$$\rho_{,t} + (\rho v^i)_{;i} = 0 \tag{1}$$

$$(\rho v_j)_{,t} + (\rho v^i v_j + P\delta^i_j)_{;i} = -\rho\phi_{,j} + c^{-1}\sum_s \int_0^\infty (\kappa_{s\varepsilon} + \sigma^{\mathrm{tr}}_{s\varepsilon})F_{s\varepsilon j} d\varepsilon \tag{2}$$

$$\left[\rho\left(e + \frac{1}{2}\|v\|^2\right)\right]_{,t} + \left[\rho v^i\left(e + \frac{1}{2}\|v\|^2 + \frac{P}{\rho}\right)\right]_{;i} = -\rho v^i \phi_{,i} \tag{3}$$

$$- \sum_s \int_0^\infty \left(j_{s\varepsilon} - c\kappa_{s\varepsilon}E_{s\varepsilon} - \frac{v^i}{c}(\kappa_{s\varepsilon} + \sigma^{\mathrm{tr}}_{s\varepsilon})F_{s\varepsilon i}\right) d\varepsilon \tag{4}$$

$$(\rho Y_e)_{,t} + (\rho Y_e v^i)_{;i} = \sum_s \int_0^\infty \xi_{s\varepsilon}(j_{s\varepsilon} - c\kappa_{s\varepsilon}E_{s\varepsilon}) d\varepsilon \tag{5}$$

$$E_{s\varepsilon,t} + (F^i_{s\varepsilon} + v^i E_{s\varepsilon})_{;i} - v^i_{;j}\frac{\partial}{\partial \ln\varepsilon}P^j_{s\varepsilon i} = j_{s\varepsilon} - c\kappa_{s\varepsilon}E_{s\varepsilon} \tag{6}$$

$$F_{s\varepsilon j,t} + (c^2 P^i_{s\varepsilon j} + v^i F_{s\varepsilon j})_{;i} + v^i_{;j}F_{s\varepsilon i} - v^i_{;k}Q^k_{s\varepsilon ji} - v^i_{;k}\frac{\partial}{\partial \ln\varepsilon}Q^k_{s\varepsilon ji} = -c(\kappa_{s\varepsilon} + \sigma^{\mathrm{tr}}_{s\varepsilon})F_{s\varepsilon j} \tag{7}$$

where e is the specific internal energy, $s \in \{\nu_e, \bar{\nu}_e, \nu_x\}$,

$$\xi_{s\varepsilon} = \begin{cases} -(N_A\varepsilon)^{-1} & s = \nu_e, \\ (N_A\varepsilon)^{-1} & s = \bar{\nu}_e, \\ 0 & s = \nu_x, \end{cases} \tag{8}$$

$P = P(\rho, e, Y_e)$, and we use the M1 closure to truncate the radiation moment hierarchy by specifying the second and third moments in terms of the zeroth and first.

1 Algorithm Steps

The following describes an Euler push (i.e. 1st-order in time). The generalization to 2nd-order Runge-Kutta schemes is straightforward.

1.1 Step 1

Advance this subsystem by Δt:

$$\rho_{,t} + (\rho v^i)_{;i} = 0 \tag{9}$$

$$(\rho v_j)_{,t} + (\rho v^i v_j + P\delta^i_j)_{;i} = -\rho\phi_{,j} \tag{10}$$

$$\left[\rho\left(e + \frac{1}{2}\|v\|^2\right)\right]_{,t} + \left[\rho v^i\left(e + \frac{1}{2}\|v\|^2 + \frac{P}{\rho}\right)\right]_{;i} = -\rho v^i \phi_{,i} \tag{11}$$

$$(\rho Y_e)_{,t} + (\rho Y_e v^i)_{;i} = 0 \tag{12}$$

$$E_{s\varepsilon,t} + (F^i_{s\varepsilon} + v^i E_{s\varepsilon})_{;i} = 0 \tag{13}$$

$$F_{s\varepsilon j,t} + (c^2 P^i_{s\varepsilon j} + v^i F_{s\varepsilon j})_{;i} + v^i_{;j}F_{s\varepsilon i} - v^i_{;k}Q^k_{s\varepsilon ji} = 0 \tag{14}$$

1.2 Step 2

Advance this subsystem by Δt:

$$E_{s\varepsilon,t} - v^i_{;j}\frac{\partial}{\partial \ln\varepsilon}P^j_{s\varepsilon i} = 0 \tag{15}$$

$$F_{s\varepsilon j,t} - v^i_{;k}\frac{\partial}{\partial \ln\varepsilon}Q^k_{s\varepsilon ji} = 0 \tag{16}$$

1.3 Step 3

Advance this subsystem by Δt:

$$(\rho v_j)_{,t} = c^{-1} \sum_s \int_0^\infty (\kappa_{s\varepsilon} + \sigma_{s\varepsilon}^{\mathrm{tr}}) F_{s\varepsilon j} d\varepsilon \tag{17}$$

$$\left[\rho \left(e + \frac{1}{2}\|v\|^2\right)\right]_{,t} = \sum_s \int_0^\infty \left(\frac{v^i}{c}(\kappa_{s\varepsilon} + \sigma_{s\varepsilon}^{\mathrm{tr}}) F_{s\varepsilon i}\right) d\varepsilon \tag{18}$$

$$F_{s\varepsilon j,t} = -c(\kappa_{s\varepsilon} + \sigma_{s\varepsilon}^{\mathrm{tr}}) F_{s\varepsilon j} \tag{19}$$

1.4 Step 4

Advance this subsystem by Δt:

$$(\rho e)_{,t} = -\sum_s \int_0^\infty (j_{s\varepsilon} - c\kappa_{s\varepsilon} E_{s\varepsilon})\, d\varepsilon \tag{20}$$

$$(\rho Y_e)_{,t} = \sum_s \int_0^\infty \xi_{s\varepsilon} (j_{s\varepsilon} - c\kappa_{s\varepsilon} E_{s\varepsilon}) d\varepsilon \tag{21}$$

$$E_{s\varepsilon,t} = j_{s\varepsilon} - c\kappa_{s\varepsilon} E_{s\varepsilon} \tag{22}$$