1}/z_{t+1}. Intuitively, the more important CBDC is as a source of liquidity for the household, the larger the impact of its cost on liquidity's average cost.

Secondly, the CBDC spread affects the cost of liquidity through the deposit side, given by the second term. The strength of this indirect effect is increasing in the relative liquidity weight of deposits, γ, and decreasing in how costly deposits are relative to the average, $\chi^n_{t+1}/\chi^z_{t+1}$. Comparing the indirect effect with the household's demand for deposits (10), we see that it is equal to the product of the share of deposits in the total stock of liquid, n_{t+1}/z_{t+1}, and the change in the deposit spread caused by a change in the CBDC spread, $\partial\chi^n_{t+1}/\partial\chi^m_{t+1}$. Analogous to the direct effect, the more important deposits are as a source of liquidity the larger is this indirect effect. However, the sign and the magnitude of the second effect also depend on how the banking sector responds to an increasing CBDC spread, captured by $\partial\chi^n_{t+1}/\partial\chi^m_{t+1}$.

In this regard, the optimality condition (16) shows that the CBDC spread can influence the deposit spread through the bank's marginal benefit of deposit issuance (left-hand side). Specifically, CBDC spread affects the elasticity of demand for deposits that the bank faces, given by (18). As we discussed previously, the demand elasticity depends on a weighted average of the household's elasticities of substitution to consumption, $1/\psi$, and CBDC, $1/\epsilon$. The CBDC spread determines this average through the relative weight s_t, given by

[2] For simplicity, we assume here that the reserve spread is constant.

(19). Taking the partial derivative of the demand elasticity (18) with respect to the CBDC spread, we get

$$\frac{1}{N}\left(\frac{\partial s_t}{\partial \chi^m_{t+1}}\right)\left(\frac{1}{\psi}-\frac{1}{\epsilon}\right) \text{ with } \frac{\partial s_t}{\partial \chi^m_{t+1}} = -\frac{1-\epsilon}{\epsilon}\frac{s_t(1-s_t)}{\chi^m_{t+1}} < 0 \; . \tag{28}$$

The partial derivative shows that the marginal impact of CBDC spread is non-zero only if $\psi \neq \epsilon$. Intuitively, banks collectively face competition from CBDC and consumption for the household's resources. Therefore, any outflow from deposits depends on the household's elasticities of substitution to CBDC and consumption. The CBDC spread only influences the relative importance of these two sources of deposit outflow, indicated by s_t. If the household finds it as easy to substitute from deposits to consumption as it does to CBDC, i.e. $\psi = \epsilon$, then the two sources of competition for the banks are equally important and the CBDC spread does not influence the banks' deposit spread. In such a case, the equilibrium spread is set equal to the marginal cost of deposit issuance plus a constant markup, similar to the case where banks are monopsonistically competitive.

In general, it seems reasonable to expect that deposits will be more substitutable with CBDC than with consumption, i.e. $\psi > \epsilon$. Then, an increase in the CBDC spread makes the demand elasticity for deposits (18) less negative in value and, in turn, decreases the marginal benefit of deposit issuance. The intuition is that when its spread widens, CBDC becomes a comparatively expensive source of liquidity and a larger fraction of potential substitution out of deposits will go to consumption (indicated by a decrease in s_t and more weight being put on $1/\psi$). The elasticity of demand moves closer to $1/\psi$, which is smaller than $1/\epsilon$, and thus decreases in absolute value. Therefore, an increase in the CBDC spread makes the household's demand for deposits less elastic. For banks with market power, a less elastic demand means that in order to attract additional deposits from the household, the spread needs to be lowered by more than before. That is, the marginal benefit of deposit issuance decreases. Given a fixed marginal cost, this implies that the equilibrium deposit spread increases. In other words, an increase in the CBDC spread is akin to giving banks more market power. Banks take advantage of this and charge a higher spread on deposits in equilibrium.

As we alluded to previously, market conditions in the deposit market also play a central role. If deposits at different banks are perfect substitutes or the deposit market is perfectly dispersed, the equilibrium deposit spread is determined without the influence of the CBDC spread. If the household does not differentiate between banks, each individual bank's choice of how much deposits to issue does not matter for the equilibrium spread, which will equal the marginal cost of deposit issuance (20): the market is competitive. Similarly, if the deposit market is perfectly dispersed and the only source of market power is differentiation, the impact of each individual bank's spread on the aggregate deposit spread goes to zero. The deposit market becomes monopsonistically competitive with a constant markup over marginal cost solely depending on the substitutability between banks, given by (21). In both cases, the government cannot use the CBDC spread to influence the banking sector.

To sum up, when the government decreases the CBDC rate and widens the CBDC spread, it directly increases the household's average cost of liquidity and affects allocation. Moreover, a higher CBDC spread increases the spread on bank deposits, provided that banks have sufficient market power, which raises the household's cost of liquidity further. The transmission of the CBDC rate through the banking sector is similar to the deposit channel of monetary policy proposed by Drechsler et al. (2017). The authors describe a situation in which the household holds cash issued by the government and deposits issued by banks with market power. Policy-makers can induce an increase in the deposit spread by increasing the household's opportunity cost of holding cash, captured by the nominal interest rate on risk-free bonds. In our model, instead, the alternative to bank deposits is CBDC. The government can similarly affect banks' deposit spread by changing the household's opportunity cost of holding this alternative, i.e. its spread.

3.3 Interest on reserves

While interest rates on central bank reserves have traditionally not been a particularly salient policy tool (for example, the Fed only started to remunerate reserves in 2008), we briefly analyse them as a stand-in for monetary policy instruments directed at the banking sector: Since the reserve rate shock effectively increases the banks' cost of deposit provision, we expect that other (unmodeled) monetary policy shocks that affect consumers only through the banking system to be affected by deposit market power in a qualitatively similar way.

In particular, the interest on reserves affects the household's average cost of liquidity only through its impact on the deposit spread. Suppose the government decreases the reserve rate so that the reserve spread increases.[3] Taking the first derivative of the average cost of liquidity with respect to the reserve spread, we get

$$\frac{\partial \chi^z_{t+1}}{\partial \chi^r_{t+1}} = \gamma^{\frac{1}{\epsilon}} \left(\frac{\chi^n_{t+1}}{\chi^z_{t+1}} \right)^{-\frac{1}{\epsilon}} \frac{\partial \chi^n_{t+1}}{\partial \chi^r_{t+1}}.$$

Notice that the marginal impact of the reserve spread is very similar to the indirect effect of the CBDC spread in (27). This is not surprising since both effects work through the banking sector. The impact of the reserve spread is the product of the share of deposits in the total stock of liquid, n_{t+1}/z_{t+1}, and the change in the deposit spread caused by the change in the reserve spread, $\partial \chi^n_{t+1}/\partial \chi^r_{t+1}$. Again, the more important deposits are as a source of liquidity, the larger is this effect. But, its sign and the magnitude also depend on how the banking sector responds to an increasing reserve spread, $\partial \chi^n_{t+1}/\partial \chi^r_{t+1}$. This key term is in turn determined by how banks react to the higher cost of deposit provision induced by χ^r and depend to what extent potential outflows to CBDC are considered in banks' deposit rate setting.

4 Calibration

To gauge the importance of deposit market concentration for the efficacy of a CBDC rate as a policy instrument, we calibrate our model to conduct various numerical exercises below. A detailed description of our procedure is provided in Appendix A.5.

[3] For simplicity, we assume here that the CBDC spread is constant.

A period is interpreted as a quarter. Following Niepelt (2024), we adopt the case with a monopsonist bank as a benchmark but will compare it with the case $N = 3$ below: Given the symmetric banks, this value implies a Hirschman-Herfindahl-Index (HHI) of 1/3, close to the average county-level HHI of 0.35 estimated by Drechsler et al. (2017) for the U.S. over the period from 1994 to 2013.

We start with exogenously setting several parameters to values from the literature: In line with standard convention, we choose the household's risk aversion parameter σ to be 2, the capital share α to be 1/3 and the depreciation rate to be 2.5%. We normalize $\bar{l} = 1/3$. Additionally, we follow Bacchetta and Perazzi (2021) and assume for our benchmark exercises that CBDC is designed so that its elasticity of substitution with respect to the deposit aggregate is $\epsilon = 1/6$. Given the uncertainty about whether this will be the practically relevant magnitude for any actual CBDC, we also consider different values of ϵ in the robustness exercises in Section 7.

We also exogenously set the initial steady state's policy: Firstly, we assume that the central bank chooses to pay a nominal interest rate of 0 on the CBDC, i.e. it has the same return as cash. This is in line with many central seeming reluctant to remunerate a potential CBDC, and, assuming 2% trend inflation, implies a real annual gross return of 0.98 and $R^m = 0.98^{1/4}$. We furthermore assume the annual gross return on reserves to amount to $R^r = 0.99^{1/4}$, implying a nominal reserve rate of 1% annually: The Fed started to pay nominal interest rates on reserves only in 2008 and they averaged roughly 1% in the time since.

Now, to be able to clearly identify the effect of deposit market concentration in our model, we restrict the model versions with $N = 1$ and $N = 3$ to be identical in all other dimensions: In particular, the CBDC is designed so that aggregate CBDC holdings amount to just 12% of deposit holdings while we induce the consumption velocity c/z to be equal to 1.2. The former target reflects that in many jurisdictions considering the implementation of a CBDC, policymakers seem reluctant to induce substantial disintermediation of the banking sector.[4] It also implies that the total amount of CBDC held is in line with just physical currency being replaced, which typically amounted to approx. 12% of aggregate deposit holdings in the post-war US.[5] In contrast, the c/z target ensures that even after the introduction of a CBDC, the overall liquidity velocity is similar to current levels.[6] The targets are achieved by setting $\gamma = 0.5938$ and $\upsilon = 0.0252$. Finally, as in Niepelt (2024), we aim to induce a deposit markdown of 1.5, which we also induce by choosing ψ accordingly in the $N = 1$ version. To achieve the same in the model version with $N = 3$, we additionally use the parameter η governing the substitutability between different banks' deposits. This results in $\psi = 0.3774$ and $\eta = 0.33$, respectively.[7]

[4]For example, Federal Reserve Governor Michelle Bowman voiced related concerns, stating that "a CBDC, if not properly designed, could disrupt the banking system and lead to disintermediation, potentially harming consumers and businesses, and could present broader financial stability risks" (Bowman, 2023).

[5]This statement is based on the series MBCURRCIR and DPSACBW027SBOG from FRED.

[6]According to FRED (Series: M2V), M2 velocity in the US is typically between 1.4 and 1.8, while aggregate consumption is usually around 60-70% of output. Thus, in the current situation without CBDC, a c/z around 1.2 seems reasonable.

[7]Note that with $N = 1$, the parameter η plays no role. Niepelt (2024) calibrates a very similar value for ψ, with the difference being due to a slightly different functional form assumption.

Regarding the costs of providing different forms of liquidity, Niepelt (2024) discusses various evidence suggesting that the annual cost of deposit- and reserve provision may amount to up to 1.2% and 0.05%, respectively. We thus choose $\omega = 0.003$ and $\rho = 1.3 \times 10^{-4}$ for our quarterly calibration. We furthermore restrict $\mu = \omega + \rho$, implying that the costs of providing CBDC are equivalent to the government operating a narrow bank. Again following Niepelt (2024), we set $\varphi = 1.5$: Note that this parameter will effectively only matter for exercises with changing reserve rates, as we always choose ϕ to induce a reserve-to-deposit ratio of 0.1945, the midpoint of the range considered in said paper. This is achieved by $\phi = 0.0021$.

Table 1: Baseline calibration

Parameter	Value	Source/Target
Household		
β	$1.04^{-1/4}$	Annual $R^f = 4\%$
σ	2	Standard
$\bar{l}$	1/3	Normalization
υ	0.0252	$c/z = 1.2$
ψ	0.3774	See text
ϵ	1/6	Bacchetta and Perazzi (2021)
η	0.33	See text
γ	0.5938	$m/n = 0.12$
Banks		
ω	0.003	Niepelt (2024)
φ	1.5	Niepelt (2024)
ϕ	0.0021	$\zeta = 0.1945$ (Niepelt, 2024)
Firms		
α	1/3	Standard
δ	0.025	Standard
Government		
ρ	1.3×10^{-4}	Niepelt (2024)
μ	0.0031	$\mu = \omega + \rho$
R^r	$0.99^{1/4}$	1% nom. return
R^m	$0.98^{1/4}$	0% nom. return

5 Short-run analysis

Armed with the calibrated model, we can now assess the implications of deposit market power for the CBDC rate as a policy tool, and, in particular, its role in shaping the transmission of policy through the direct and indirect effects outlined above. We start by analyzing to what extent it matters if a central bank aims to use the CBDC rate as a tool to influence business cycle fluctuations, for which we compute linearized Impulse Response Functions (IRFs) of the economy to shocks to the CBDC and reserve rates under the different assumptions on deposit market power (i.e. the cases with $N = 1$ and $N = 3$). Note that according to the Blanchard-Kahn criterion, all model versions feature locally unique equilibria.

5.1 Policy Shocks

When analyzing the impact of a CBDC rate shock numerically below, we assume it to follow a log AR(1) process

$$\log(R^m_{t+1}) = (1 - \rho^m) \log(R^m) + \rho^m \log(R^m_t) + e^m_t,$$

where ρ^m is the persistence parameter, R^m is the steady state CBDC rate, and e^m_t is the exogenous shock. The exogenous shock is non-zero in the first period of the simulation and returns to zero afterward. In order to properly isolate the effect, when analyzing the CBDC rate, we assume that the reserve rate is set so that the reserve spread is constant at its steady state level, i.e. it fulfills

$$R^r_{t+1} = R^f_{t+1}(\beta R^r),$$

where R^r is the steady state reserve rate. When we discuss the case of the reserve rates, equivalent assumptions are made, effectively interchanging the processes for R^r and R^m.

5.2 Impulse responses

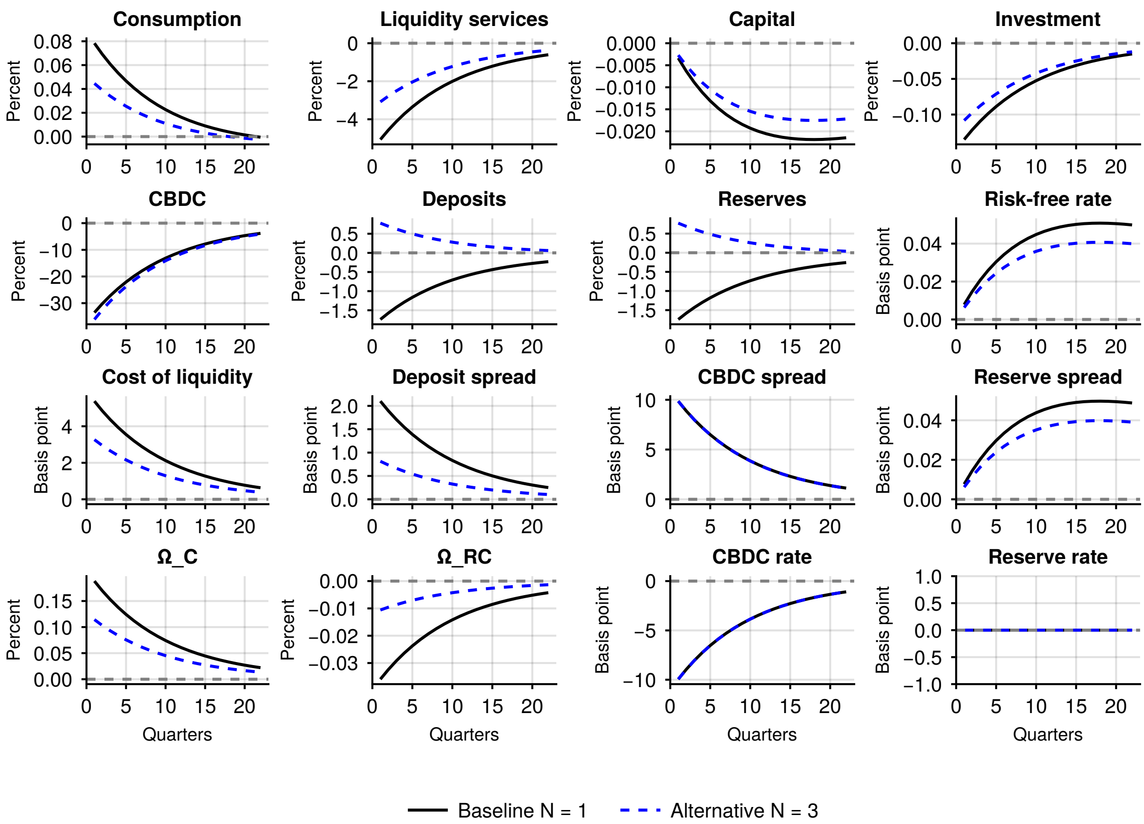
5.2.1 Response to a CBDC rate shock

Figure 1 shows the IRFs, as deviations from the non-stochastic steady state, to a negative 10 basis points shock to the quarterly CBDC rate. Naturally, the decrease in the CBDC rate immediately widens the CBDC spread by essentially the same magnitude as the aggregate capital stock and the risk-free rate changes little. The increasing CBDC spread raises the household's average cost of liquidity in both the baseline $N = 1$ and the alternative $N = 3$ cases so that households choose to enjoy less liquidity services. This decreases the household's demand for liquidity services but increases the household's current marginal utility of consumption, reflected in a higher Ω^c_{t+1}. This is due to our calibration featuring $\psi < \sigma$. In other words, the household's opportunity cost of saving, in utility terms, goes up. The household is incentivized to save less and increase current consumption.[8] Overall, the effect of the cut is not overly strong in either case, reflecting its size and the assumed scenario of limited CBDC adoption.

Nevertheless, there is a substantial difference in the relative magnitudes of the responses, with e.g. the overall impact of the shock on aggregate consumption being almost twice as large in the baseline case with $N = 1$. As outlined above, in our model, the real effects of a CBDC rate are transmitted via its effect on the liquidity cost χ^z and the impact of the

[8]Note that in the absence of a labor supply margin, the response of output is determined by the response of the capital shock, implying that the rate shocks do not induce the positive comovement of consumption

Figure 1: Impulse responses to 10 basis points decrease in CBDC rate



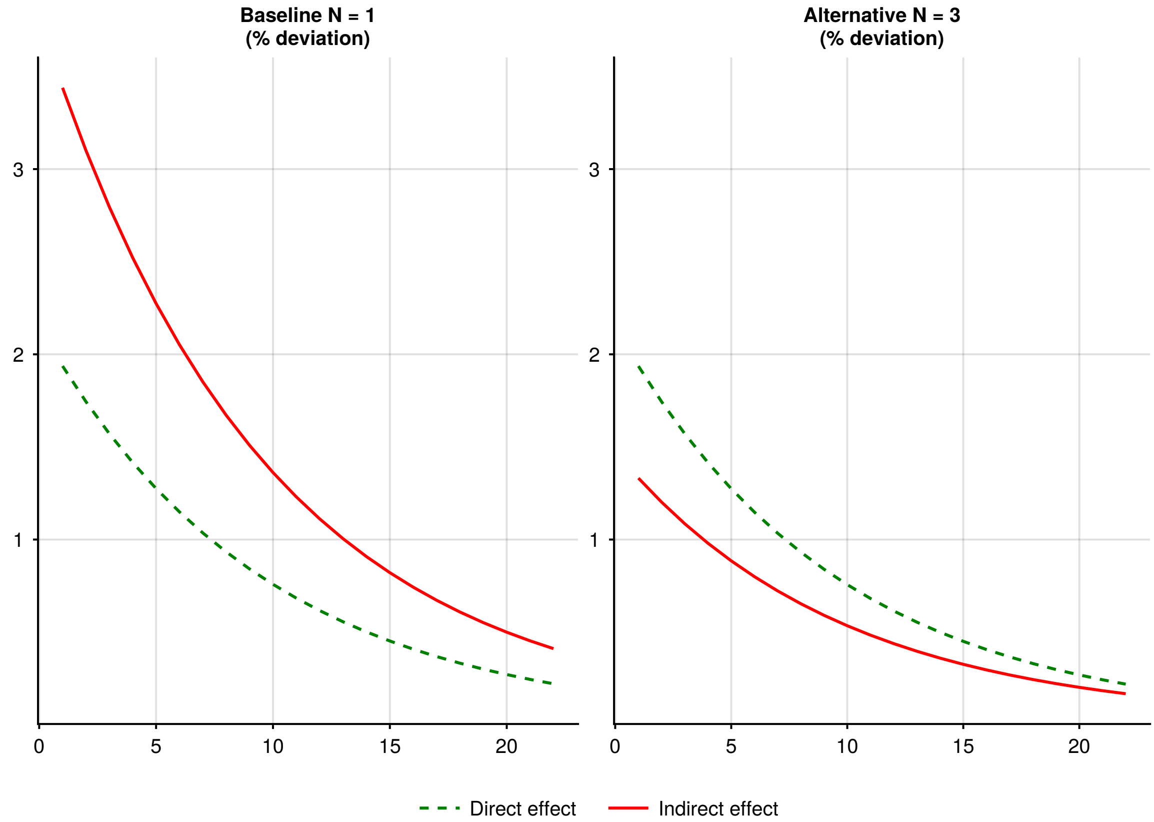
CBDC spread on this term can be decomposed into a direct effect and an indirect effect, shown in (27). Now, Figure 2 displays this decomposition of the responses of the liquidity cost: The green dashed lines show the direct effects and the red solid lines show the indirect effects. The sum of the lines equals the original impulse responses of the cost of liquidity in figure 1.

We immediately notice that in the case with $N = 1$, the indirect effect dominates the direct one and is responsible for the bulk of the χ^z response. In contrast, with $N = 3$, the indirect effect is much smaller and dominated by the direct one, resulting in an overall weaker response from the cost of liquidity and consumption.

Recall that the direct effect is equivalent to the ratio of CBDC to liquidity services, m_{t+1}/z_{t+1}. Since CBDC constitutes only a small fraction of the household's portfolio, the direct effect is small. However, the indirect effect is the product of the ratio of deposits to liquidity services, n_{t+1}/z_{t+1}, and the marginal change in the deposit spread induced by the CBDC spread, $\partial\chi^n_{t+1}/\partial\chi^m_{t+1}$. If deposit market concentration is large and the key determinant of bank market power, the impact of each bank's action on the aggregate is larger. In effect, bank(s) compete less with each other and more with CBDC: Changes in the CBDC spread enter into the banks' competitive considerations to a much larger extent and the deposit

and output typical for e.g. New Keynesian models.

Figure 2: Decomposition of response of cost of liquidity



spread is much more sensitive to the CBDC spread. However, in the alternative case where deposit market concentration is low, the "threat" from CBDC matters little for the banks. The equilibrium deposit spread is to a larger extent determined by the differentiation margin, captured by $1/\eta$.

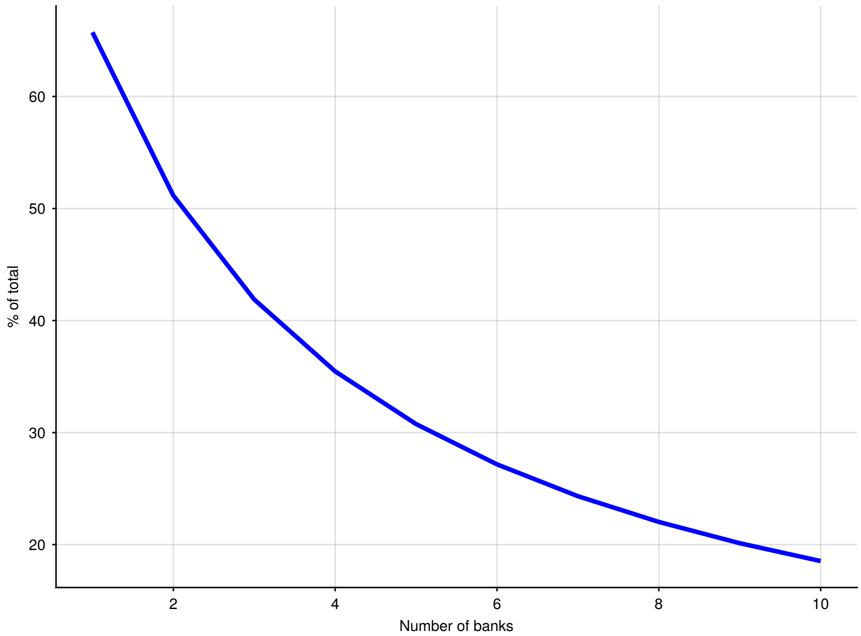
Interestingly, the different assumptions on deposit market power even lead to qualitatively different predictions on how the banking sector is affected by the changing CBDC rates: In the monopsonist $N = 1$ setting, its decrease induces the bank to raise the deposit spread by so much that their resulting lower return induces the households to not only hold less CBDC but also *less deposits*. This is reminiscent of Chiu et al. (2023), whose theoretical model predicts decreased financial intermediation for lower CBDC rates in the presence of bank market power.[9]
With $N = 3$, though, the relatively smaller reaction of the deposit spread instead results in the representative agent choosing to hold *more deposits* after the shock increased the opportunity cost of CBDC. This suggests that the *sources* of bank market power is key for the equilibrium impact of CBDC rates: Without knowledge of whether they are driven by concentration or differentiation, measures of bank competition such as deposit markdowns are not informative about the effect of a given CBDC rate on aggregate deposit holdings and

[9]In our model, lower deposit holdings also imply that a lower share of the aggregate capital stock is held by banks, which can be interpreted as dis-intermediation.

bank intermediation.[10]

To further illustrate how deposit market concentration affects the model economy's response to the CBDC rate, Figure 3 displays the indirect effect's relative contribution to the change of χ^z upon impact of the same CBDC rate shock for different N's: As in the cases analyzed more extensively above, all these model versions are calibrated to be consistent with the same steady state moments, including the deposit markdown. As we already know from figure 2, the indirect effect dominates for the case of a banking monopoly ($N = 1$) but declines relatively quickly when allowing for more deposit providers. Nevertheless, even with a larger number of banks, such as $N = 10$, the indirect effect's contribution remains firmly positive and of significant relative size.

Figure 3: Relative sizes of the indirect effect by bank concentration



Overall, the above exercises thus support our assertion that the transmission of CBDC rate changes should depend importantly on *indirect effects* shaped by the degree and type of deposit market power.

5.2.2 Response to a reserve rate shock

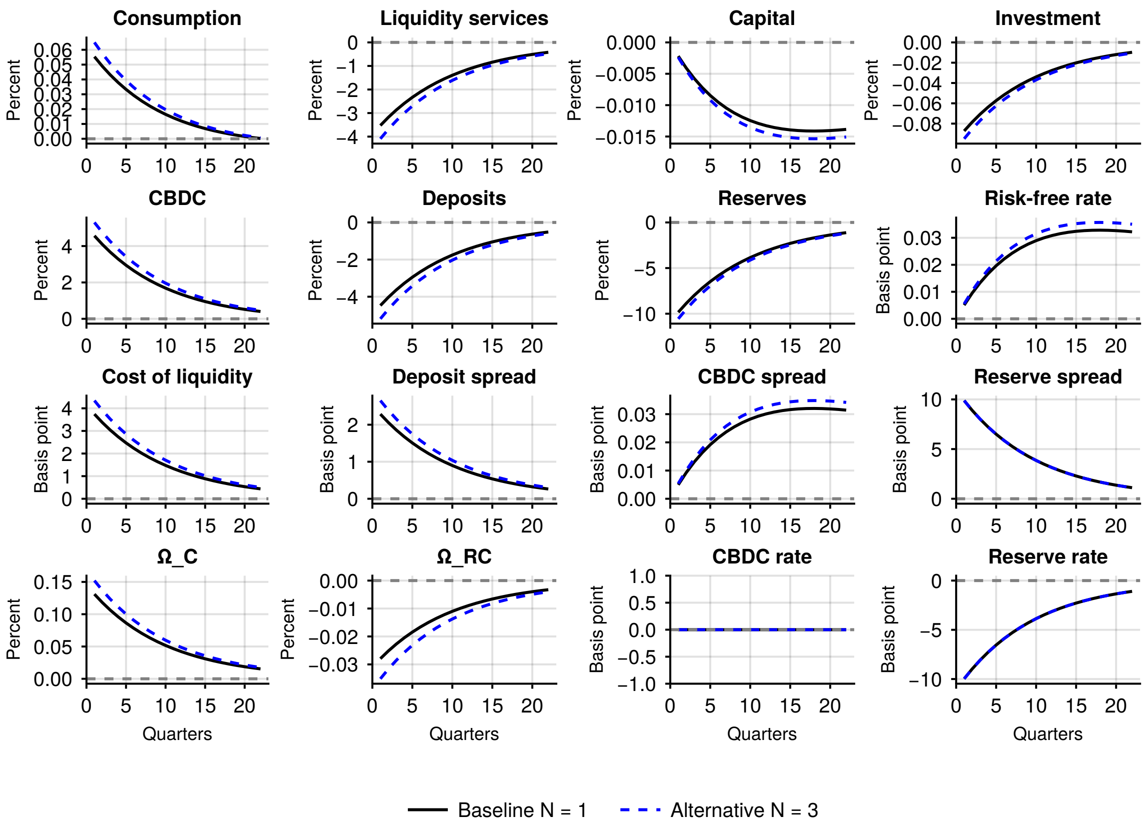
To now also check how deposit market concentration affects the transmission of monetary policy tools directed at the banking sector, we briefly analyze the aggregate effects of a reserve rate shock: Figure 4 shows the IRFs of the economy to a 10 basis points decrease in the reserve rate. This effectively increases banks' costs of deposit provision, causing them

[10]Recall that steady state deposit spreads and -markdowns in the $N = 1$ and $N = 3$ versions are equal by construction.

to also increase their spreads which in turn raises the cost of liquidity in both specifications. The higher cost of liquidity then affects the allocation through the same mechanisms as described in the CBDC case above.

As we have shown earlier, the impact of the reserve spread on the cost of liquidity is the product of the ratio of deposits to liquidity services, n_{t+1}/z_{t+1}, and the marginal change in the deposit spread induced by shock, $\partial\chi^n_{t+1}/\partial\chi^r_{t+1}$. We see in Figure 4 that the increasing reserve spread pushes up the deposit spread, and in contrast to the CBDC shock, the aggregate effect is actually slightly more pronounced with *lower* deposit market concentration. In that case, the banks' marginal benefit of deposit issuance (left-hand side of (16)) is less sensitive to changes in the deposit spread. When a shock increases the marginal cost of deposit provision (right-hand side of (16)), the banks then increase their deposit spread by more than in a case with higher market concentration. Intuitively, when deposit provision is less concentrated, the "price" banks charge on deposits is to a greater extent dictated by their marginal costs.[11] The deposit spread is then more sensitive to changes in the reserve spread or alternative shocks with similar effect. Thus, the results above suggest that deposit market concentration has quite different implications for the transmission of traditional monetary policy tools compared to CBDC.

Figure 4: Impulse responses to 10 basis points decrease in reserve rate



[11]In the limit case with monopsonistic competition, besides the constant markup, the deposit spread would end up being entirely determined by banks' cost.

6 Long-run analysis

After having investigating transitory CBDC policies and their business cycle effects above, it is equally (or even more) important to ask to what extent deposit market concentration and its implications matter for the efficacy of CBDC policy in the long run: Here, we are particularly interested in its effect on steady state welfare, which the CBDC rate affects by inducing more or less efficient aggregate liquidity mixes.

6.1 Optimal policy

To start, we briefly discuss the Ramsey optimal policy in the context of our model. As many of the results are similar to those in Niepelt (2024), we just summarize the key takeaways here and relegate a more detailed discussion and derivations to Appendix B.

In our framework, for a given CBDC design, the long-run (steady state) socially optimal policy is mainly concerned with inducing an efficient liquidity mix, trading off the utility benefits of bank deposits and CBDC with the resource costs of providing them. Assuming that the Ramsey planner has access to a sufficiently rich set of tools, it sets policy rates so that the resulting spreads equal the unit costs of providing CBDC and reserves, i.e. $\chi^m = \mu$ and $\chi^r = \rho$, respectively. At the same time, deposit market power is addressed using a *deposit subsidy*. Thus, the optimal CBDC rate is independent of deposit market concentration if the government has another tool to influence the latter. The deposit subsidy, however, will generically depend on the number of banks N and the parameter η governing how easily households can substitute between banks. This implies the planner still needs to consider deposit market power and its different sources.

What if the planner does not have access to the deposit subsidy, a theoretical policy tool without obvious real world equivalent? If deposits and CBDC are imperfect substitutes, the planner will in general not be able to achieve the first best allocation, as Niepelt (2024) demonstrates for the special case with $N = 1$. While our model does not provide for a closed-form solution to the restricted planner problem, we searched numerically for steady state welfare-maximizing policies: In the baseline model with $N = 1$, our optimization routine suggests $\chi^{m*,1} \approx 0.0032$ and $\chi^{r*,1} \approx 0.7 \times 10^{-4}$ as optimal policy. In contrast, if $N = 3$ and bank market power is also due to *differentiation*, we obtain $\chi^{m*,3} \approx 0.0031$ and $\chi^{r*,3} \approx 0.3 \times 10^{-4}$.[12]

The result that $\chi^{r*} < \rho$ in either case seems intuitive: the planner can partly compensate the missing deposit subsidy by subsidizing the banks indirectly using the return on reserves, which she chooses to remunerate more generously in the restricted solution. Furthermore, we notice that this indirect subsidy is smaller in the high concentration scenario, i.e. $\chi^{r*,1} > \chi^{r*,3}$. At the same time, the planner pays a higher return on money in that scenario, $\chi^{m*,1} < \chi^{m*,3}$. Again, this makes sense: if concentration is low and differentiation is an important determinant of the deposit markdown, a higher CBDC return has a smaller influence on the deposit rates set by the banks and the consolidated government has to rely more on

[12] We used a quasi-Newton algorithm as implemented by the Matlab-function `fminunc`.

the indirect subsidy through reserves. Hence, the reserve spread deviates more from the unconstrained optimum and the CBDC spread less. In the high concentration $N = 1$ case, we just have the reverse.

In conclusion, the above analysis indicates that the source of bank market power is also a pivotal determinant of how a constrained planner would use CBDC- and reserve policy rates if she cannot subsidize deposit provision directly. Naturally, similar as for direct deposit subsidies, an indirect bank subsidy through generous reserve rates may not be feasible due to political economy reasons (or only to some extent): Either increases bank profits, an outcome one could imagine to be unpopular with the public.[13] Thus, it remains interesting to check to what extent interest payments on CBDC can improve long-run welfare and through which ways.

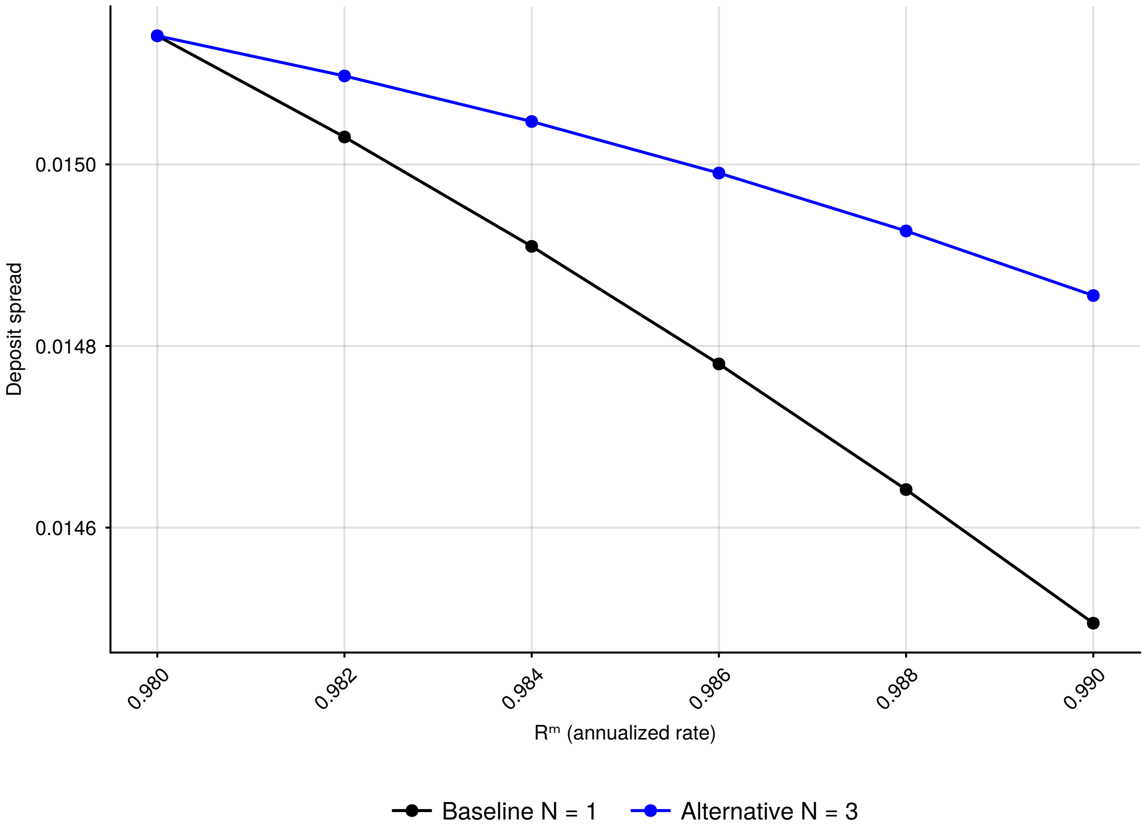
6.2 Long-run effects of CBDC rates

We now turn to investigating how paying (higher) interest on CBDC affects steady state efficiency and welfare. For the purpose of making the related analysis more practically relevant, we divorce it from the optimal policy analysis in the subsection above: There may be various reasons why policymakers may not be able or willing to subsidize banks directly or indirectly, e.g. equity considerations. Instead, we just consider the long-run effects of a higher CBDC return compared to our baseline steady state(s). Recall that we calibrated the latter to be in line with what currently appears to be a plausible scenario for CBDC introduction, in that the digital currency does not pay a nominal return and is designed so that households choose to hold a CBDC amount comparable to physical currency in circulation. Thus, we effectively answer how steady state welfare would be affected if the government were to actually pay a higher real return on the digital currency in that scenario and how they come about.

As a starting point for analyzing how the CBDC rate can affect the long-run aggregate allocation and efficiency, Figure 5 briefly affirms the (by now perhaps obvious) point that indirect effects also matter in the long run and more so with higher deposit market concentration: It simply plots the long-run equilibrium deposit rate for different (gross) CBDC rates, starting from the initial steady states calibrated for Section 4. In these, deposit spreads are equal by construction for $N = 3$ and $N = 1$. The downward curves are consistent with what we have discussed so far: a higher R^m makes CBDC more attractive, exerting downward pressure on the deposit spread which is substantially more pronounced in the high concentration ($N = 1$) case. Again, this is because the large bank competes only with CBDC without interbank pressure, causing it to adjust its deposit spread more in case the central bank increases the CBDC rate.

[13] As an example, De Grauwe and Ji (2023) recently argued that reserve remuneration may result in overly large bank profits.

Figure 5: Deposit spread against different CBDC rate changes



Now, to understand how this matters for efficiency, we calculate corresponding consumption equivalent (CE) welfare gains achieved by the household, which measure how much percent of consumption the representative household would be willing to sacrifice to live in the steady states with higher R^m compared to the original one. To further assess what fraction of the gains is due to indirect effects through the banking sector, we also calculate the CE gain under the counterfactual assumption that banks' deposit spreads are kept fixed at the level calibrated in the baseline steady state with $R^m = 0.98$: The difference between the true and counterfactual CE gains then isolates the contribution of the indirect effects to welfare.[14]

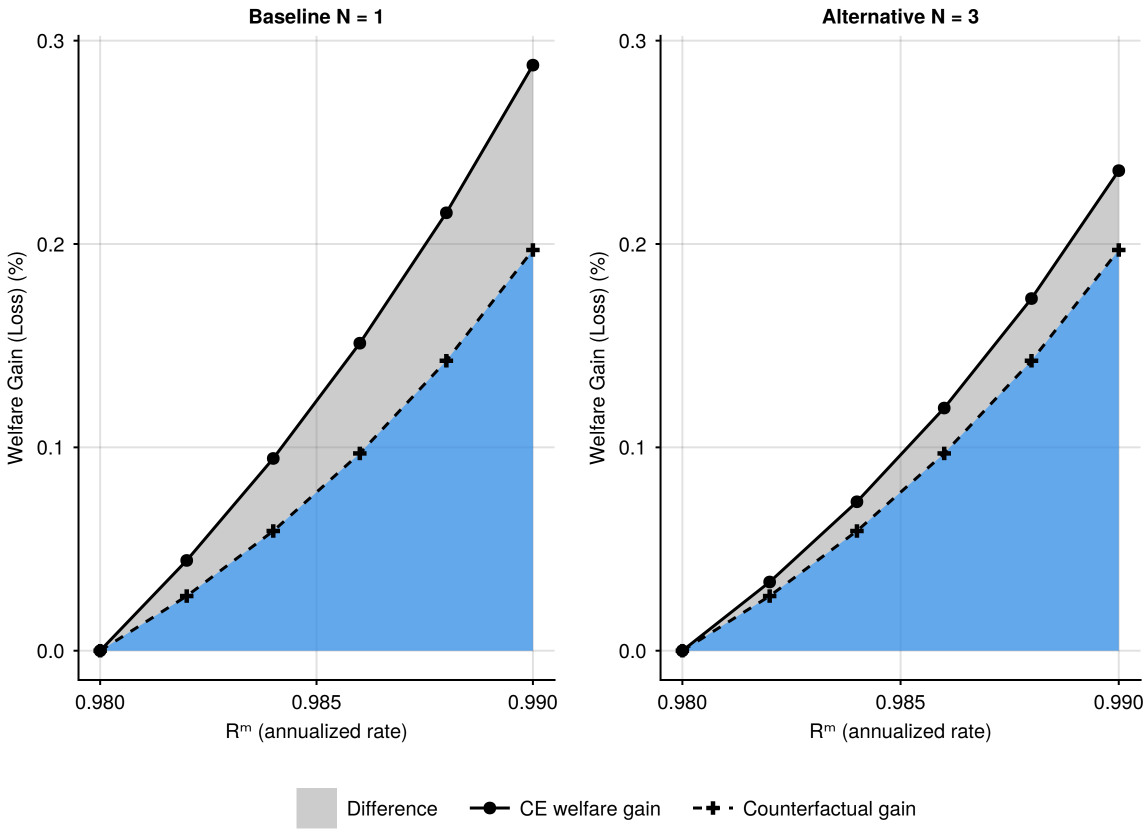
Figure 6 illustrates these CE welfare effects for the by now familiar cases of a monopsonistic bank ($N = 1$) and the oligopsonistic environment with three banks ($N = 3$). Clearly, the model suggests that paying a higher real return on CBDC has welfare benefits, which reflects the results by Niepelt (2024) who finds CBDC to be a very efficient means of liquidity provision: As the higher rate on CBDC induces the representative agent to hold relatively more of it, the aggregate costs of liquidity provision are reduced. Overall, the contribution of the indirect effect to aggregate welfare is substantial and can account for almost a third of the overall CE gains with $N = 1$. With $N = 3$, the share is smaller but still noticeable. Of

[14]Note that this decomposition differs from the one used in the previous Section 5, which applied to the cost of liquidity χ^z.

course, this is again because with differentiation being the more important source of deposit market power, the smaller banks react *less* to the decreasing CBDC spreads.

Considering the results above, it seems that indirect effects through the banking sector can be quantitatively important for the welfare gains a central bank can achieve by paying positive interest on CBDC. This is particularly the case if deposit market power is determined by *concentration* and reiterates the point that the source of bank market power is an important determinant of the model effects of CBDC policy. Interestingly, given that welfare gains are higher for $N = 1$ than for $N = 3$, the exercise above also suggests that for given policy rates, moving from low to high banking concentration can potentially increase aggregate efficiency as the differing deposit rates change the composition of the aggregate liquidity mix.

Figure 6: CE welfare gain vs. Counterfactual welfare gain



7 Robustness tests

While we discipline most parameters of our model in line with the literature, a key uncertainty is the substitutability between CBDC and deposits, captured by ϵ. Since for most countries, CBDC still largely remains a theoretical possibility, we are left to speculate regarding some aspects of the relationship. Therefore, we test the robustness of the main insights in our paper by changing the substitutability between CBDC and deposits. The main specifications assume a "medium" degree of substitutability between CBDC and deposits, i.e. $\epsilon = 1/6$, following Bacchetta and Perazzi (2021). The test is then changing the degree of substitutability to $\epsilon = 1/4$ and $\epsilon = 1/10$ and repeating the exercises above: Note that this entails re-calibrating parameters such as ψ and γ to achieve the same steady state targets as in the baseline model, ensuring that the resulting differences reflect only the differing choice of ϵ.

7.1 Short-run analysis

Figures 7 and 8 in Appendix C show the impulse responses to a 10 basis points decrease in the CBDC rate with the alternative specifications described above. We see that the main takeaways from Section 5 still stand: An increase in the CBDC spread increases consumption and lowers capital investment. The extent to which deposit market power is shaped by *concentration* still has a noticeable impact: Higher market concentration noticeably amplifies the impulse responses, although both the aggregate impact of the shock as well as the relative contribution of concentration decrease (increase) for higher (lower) ϵ. Naturally, if the representative agent finds it harder to substitute deposits with CBDC, banks need to be less concerned with the impact of its return R^m on deposit demand, which dampens the indirect effect and in turn the strength of the overall response. If there is thus less scope overall for indirect effects due to lower substitutability, the impact of deposit market concentration on the effects of the shock is lower.

We also consider how the response to the reserve shock is shaped by η, for which Appendix Figures 9 and 10 display the IRFs for again the cases $\epsilon = 4$ and $\epsilon = 10$, respectively. In contrast to CBDC, lower (higher) substitutability now increases (decreases) the aggregate effects of the shock but also dampens (strengthens) the impact of concentration N. With less scope for substitution between CBDC and deposits, banks have more scope to react to their increases in costs due to the lower reserve rates, amplifying their aggregate effects.

Overall, the above results imply that the attention central banks will need to pay to deposit market power and its sources is going to depend on how they design their potential CBDC: In case it becomes rather easy for consumers to switch between deposits and the digital currency, it is going to matter more for the effects of monetary policy.

7.2 Long-run analysis

Similar considerations as for the short-run analysis above apply to the long-run effects of the CBDC rate: Figures 11 and 12 illustrate the CE welfare gains for different levels of the elasticity of substitution between CBDC and bank deposits for the cases of a monopsonistic bank ($N = 1$) and the oligopsonistic setting ($N = 3$), respectively. In either case, a higher elasticity of substitution (lower value of ϵ) corresponds to overall stronger welfare effects of the CBDC rate in the long run, as it ends up affecting deposit spreads and hence the aggregate liquidity mix more.

8 Conclusion

This paper has studied the transmission and effects of interest rates on CBDC, a potential new policy lever at central bankers' disposal. The analysis focused on bank market power and highlighted that if the deposit market is concentrated, a CBDC rate will in general affect the aggregate economy through both *direct* and *indirect* effects: A higher return for holding the digital currency not only affects households' saving- and portfolio decisions by itself, but also incentivizes the non-competitive banks to adjust their deposit spreads, which may in turn strengthen the aggregate effect of the policy innovation.

Our simple theoretical model implies that the latter indirect channel has the potential to substantially amplify the general equilibrium consequences of CBDC policy, but only if *concentration* is the key determinant of deposit market power. If deposit markdowns are instead due to banks providing *differentiated* liquidity services and concentration is lower, the direct channel typically dominates. We establish these insights both for the short run, by studying policy shocks, as well as the long run, by comparing steady states. Regarding the latter, we also extend the insights of Niepelt (2024) on optimal policy, which depends on deposit market concentration and the potential scope for indirect effects particularly if the government cannot subsidize banks directly. Moreover, deposit market concentration is also relevant for other monetary policy tools directed at the banking sector (such as reserve rates) but tends to affect them in the opposite way, weakening their efficacy.

We view our findings as relevant for both practice and theory: Regarding the former, it means that using interest rates on CBDC as a policy tool necessitates a detailed understanding of the deposit market market and competition thereon. For modeling, in turn, they reveal that specific assumptions on bank competition can importantly affect results for models of CBDC- and monetary policy, even if consistency with the same aggregate moments is ensured. It thus seems interesting for future research to explore this further by incorporating oligopsonistic banks into quantitative models of CBDC allowing for richer frictions and shocks.

References

Agur, Itai, Anil Ari, and Giovanni Dell'Ariccia (2022). "Designing Central Bank Digital Currencies," *Journal of Monetary Economics*, 125: 62–79.

Andolfatto, David (2021). "Assessing the Impact of Central Bank Digital Currency on Private Banks," *The Economic Journal*, 131(634): 525–540.

Atlantic Council (2024). "Central Bank Digital Currency Tracker," `https://www.atlanticcouncil.org/cbdctracker`.

Auer, Raphael, Jon Frost, Leonardo Gambacorta, Cyril Monnet, Tara Rice, and Hyun Song Shin (2022). "Central Bank Digital Currencies: Motives, Economic Implications and the Research Frontier," *Annual Review of Economics*, 14: 697–721.

Bacchetta, Philippe and Elena Perazzi (2021). "CBDC as Imperfect Substitute for Bank Deposits: A Macroeconomic Perspective," Swiss Finance Institute Research Paper Series 21-81, Swiss Finance Institute.

Bank for International Settlement (2020). "Central Bank Digital Currencies: Foundational Principles and Core Features," Technical report, Bank for International Settlement.

Bowman, Michelle W. (2023). "Considerations for a Central Bank Digital Currency," Speech presented at the Georgetown University McDonough School of Business, Psaros Center for Financial Markets and Policy, retrieved from `https://www.federalreserve.gov/newsevents/speech/bowman20230418a.htm`.

Chiu, Jonathan, Seyed Mohammadreza Davoodalhosseini, Janet Jiang, and Yu Zhu (2023). "Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment," *Journal of Political Economy*, 131(5): 1213–1248.

Choi, Michael and Guillaume Rocheteau (2023). "A Model of Retail Banking and the Deposits Channel of Monetary Policy," *Journal of Monetary Economics*, 139(C): 127–147.

Corbae, Dean and Pablo D'Erasmo (2020). "Rising Bank Concentration," *Journal of Economic Dynamics and Control*, 115: 103877.

De Grauwe, Paul and Yuemei Ji (2023). "The extraordinary generosity of central banks towards banks: Some reflections on its origin," Available at VoxEU: https://cepr.org/voxeu/columns/extraordinary-generosity-central-banks-towards-banks-some-reflexions-its-origin.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl (2017). "The Deposits Channel of Monetary Policy," *The Quarterly Journal of Economics*, 132(4): 1819–1876.

Engert, Walter and Ben Siu-Cheong Fung (2017). "Central Bank Digital Currency: Motivations and Implications," Bank of Canada Staff Discussion Paper 2017-16.

Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig (2021). "Central Bank Digital Currency: Central Banking for All?" *Review of Economic Dynamics*, 41: 225–242.

Garratt, Rodney, Jiaheng Yu, and Haoxiang Zhu (2022). "How Central Bank Digital Currency Design Choices Impact Monetary Policy Pass-Through and Market Composition," Available at SSRN: https://ssrn.com/abstract=4004341 or http://dx.doi.org/10.2139/ssrn.4004341.

Jiang, Janet Hua and Yu Zhu (2021). "Monetary Policy Pass-Through with Central Bank Digital Currency," Bank of Canada Staff Working Paper 2021-10.

Keister, Todd and Daniel Sanches (2023). "Should Central Banks Issue Digital Currency?" *The Review of Economic Studies*, 90(1): 404–431.

Niepelt, Dirk (2024). "Money and Banking with Reserves and CBDC," *Journal of Finance*, 79: 2505–2552.

Piazzesi, Monika and Martin Schneider (2022). "Credit Lines, Bank Deposits or CBDC? Competition and Efficiency in Modern Payment Systems," Unpublished, Stanford University.

Sidrauski, Miguel (1967). "Rational Choice and Patterns of Growth in a Monetary Economy," *The American Economic Review*, 57(2): 534–544.

Wang, Yifei, Toni M Whited, Yufeng Wu, and Kairong Xiao (2022). "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation," *The Journal of Finance*, 77(4): 2093–2141.

Williamson, Stephen D (2022). "Central Bank Digital Currency and Flight to Safety," *Journal of Economic Dynamics and Control*, 142: 104146.

A Derivations

A.1 Households

The household, taking prices, profits and taxes as given, solves

$$
\begin{aligned}
&\max_{\{c_t,k^h_{t+1},m_{t+1},n^i_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, z_{t+1}) \\
&\text{s.t.} \quad c_t + k^h_{t+1} + m_{t+1} + \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1} + \tau_t = w_t \bar{l} + \pi_t + k^h_t R^k_t + m_t R^m_t + \frac{1}{N}\sum_{i=1}^{N} n^i_t R^{n,i}_t, \\
&\quad\quad k^h_{t+1}, m_{t+1}, n^i_{t+1} \geq 0.
\end{aligned}
$$

Focusing on the interior solution, the first-order conditions with respect to capital, CBDC and deposits are

$$
k^h_{t+1}: \quad 1 = \mathbb{E}_t\left[\Lambda_{t+1} R^k_{t+1}\right] \tag{A.1}
$$

$$
m_{t+1}: \quad \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \chi^m_{t+1} \tag{A.2}
$$

$$
n^i_{t+1}: \quad \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} = \frac{1}{N}\chi^{n,i}_{t+1} \tag{A.3}
$$

where $f_{a,t}$ denotes the partial derivative of function f with respect to its argument a, Λ_{t+1} is the household's stochastic discount factor

$$
\Lambda_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}},
$$

χ^m_{t+1} and $\chi^{n,i}_{t+1}$ are the CBDC spread and deposit spread at bank i, respectively,

$$
\chi^m_{t+1} = 1 - \frac{R^m_{t+1}}{R^f_{t+1}}, \quad \chi^{n,i}_{t+1} = 1 - \frac{R^{n,i}_{t+1}}{R^f_{t+1}},
$$

and the risk-free rate is defined as

$$
R^f_{t+1} = \frac{1}{\mathbb{E}_t[\Lambda_{t+1}]}.
$$

A.1.1 Demand for individual bank deposits

Household's first-order condition (A.3) with respect to deposits at any bank i can be written as

$$
\frac{u_{z,t} z_{n,t+1}}{u_{c,t}} \left(\frac{n_{t+1}}{n^i_{t+1}}\right)^{\eta} = \chi^{n,i}_{t+1}. \tag{A.4}
$$

Since the last expression holds for any bank, it means that for any two banks i and j

$$
\chi^{n,i}_{t+1} \left(\frac{n^i_{t+1}}{n_{t+1}}\right)^{\eta} = \chi^{n,j}_{t+1} \left(\frac{n^j_{t+1}}{n_{t+1}}\right)^{\eta},
$$

from which we find the demand for bank deposits j

$$n_{t+1}^{j}=\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n,j}}\right)^{\frac{1}{\eta}}n_{t+1}^{i}. \tag{A.5}$$

Let T denote the sum of deposit spreads that the household incurs, and insert (A.5) into the expression,

$$T=\frac{1}{N}\sum_{i=1}^{N}n_{t+1}^{i}\chi_{t+1}^{n,i}=\frac{1}{N}\sum_{j=1}^{N}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n,j}}\right)^{\frac{1}{\eta}}n_{t+1}^{i}\chi_{t+1}^{n,j},$$

to find an expression for n_{t+1}^{i}

$$n_{t+1}^{i}=\frac{NT\left(\chi_{t+1}^{n,i}\right)^{-\frac{1}{\eta}}}{\sum_{j=1}^{N}\left(\chi_{t+1}^{n,j}\right)^{\frac{\eta-1}{\eta}}}. \tag{A.6}$$

We plug equation (A.6) into the definition of aggregate deposit, given by (1),

$$n_{t+1}=N^{\frac{\eta}{\eta-1}}T\left(\sum_{i=1}^{N}\left(\chi_{t+1}^{n,i}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{1-\eta}}. \tag{A.7}$$

Let χ_{t+1}^{n} be the spread associated with one unit of aggregate deposit, n_{t+1}. By setting $n_{t+1}=1$, we see from equation (A.7) that

$$\chi_{t+1}^{n}=\left(\frac{1}{N}\sum_{i=1}^{N}\left(\chi_{t+1}^{n,i}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}. \tag{A.8}$$

Given equation (A.8), we see that equation (A.6) can be written as

$$n_{t+1}^{i}=\frac{T}{\chi_{t+1}^{n}}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\right)^{-\frac{1}{\eta}}, \tag{A.9}$$

and inserting the resulting expression into (1), we get

$$n_{t+1}=\left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{T}{\chi_{t+1}^{n}}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\right)^{-\frac{1}{\eta}}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}=\frac{T}{\chi_{t+1}^{n}}. \tag{A.10}$$

Combining the expression for T and (A.10) we see that

$$T=\frac{1}{N}\sum_{i=1}^{N}n_{t+1}^{i}\chi_{t+1}^{n,i}=n_{t+1}\chi_{t+1}^{n}. \tag{A.11}$$

Lastly, inserting equation (A.11) into (A.9), we get the household's demand for deposits at bank i

$$n_{t+1}^{i}=n_{t+1}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\right)^{-\frac{1}{\eta}}. \tag{A.12}$$

Combining the household's demand schedule with the first-order condition (A.4), we see that (A.4) can be expressed as

$$\frac{u_{z,t} z_{n,t+1}}{u_{c,t}} = \chi_{t+1}^{n,i} \left(\frac{n_{t+1}}{n_{t+1}^{i}} \right)^{-\eta} = \chi_{t+1}^{n}. \tag{A.13}$$

A.1.2 Optimality conditions

Given the functional form assumptions, the household's first-order conditions (A.2) and (A.13) become

$$m_{t+1}: \quad \frac{\upsilon z_{t+1}^{-\psi}}{(1-\upsilon) c_t^{-\psi}} (1-\gamma) \left(\frac{z_{t+1}}{m_{t+1}} \right)^{\epsilon} = \chi_{t+1}^{m} \tag{A.14}$$

$$n_{t+1}^{i}: \quad \frac{\upsilon z_{t+1}^{-\psi}}{(1-\upsilon) c_t^{-\psi}} \gamma \left(\frac{z_{t+1}}{n_{t+1}} \right)^{\epsilon} = \chi_{t+1}^{n}. \tag{A.15}$$

We combine (A.14) and (A.15) to get the ratio

$$\frac{m_{t+1}}{n_{t+1}} = \left(\frac{(1-\gamma)\chi_{t+1}^{n}}{\gamma \chi_{t+1}^{m}} \right)^{\frac{1}{\epsilon}}. \tag{A.16}$$

We plug equation (A.16) into CES function for z_{t+1} and solve for the ratio of z_{t+1} to n_{t+1}

$$\frac{z_{t+1}}{n_{t+1}} = \left(\frac{\left(\left((1-\gamma) \left(\chi_{t+1}^{n} \right)^{1-\epsilon} \right)^{\frac{1}{\epsilon}} + \left(\gamma \left(\chi_{t+1}^{m} \right)^{1-\epsilon} \right)^{\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}}}{\gamma \chi_{t+1}^{m}} \right)^{\frac{1}{\epsilon}}. \tag{A.17}$$

Inserting (A.17) into equation (A.15) and solve for z_{t+1}, we get the household's optimal demand for liquidity

$$z_{t+1} = c_t \left(\frac{\upsilon}{1-\upsilon} \frac{1}{\chi_{t+1}^{z}} \right)^{\frac{1}{\psi}}, \tag{A.18}$$

where χ_{t+1}^{z} is the average cost of liquidity faced by the household

$$\chi_{t+1}^{z} = \frac{\chi_{t+1}^{m} \chi_{t+1}^{n}}{\left((1-\gamma)^{\frac{1}{\epsilon}} \left(\chi_{t+1}^{n} \right)^{\frac{1-\epsilon}{\epsilon}} + \gamma^{\frac{1}{\epsilon}} \left(\chi_{t+1}^{m} \right)^{\frac{1-\epsilon}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}}}.$$

Given household's optimal demand for z_{t+1}, we find the household's demand for m_{t+1} and n_{t+1}

$$m_{t+1} = z_{t+1} \left((1-\gamma) \frac{\chi_{t+1}^{z}}{\chi_{t+1}^{m}} \right)^{\frac{1}{\epsilon}}$$

$$n_{t+1} = z_{t+1} \left(\gamma \frac{\chi_{t+1}^{z}}{\chi_{t+1}^{n}} \right)^{\frac{1}{\epsilon}}. \tag{A.19}$$

Plugging optimal z_{t+1}, given by (A.18), into the first-order condition (A.1), we find the household's Euler equation

$$c_t^{-\sigma}\Omega_t^c = \beta\mathbb{E}_t\left[R_{t+1}^k c_{t+1}^{-\sigma}\Omega_{t+1}^c\right] \tag{A.20}$$

where Ω_t^c is given by

$$\Omega_t^c = (1-\upsilon)^{\frac{1-\sigma}{1-\psi}}\left(1+\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\psi}}\left(\chi_{t+1}^z\right)^{1-\frac{1}{\psi}}\right)^{\frac{\psi-\sigma}{1-\psi}}.$$

A.2 Banks

The date-t program of a typical bank is

$$\begin{aligned}
\max_{r_{t+1}^i, R_{t+1}^{n,i}} \quad & -n_{t+1}^i\nu_t^i + \mathbb{E}_t\left[\Lambda_{t+1}\left(k_{t+1}^i R_{t+1}^k + r_{t+1}^i R_{t+1}^r - n_{t+1}^i R_{t+1}^{n,i}\right)\right] \\
\text{s.t.} \quad & n_{t+1}^i = n_{t+1}\left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^n}\right)^{-\frac{1}{\eta}} \\
& k_{t+1}^i = n_{t+1}^i - r_{t+1}^i,
\end{aligned}$$

where

$$\nu_t^i\left(\zeta_{t+1}^i\right) = \omega + \phi\left(\zeta_{t+1}^i\right)^{1-\varphi}, \quad \zeta_{t+1}^i = \frac{r_{t+1}^i}{n_{t+1}^i}.$$

The first-order conditions for bank i with respect to its deposit rate and reserve holdings are, respectively,

$$R_{t+1}^{n,i}: \quad \chi_{t+1}^{n,i} + \frac{\chi_{t+1}^{n,i}}{e_{t+1}^{n,i}} = \nu_t^i - \nu_{\zeta,t}^i\zeta_{t+1}^i \tag{A.21}$$

$$r_{t+1}^i: \quad -\nu_{\zeta,t}^i = \chi_{t+1}^r, \tag{A.22}$$

where $\chi_{t+1}^r = 1 - R_{t+1}^r/R_{t+1}^f$ and $e_{t+1}^{n,i}$ denotes the elasticity of demand for deposits at bank i with respect to its deposit spread, $\chi_{t+1}^{n,i}$, which in a symmetric industry equilibrium can be shown to be

$$e_{t+1}^{n,i} = \frac{\partial n_{t+1}^i}{\partial\chi_{t+1}^{n,i}}\frac{\chi_{t+1}^{n,i}}{n_{t+1}^i}.$$

Given functional form assumptions, the first-order condition (A.21) becomes

$$\chi_{t+1}^{n,i}\left(1+\frac{1}{e_{t+1}^{n,i}}\right) = \omega + \varphi\phi\left(\zeta_{t+1}^i\right)^{1-\varphi},$$

where bank i's optimal reserves-to-deposits ratio is given by the first-order condition (A.22)

$$\zeta_{t+1}^{i} = \left(\frac{\chi_{t+1}^{r}}{\phi(\varphi-1)}\right)^{-\frac{1}{\varphi}}.$$

To find the demand elasticity, $e_{t+1}^{n,i}$, we differentiate the household's demand for deposit at bank i, equation (A.12) with respect to $\chi_{t+1}^{n,i}$ and multiply it with the ratio $\chi_{t+1}^{n,i}/n_{t+1}^{i}$

$$\begin{aligned} e_{t+1}^{n,i} &= \left(-\frac{1}{\eta}\frac{n_{t+1}}{\chi_{t+1}^{n,i}}\left(\frac{\chi_{t+1}^{n}}{\chi_{t+1}^{n,i}}\right)^{\frac{1}{\eta}} + \frac{1}{\eta}\frac{n_{t+1}}{\chi_{t+1}^{n}}\left(\frac{\chi_{t+1}^{n}}{\chi_{t+1}^{n,i}}\right)^{\frac{1}{\eta}}\frac{\partial\chi_{t+1}^{n}}{\partial\chi_{t+1}^{n,i}} + \left(\frac{\chi_{t+1}^{n}}{\chi_{t+1}^{n,i}}\right)^{\frac{1}{\eta}}\frac{\partial n_{t+1}}{\partial\chi_{t+1}^{n}}\frac{\partial\chi_{t+1}^{n}}{\partial\chi_{t+1}^{n,i}}\right)\frac{\chi_{t+1}^{n,i}}{n_{t+1}^{i}} \\ &= -\frac{1}{\eta} + \frac{1}{\eta}\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\frac{\partial\chi_{t+1}^{n}}{\partial\chi_{t+1}^{n,i}} + \left(\frac{\chi_{t+1}^{n,i}}{\chi_{t+1}^{n}}\right)^{\frac{1}{\eta}}\frac{\chi_{t+1}^{n,i}}{n_{t+1}^{n,i}}\frac{\partial n_{t+1}}{\partial\chi_{t+1}^{n}}\frac{\partial\chi_{t+1}^{n}}{\partial\chi_{t+1}^{n,i}} \end{aligned} \tag{A.23}$$

In a symmetric industry equilibrium, where $\chi_{t+1}^{n,i} = \chi_{t+1}^{n,j}$ and $n_{t+1}^{i} = n_{t+1}^{j}$ for any bank i and j,

$$\begin{aligned} \chi_{t+1}^{n} &= \left(\frac{1}{N}\sum_{i=1}^{N}\left(\chi_{t+1}^{n,i}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} = \chi_{t+1}^{n,i} \\ n_{t+1} &= \left(\frac{1}{N}\sum_{i=1}^{N}\left(n_{t+1}^{i}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} = n_{t+1}^{i} \end{aligned}$$

Then, equation (A.23) reduces to

$$e_{t+1}^{n,i} = \frac{1}{N}\left(\frac{\partial n_{t+1}}{\partial\chi_{t+1}^{n}}\frac{\chi_{t+1}^{n}}{n_{t+1}}\right) - \left(1-\frac{1}{N}\right)\frac{1}{\eta}.$$

To find the aggregate demand elasticity, we differentiate household's optimal deposit demand, equation (A.19), with respect to the liquidity premium on deposits, χ_{t+1}^{n},

$$\frac{\partial n_{t+1}}{\partial\chi_{t+1}^{n}} = \frac{\partial z_{t+1}}{\partial\chi_{t+1}^{z}}\frac{\partial\chi_{t+1}^{z}}{\partial\chi_{t+1}^{n}}\left(\frac{\gamma\chi_{t+1}^{z}}{\chi_{t+1}^{n}}\right)^{\frac{1}{\epsilon}} + \frac{z_{t+1}\gamma}{\epsilon\chi_{t+1}^{n}}\frac{\partial\chi_{t+1}^{z}}{\partial\chi_{t+1}^{n}}\left(\frac{\gamma\chi_{t+1}^{z}}{\chi_{t+1}^{n}}\right)^{\frac{1-\epsilon}{\epsilon}} - \frac{z_{t+1}\gamma\chi_{t+1}^{z}}{\epsilon\left(\chi_{t+1}^{n}\right)^{2}}\left(\frac{\gamma\chi_{t+1}^{z}}{\chi_{t+1}^{n}}\right)^{\frac{1-\epsilon}{\epsilon}}$$

and multiply the last expression with the ratio χ_{t+1}^{n}/n_{t+1}

$$\frac{\partial n_{t+1}}{\partial\chi_{t+1}^{n}}\frac{\chi_{t+1}^{n}}{n_{t+1}} = -\frac{1}{\psi}\gamma^{\frac{1}{\epsilon}}\left(\frac{\chi_{t+1}^{z}}{\chi_{t+1}^{n}}\right)^{\frac{1-\epsilon}{\epsilon}} - \frac{1}{\epsilon}(1-\gamma)^{\frac{1}{\epsilon}}\left(\frac{\chi_{t+1}^{z}}{\chi_{t+1}^{m}}\right)^{\frac{1-\epsilon}{\epsilon}}.$$

Lastly, we write the optimality condition as it applies to a representative bank (and dropping the individual superscript i)

$$\chi_{t+1}^{n} + \chi_{t+1}^{n}\left(\frac{1}{N}\left(-\frac{1-s_t}{\psi} - \frac{s_t}{\epsilon}\right) - \left(1-\frac{1}{N}\right)\frac{1}{\eta}\right)^{-1} = \omega + \varphi\phi\zeta_{t+1}^{1-\varphi}, \tag{A.24}$$

where

$$\zeta_{t+1} = \left(\frac{\chi^r_{t+1}}{\phi(\varphi - 1)} \right)^{-\frac{1}{\varphi}} \tag{A.25}$$

and $s_t \in [0, 1]$ is

$$s_t = (1-\gamma)^{\frac{1}{\epsilon}} \left(\frac{\chi^z_{t+1}}{\chi^m_{t+1}} \right)^{\frac{1-\epsilon}{\epsilon}}.$$

A.3 Aggregate resource constraint

To find the aggregate resource constraint, we start by inserting total profit, π_t, into the household's budget constraint, imposing market clearing for labor and capital and rearranging

$$\begin{aligned} k^h_{t+1} = & \, a_t k^\alpha_t \bar{l}^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1} - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1} - \tau_t \\ & - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1}\nu^i_t + \frac{1}{N}\sum_{i=1}^{N} r^i_t R^r_t - k^g_t R^k_t + m_t R^m_t. \end{aligned}$$

Next, from the government's budget constraint (24) we find an expression for k^g_{t+1}

$$k^g_{t+1} = m_{t+1}(1-\mu) + \frac{1}{N}\sum_{i=1}^{N} r^i_{t+1}(1-\rho) + k^g_t R^k_t + \tau_t - m_t R^m_t - \frac{1}{N}\sum_{i=1}^{N} r^i_t R^r_t.$$

We iterate forward capital market clearing condition

$$k_{t+1} = k^h_{t+1} + k^g_{t+1} + \frac{1}{N}\sum_{i=1}^{N} \left(n^i_{t+1} - r^i_{t+1} \right)$$

and plug in the expressions for k^h_{t+1} and k^g_{t+1} to get the aggregate resource constraint

$$k_{t+1} = a_t k^\alpha_t \bar{l}^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1}\mu - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1}\nu^i_t - \frac{1}{N}\sum_{i=1}^{N} r^i_{t+1}\rho. \tag{A.26}$$

In a symmetric industry equilibrium, all banks choose the same balance sheet positions and $n_{t+1} = n^i_{t+1}$ and $r_{t+1} = r^i_{t+1}$, then the resource constraint becomes

$$k_{t+1} = a_t k^\alpha_t \bar{l}^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1}\mu - n_{t+1}\nu_t - r_{t+1}\rho,$$

where

$$\nu_t = \omega + \phi\zeta^{1-\varphi}_{t+1}, \quad \zeta_{t+1} = \frac{r_{t+1}}{n_{t+1}}.$$

We can rewrite the resource constraint, using the definition of ζ_{t+1}, as

$$k_{t+1} = a_t k^\alpha_t \bar{l}^{1-\alpha} + k_t(1-\delta) - c_t \Omega^{rc}_t, \tag{A.27}$$

where

$$\Omega^{rc}_t = 1 + \left(\frac{\upsilon}{1-\upsilon} \frac{1}{\chi^z_{t+1}} \right)^{\frac{1}{\psi}} \left(\frac{m_{t+1}}{z_{t+1}}\mu + \frac{n_{t+1}}{z_{t+1}} \left(\omega + \phi\zeta^{1-\varphi}_{t+1} + \zeta_{t+1}\rho \right) \right).$$

A.4 Steady state

Following the standard convention for the analysis of business cycle models, we analyze the effects of monetary policy by studying small policy perturbations around the economy's non-stochastic steady state, which we characterize here.

We denote steady state variables by dropping the time subscripts. In the steady state, the capital return and the risk-free rate are equal and given by the household's discount factor

$$R^k = R^f = \frac{1}{\beta}.$$

Conditional on policy, the CBDC and reserve spreads, χ^m and χ^r, are known. Then, the steady state deposit spread, χ^n, and reserves-to-deposits ratio, ζ, can be found using the bank optimality condition (A.24) and equation (A.25). Given the CBDC and deposit spreads, the cost of liquidity χ^z and the quantities Ω^c and Ω^{rc} are also known.

Note that we consider a model with fixed labor, i.e. $l_t = \bar{l}$. We divide the expression for capital return (22) by labor supply, $\bar{l}$, to find the steady state capital-labor ratio in terms of primitives

$$\frac{k}{\bar{l}} = \left(\frac{1}{a\alpha}\left(R^k - 1 + \delta\right)\right)^{\frac{1}{\alpha-1}}.$$

Notice that the steady state capital-labor ratio is identical to one that would have resulted in a baseline non-monetary RBC model. We divide resource constraint (A.27) by labor supply, $\bar{l}$, to find the steady state consumption-labor ratio

$$\frac{c}{\bar{l}} = \left(a\left(\frac{k}{\bar{l}}\right)^{\alpha} - \delta\left(\frac{k}{\bar{l}}\right)\right)\frac{1}{\Omega^{rc}}.$$

The steady state wage rate is also a function of the capital-labor ratio

$$w = a(1-\alpha)\left(\frac{k}{\bar{l}}\right)^{\alpha}.$$

Given the fixed labor supply $\bar{l}$, it is straightforward to back out the rest of the allocation and asset holdings: k, c, z, m, n, and r.

A.5 Calibration

We calibrate parameters ϕ, γ, υ, ψ, and η in the following way. First, the banks' optimal reserves-to-deposits ratio is given by

$$\zeta = \left(\frac{\chi^r}{\phi(\varphi-1)}\right)^{-\frac{1}{\varphi}},$$

from which we find an expression for ϕ

$$\phi = \frac{\chi^r}{\zeta^{-\varphi}(\varphi-1)}.$$

Given the target for deposit markdown, we can pin down the deposit spread from the bank's optimality condition (16) as follows:

$$\chi^n = \text{deposit markdown} \times (\omega + \varphi\phi\zeta^{1-\varphi}),$$

which can be used to calculate the average liquidity cost $\chi^z(\chi^m, \chi^n)$.

The household's demand for CBDC and deposits yields the expression

$$\frac{1-\gamma}{\gamma} = \left(\frac{m}{n}\right)^{\epsilon} \frac{\chi^m}{\chi_n},$$

which can be used to obtain γ, given $m/n, \chi^m$, and χ^n.

In the baseline case $N = 1$, the parameter η plays no role and we calibrate ψ to match the targeted deposit markdown, given known ϵ, γ, χ^m, and χ^n.

Next, the household's demand for liquidity services is given by

$$z = c\left(\frac{\upsilon}{1-\upsilon}\frac{1}{\chi^z}\right)^{\frac{1}{\psi}}.$$

Knowing the cost of liquidity, χ^z, and the desired inverse velocity, z/c, we find υ

$$\upsilon = \frac{\left(\frac{z}{c}\right)^{\psi}\chi^z}{1+\left(\frac{z}{c}\right)^{\psi}\chi^z}.$$

To calibrate parameter η for the case $N = 3$, we target the same deposit markdown and set ψ equal to the value calibrated in the baseline case $N = 1$.

B Optimal policy rules

A Ramsey government can implement the first-best equilibrium allocation by setting policy instruments appropriately. In this section, we show the optimal policy rules and discuss the extent to which deposit market concentration impacts these rules.

The social planner maximizes the household's utility subject to the aggregate resource constraint (A.26)

$$\max_{\{c_t, k_{t+1}, m_{t+1}, n^i_{t+1}, r^i_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, z_{t+1})$$

$$\text{s.t.} \quad k_{t+1} = a_t k_t^{\alpha} \bar{l}^{1-\alpha} + k_t(1-\delta) - c_t - m_{t+1}\mu - \frac{1}{N}\sum_{i=1}^{N} n^i_{t+1}\nu^i_t - \frac{1}{N}\sum_{i=1}^{N} r^i_{t+1}\rho.$$

The relevant first-order conditions are

$$
\begin{aligned}
k_{t+1}: &\quad 1 = \mathbb{E}_t \left[\Lambda_{t+1} \left(a_{t+1} f_k(k_{t+1}, \bar{l}) + 1 - \delta\right)\right] \\
m_{t+1}: &\quad \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \mu && \text{(B.1)} \\
n^i_{t+1}: &\quad \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} = \frac{1}{N}(\nu^i_t - \nu^i_{\zeta,t} \zeta^i_{t+1}) && \text{(B.2)} \\
r^i_{t+1}: &\quad -\nu^i_{\zeta,t} = \rho. && \text{(B.3)}
\end{aligned}
$$

Note that the social planner conditions are directly comparable to the first-order conditions of the household and banks.

We start by comparing the first-order condition with respect to CBDC in the household problem (A.2) and its social planner counterpart (B.1),

$$
\begin{aligned}
\text{Household}: &\quad \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \chi^m_{t+1} \\
\text{Social planner}: &\quad \frac{u_{z,t} z_{m,t+1}}{u_{c,t}} = \mu.
\end{aligned}
$$

In order to replicate the social planner condition, the government should keep the spread on CBDC, χ^m_{t+1}, equal to the government's per unit cost of issuing (and managing) CBDC, μ, at all times

$$\chi^{m*}_{t+1} = \mu.$$

Then, the CBDC rate should be set such that its spread, which is the household's opportunity cost of holding CBDC, is equal to the government's (societal) cost of issuing CBDC. Given the optimal target for the CBDC spread, we find the optimal rule for the CBDC rate

$$R^{m*}_{t+1} = R^f_{t+1}(1 - \mu).$$

Next, we compare the first-order condition with respect to the reserve holdings of any bank i, r^i_{t+1}, in the bank problem (A.22) and its social planner counterpart (B.3),

$$
\begin{aligned}
\text{Bank}: &\quad -\nu^i_{\zeta,t} = \chi^r_{t+1} \\
\text{Social planner}: &\quad -\nu^i_{\zeta,t} = \rho.
\end{aligned}
$$

To replicate the social planner condition, the government should ensure that the reserve spread, χ^r_{t+1}, is equal to the government's per unit cost of issuing (and managing) reserves, ρ,

$$\chi^{r*}_{t+1} = \rho. \tag{B.4}$$

The optimal rule for the reserve rate is then

$$R^{r*}_{t+1} = R^f_{t+1}\left(1 - \rho\right).$$

Lastly, consider the first-order condition with respect to the deposit of bank i, n^i_{t+1}, in the household problem (A.3) and its social planner counterpart (B.2),

$$\begin{aligned}\text{Household}: \quad \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} &= \frac{1}{N} \chi^{n,i}_{t+1} \\ \text{Social planner}: \quad \frac{u_{z,t} z_{n^i,t+1}}{u_{c,t}} &= \frac{1}{N} (\nu^i_t - \nu^i_{\zeta,t} \zeta^i_{t+1}),\end{aligned}$$

which we see are equalized if

$$\chi^{n^i *}_{t+1} = \nu^i_t - \nu^i_{\zeta,t} \zeta^i_{t+1}.$$

We recast the expression as it applies to a representative bank and get the expression for the optimal deposit spread

$$\chi^{n*}_{t+1} = \omega + \varphi \phi \left(\zeta^*_{t+1}\right)^{1-\varphi}, \tag{B.5}$$

where, given the optimal reserve spread, the banks' optimal reserves-to-deposits ratio is

$$\zeta^*_{t+1} = \left(\frac{\rho}{\phi(\varphi - 1)} \right)^{-\frac{1}{\varphi}}.$$

However, the government cannot control the deposit spread directly as it is determined by the banking sector. Specifically, it is determined by the bank optimality condition (A.24). The government can, nevertheless, offer banks a subsidy per unit of their deposit issuance, θ_t. The deposit subsidy should be set such that the last equation is fulfilled. Equating bank optimality condition (A.24) and the last expression for the optimal spread on deposits, we find the optimal level of subsidy

$$\theta^*_t = \chi^{n*}_{t+1} \left(\frac{1}{N} \left(\frac{1 - s_t}{\psi} + \frac{s_t}{\epsilon} \right) + \left(1 - \frac{1}{N} \right) \frac{1}{\eta} \right)^{-1},$$

which is the product of the optimal deposit spread and the inverse of the household's elasticity of demand for deposits (in absolute value).

The qualitative insights of the policy rules we derived are similar to those in Niepelt (2024). The spreads on CBDC and reserves should be targeted so that the opportunity costs of holding CBDC and reserves are equal to the societal costs of providing them. A deposit subsidy should be offered to the banks to eliminate distortion caused by bank market power. The key difference, however, is the importance of market concentration in our model. As we discussed before, with a more concentrated deposit market the household's demand for deposits is also less elastic. Then, banks have more market power and the optimal subsidy that corrects for that should also be larger. The same mechanism is also at work if the substitutability between banks is lower.

C Additional Figures

Figure 7: Lower CBDC-deposits substitutability $\epsilon = \frac{1}{4}$

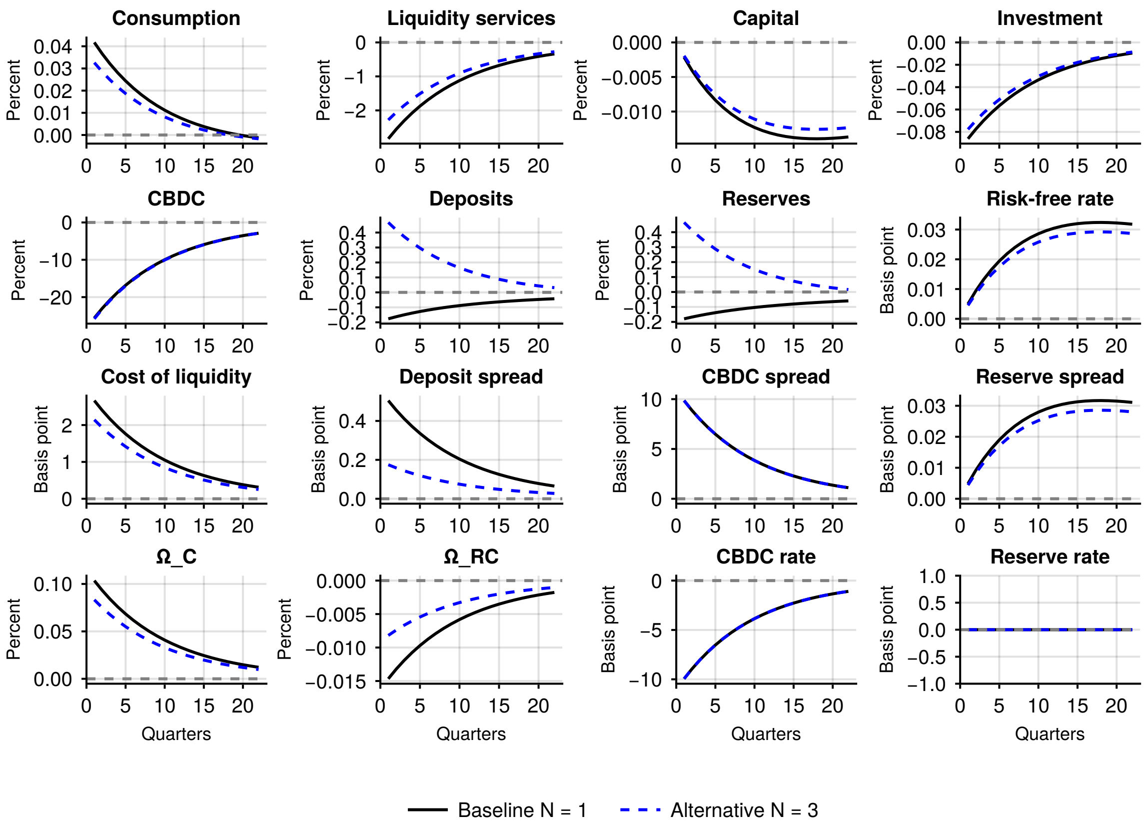


Figure 8: Higher CBDC-deposits substitutability $\epsilon = \frac{1}{10}$

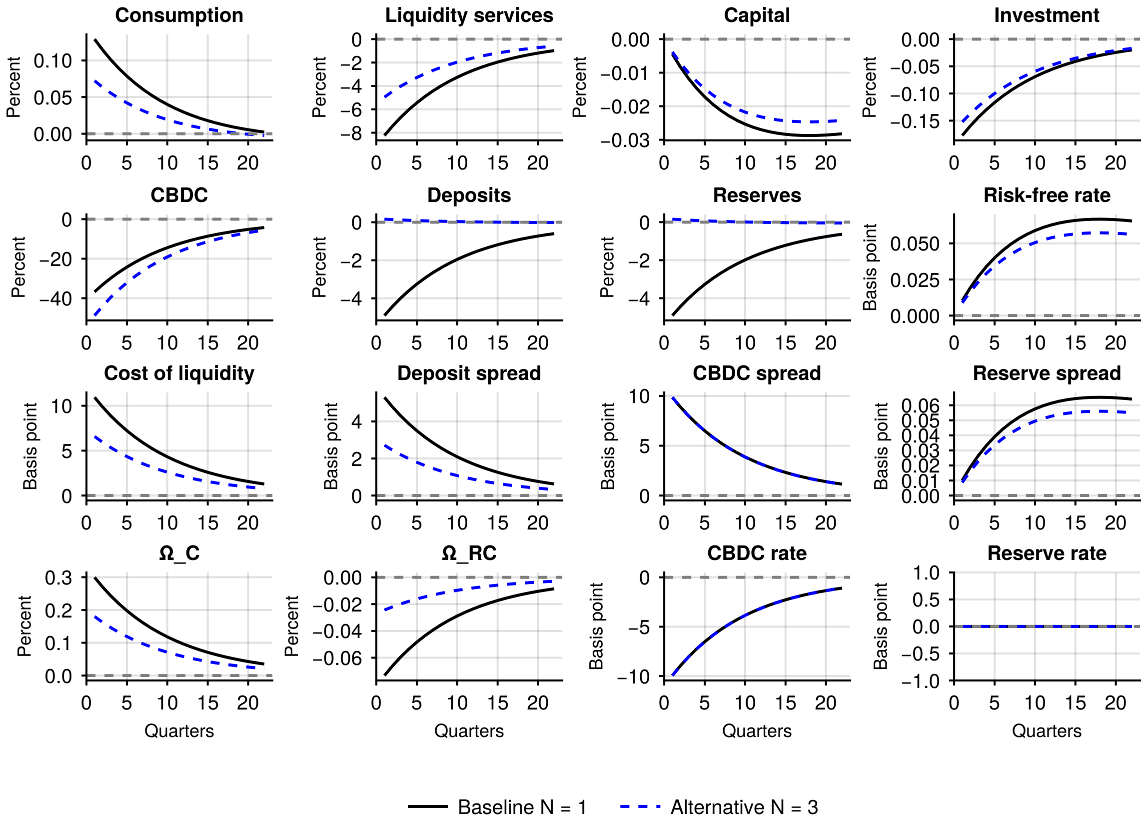


Figure 9: Lower CBDC-deposits substitutability $\epsilon = \frac{1}{4}$

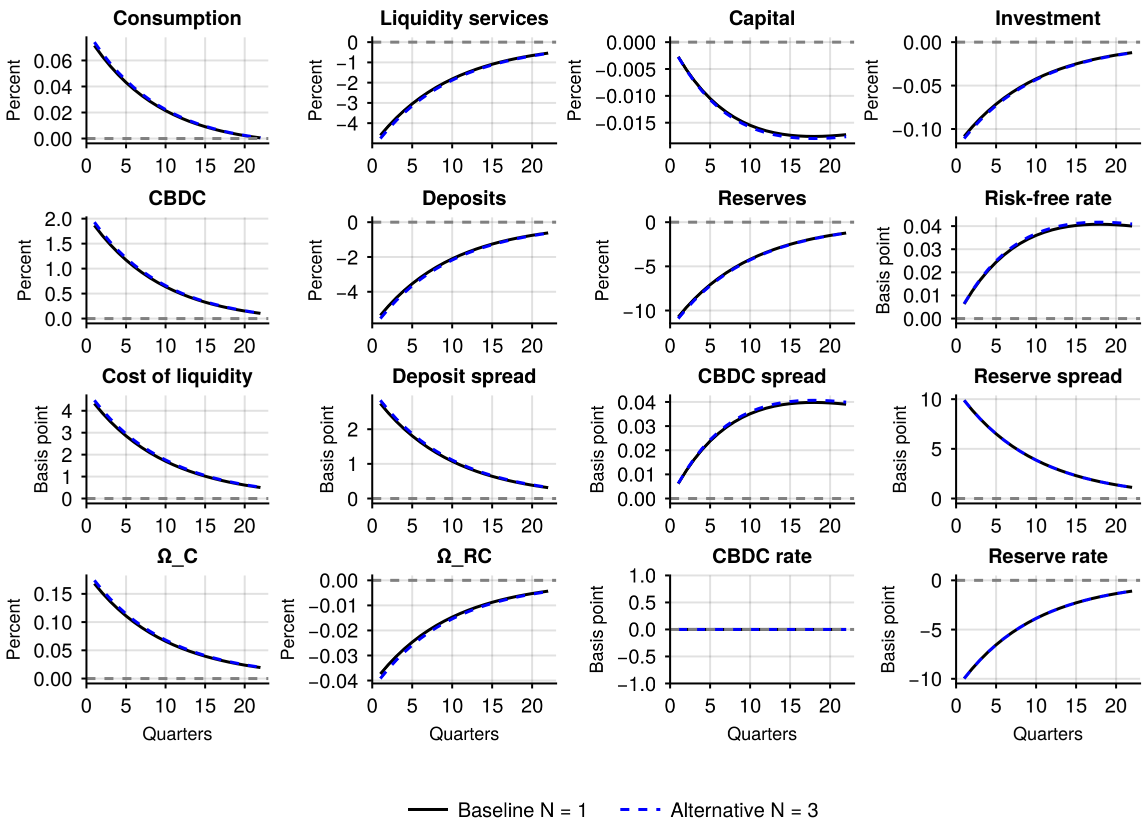


Figure 10: Higher CBDC-deposits substitutability $\epsilon = \frac{1}{10}$

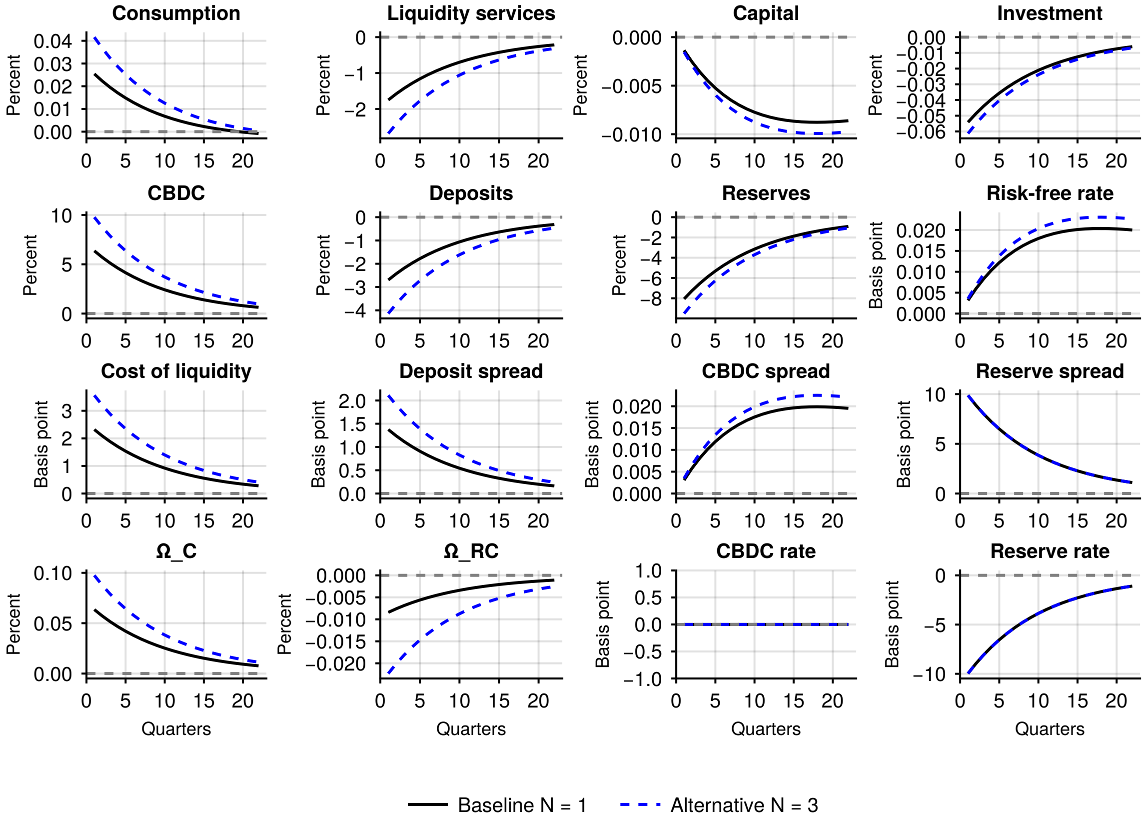


Figure 11: CE welfare gain (loss) with different scenarios ($N = 1$)

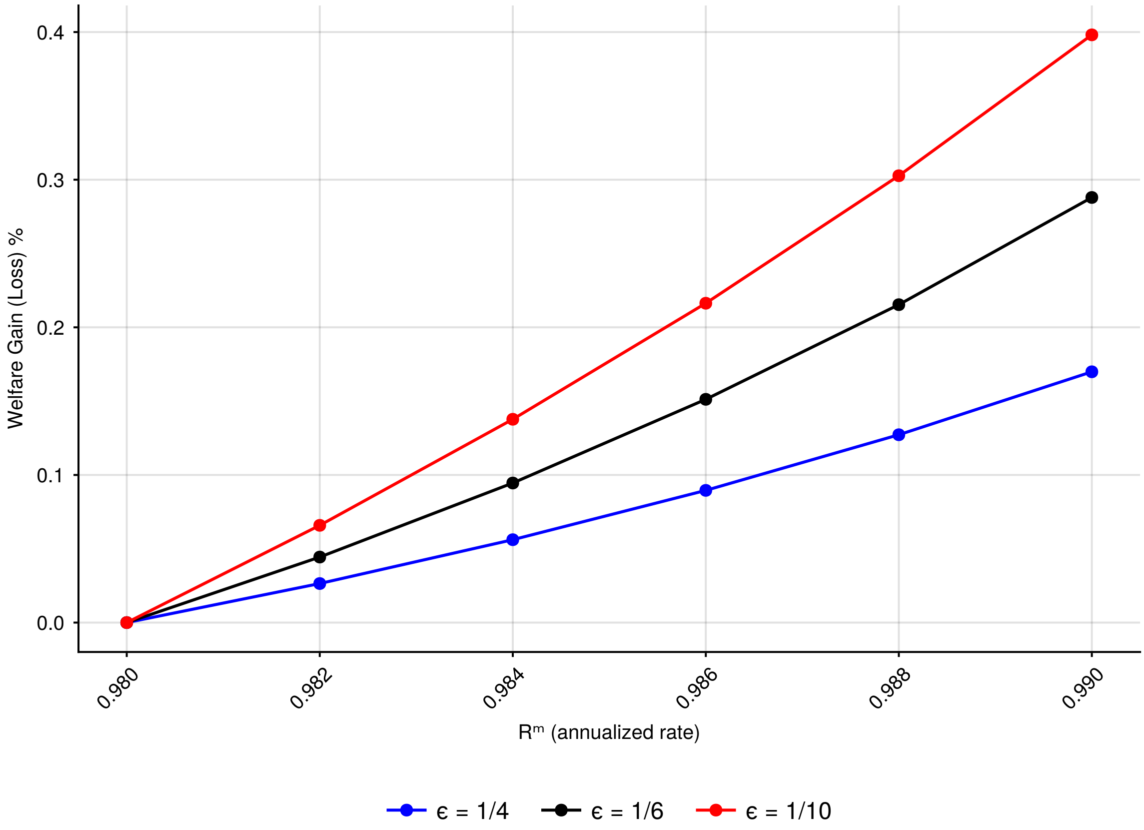


Figure 12: CE welfare gain (loss) with different scenarios ($N = 3$)