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FORNAX: A FLEXIBLE CODE FOR MULTIPHYSICS ASTROPHYSICAL SIMULATIONS

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ABSTRACT

The abstract.
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1. INTRODUCTION

In recent years, there has been an explosion of techniques and codes to solve the equations of radiation hydrodynamics in astrophysical environments [REFS].

2. FORMULATION OF THE EQUATIONS

In Fornax, we have decided to make use of a metric tensor to formulate our equations in a way that makes no reference to any particular geometry or set of coordinates. Perhaps the biggest advantage of this approach is flexibility; as we discuss below, switching geometries and coordinates in Fornax is straightforward, quick, and much less error prone than in codes that explicitly express these choices in the equations to be solved.

2.1. *Hydrodynamics*

As is standard practice, we denote contravariant components of a vector with raised indices, covariant components with lowered indices, covariant differentiation with a semicolon, partial differentiation with a comma, and make use of Einstein notation for summation over repeated indices. Here, and throughout this work, we adopt a coordinate basis. In this notation, the equations of hydrodynamics can be written

$$\rho_{,t} + (\rho v^i)_{;i} = 0 \tag{1}$$

$$(\rho v_j)_{,t} + (\rho v^i v_j + P\delta^i{}_j)_{;i} = S_j \tag{2}$$

$$(\rho e)_{,t} + \left[\rho v^i \left(e + \frac{P}{\rho}\right)\right]_{;i} = S_E \tag{3}$$

$$(\rho X)_{,t} + (\rho X v^i)_{;i} = S_X \,, \tag{4}$$

where e is the specific total energy of the gas, X is a an arbitrary scalar that may represent, for example, composition, and S_j, S_E, and S_X are source terms that account for additional physics. In a coordinate basis, the covariant derivatives can be expanded yielding

$$\rho_{,t} + \frac{1}{\sqrt{g}}(\sqrt{g}\rho v^i)_{,i} = 0 \tag{5}$$

$$(\rho v_j)_{,t} + \frac{1}{\sqrt{g}}\left[\sqrt{g}(\rho v^i v_j + P\delta^i{}_j)\right]_{,i} = \Gamma^l{}_{jk}T^k_l + S_j \tag{6}$$

$$(\rho e)_{,t} + \frac{1}{\sqrt{g}}\left[\sqrt{g}\rho v^i \left(e + \frac{P}{\rho}\right)\right]_{,i} = S_E \tag{7}$$

$$(\rho X)_{,t} + \frac{1}{\sqrt{g}}(\sqrt{g}\rho X v^i)_{,i} = S_X \,, \tag{8}$$

where g is the determinant of the metric, $\Gamma^l{}_{jk}$ are the Christoffel symbols, defined in terms of derivatives of the metric, and $T^k_l = \rho v^i v_j + \delta^i{}_j P$ is the fluid stress tensor.

Note that we have chosen to express the momentum equation as a conservation law for the covariant compenents of the momentum. There is good reason to do so. Written this way, the geometric source terms, $\Gamma^l{}_{jk}T^k_l$, vanish identically for components associated with ignorable coordinates in the metric. A good example is in spherical (r, θ, ϕ) coordinates where, since ϕ does not explicitly enter into the metric, the geometric source terms vanish for the ρv_ϕ equation. Physically, ρv_ϕ is the angular momentum, so we are left with an explicit expression of angular momentum conservation that the numerics will satisfy to machine precision, rather than to the level of truncation error. In general, the covariant expression of the momentum equation respects the geometry of the problem without special consideration or coordinate specific modifications of the code.

2.2. *Radiation*

Fornax evolves the zeroth and first moments of the frequency-dependent radiation transport equation. Keeping all terms to $\mathcal{O}(v/c)$ and dropping terms proportional to the fluid acceleration, the evolution equations can be written

$$E_{\varepsilon,t} + (F^i_\varepsilon + v^i E_\varepsilon)_{;i} - v^i_{;j}\frac{\partial P^j_{\varepsilon i}}{\partial \ln \varepsilon} = R_{\varepsilon E} \tag{9}$$

$$F_{\varepsilon j,t} + (c^2 P^i_{\varepsilon j} + v^i F_{\varepsilon j})_{;i} + v^i_{;j}F_{\varepsilon i} - v^i_{;k}\frac{\partial \varepsilon Q^k_{\varepsilon ji}}{\partial \varepsilon} = R_{\varepsilon j} \,. \tag{10}$$

Here, E_ε and $F_{\varepsilon j}$ are the monochromatic energy density and flux of the radiation field at energy ε in the comoving frame, $P^j_{\varepsilon i}$ is the radiation pressure tensor ($2^{\rm nd}$ moment),

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$Q^k_{\varepsilon ji}$ is the heat-flux tensor (3$^{\rm rd}$ moment), and $R_{\varepsilon E}$ and $R_{\varepsilon j}$ are the collision terms that account for interactions between the radiation and matter. The interaction terms are written

$$R_{\varepsilon E} = j_\varepsilon - c\kappa_\varepsilon E_\varepsilon \tag{11}$$

$$R_{\varepsilon j} = -c(\kappa_\varepsilon + \sigma^{\rm tr}_\varepsilon)F_{\varepsilon j}\,, \tag{12}$$

where j_ε is the emissivity, κ_ε is the absorption coefficient, and $\sigma^{\rm tr}_\varepsilon$ is the scattering coefficient. Correspondingly, there are energy and momentum source terms in the fluid equations:

$$S^{\rm rad}_j = -\frac{1}{c^2}\int_0^\infty R_{\varepsilon j} d\varepsilon \tag{13}$$

$$S^{\rm rad}_E = -\int_0^\infty (R_{\varepsilon E} + \frac{v^i}{c^2}R_{\varepsilon i}) d\varepsilon\,. \tag{14}$$

As in the fluid sector, we rewrite these covariant derivatives as partial derivative, introducing geometric source terms in the (radiation) momentum equation.

Fornax can treat either photon or neutrino radiation fields. For neutrinos, Eqs. 9–12 are solved separately for each species and Eqs. 13 & 14 are summed over species. Additionlly, the electron fraction is evolved according to Eq. 8, with $X = Y_e$ and

$$S_X = \sum_s \int_0^\infty \xi_{s\varepsilon}(j_{s\varepsilon} - c\kappa_{s\varepsilon}E_{s\varepsilon})d\varepsilon\,, \tag{15}$$

where s refers to the neutrino species and

$$\xi_{s\varepsilon} = \begin{cases} -(N_A\varepsilon)^{-1} & s = \nu_e, \\ (N_A\varepsilon)^{-1} & s = \bar{\nu}_e, \\ 0 & s = \nu_x, \end{cases}\,. \tag{16}$$

2.3. *Self-gravity*

Motivated by our first applications, we have limited our implementation of self-gravity to spherical geometries. In such cases (core-collapse supernova models, for example), we use a multipole expansion of the gravitional potential, centered on the origin of the coordinate system. More text (Aaron?).

2.4. *Reactions*

Should we include this?

3. NUMERICAL DISCRETIZATION

We adopt a finite volume discretization of the equations presented in Sec. 2. Independent of geometry or coordinates, the volume element can always be expressed $dV = \sqrt{g}d^3x$, for arbitrary coordinates x. Similarly, the area element is $dA = \sqrt{g}d^2x$. Integrating the equations over control volumes (cells) and diving by these same volumes leads to a set of exact equations describing the evolution of cell-volume averaged quantities. Applying the divergence theorem, we express the divergence terms in each equation as a net flux through the faces. For example, the density in cell (i, j, k) evolves according to

$$\begin{aligned}\frac{d\rho_{i,j,k}}{dt} = -\frac{1}{V_{i,j,k}}&((\rho v^0 A_0)_{i+1/2,j,k} - (\rho v^0 A_0)_{i-1/2,j,k} \\ &+(\rho v^1 A_1)_{i,j+1/2,k} - (\rho v^1 A_1)_{i,j-1/2,k} \\ &+(\rho v^2 A_2)_{i,j,k+1/2} - (\rho v^2 A_2)_{i,j,k-1/2})\end{aligned} \tag{17}$$