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Responsible investing: The ESG-efficient frontier☆

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ABSTRACT

We propose a theory in which each stock's environmental, social, and governance (ESG) score plays two roles: (1) providing information about firm fundamentals and (2) affecting investor preferences. The solution to the investor's portfolio problem is characterized by an ESG-efficient frontier, showing the highest attainable Sharpe ratio for each ESG level. The corresponding portfolios satisfy four-fund separation. Equilibrium asset prices are determined by an ESG-adjusted capital asset pricing model, showing when ESG raises or lowers the required return. Combining several large data sets, we compute the empirical ESG-efficient frontier and show the costs and benefits of responsible investing. Finally, we test our theory's predictions using proxies for E (carbon emissions), S, G, and overall ESG.

1. Introduction

Asset owners and portfolio managers overseeing trillions of dollars seek to incorporate environmental, social, and governance (ESG) considerations into their investment process.[1] Meanwhile, investors have little guidance in how to incorporate ESG in portfolio choice and, worse, opinions

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[1] For example, the *2018 Global Sustainable Investment Review* reports over $30 trillion invested with explicit ESG goals as of the beginning of 2018. The 2017–2018 annual report of the Principles for Responsible

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differ dramatically across academics and practitioners about whether ESG will help or hurt their performance. Some argue that ESG considerations must necessarily lower expected returns (e.g., Hong and Kacperczyk, 2009), and others argue that the "outperformance of ESG strategies is beyond doubt" (Financial Times, 2017).[2]

To reconcile these opposing views, we develop a theory that illuminates both the potential costs and benefits of ESG-based investing. Our theory explains how the increasingly widespread adoption of ESG affects portfolio choice and equilibrium asset prices. Further, we estimate the magnitude of these effects empirically.

Our conclusions are fivefold. (1) Theoretically, we show that an investor optimally chooses a portfolio on the ESG-efficient frontier. (2) The portfolios that span the frontier are all combinations of the risk-free asset, the tangency portfolio, the minimum-variance portfolio, and what we call the ESG-tangency portfolio (four-fund separation). (3) Equilibrium asset returns satisfy an ESG-adjusted capital asset pricing model (CAPM), showing when higher ESG assets have lower or higher equilibrium expected returns. (4) We estimate the costs and benefits of responsible investing via the empirical ESG-efficient frontier based on environmental (E) and governance (G) measures and show how ESG screens can have surprising effects. (5) We test the theory's equilibrium predictions using four ESG proxies, providing a rationale for why certain ESG measures predict returns positively (some aspects of governance) and others negatively (non-sin stocks, a measure of S) or close to zero (low carbon emissions, an example of E, and commercial ESG measures).

We consider three types of investors. Type-U (ESG-unaware) investors are unaware of ESG scores and simply seek to maximize their unconditional mean-variance utility. Type-A (ESG-aware) investors also have mean-variance preferences, but they use assets' ESG scores to update their views on risk and expected return. Type-M (ESG-motivated) investors use ESG information and also have preferences for high ESG scores. In other words, M investors seek a portfolio with an optimal trade-off between a high expected return, low risk, and high average ESG score. While optimizing across three characteristics (risk, return, ESG) can seem challenging, we show that the investor's problem can be reduced to a trade-off between ESG and Sharpe ratio. In other words, risk and return can be summarized by the Sharpe ratio.

Specifically, for each level of ESG, we compute the highest attainable Sharpe ratio (SR). We denote this connection between ESG scores and the highest SR by the ESG-SR frontier, as seen in Fig. 1, Panel A. The ESG-SR frontier is a useful way to illustrate the investment opportunity set when people care about risk, return, and ESG. This frontier depends only on security characteristics; that is, it is independent of investor preferences. Hence, an investment staff can first mechanically compute the frontier and then the investment board can choose a point on the frontier based on the board's preferences. Further, investors with the same information should agree on the frontier even if they prefer different portfolios on the frontier. This separation property resembles that of the standard mean-variance frontier, which also depends only on security characteristics, so investors can mechanically compute the frontier and then choose their portfolio's placement on the frontier based on risk aversion.

To understand why the ESG-SR frontier is hump-shaped, consider first the tangency portfolio known from the standard mean-variance frontier, shown in Fig. 1, Panel B. This tangency portfolio has the highest SR among all portfolios, so its ESG score and SR define the peak in the ESG-SR frontier. Further, the ESG-SR frontier is hump-shaped because restricting portfolios to have any ESG score other than that of the tangency portfolio must yield a lower maximum SR, as illustrated in Panel B.

Type-A investors choose the portfolio with the highest SR, that is, the tangency portfolio using ESG information in Fig. 1, Panel A. Type-M investors have a preference for higher ESG, so they choose portfolios to the right of the tangency portfolio, on the ESG-efficient frontier. Choosing portfolios below or to the left of the efficient frontier is suboptimal because, in this case, the investor can improve one or both of the ESG score and the SR, without reducing the other. Nevertheless, type-U investors may choose a portfolio below the frontier, because they compute the tangency portfolio while ignoring the security information contained in ESG scores (they condition on less information). Type-M investors with a small preference for ESG choose a portfolio just to the right of peak with nearly the maximum SR (higher than the SR achieved by type-U in the example depicted in Fig. 1), and type-M investors with strong preferences for ESG choose portfolios on the far right of the ESG-efficient frontier (possibly with lower Sharpe ratios than U investors).

We also derive the equilibrium security prices and returns. We show that expected returns are given by an ESG-adjusted CAPM, as seen in Fig. 2. When there are many type-U investors and when high ESG predicts high future profits, we show that high-ESG stocks deliver high expected returns.[3] This is because high-ESG stocks are profitable, yet their prices are not bid up by type-U investors, leading to high future returns. When the economy has many type-A investors, then these investors bid up the prices of high-ESG stocks to reflect their expected profits, thus eliminating the connection between ESG and expected returns. Further, if the economy has many type-M investors, then high-ESG stocks actually deliver low expected returns, because ESG-motivated investors are willing to accept a lower return for a higher ESG portfolio.

Investments, a proponent of ESG supported by the United Nations, states that its signatories manage close to $90 trillion in assets.

[2] See also Edmans (2011, p. 621), who finds that "certain socially responsible investing (SRI) screens may improve investment returns," and Nagy et al. (2015, p. 3), who find that portfolios that incorporate ESG as an investment signal "outperformed the MSCI World Index over the sample period while also increasing their ESG profile."

[3] High-ESG firms are more profitable if such firms benefit from being less wasteful, having more motivated employees, being better governed, or having customers who are willing to pay a higher price for their products. See also the literature on corporate social responsibility, e.g., Baron (2009), Benabou and Tirole (2010), Hart and Zingales (2017), and Oehmke and Opp (2020).

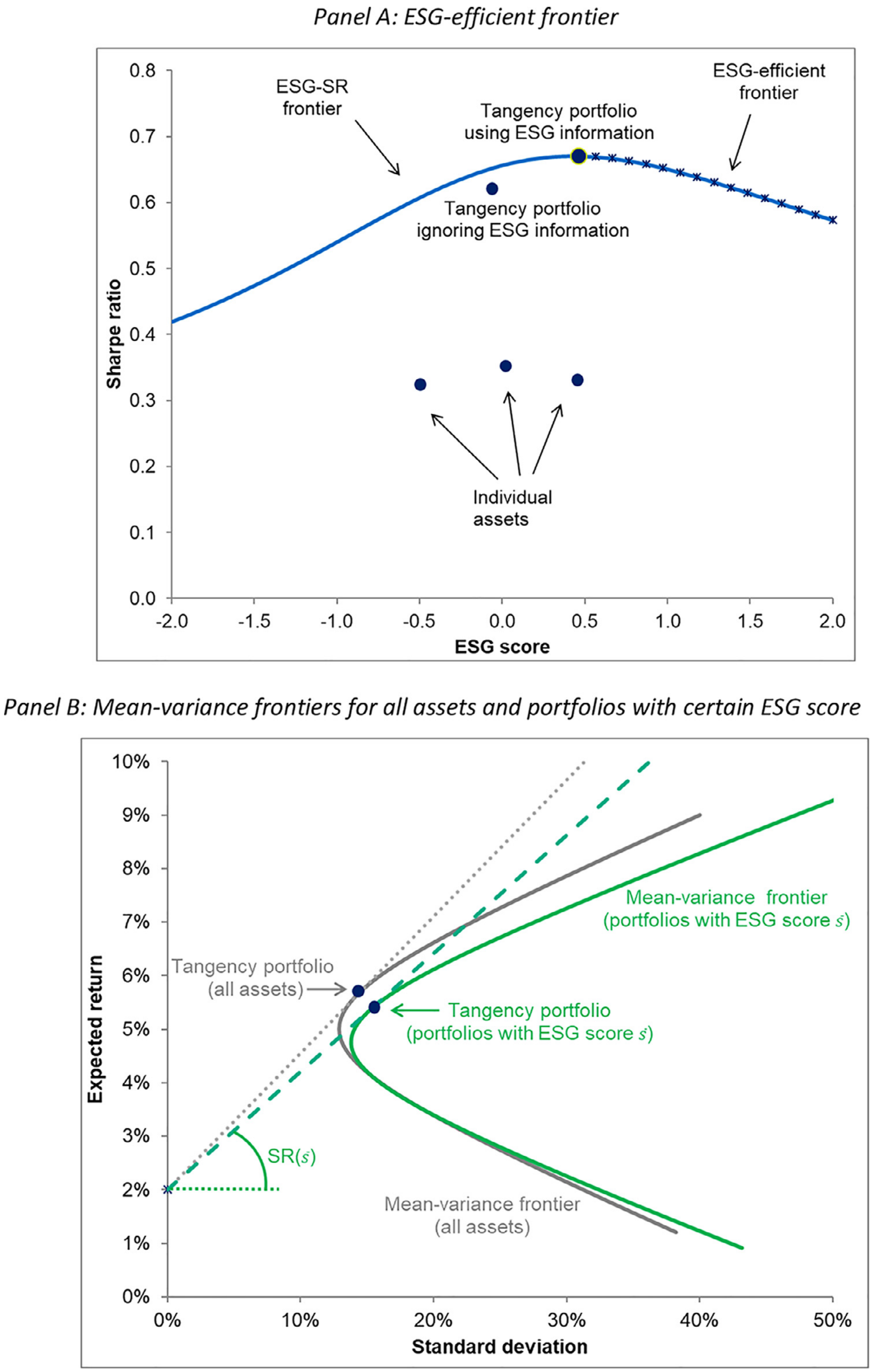

Fig. 1. Environmental, social, and governance (ESG)–efficient frontier and relation to mean-variance frontier. Panel A shows the ESG-SR frontier; that is, the maximum Sharpe ratio (on the y-axis) that can be achieved for all portfolios with a given ESG score (on the x-axis). The peak of the ESG-SR frontier is the Sharpe ratio (SR) of the standard tangency portfolio. Investors who care about both SR and ESG should choose a frontier portfolio to the right of this portfolio, on the ESG-efficient frontier. Panel B shows the standard mean-variance frontier and the corresponding standard tangency portfolio (denoted "all assets"). The slope of the line from the risk-free rate to the tangency portfolio is the maximum SR. Panel B also shows the mean-variance frontier built exclusively for portfolios with a certain ESG score, $\bar{s}$. This frontier is a hyperbola that lies inside (i.e., to the right of) the standard hyperbola, and it has its own tangency portfolio with corresponding Sharpe ratio $SR(\bar{s})$. This Sharpe ratio defines a point on the ESG-SR frontier: $\{\bar{s}, SR(\bar{s})\}$.

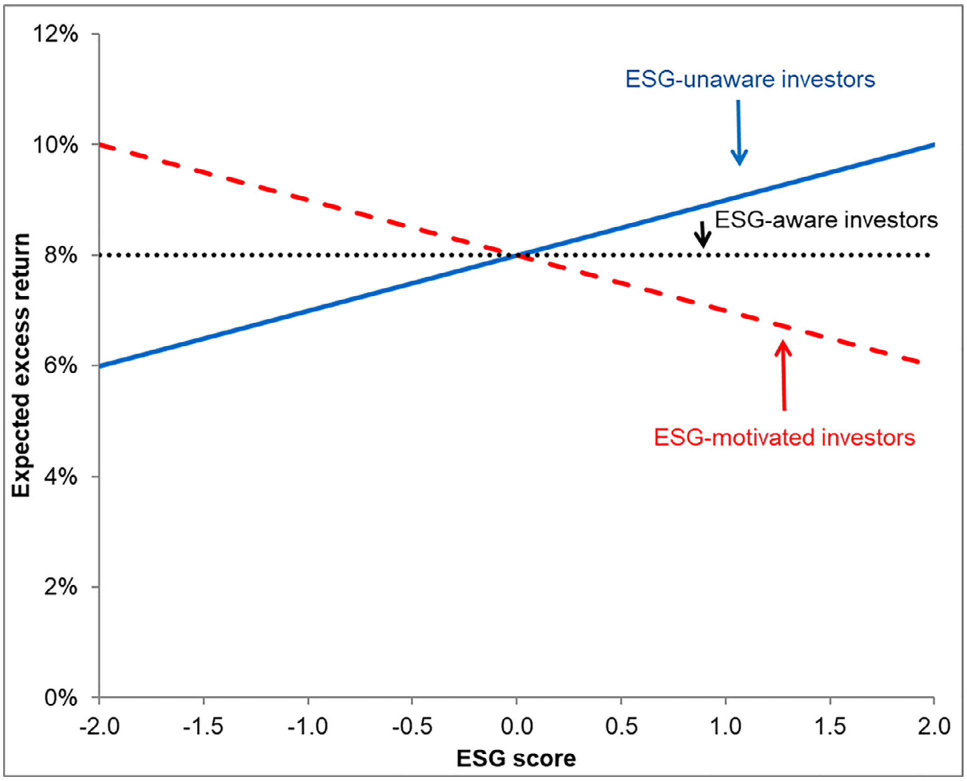

Fig. 2. Environmental, social, and governance–adjusted capital asset pricing model (ESG-CAPM).

To illustrate how the theory can be used in practice and investigate its testable implications, we consider empirical proxies for E, S, G, and overall ESG. As a measure of E (i.e., how green a company is), we compute each company's carbon intensity. As a measure of S, we use the sin stock indicator defined as in Hong and Kacperczyk (2009). As a measure of G, we compute how (un)aggressive a company is in its accounting choices based on the accruals in the financial statements (Sloan, 1996). As a measure of overall ESG, we use the aggregate ESG score produced by MSCI, a leading provider of ESG ratings.

We begin by empirically estimating the ESG-efficient frontier for some of these ESG proxies. The shape of the empirical frontier naturally depends on whether ESG predicts returns. Hence, we consider a frontier for a proxy that predicts returns in our sample (G) and one that does not (E). Given that G predicts returns, both benefits and costs accrue to ESG investing using this proxy. Starting with the benefit of ESG *information*, we find that the maximum SR that incorporates this ESG proxy is about 12% higher than the maximum SR that ignores such information (corresponding to the vertical difference between the two tangency portfolios in Fig. 1, Panel A). For the cost of ESG *preferences*, doubling the average ESG score relative to the level that maximizes the SR leads to a reduction in SR of only 3%.

When we estimate the ESG-SR frontier using E (carbon), we find little ex post improvement to the Sharpe ratio of an investor who incorporates such information in her portfolio decision. The frontier is still useful, however, because it shows the SR cost of tilting toward a less carbon intensive portfolio, a cost that is empirically small even for a significant reduction in carbon. In summary, these frontiers show a responsible investor's opportunity set, quantifying the costs and benefits of using ESG in investing.

We also study a common way of incorporating ESG into a portfolio: restricting the investment universe by removing the assets with the weakest ESG scores. We find a seemingly counterintuitive result that investors who screen out assets with the worst ESG characteristics may build optimal portfolios that have *lower* aggregate ESG scores than portfolios of investors who do not impose ESG-type restrictions. This happens because unconstrained investors can short poor ESG assets to hedge out risks or to finance larger positions in high-ESG assets. Not surprisingly, limiting the breadth of the investment universe detracts from financial outcomes as well. The ESG-SR frontier for investors who screen out poor ESG stocks is strictly dominated by the unconstrained frontier.

Finally, we carry out a series of theory-motivated empirical tests that help explain how the four ESG proxies we consider correlate with returns. To help explain why our measure of G predicts returns, we first show that this aspect of governance positively predicts future profitability. We also observe some increase in investor demand for stocks of this type, but not to the point of making them more expensive compared with other stocks. In fact, stocks with attractive G trade at relatively cheaper Tobin's q. So, G could predict returns in our sample because investors did not fully appreciate that G predicts profitability. Our measure of S (not being a sin stock) predicts returns negatively as shown by Hong and Kacperczyk (2009), although the statistical significance is limited in our tests. To understand why, we show that this measure of S predicts profits negatively and high S is associated with stronger investor demand. Finally, we find that our two remaining proxies, E (carbon intensity) and overall ESG (from MSCI), correlate positively with investor demand and high valuations. These proxies do not have a statistically significant link to returns in our data, perhaps because of the much shorter sample periods.

We contribute to the literature both theoretically and empirically. A growing theoretical literature on ESG follows Merton (1987) and assumes that ESG-sensitive investors refuse to hold certain assets. For example, Heinkel et al. (2001), Luo and Balvers (2017), and Zerbib (2020) show that, in equilibrium, such market segmentation leads to higher expected returns to non-green companies.

Besides allowing such segmentation, we explicitly model many assets characterized by ESG scores in addition to the standard risk-return characteristics.[4] Based on this general setting, we derive several interesting properties of the solution to the portfolio problem with parallels to the classic Markowitz solution, including the novel result that the ESG-SR frontier characterizes the solution, under certain conditions. Further, we show when ESG should predict returns positively or negatively in equilibrium.

Empirically, our research bridges the gap between papers arguing that ESG hurts performance and those arriving at the opposite conclusion. The former group, based on the segmentation theories, is supported by empirical literature showing that sin stocks (alcohol, tobacco, and gaming, which can be seen as a poor S in ESG) generate positive abnormal returns (Hong and Kacperczyk, 2009). The sin premium parallels the finding of Baker et al. (2018) that “green municipal bonds are issued at a premium to otherwise similar ordinary bonds.” Papers in the latter group show that stocks with good governance (the G in ESG) generate positive abnormal returns (Sloan, 1996; Gompers et al., 2003) as do stocks with higher employee satisfaction (part of the S of ESG) (Edmans, 2011). Our model and empirical results help explain these opposing findings. We submit that ESG is a positive return predictor if ESG is a positive predictor of future firm profits and the value of ESG is not fully priced in the market. Further, the model predicts that this relation can be weakened with ESG becoming a neutral return predictor when most investors see the value in ESG and even flips sign, with ESG becoming a negative predictor of returns, when investors are willing to accept lower returns for more responsible stocks. So, according to our model, the results of Hong and Kacperczyk (2009) arise because their measure of sin stocks (belonging to the industries related to alcohol, tobacco, and gaming) is associated with low investor demand, while the ESG measures of Gompers et al. (2003) and Edmans (2011) are related to higher firm profits in a way that the market has not fully appreciated.[5]

Our paper is also linked to the economic theories of discrimination: taste-based discrimination (Becker, 1957) and statistical discrimination (Phelps, 1972). Indeed, ESG scores play a dual role in our model because ESG affects investor preferences both directly (a kind of taste-based discrimination) and indirectly because ESG scores are informative of risk and expected returns (a form of statistical discrimination). In equilibrium, the interplay between these two dimensions allows for a variety of potential outcomes. This flexibility is important, because the empirical literature suggests that the link between ESG and returns is not trivial. Certain ESG measures predict returns positively while others predict negatively, which highlights the need for a theoretical framework that allows for a similar flexibility in outcomes, with testable predictions of when each applies.

2. Portfolio choice with ESG: the ESG-efficient frontier

2.1. Model: Markowitz meets sustainability goals

We examine an investor's problem of choosing a portfolio of n risky assets and a risk-free security. The risk-free return is r^f, and the risky assets have excess returns collected in the vector of random variables denoted by $r = (r^1, .., r^n)'$. The assets have an ESG scores given by $s = (s^1, .., s^n)'$.

We consider three types of investors. Type-U investors are uninterested or unaware of ESG scores. They take expected excess returns to be $E(r)$ with risk given by the variance-covariance matrix, $\text{var}(r)$. Type-A (ESG-aware) investors use ESG scores to update their views on risk and expected return. They use assets' expected excess return, $\mu = \text{E}(r|s)$, conditional on the ESG information s, and the conditional variance-covariance matrix of excess returns $\Sigma = \text{var}(r|s)$.[6] Type-M (ESG-motivated) investors use ESG information and also have preferences for high ESG scores. The portfolio problem for U and A investors has the stan-

[4] In our model, ESG-motivated investors have a preference for stocks with high ESG, but, mathematically, these investors' utility could in principle capture a preference for any security characteristic. The only other models of this form with many assets that we are aware of are provided by Fama and French (2007), who consider a model of investor “taste”, Baker et al. (2018), who consider a model in which some investors prefer green bonds, and Pastor et al. (2019) and Zerbib (2020), who consider ESG scores. These papers assume that the relevant characteristic, e.g., ESG, has a linear effect on utility, essentially changing expected returns, whereas we consider more general ESG preferences. Further, these papers do not derive the ESG-SR frontier or our other theoretical results, except the finding that the preferred assets could have lower expected returns in equilibrium. See also Gollier and Puget (2014) and Friedman and Heinle (2016), who consider a single risky asset to study issues related to corporate engagement of responsible investors.

[5] Bebchuk et al. (2013) find that the return predictability associated with the governance indicator of Gompers et al. (2003) has disappeared, conjecturing an explanation based on investor learning. We find that the governance metric of Sloan (1996) based on accruals has continued to predict returns post-publication.

[6] An active debate is ongoing about whether ESG has an effect on valuations and, even more so, whether it is relevant to future risks or returns. For example, Flammer (2015) and Kruger (2015) provide supportive evidence for valuations and returns, and Dunn et al. (2018), Ilhan et al. (2018), and Hoepner et al. (2019) show that ESG correlates with risks.

dard Markowitz solution, so we focus here on the solution for type-M investors. Section 3 discusses equilibrium asset prices with all three types of investors.

Investor M starts with a wealth of W and chooses a portfolio of risky assets, $x = (x^1, \dots, x^n)'$, where x^i is the fraction of capital invested in security i or, said differently, the investor buys $x^i W$ dollars' worth of security i. The investor's utility depends on her future wealth and the ESG characteristics of the portfolio. Given her portfolio choice, the investor's future wealth is

$$\widehat{W} = W\left(1 + r^f + x'r\right). \tag{1}$$

The investor seeks to maximize her utility U over final wealth W and average ESG score, $\bar{s} = \frac{x's}{x'1}$, given the extended mean-variance framework

$$U = E\left(\widehat{W}|s\right) - \frac{\bar{\gamma}}{2} Var\left(\widehat{W}|s\right) + W f(\bar{s}). \tag{2}$$

Here, $\bar{\gamma}$ is the absolute risk-aversion parameter and $f : \mathbb{R} \to \mathbb{R} \cup \{-\infty\}$ is the ESG preference function.[7] The ESG preference function depends on the average ESG score among the risky asset positions (i.e., $\bar{s}$ is the weighted sum of ESG scores, scaled by the total position in risky assets, $x'1$), meaning that the investor gets no ESG utility from investing in the risk-free asset. We consider more general ESG preference functions in Section 2.4. The overall utility can be written as

$$\begin{aligned} U &= W\left(1 + r^f + x'\mu\right) - \frac{\bar{\gamma}}{2} W^2 x'\Sigma x + W f\left(\frac{x's}{x'1}\right) \\ &= W\left(1 + r^f + x'\mu - \frac{\gamma}{2} x'\Sigma x + f\left(\frac{x's}{x'1}\right)\right), \end{aligned} \tag{3}$$

where $\gamma = \bar{\gamma} W$ is the relative risk aversion. Hence, by dropping constant terms, the utility maximization problem is

$$\max_{x \in X} \left(x'\mu - \frac{\gamma}{2} x'\Sigma x + f\left(\frac{x's}{x'1}\right)\right), \tag{4}$$

where the set of feasible portfolios is $X = \{x \in \mathbb{R}^n | x'1 > 0\}$, that is, all long-biased portfolios (generalized sets of allowed portfolios are discussed in Section 2.3). We consider portfolios that invest at least as much long as short because defining the overall ESG characteristic for a portfolio that is short overall is difficult, but, in principle, the framework can be applied more generally.

2.2. Solution: ESG-SR frontier

We now solve an ESG-motivated investor's portfolio problem. Because the objective function depends on the ESG scores, s, the optimal portfolio depends on these scores.

In a standard mean-variance analysis, the investor optimally combines the tangency portfolio with the risk-free security. The tangency portfolio is the portfolio that maximizes the Sharpe ratio, namely, the expected excess return divided by the standard deviation of excess returns. To generalize this idea, we consider the maximum SR for each level of ESG score. The maximum SR that can be achieved with an ESG score of $\bar{s}$ is denoted the ESG-SR frontier, $SR(\bar{s})$:

$$SR(\bar{s}) = \max_{\substack{x \in X \\ \text{s.t. } \bar{s} = \frac{x's}{x'1}}} \left(\frac{x'\mu}{\sqrt{x'\Sigma x}}\right) = \max_{\substack{x \\ \text{s.t. } x'1 = 1 \\ \text{and } x's = \bar{s}}} \left(\frac{x'\mu}{\sqrt{x'\Sigma x}}\right). \tag{5}$$

In order to use this definition of the highest Sharpe for each ESG level, we first rewrite the utility maximization problem Eq. (4) as

$$\max_{\bar{s}} \left[\max_{\sigma} \left\{ \max_{\substack{x \in X \\ \text{s.t. } \bar{s} = \frac{x's}{x'1} \\ \sigma^2 = x'\Sigma x}} \left(x'\mu - \frac{\gamma}{2}\sigma^2 + f(\bar{s})\right) \right\} \right]. \tag{6}$$

This expression means that the investor's problem can be thought of as first choosing the best portfolio given a level of risk σ and an ESG score $\bar{s}$ and then maximizing over σ and $\bar{s}$. The former problem is solved by choosing the portfolio with the highest SR for the given ESG score (a more detailed proof is given in the Appendix), which yields

$$\max_{\bar{s}} \left[\max_{\sigma} \left\{ SR(\bar{s})\sigma - \frac{\gamma}{2}\sigma^2 + f(\bar{s}) \right\} \right]. \tag{7}$$

The optimal level of risk is given by $\sigma = SR(\bar{s})/\gamma$. Inserting this risk level and simplifying the expression results in Proposition 1.

Proposition 1 (ESG-SR trade-off). *The investor should choose her average ESG score $\bar{s}$ to maximize the following function of the squared Sharpe ratio and the ESG preference function f:*

$$\max_{\bar{s}} \left[(SR(\bar{s}))^2 + 2\gamma f(\bar{s}) \right]. \tag{8}$$

This proposition shows how investors optimally trade off ESG and Sharpe ratios. Not surprisingly, ESG affects the optimal portfolio choice, given that ESG is in the utility function, but the interesting result here is that we can analyze this trade-off using a part that depends only on securities [the ESG-SR frontier, $SR(\bar{s})$] and another part that depends only on preferences [$2\gamma f(\bar{s})$]. In other words, just like the standard Markowitz theory is powerful because the mean-variance frontier can be computed independent of preference parameters and then decisions about what portfolio to pick are based on risk aversion, the ESG-SR frontier can be computed independent of preferences and then the investor can decide in the end where on the frontier to place herself. Put differently, the ESG-SR frontier summarizes all security-relevant information. The investor's problem is to first place herself on the ESG-SR frontier and then decide on the amount of risk. This method works because investors care about the average

[7] Economists generally hesitate to add arguments to the utility function because this flexibility means that almost any outcome can be justified, but, here, we simply formalize the intentions of investors who control trillions of dollars, as discussed in the Introduction. We allow that the ESG preference function takes the value $-\infty$ to capture screens, as discussed in Section 2.3.

ESG, which does not change when the investor chooses the risk level in the second step by choosing her cash holding. If investors care about total ESG, $x's$, instead of average ESG, then the investor's problem cannot be summarized as the ESG-SR frontier, which also shows that our frontier results are not trivial.

Understanding the ESG-SR frontier shows how differences in risk aversion and differences in ESG preferences can be distinguished. If a group of investors have no direct preferences for ESG ($f \equiv 0$) but differ in their risk aversion γ, then all these investors should invest in the same portfolio of risky assets (i.e., with the same Sharpe ratio and average ESG score), but the more risk tolerant should put a larger fraction of their wealth in this portfolio (i.e., own less cash instruments). If a group of investors have the same risk aversion but differ in their ESG preferences, then investors with stronger ESG preferences should buy a portfolio with lower SR, but higher average ESG score. Interaction effects also exist. If a group of investors care equally about ESG but differ in their risk aversion, then an investor with higher risk aversion not only puts more money in the risk-free asset, but she also tilts her portfolio toward higher ESG and lower SR. Mathematically, this behavior is due to the fact that the second term in Eq. (8) is $\gamma f(\bar{s})$, and, economically, this interaction is due to the fact that SR matters less when an investor is more risk averse (because she knows that she will take less risk anyway), so, in relative terms, ESG becomes more important. More generally, observing an investor's portfolio of risky assets and its placement on the ESG-SR frontier is revelatory about $\gamma f(\bar{s})$; observing the investor's cash position (or leverage), about the risk aversion γ.

We next characterize how the maximum Sharpe ratio depends on the ESG score. We use the notation $c_{ab} = a'\Sigma^{-1}b \in \mathbb{R}$ for any vectors $a, b \in \mathbb{R}^n$.

Proposition 2 (ESG-SR frontier). *The maximum Sharpe ratio, $SR(\bar{s})$, that can be achieved with an ESG score of $\bar{s}$ is*

$$SR(\bar{s}) = \sqrt{c_{\mu\mu} - \frac{\left(c_{s\mu} - \bar{s}c_{1\mu}\right)^2}{c_{ss} - 2\bar{s}c_{1s} + \bar{s}^2 c_{11}}}. \tag{9}$$

The maximum Sharpe ratio across all portfolios is $SR(s^*) = \sqrt{c_{\mu\mu}}$, which is attained with an ESG score of $s^* = c_{s\mu}/c_{1\mu}$. Increasing the ESG score locally around s^* leads to nearly the same Sharpe ratio, $SR(s^* + \Delta) = SR(s^*) + o(\Delta)$, because the first-order effect is zero, $\frac{dSR(s^*)}{ds} = 0$.

We next consider the nature of the optimal portfolio weights for an ESG-aware investor.

Proposition 3 (four-fund separation). *Given an average ESG score $\bar{s}$, the optimal portfolio is*

$$x = \frac{1}{\gamma}\Sigma^{-1}(\mu + \pi(s - 1\bar{s})) \tag{10}$$

as long as $x'1 > 0$, where

$$\pi = \frac{c_{1\mu}\bar{s} - c_{s\mu}}{c_{ss} - 2c_{1s}\bar{s} + c_{11}\bar{s}^2}. \tag{11}$$

The optimal portfolio is therefore a combination of the risk-free asset, the tangency portfolio, $\Sigma^{-1}\mu$, the minimum-variance portfolio, $\Sigma^{-1}1$, and the ESG-tangency portfolio, $\Sigma^{-1}s$.

The optimal portfolio looks the same as the standard Markowitz solution, except that the expected excess returns μ have been adjusted. In other words, the optimal portfolio can be found as follows. The investor first compute ESG-adjusted expected returns, $\mu + \pi(s - 1\bar{s})$, in the sense that each stock's expected excess return is increased if its ESG score s_i is above the desired average score $\bar{s}$; otherwise, it is lowered. The amount of adjustment depends on the scaling parameter π, or the strength of the preference for ESG.[8] Next, the investor compute the optimal portfolio found in the standard way, that is, multiplying by $\frac{1}{\gamma}\Sigma^{-1}$. Therefore, all investors, regardless of their risk aversion and ESG preferences, should choose a combination of four portfolios (or funds): the risk-free asset, the standard tangency portfolio, the minimum variance portfolio, and the portfolio that we call the ESG-tangency portfolio. The ESG-tangency portfolio is the tangency portfolio if we replace the expected excess returns with the ESG scores.

2.3. Example: how investors choose portfolios using the ESG-SR frontier

Fig. 3, Panel A, illustrates how the ESG-motivated investor M chooses her portfolio using the ESG–Sharpe ratio frontier. For every ESG level, she finds the portfolio with the highest SR. One way to think about this step is that the investor computes a standard mean-variance frontier for all portfolios with this level of ESG as illustrated in Fig. 1, Panel B. Then, the investor computes the maximum Sharpe ratio as the slope of the line that goes from the risk-free security to the tangency portfolio (again, based only on portfolios with this ESG level). The investor collects all these Sharpe ratios and plots them against the ESG levels as seen in Fig. 3, Panel A. The Appendix further explains the connection between the standard mean-variance frontier and the ESG-SR frontier.

Panel A also shows investor M's indifference curves. These curves slope down because investor M likes high Sharpe ratios and high ESG scores and can trade off one versus the other to remain indifferent about all portfolios on each indifference curve. Investor M's utility is maximized at the point where her indifference curve is tangent to the ESG-SR frontier. This solution is not the global maximum of the Sharpe ratio, as the investor optimally chooses a higher level of ESG to satisfy her nonfinancial preference for ESG.

This solution contrasts with that of our ESG-aware investor A, depicted in Fig. 3, Panel B. Investor A also considers ESG information to build a better forecast of returns but does not have any direct (nonfinancial) preference for ESG. That is, he would tilt toward portfolios with high ESG (or, for that matter, with low ESG) only in as much as they help maximize the investment outcome. This means that the investor has horizontal indifference curves, illustrating that his preference depends only on the Sharpe ratio.

[8] When $\pi = 0$, portfolio choice simplifies to the traditional mean-variance optimization.

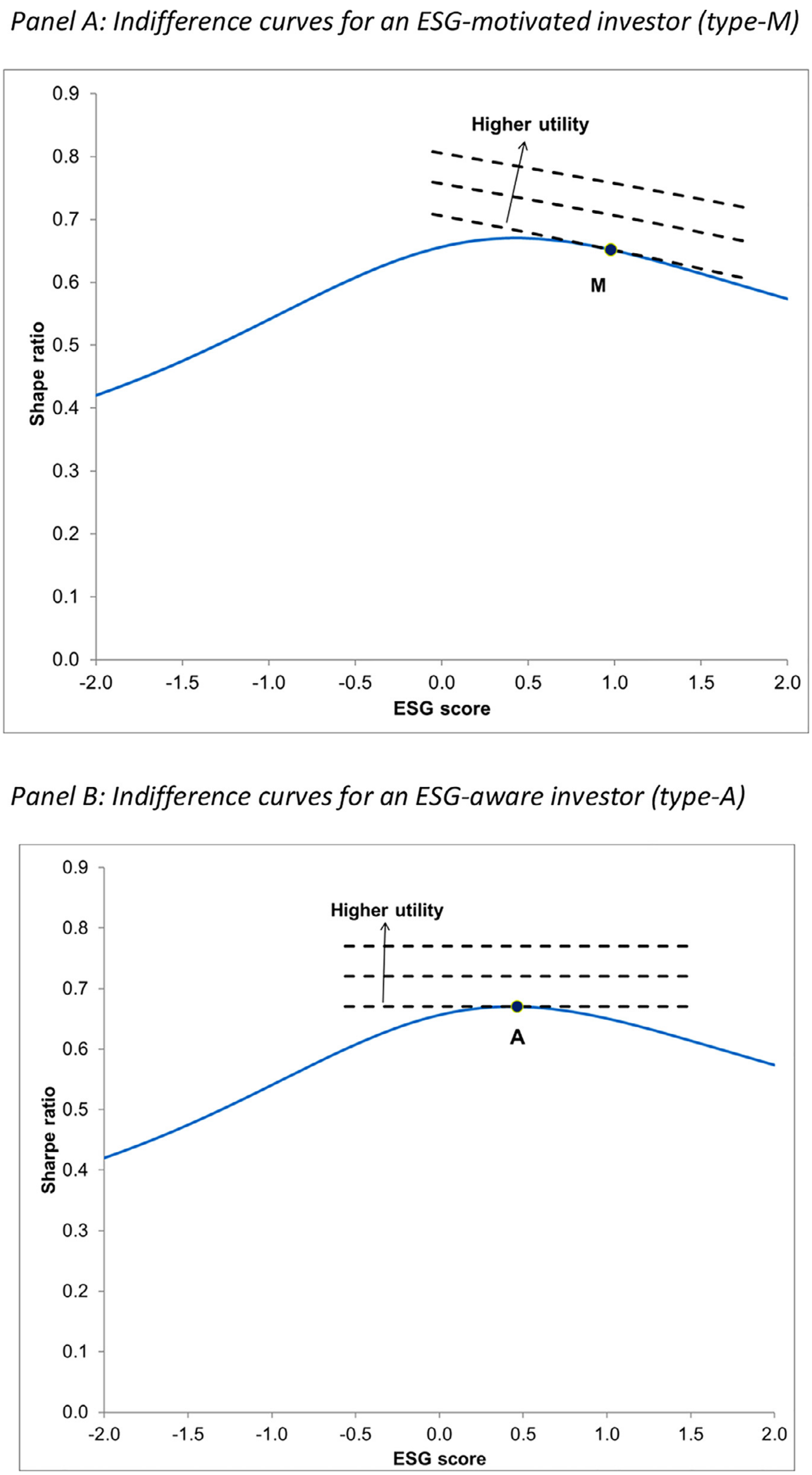

Fig. 3. ESG–efficient frontier and investor indifference curves. This figure shows examples of an ESG–Sharpe ratio frontier (solid line) and an investor's indifference curves (dashed lines). Panel A draws an ESG-motivated investor's indifference curves. This type-M investor's utility increases in both the Sharpe ratio and the ESG score of her portfolio, yielding a trade-off illustrated by the downward-sloping indifference curves. Panel B draws an ESG-aware investor's indifference curves, which are horizontal because this type of investor does not derive direct utility from ESG.

We can also imagine that this investor considers the ESG–Sharpe ratio frontier but would always choose the portfolio with the highest possible Sharpe.

Finally, investor U solves a standard mean-variance optimization just like investor A, except that U computes potentially different estimates of risk and expected returns. We illustrate this when we estimate the empirical ESG-SR frontier in Section 4.2.

2.4. Generalized ESG preferences

Some investors use screens to help implement their ESG views. For example, an investor can screen out any stock with a low ESG score, for example, $s^i < 0.2$. The previous analysis naturally holds for the subset of non-screened stocks. We can also incorporate such screens more directly by changing the set of allowed portfolios to $X = \{x \in \mathrm{R}^n | x'1 > 0, \forall i\; x^i = 0 \text{ if } s^i < s^*\}$. Zerbib (2020) also models screens combined with ESG preferences and empirically analyzes their effects.

Some investors prefer to exclude short positions, which can be captured by $X = \{x \in \mathrm{R}^n_+\}$, or both short positions and screened stocks $X = \{x \in \mathrm{R}^n_+ | \forall i\; x^i = 0 \text{ if } s^i < s^*\}$. Investors can achieve a better risk-return trade-off if they allow shorting, and shorting low-ESG stocks could be consistent with ESG preferences.[9] Hence, investors can require that their position in low-ESG stocks be zero or negative, that is, $X = \{x \in \mathrm{R}^n | x'1 > 0, \forall i\; x^i \leq 0 \text{ if } s^i < s^*\}$. For any of these restrictions, we can use the following result because all these portfolio sets are cone-shaped. We say that X is cone-shaped if $x \in X$ implies that $ax \in X$ for all $a > 0$ (said differently, X depends only on $x/x'1$).

Proposition 4 (ESG-SR frontier with screens). *The conclusion of Proposition 1 continues to hold for any cone-shaped X.*[10]

We can consider even more general ESG utility functions of the form $e(x, s) : X \times \mathrm{R}^n \to \mathrm{R} \cup \{-\infty\}$, where $X \subseteq \mathrm{R}^n$ is a cone-shaped set of allowed portfolios. We assume that the ESG utility function is homogeneous of degree zero with respect to portfolios, that is, $e(ax, s) = e(x, s)$ for any $a > 0$. This is a natural assumption because it means that the cash holding does not affect the ESG utility. For example, the portfolio $x = (0.2, 0.2)$ means that 20% of assets are put in each risky asset and the rest, 60%, is in cash, and the portfolio $2x = (0.4, 0.4)$ means that twice as much money is put in the same portfolio of risky assets, leaving only 20% in cash. Homogeneity means that the same ESG utility results because the risky portfolio is the same. This homogeneity is what allows the investor to first focus on the optimal combination of the Sharpe ratio and portfolio-level ESG score and then decide on the amount of risk.

One interesting example is $e(x, s) = f(\frac{x's}{\sqrt{x'\Sigma x}})$, where the investor cares about how much ESG she gets per unit of risk. This specification has the advantage that it also works for long-short portfolios with $x'1 = 0$ and it retains much of the tractability of the specification considered earlier.

The generalized ESG preference function can capture screens by having $e(x, s) = -\infty$ for all portfolios, where $x_i \neq 0$ for any security with $s_i < 0.2$. A screen can be seen as an extreme version of nonlinear preferences across the stocks' ESG scores. In other words, an investor perhaps does not view a portfolio of three stocks with ESG scores of (0.1, 0.8, 0.9) the same as one with (0.6, 0.6, 0.6) even if they have the same average, because the former has one very low-ESG stock. Instead of capturing this idea with a screen, a less extreme (and still tractable) version would be $e(x, s) = e_1 \frac{x's}{x'1} - e_2 \frac{x' \operatorname{diag}(\frac{1}{s_1}, \ldots, \frac{1}{s_n}) x}{(x'1)^2}$, where $e_1, e_2 \in \mathrm{R}$ are parameters. Here, the utility is more penalized if the investor has a stock with an ESG score close to zero. In any event, the investor can still think in terms of an ESG-SR frontier as seen from Proposition 5.

Proposition 5 (generalized ESG-SR frontier). *If the investor has generalized ESG preferences $e(x, s)$, then the investor's problem is*

$$\max_{\bar{e}} \left[\frac{(SR(\bar{e}))^2}{2\gamma} + \bar{e} \right], \tag{12}$$

where $SR(\bar{e})$ is the maximum Sharpe ratio for a given level of ESG utility:

$$SR(\bar{e}) = \max_{\substack{x \in X \\ \text{s.t. } \bar{e} = e(x,s)}} \left(\frac{x'\mu}{\sqrt{x'\Sigma x}} \right). \tag{13}$$

Finally, the theory can also work if each security has a multidimensional ESG score (e.g., one score for environmental concerns, another for social, and a third for governance, with investors having preferences over such combinations).

Having characterized the solution to the ESG-aware portfolio problem in a variety of cases, we note that such a solution exists under certain conditions.[11] Instead of going into theoretical details, the empirical Section 4.2 shows the practical applicability of the framework.

3. Equilibrium asset pricing with ESG

3.1. ESG-adjusted CAPM

Having solved the Markowitz problem with ESG investors, we next endogenously derive security prices and returns. We consider an overlapping-generations (OLG) economy in which, at time t, security prices are $p_t = (p^1_t, .., p^n_t)'$ and excess returns from time $t-1$ to t are $r_t = (r^1_t, .., r^n_t)'$. The exogenous variables are the ESG scores

[9] In the approach based on the average ESG score, the optimal portfolio can include short positions, and this approach gives the investor credit if the short positions have lower ESG scores than the long ones. Fitzgibbons et al. (2018) argue that ESG-sensitive investors should be willing to short low-ESG stocks.

[10] The definition [Eq. (4)] of the SR function must depend on the same set of allowed portfolios, X.

[11] A sufficient condition for existence is that the ESG preference function f is continuous, we consider a compact space of ESG levels, $\bar{s} \in [s_{min}, s_{max}]$, and for all such ESG levels, the portfolio x in Eq. (10) satisfies $x'1 > 0$. In this case, for any $\bar{s}$, an optimal portfolio is given in Eq. (10) with a resulting objective function Eq. (8) that is continuous in $\bar{s}$, and any continuous function attains its maximum on a compact space.

s, the risk-free rate r^f, the security dividend payoffs $v_t = (v_t^1, .., v_t^n)'$, and the shares outstanding of each stock, normalized to one. We denote the total market dividend by $v_t^m = v_t^1 + \ldots + v_t^n$ and assume that dividends are independent and identically distributed (i.i.d.) over time. We model the informational value of ESG scores as $\mathrm{E}(v_t|s) = \hat{\mu} + \lambda(s - s^m)$, where $s^m = \sum_i m^i s^i$ is the weighted-average ESG score of the market portfolio, $m^i = p^i / \sum_j p^j$ is the weight of the market portfolio in stock i, and the parameter $\lambda \in \mathrm{R}$ determines how informative ESG scores are for future profits. A positive λ means that more ESG friendly firms are also more profitable on average, and a negative λ has the reverse interpretation.

Recall that the economy has three types of investors. Type-U investors do not use ESG information at all: They have no preference for ESG (i.e., their ESG preference function is $f_U \equiv 0$), and they ignore the informational value of ESG signals s, assuming that the best forecast of future dividends is the unconditional mean $\hat{\mu} = \mathrm{E}(v)$ and payoff risk is taken to be $\hat{\Sigma} = \mathrm{var}(v)$. ESG-aware type-A investors also do not enjoy ESG utility ($f_A \equiv 0$), but they exploit ESG to update their views on securities, using $\bar{\mu} = \mathrm{E}(v|s)$ as the expected payoff and $\bar{\Sigma} = \mathrm{var}(v|s)$ to capture payoff risk. ESG-motivated type-M investors use ESG information and have a preference for a high average ESG score. A new generation of investors appears each time period, with type-U investors born with wealth W^U and similarly for types A and M, and the aggregate wealth is $W = W^U + W^A + W^M$. Investors live for one period, and market clearing requires that the total demand for shares from all young investors equals the shares outstanding.

We are looking for equilibrium prices p_t and excess returns r_t and start by noticing that these are related as

$$r_t^i = \frac{v_t^i + p_t^i}{p_{t-1}^i} - 1 - r^f. \tag{14}$$

We focus on the steady-state equilibrium in which prices (and expected returns) are constant, $p_t = p$ for all t. In such an equilibrium, excess returns are simply given by $r_t^i = \frac{v_t^i}{p^i} - r^f$, and the return variance is driven by dividend risk as prices are constant. Such a steady-state equilibrium exists because, over time, dividends are i.i.d., ESG scores are constant, and the wealth of different investor types is constant. If we did not make these assumptions, each security price would depend on its current ESG score and the current investor ESG sentiment (as summarized by the total π_t from Proposition 3), leading to interesting dynamics. For example, a security's return variance would suddenly also depend on the risk of changes in the overall ESG investor sentiment, changes in the stock's own ESG score, changes in how ESG predicts dividends (e.g., because of changes in customer demand for green products), and the covariances of all shocks. Here we focus on the steady state for simplicity.[12]

Let us consider equilibrium implications of the model, starting with the simplest cases in which all investors are of the same type. If all investors ignore ESG (i.e., all are type-U), then we are back to a standard CAPM equilibrium. All investors hold the unconditional tangency portfolio, that is, the portfolio that maximizes SR relative to their information set, which ignores ESG. The tangency portfolio equals the market portfolio, and each security's expected excess return is driven by its unconditional market beta, $\beta^i = \frac{\mathrm{cov}(r_t^i, r_t^m)}{\mathrm{var}(r_t^m)}$. What is new here is that a (small) investor who understands that ESG scores are informative can exploit this insight. Proposition 6 characterizes the equilibrium.

Proposition 6. If all investors are ESG-unaware, i.e., *of type-U ($W^A = W^M = 0$), then any security i has steady-state equilibrium price*

$$p^i = \frac{\hat{\mu}^i - \frac{\gamma}{W}\,\mathrm{cov}\left(v^i, v^m\right)}{r^f}. \tag{15}$$

Unconditional expected excess return obeys the standard unconditional CAPM:

$$E\left(r_t^i\right) = \beta^i E(r_t^m), \tag{16}$$

but conditional expected returns are given by

$$E\left(r_t^i|s\right) = \beta^i E(r_t^m) + \lambda \frac{s^i - s^m}{p^i}. \tag{17}$$

This proposition provides several intuitive results. First, the price [Eq. (15)] of any firm's equity is given by its expected cash flow payoff ($\hat{\mu}^i$) less a risk premium $[\frac{\gamma}{W}\mathrm{cov}(v^i, v^m)]$, discounted by the risk-free rate. Second, expected excess returns [Eq. (16)] are driven by market betas from the perspective of an investor who ignores ESG scores. Third, from the perspective of an investor who uses ESG scores, Eq. (17) shows that stocks returns have alphas relative to the CAPM that depend linearly on ESG. If a high-ESG score is indicative of a high future profit, that is, if $\lambda > 0$, then stocks with ESG scores above average have higher conditional expected returns than those with below-average ESG scores. This is in line with the empirical findings such as those of Gompers et al. (2003), who show that an ESG-type metric (governance) earns CAPM alphas.[13] Market prices adjust when more investors are aware that this type of information could be relevant. At the extreme, all market participants incorporate it into their decision, as in the case that we consider next.

Suppose that all investors use ESG signals, but without ESG preferences (i.e., all are ESG-aware of type-A). In this case, we get a conditional CAPM equilibrium, and investors can no longer profit from using the informational value of ESG scores because this information is already incorporated into prices. This theoretical prediction is in line with the empirical finding of Bebchuk et al. (2013), who argue that market participants have gradually learned about the usefulness of governance and have impounded it into prices. Consequently, they show that the measures from Gompers et al. (2003) do not predict abnormal returns out-of-sample.

[12] Pastor et al. (2019) consider a simplified three-period model with ESG risk, deriving an interesting two-factor model in which required returns depend on the covariance with the market and an ESG factor.

[13] The model is also consistent with $\lambda < 0$, when ESG is in conflict with financial outcomes (e.g., when corporations engage in charity).

Finally, suppose that all investors use ESG in their signals and in their identical ESG preferences (i.e., all type-M). Such ESG preferences change the equilibrium in an interesting way. To derive this equilibrium, we first note that returns [Eq. (14)] can be written in vector form as

$$r_t = \mathrm{diag}\left(\frac{1}{p^i}\right)v_t - r^f, \tag{18}$$

where $\mathrm{diag}(\frac{1}{p^i})$ means the diagonal matrix with elements $(\frac{1}{p^1}, \ldots, \frac{1}{p^n})$. Any investor clearly wants to maximize the SR for the chosen ESG score. Further, in equilibrium, all investors must choose the market portfolio, which must therefore maximize for SR among all portfolios with an ESG equal to that of the market, s^m. Based on Proposition 3, any investor buys the following portfolio:

$$\begin{aligned} x = &\frac{1}{\gamma}\mathrm{diag}\left(p^i\right)\bar{\Sigma}^{-1}\mathrm{diag}\left(p^i\right) \\ &\times\left(\mathrm{diag}\left(\frac{1}{p^i}\right)\bar{\mu} - r^f + \pi\left(s - 1s^m\right)\right). \end{aligned} \tag{19}$$

The total wealth invested in each stock is Wx, where W is the aggregate wealth, and the total dollar supply is p because shares outstanding are normalized to one. Hence, the equilibrium condition is $p = Wx$. (We derive the equilibrium in the Appendix.) All investors hold the market portfolio in this equilibrium with only type-M investors (everyone cannot be more ESG friendly than the average). Nevertheless, a security's required return is affected by its ESG as well as its conditional market beta, $\bar{\beta}^i = \frac{\mathrm{cov}(r_t^i, r_t^m|s)}{\mathrm{var}(r_t^m|s)}$, as seen in Proposition 7.

Proposition 7 (ESG-CAPM). *If all investors are ESG-motivated of type-M ($W^U = W^A = 0$), then any security i has equilibrium price*

$$p^i = \frac{\bar{\mu}^i + \lambda\left(s^i - s^m\right) - \frac{\gamma}{W}\,\mathrm{cov}(v^i, v^m|s)}{r^f - \pi\left(s^i - s^m\right)}, \tag{20}$$

where s^m is the ESG score of the market portfolio and the corresponding π is given by Eq. (11). The equilibrium conditional expected excess return is given by

$$E(r_t^i|s) = \bar{\beta}^i E(r_t^m|s) - \pi\left(s^i - s^m\right). \tag{21}$$

If all investors are ESG-aware of type-A ($W^U = W^M = 0$), the same conclusions hold with $\pi = 0$.

This proposition shows that equilibrium asset prices are different when all investors derive utility from ESG (type-M) relative to an economy dominated by investors who ignore ESG (as in Proposition 6). With such ESG-motivated investors, the price of any firm's equity depends on its ESG score in two ways. First, the ESG score affects the expected cash flow as seen in the numerator of Eq. (20). Second, a higher ESG score lowers the discount rate used in the denominator, thus increasing the price. Turning to the implications for returns in Eq. (21), the firm's cost of capital is given by the standard conditional CAPM expression $[\bar{\beta}^i E(r_t^m|s)]$ adjusted for whether the ESG score is above or below that of the market. In other words, the firm's cost of capital is lower if its ESG score is higher or, equivalently, the firm can issue shares at higher prices. This low cost of capital encourages high-ESG firms to make real investments because, using this low discount rate, more projects would have a positive net present value. While we do not explicitly model firm decisions to invest in ESG, this insight helps explain why firms can choose to increase their corporate investment in ESG or why firms with a stronger ESG profile could realize higher growth than firms with relatively weaker ESG. Recent papers emphasizing the effect of ESG investment on corporate decisions include Albuquerque et al. (2018), Landier and Lovo (2020), Oehmke and Opp (2020), and Pastor et al. (2019).

If all types of investors exist, then several things can happen. If a security has a higher ESG score, then, everything else equal, its expected return can be higher or lower. A higher ESG score increases the demand for the stock from type-M investors, leading to a higher price and, therefore, a lower required return, as seen in Proposition 7. Companies with poor ESG scores that are down-weighted by type-M investors would have lower prices and higher cost of capital.

Furthermore, the force that can increase the expected return is that the higher ESG could be a favorable signal of firm fundamentals, and, if many type-U investors ignore this, the fundamental signal perhaps would not be fully reflected in the price, as seen in Proposition 6. Whether favorable ESG characteristics signal good profitability (e.g., good governance leading to a well-run company or a social company with happy productive employees) or low profitability (e.g., a company spending shareholders' money on charities that employees and customers do not appreciate) is an empirical question; that is, the sign of λ is an empirical question. Further, it is an empirical question whether the force of Proposition 6 or 7 is stronger, that is, the extent to which ESG information is incorporated into prices and the extent to which ESG-investors' demand pressure affects required returns.

Finally, we can consider the effect of an increasing adoption of ESG investing over time (i.e., an increasing fraction of ESG-motivated investors or a stronger ESG preference among them). A future increase in ESG investing would lead to higher prices for high-ESG stocks, corresponding to a larger π in the model (as seen in Proposition 7). If these flows are unexpected (or not fully captured in the price for other reasons), then high-ESG stocks would experience a return boost during the period of this repricing of ESG. If these flows are expected, then expected returns should not be affected.

3.2. Testable predictions of the theory

To summarize, the theory makes the following predictions:

1. The trade-off between risk, expected returns, and ESG can be summarized by the ESG-SR frontier.
2. Using ESG information can increase the investor's SR by improving the ESG-SR frontier.
3. Given the investor's information set, investors with stronger ESG preferences (or higher risk aversion) choose portfolios with higher ESG scores and (marginally) lower SR.

4. Even investors with preferences for the average ESG score optimally choose portfolios with positions (long or short) in almost any security (as opposed to standard models of taste-based discrimination that imply stricter segregation).
5. ESG investors choose a combination of four portfolios (or funds): the risk-free asset, the standard tangency portfolio, the minimum-variance portfolio, and the ESG-tangency portfolio.
6. A security with a higher ESG score has
   a. A higher demand from ESG investors, which lowers the expected return;
   b. Different expected future profits, which can increase the expected return if the market underreacts to this predictability of fundamentals; and
   c. Stronger flows from investors, which can increase the price in the short term.

Many of these predictions are qualitative in nature, but it is interesting to considering the quantitative effects of ESG on returns (predictions 6.a and 6.b). Starting with 6.b (corresponding to Proposition 6), we empirically estimate how different ESG measures predict future earnings (see Section 4.4). This provides an estimate of λ in the model. Specifically, we run a regression of the form $\frac{v_t^i}{A_{t-1}^i} = \lambda s_{t-1}^i + \text{controls} + \varepsilon_t^i$, where A_{t-1}^i is assets. We empirically scale earnings by assets (instead of just using earnings as in the model) so that our variables are more stationary, but we can link the results to the model as follows. If this predictability is not already incorporated in prices, then the effect of expected returns for an investor exploiting this effect should be $E_t(r_t^i|s_{t-1}^i) = E_t(\frac{v_t^i}{p_{t-1}^i} - r^f|s_{t-1}^i) = \frac{E_t(v_t^i/A_{t-1}^i|s_{t-1}^i)}{p_{t-1}^i/A_{t-1}^i} - r^f = \frac{\lambda s_{t-1}^i}{p_{t-1}^i/A_{t-1}^i} - r^f$. To make this concrete, we can use the estimates from Table 1 (explained in more detail in Section 4.4). For example, one of the strongest predictors of future profits is our proxy for governance, which has $\lambda = 0.061$ in Table 1, Panel B, Regression 5. Coupled with the average price-to-asset of $\frac{p_{t-1}^i}{A_{t-1}^i}$ of 1.5 in our sample, this means that an increase of s_{t-1}^i of 0.22 (equivalent to moving from the 10th to the 90th percentile of this variable) could elevate returns by $\frac{0.061 \times 0.22}{1.5} = 0.89\%$. This calculation takes into account only the value of the earnings at time t; that is, prices are assumed constant in steady state. If prices also adjust, then the effect could be larger. To capture this effect, note that, when the economy is not in steady state, returns are given by $r_t^i = \frac{v_t^i + p_t^i}{p_{t-1}^i} - 1 - r^f$, so an additional effect comes from $E_t(\frac{p_t^i}{p_{t-1}^i}|s_{t-1}^i) = E_t(\frac{p_t^i/v_t^i}{p_{t-1}^i/v_{t-1}^i}\frac{A_{t-1}^i}{v_{t-1}^i}\frac{v_t^i}{A_{t-1}^i}|s_{t-1}^i) = \frac{A_{t-1}^i}{v_{t-1}^i}\lambda s_{t-1}^i$, where we assume that price-earnings ratios stay constant. So, with $\frac{A_{t-1}^i}{v_{t-1}^i} = 3.2$, which is the median of assets-to-gross profits in our data, this return effect would be $3.2 \times 0.061 \times 0.22 = 4.3\%$.

Finally, we consider the quantitative effect of ESG demand (prediction 6.a, corresponding to Proposition 7, but here looking at ESG demand from some, but not all, investors). We start with a one-period economy with a risk-free rate of $r^f = 3\%$ and $n = 10$ risky assets, which we think of as equity sectors. The final payoff of each asset is $v^i = \bar{\mu} + f + \varepsilon^i$, where $\bar{\mu} = 1$ is the expected payoff, f is a common shock, and ε^i is an idiosyncratic shock, where both shocks have zero means and volatilities $\sigma_f = \sigma_\varepsilon = 0.15$. Each asset has a supply of shares of $z^i = \frac{1}{n} = 10\%$ so that the market portfolio has payoff $v^m = \sum_i z^i v^i = \bar{\mu} + f + \sum_i \frac{1}{n}\varepsilon^i$.

One of the assets is brown, and the others are green. Type-M investors buy $b = 30\%$ of the shares outstanding of green stocks and 0% of brown stocks. This screening approach is more extreme than the ESG-integration approach that we focus on elsewhere, but it provides a simple example of how much prices change for a given change in demand. The market is cleared by type-A investors, who have risk aversion of $\gamma = 3$ and wealth $W^A = 1$ (equal to the expected future value of the market).

The difference in expected returns of brown-versus-green assets is $E(r^{\text{brown}}) - E(r^{\text{green}}) = 0.23\%$ in equilibrium, as shown in the Appendix. In a one-period model, this difference in required returns corresponds to a small difference in prices of only $\frac{p^{\text{brown}}}{p^{\text{green}}} - 1 = -0.2\%$. With many time periods, a permanent difference in required returns can have a large price effect. To see this, recall from the Gordon Growth Model (GGM) that $P = D/(k-g)$, where k is the required return and g is growth. GGM implies that

$$\frac{\partial P}{\partial k}\frac{1}{P} = -\frac{D}{(k-g)^2}\frac{1}{P} = -\frac{1}{k-g} = -\frac{P}{D}. \tag{22}$$

So, with a price-dividend ratio of $\frac{P}{D} = 30$, a permanent difference in required returns of $\partial k = 0.23\%$ is associated with a meaningful price difference of $\frac{\partial P}{P} = -\frac{P}{D}\partial k = -30 \times 0.23\% = -7\%$.

4. Empirical results

4.1. ESG measures and data

As ESG is a broad umbrella term, we consider four proxies that capture different ESG aspects, possibly followed by different investor clienteles. Our goal is not to run a horse race between them, but rather to present a discussion of how different elements of ESG can be priced in the market and an illustration of how our theory guides empirical tests for investors who want to incorporate some ESG metric into their portfolios.

1. A measure of E: low carbon intensity. As a measure of how green a company is (the E in ESG), we compute its carbon intensity (CO_2), defined as the ratio of carbon emissions in thousands of tons over sales in millions of dollars. Carbon emissions can be measured in different ways, but we use the sum of scope 1 carbon emissions (a firm's direct emissions, e.g., from the firm's own fossil fuel usage) and scope 2 carbon emissions (indirect emissions from purchased energy, e.g., electricity). We do not include scope 3 emissions (other indirect emissions) because they are rarely reported by companies and are at best noisily estimated

and inconsistent across different data providers (e.g., Busch et al., 2018). We negate the CO_2 variable so that higher values indicate better ESG (less carbon intensive, greener companies). These data are obtained from Trucost and are available from January 2009 through March 2019.
2. A measure of S: non–sin stock indicator. Stocks in certain sin industries are shunned by some ESG-conscious investors, for example, tobacco, gambling, and alcohol (related to the S in ESG). We consider a non–sin stock indicator, taking the value of zero for sin stocks and the value of one otherwise, so that higher values indicate better ESG. Sin industries are defined as in Hong and Kacperczyk (2009), and this indicator is available for our longest sample, January 1963 through March 2019.
3. A measure of G: low accruals. We use a measure of governance that can be computed over a long sample period based on accounting information. We look at each firm's accruals over assets with a sample period spanning January 1963 through March 2019. Accruals are essentially accounting income for which the related cash has not yet been received.[14] We negate accruals so that higher values indicate better ESG. The idea, coming from the accounting literature, is that low accruals indicate that a firm is conservative in its accounting of profits (e.g., Sloan, 1996) and better governed companies tend to adopt more conservative accounting processes (e.g., Kim et al., 2012). Research shows companies that are subject to Securities and Exchange Commission enforcement actions tend to have abnormally high accruals prior to such actions (e.g., Richardson et al., 2006) and companies with high accruals have a higher likelihood of earnings restatements (e.g., Richardson et al., 2002).
4. A measure of overall ESG: MSCI ESG scores. One of the most widely used ESG scores by institutional investors is computed by MSCI, and our sample for this variable is from January 2007 through March 2019.[15] The MSCI score is a comprehensive assessment of each company's ESG profile. We use the top-level ESG score that summarizes each company's E, S, and G characteristics, on an industry-adjusted basis, as a numerical score from zero (worst ESG) to ten (best ESG).

We merge these data sets with the XpressFeed database for stock returns and market values, the Compustat database to compute firm fundamentals, institutional holdings from 13f holdings reports (as aggregated by Thomson Reuters), signed order flow computed from intraday data, and the risk model of Barra US Equity (USE3L) that is used in the computation of the empirical EGS efficient frontier.[16]

4.2. Empirical ESG-SR frontier

To compute the ESG–Sharpe ratio frontier implied by our theory, investors must first choose their investment universe and compute risk and expected returns. We consider monthly returns of stocks in the Standard & Poor's (S&P) 500 index, which makes the analysis conservative in the sense that we focus on a liquid and realistic investment universe with high data coverage, ruling out that our results are driven by microcap stocks. To compute risk (i.e., the variance-covariance matrix of the S&P 500 stocks), we assume that all investors use Barra's US Equity risk model (Barra USE3L model), an industry standard for use in portfolio management.[17] ESG-unaware investors and ESG-aware investors compute expected returns in different ways. U investors focus on the general equity risk premium and the traditional value factor, book-to-market, while A investors also use ESG information.[18]

To compute the annualized expected return of any stock i in any month t, U investors use

$$E_t^{\mathrm{U}}(r_{i,t+1}) = \overline{MKT}_t + bm_{i,t}\ \overline{BM}_t, \tag{23}$$

where $\overline{MKT}_t$ is the equity risk premium, $bm_{i,t}$ is stock i's cross-sectional book-to-market z-score (i.e., the stock's book-to-price ratio minus the cross-sectional mean, divided by the cross-sectional standard deviation), and $\overline{BM}_t$ is the return premium of the value factor. For each factor, the return premium at time t is its constant Sharpe ratio, multiplied by its volatility as estimated using the Barra model. Details on the estimation method are given in the Appendix.

Similarly, A and M investors compute the annualized expected return of stock i as

$$E_t^{\mathrm{A}}(r_{i,t+1}) = \overline{MKT}_t + bm_{i,t}\ \overline{BM}_t + s_{i,t}\ \overline{ESG}_t, \tag{24}$$

where $s_{i,t}$ is the stock's ESG score at time t and $\overline{ESG}_t$ is the return premium of the ESG factor, based on one of the proxies listed in Section 4.1. The ESG score $s_{i,t}$ is computed as the cross-sectional z-score of the raw ESG metric. Because a stock's ESG score s_i is normalized as a cross-sectional z-score, we get the intuitive interpretation that an ESG score of zero means an average stock in terms of the ESG measure, a score of two means that the stock has ESG characteristics two standard deviations better than the average stock, and so on. For a portfolio, the average ESG score is computed as in the theory Section 2.1, $\bar{s} = \frac{x's}{x'1}$, which provides a similar intuition for long-only portfo-

[14] We measure accruals as in Sloan (1996): (change in current assets minus change in cash) minus (change in current liabilities minus change in debt included in current liabilities minus change in taxes payable) minus (depreciation and amortization expense).

[15] The MSCI website states that, as of August 2018, "MSCI ESG Research is used by 46 of the top 50 asset managers and over 1,200 investors worldwide" (https://www.msci.com/esg-ratings, accessed July 7, 2019).

[16] The variables related to signed order flow are defined as in Chordia et al (2002) and Chordia and Subrahmanyam (2004) and are available between January 1993 and December 2012. We thank Tarun Chordia for kindly making these variables available to us.

[17] Estimating the covariance matrix is not a contribution of this paper, so we use a third-party risk model for convenience. For details about the risk model, see Barra documentation, available, for example, at http://www.alacra.com/alacra/help/barra_handbook_US.pdf.

[18] We design our empirical setup to be as simple as possible, with a single non-ESG factor, value. Of course, investors may consider other factors as well. In such cases, we would expect similar patterns to those discussed here, although including or not including ESG could matter relatively less for investment outcomes. Unless ESG has meaningfully better performance or diversification properties than other factors, we would expect that as one adds more factors, the optimal weight on ESG, and its incremental impact, to decrease.

lios, but long-short portfolios can in principle attain an unbounded range of ESG scores.

Using the above methodology, we compute the ESG-SR frontiers for two ESG proxies: E and G. We do not build the frontier for S because this proxy is binary (sin or non-sin), which corresponds to screening (something we consider in Section 4.3). For brevity, we leave out the frontier for overall ESG because it resembles the E frontier.

Starting with the ESG-SR frontier for the environmental proxy based on CO_2 emissions, Fig. 4 shows the frontier both from the perspective of ESG-unaware and ESG-aware investors (solid and dashed lines, respectively). Further, we distinguish what we call the ex ante perceived frontier (Panel A) and the realized frontier (Panel B). For the former, each month, the investor computes risk and expected returns as defined previously and then derives the ESG-SR frontier and the corresponding frontier portfolios. Panel A simply shows the time series average of these perceived frontiers. The ex post frontiers in Panel B show the realized Sharpe ratios of these portfolios.

The two ESG-SR frontiers in Panel A are close together, suggesting that the environmental proxy we use here is not very helpful in explaining average returns. This is also confirmed by the fact that the two frontiers peak around a carbon score of zero, suggesting that the typical stock in investor's A and B tangency portfolio is about average in its emissions footprint (we further confirm this in the regression framework in Section 4.6). This finding is even more striking when looking at Panel B: The two frontiers sit on top of each other, meaning that the realized Sharpe ratios of the portfolios on the two frontiers are essentially identical for any given level of carbon intensity.

The ESG-SR frontier remains useful even when the ESG proxy is a weak predictor of returns (as is the case in Fig. 4). For example, the frontier can be used to quantify the trade-off faced by type-M investors, who are willing to sacrifice some of the Sharpe ratio to improve their portfolios' ESG profile. In the context of Panel B, such ESG-motivated investors seek portfolios with less carbon emissions (greener portfolios). Moving two units to the right from the tangency portfolio (i.e., moving toward greener portfolios, so that the typical stock in the portfolio is two standard deviations greener) reduces the optimal Sharpe ratio by about 3%. This modest reduction in SR could be an acceptable price to pay for some ESG-motivated investors for such a large reduction in CO_2. Pushing further toward greener portfolios is increasingly costly; for example, moving from the peak to the portfolio score four units greener reduces the Sharpe ratio by about 10%.

Fig. 5 presents the ex ante and ex post frontiers, built similarly as in Fig. 4, but using our governance proxy. These frontiers are interesting because the frontiers for the ESG-unaware differ significantly from those of the ESG-aware investor. This difference arises because our G proxy predicts returns in our sample (as discussed further in Section 4.6). To understand Fig. 5, Panel A, note that the ESG-unaware investor U maximizes the Sharpe ratio for the ESG score of 0.25, meaning that a typical stock in her portfolio is close to average for this ESG measure. This near-neutrality to ESG is not surprising because the U investor uses information only on book-to-market ratios, and any exposure to G happens incidentally through the weak correlation between book-to-market and G. Moreover, the frontier is relatively symmetric in the neighborhood of zero, meaning that this investor perceives the cost of targeting a positive G score to be similar to the cost of targeting a same-magnitude negative tilt on ESG. For example, targeting a G score two standard deviations higher than optimal (i.e., moving from 0.25 to 2.25) lowers investor U's perceived Sharpe ratio by about 9% and targeting a G score two standard deviations lower than optimal (−1.75) degrades the perceived Sharpe ratio by 7%.

The ESG-aware investor's perceived frontier looks very different, as seen in Fig. 5, Panel A. The frontier peaks at a G score of 2.25; that is, for the ESG-aware investor, maximizing the Sharpe ratio means targeting a portfolio with a significantly higher G score than the market. Moreover, the frontier is clearly asymmetric, in a way that suggests that decreasing a portfolio's G score would be meaningfully more costly to the Sharpe ratio than increasing it. For example, a two standard deviation increase from the optimal point (2.25 to 4.25) reduces the Sharpe ratio by about 3%. The penalty for a similar move in the opposite direction (2.25 to 0.25) is three times as high, 9%.

The perceived frontiers in Fig. 5, Panel A, intersect because forcing a negative ESG score is seen as more costly by investor A than by investor U given that A takes into account that G positively predicts returns. The two curves cross at a G score of approximately zero, which is also intuitive. At this point, the optimal portfolio is essentially the same for both investors because none of them can get exposure to the G score that they disagree about.

Finally, Panel B of Fig. 5 shows the realized Sharpe ratios of the portfolios that underlie the frontiers in Panel A. A's (ex post) realized frontier is similar to A's ex ante perceived frontier, because the ESG score that drives the frontier is explicitly incorporated into A's returns forecast and because our model of ex ante risk and expected returns captures well the ex post realized returns.

U's realized frontier in Panel B has a different shape than U's perceived frontier in Panel A because U ignores that G predicts returns. The realized ESG-SR frontier looks fairly similar to that of investor A for ESG scores close to zero because their portfolios are more similar in that range. U's frontier is otherwise below because, for any ESG target, investor U chooses a portfolio with a suboptimal trade-off between market exposure, value, and G.

Fig. 5, Panel B, shows the costs and benefits of using ESG investing based on governance. The benefit of using G information can be measured by looking at the realized SR of the ESG-aware investor, which is 11% higher than the realized SR of the ESG-unaware investor (ex ante, in Fig. 5, Panel A, it is 12% higher). The cost of an ESG-motivated investor's preferences can be measured as the reduction in SR that occurs when targeting an even higher ESG score than that of an A investor.

4.3. Impact of restrictions: screening out the worst ESG stocks

Our empirical application has so far allowed investors to deploy their capital in unconstrained portfolios, going

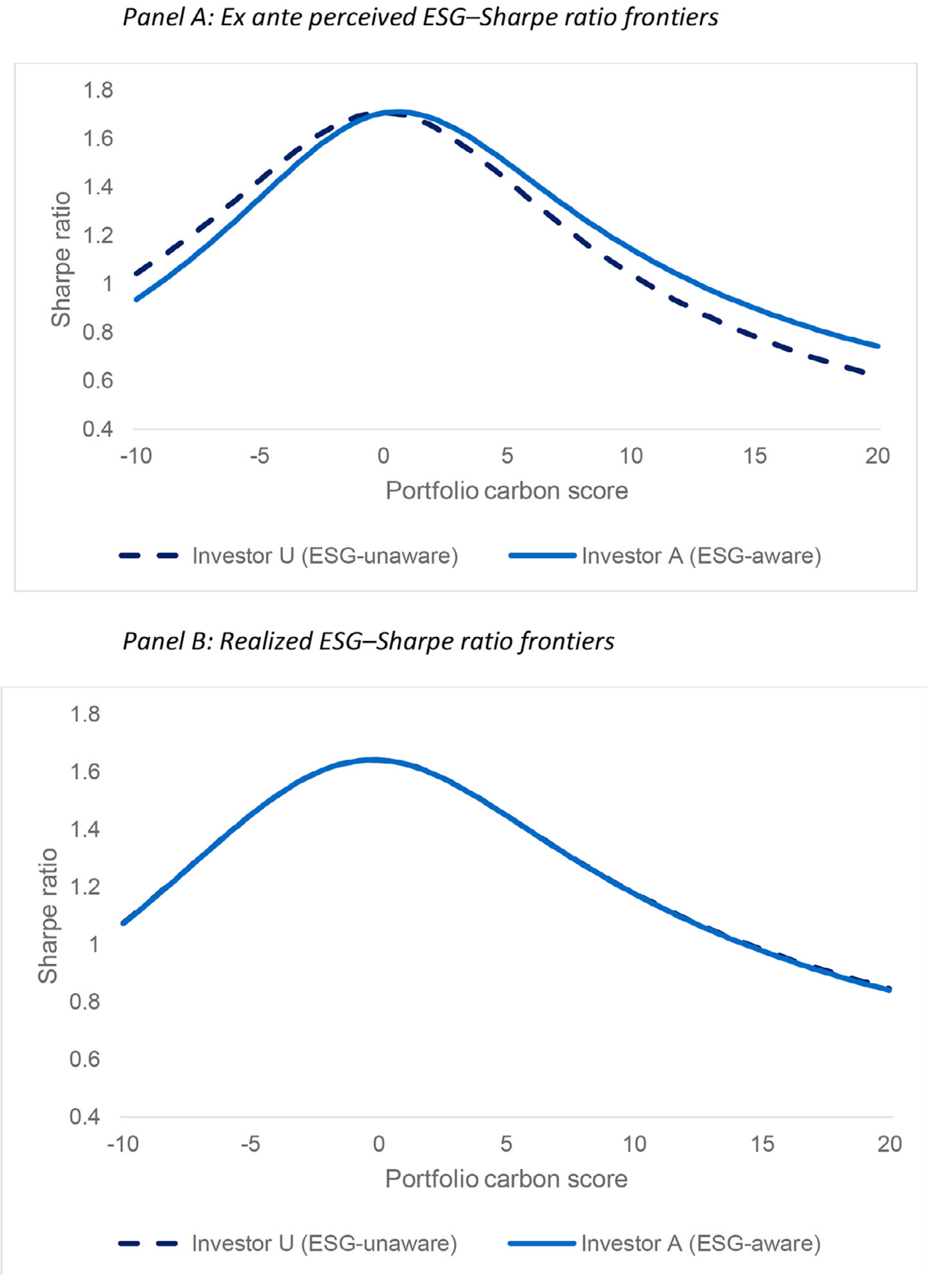

Fig. 4. Empirical ESG–efficient frontier using carbon emissions as a proxy for E. We estimate the ESG–Sharpe ratio frontier for Standard & Poor's (S&P) 500 stocks, with returns driven by valuation (measured by each stock's book-to-market ratio) and a proxy for E (measured by each stock's CO_2 emissions-to-sales ratio). The figure shows annualized maximum Sharpe ratios attainable for each level of ESG constraint. The ESG-unaware investor U (dashed line) solely utilizes book-to-market to estimate expected returns. The ESG-aware investor A (solid line) uses both book-to-market and a measure of governance (the G in ESG) based on accruals to estimate expected returns. Panel A presents the perceived frontier, built using the ex ante estimates from each investor. Panel B presents the ex post frontier using the realized Sharpe ratios of the portfolios from Panel A.

long and short any stock in the investment universe. Also of interest is to consider realistic constraints faced by many ESG-sensitive investors. Among such constraints, undoubtedly the most popular one is screening out stocks with the weakest ESG characteristics (i.e., removing such stocks from the investable universe). Fig. 6 shows how the ESG-SR frontier is affected by screens using the governance-related proxy we utilize in Fig. 5. Fig. 6 shows three different frontiers: one for the unconstrained investor A (exactly as in Fig. 5, Panel A), another obtained when the investor removes the 10% of stocks with the lowest ESG characteristics, and a third frontier with a 20% screen.

The first observation is perhaps the most obvious: Constraints reduce a portfolio's expected performance. Not sur-

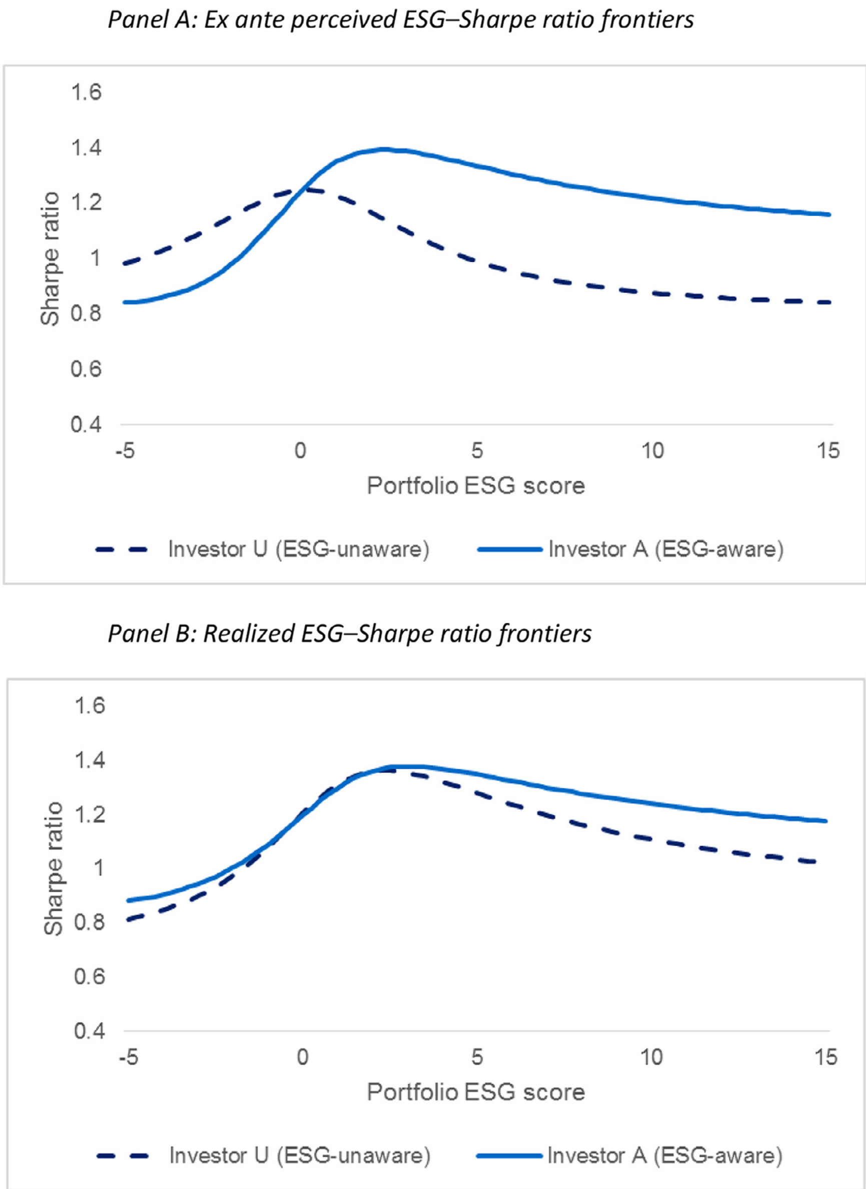

Fig. 5. Empirical ESG–efficient frontier using accruals as a proxy for G. We estimate the ESG–Sharpe ratio frontier for Standard & Poor's (S&P) 500 stocks, with returns driven by valuation (measured by each stock's book-to-market ratio) and ESG (measured by each stock's accruals-to-assets ratio, a measure related to governance). The figure shows annualized maximum Sharpe ratios attainable for each level of ESG constraint. The ESG-unaware investor U (dashed line) solely utilizes book-to-market to estimate expected returns. The ESG-aware investor A (solid line) uses both book-to-market and a measure of governance (the G in ESG) based on accruals to estimate expected returns. Panel A presents the perceived frontier, built using the ex ante estimates from each investor. Panel B presents the ex post frontier using the realized Sharpe ratios of the portfolios from Panel A.

prisingly, the frontier with the 10% screen is strictly below the unconstrained one, and the frontier with a 20% screen is lower still. This means that, for any desired level of the ESG score, the maximum attainable Sharpe ratio is lower in a screened universe than in the unrestricted one.

What is perhaps more interesting is the magnitude by which the Sharpe ratio decreases. To benchmark the reduction, a useful rule of thumb is that, under certain assumptions, the Sharpe ratio is approximately linear in the square root of investment breadth (e.g., Grinold and Kahn, 1995). This implies that a 10% (20%) reduction in breadth should lower the Sharpe ratio roughly by 5% (10%). The reductions are roughly the magnitudes of the decrease for ESG scores below about –0.5. The penalty is about half as small closer to the ESG score of zero, perhaps because around that value the optimal portfolio does not invest in extremely weak ESG stocks (or, presumably, in extremely strong ESG stocks). For the values of ESG score meaningfully above zero, the magnitude of the penalty is sharply higher than what could be inferred from the square root of breadth rule of thumb. For example, removing the 20% of stocks with the lowest ESG reduces the Sharpe ratio by

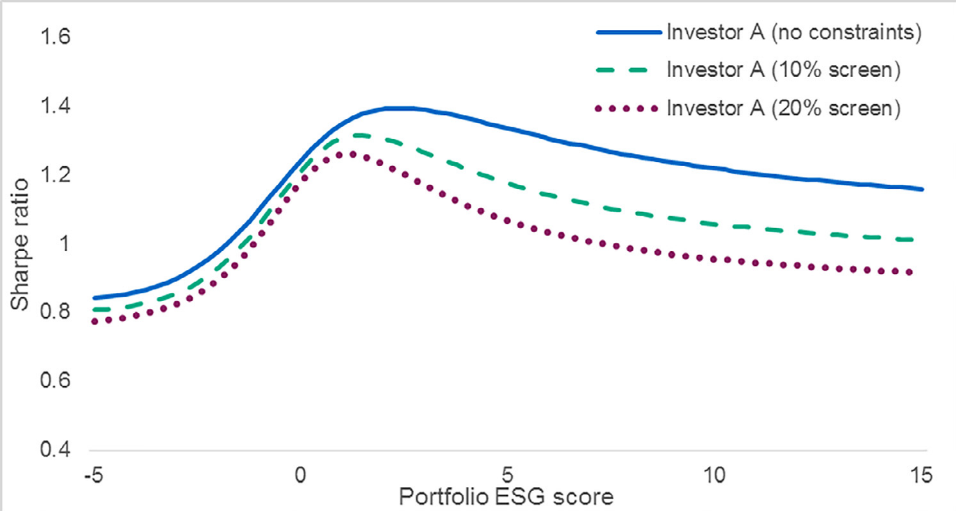

Fig. 6. The Impact of screening on the ESG–efficient frontier. This figure shows an ESG-aware investor's perceived ESG–Sharpe ratio frontier (solid line; the same as the solid line in Fig. 5, Panel A) as well as two frontiers for an investor who allows herself to use only a screened investment universe: removing 10% of stocks with the lowest ESG scores (dashed line) or removing 20% of stocks (dotted line). The ESG proxy used here is G, based on negated accruals scaled by assets.

over 25% when the investor seeks to achieve high portfolio ESG scores, due in part to the benefits of shorting low-ESG stocks.

A related finding from Fig. 6 is that the portfolio with the highest Sharpe ratio (the tangency portfolio) has a lower ESG score when the worst ESG stocks are removed. The unconstrained investor A optimizes the Sharpe ratio at the portfolio ESG score of 2.25. After removing 10% of weakest ESG stocks, the Sharpe ratio is maximized at the ESG score of 1.5; after removing 20%, the optimum is an ESG score of one.

This finding is surprising since it means that investors who exclude low-ESG assets from their investment universes may optimally build portfolios with lower ESG scores than investors who allow for such low-ESG assets. The intuition behind this finding is that low-ESG assets are effectively funding sources, allowing the unconstrained investor to short them to build larger long positions in high-ESG securities. Moreover, low-ESG assets can be useful hedging instruments for high-ESG assets and could help the investor improve the Sharpe ratio of the overall portfolio, potentially by increasing their investment in high-ESG securities. With screening, the investor may optimally choose not to take such a large position in high-ESG assets.

4.4. Does ESG predict future fundamentals?

A necessary condition for ESG-type information to generate positive abnormal returns is that it correlates with future fundamentals.[19] To test for this possibility, we relate our ESG proxies to future fundamentals. We consider two measures of fundamentals in Table 1. Panel A reports results based on the accounting rate of returns, defined as the return on net operating assets as in Richardson et al. (2006) and Panel B based on gross profitability over assets, defined as revenue minus cost of goods sold over total assets as in Novy-Marx (2013). In both panels, these firm fundamentals are measured 12 months after the ESG variables. For each of our four ESG proxies defined in Section 4.1, we present two specifications, one based on a pooled sample with month fixed effects and with standard errors clustered at the firm level and the other using the Fama-MacBeth procedure with Newey-West standard errors. We also control for firm beta, size, and book-to-market, although these control variables are not critical for our results.

Regressions 1 and 2 in Table 1 use our E proxy. Negated carbon emissions predict higher accounting returns in Panel A but are insignificant predictors of gross profitability in Panel B. We conclude that our E proxy perhaps is not robustly related to fundamentals. We find somewhat mixed results for our S proxy. The negative estimates in Regressions 3 and 4, in both panels, indicate that sin stocks have relatively stronger future fundamentals, consistent with Blitz and Fabozzi (2017), but these estimates are only borderline significant. Regressions 7 and 8 show that the overall ESG score from MSCI is positively related to future fundamentals, but with statistical significance only in Panel B.

The results are the strongest for our governance proxy (based on low accruals) in Regressions 5 and 6. In Panel A, the highly statistically significant also have a large economic magnitude. A one standard deviation increase in negated accruals predicts a corresponding increase of 0.02 in the accounting rate of returns, or 20% of its average level of 0.1. This finding opens up the possibility, which we confirm later, that accruals contain information about future fundamentals that may not be fully priced into the market (similar to findings of Richardson et al., 2006). The cor-

[19] ESG could lead to price increases even without a fundamentals channel if investor demand for ESG characteristics goes up. This is perhaps more likely over short periods and does not lead to a consistent return premium over the long term.

Table 1
Does environmental, social, and governance (ESG) score predict firm profits?

This table reports the regression of future profitability on current ESG scores, where profitability is measured 12 months into the future. Profitability is computed as the accounting return (return on net operating assets, RNOA) in Panel A and as gross profit over assets in Panel B. We consider four ESG metrics [E (negated CO_2 intensity), S (a non–sin stock indicator), G (negated accruals over assets), and overall ESG (using MSCI ESG scores)] and three control variables (market beta, the logarithm of market capitalization, and the logarithm of the book-to-price ratio). The estimation method is either a pooled regression with month fixed effects (pooled) or Fama-MacBeth (FM). Robust t-statistics are in parentheses and are clustered at the stock level in pooled regressions or adjusted using a Newey-West weighting scheme in Fama-MacBeth regressions.

Panel A: Predicting RNOA

Dependent variable	*RNOA* (t + 12)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
E (low CO2)	0.006***	0.006***						
	(4.91)	(7.34)						
S (non-sin)			−0.008*	−0.006***				
			(−1.94)	(−2.88)				
G (low accruals)					0.208***	0.193***		
					(23.26)	(28.64)		
ESG (MSCI)							0.0001	0.0001
							(0.15)	(0.24)
Beta	−0.068***	−0.068***	−0.064***	−0.067***	−0.060***	−0.062***	−0.052***	−0.040***
	(−17.90)	(−10.24)	(−33.77)	(−20.69)	(−31.79)	(−19.43)	(−11.62)	(−4.40)
Ln market cap	0.011***	0.011***	0.015***	0.015***	0.014***	0.014***	0.008***	0.006***
	(12.45)	(23.91)	(32.71)	(26.55)	(30.14)	(26.85)	(6.54)	(4.89)
Ln(P/B)	0.014***	0.015***	0.027***	0.028***	0.028***	0.028***	0.026***	0.038***
	(6.72)	(6.98)	(22.59)	(22.01)	(23.73)	(22.11)	(9.27)	(11.94)
RNOA(t)	0.763***	0.765***	0.710***	0.707***	0.725***	0.720***	0.756***	0.734***
	(88.59)	(97.48)	(167.53)	(118.95)	(169.65)	(128.80)	(63.53)	(61.25)
Constant	0.020***	0.021**	−0.005	0.003	−0.019***	−0.009	0.002	0.001
	(2.78)	(2.32)	(−0.95)	(0.47)	(−6.59)	(−1.56)	(0.19)	(0.06)
Number of observations	239,440	239,440	1374,620	1374,620	1354,499	1354,499	116,130	116,130
R-squared	0.708	0.712	0.631	0.631	0.636	0.635	0.723	0.727
Estimation method	Pooled	FM	Pooled	FM	Pooled	FM	Pooled	FM

Panel B: Predicting profitability

Dependent variable	*Gross profit over assets* (t + 12)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
E (low CO2)	−0.005	−0.006*						
	(−0.96)	(−1.79)						
S (non-sin)			−0.002	−0.003*				
			(−0.89)	(−1.79)				
G (low accruals)					0.061***	0.070***		
					(7.66)	(14.46)		
ESG (MSCI)							0.001**	0.001***
							(2.49)	(3.02)
Beta	−0.022***	−0.014**	−0.025**	−0.013***	−0.009***	−0.008***	−0.017***	−0.015***
	(−4.89)	(−2.29)	(−2.38)	(−5.91)	(−5.15)	(−3.59)	(−6.98)	(−3.30)
Ln market cap	−0.005	−0.004**	−0.001	−0.002***	−0.001*	−0.001***	−0.001*	−0.001**
	(−1.43)	(−2.37)	(−1.39)	(−3.84)	(−1.85)	(−4.13)	(−1.93)	(−2.24)
Ln(P/B)	0.036	0.038**	0.012	0.014***	0.002**	0.002***	0.006***	0.006***
	(1.32)	(2.25)	(1.53)	(3.42)	(2.21)	(3.34)	(4.83)	(8.58)
GPOA(t)	1.026***	1.017***	0.978***	0.980***	0.960***	0.960***	0.954***	0.948***
	(25.35)	(63.36)	(49.31)	(132.34)	(160.64)	(252.59)	(102.37)	(177.07)
Constant	0.019***	0.010	0.028***	0.023***	0.020***	0.023***	0.026***	0.028***
	(3.11)	(1.13)	(7.32)	(7.92)	(8.73)	(7.91)	(5.26)	(3.51)
Number of observations	361,540	361,540	1877,268	1877,268	1521,202	1521,202	171,284	171,284
R-squared	0.087	0.684	0.267	0.686	0.712	0.747	0.866	0.892
Estimation method	Pooled	FM	Pooled	FM	Pooled	FM	Pooled	FM

responding regressions in Panel B replicate the result for gross profitability. Again, higher G scores predict an increase in future profitability, but this time by a relatively smaller amount. A one standard deviation move of accruals is associated with a 0.006 move in gross profitability, or about 2% of its average level of 0.3.

The results for the G proxy are robust to a variety of controls. For example, differences could exist in accruals across industries, but the addition of industry dummy variables to Regression 5 does not change the coefficient (it slightly increases from 0.208 to 0.209, with a t-statistic of 22.6 versus 23.3). Similarly, running the regressions without controls for firm size, book-to-market, or beta, or without date fixed effects, has little effect on the result. Lastly, a strong positive effect exists on accounting returns and on profitability even 24 or 36 months after we measure accruals. We conclude that there is strong evidence that accruals correlate with future profitability.

4.5. Does ESG predict investor demand?

As we explain in the theory section, correlation with future fundamentals is not enough in itself to determine whether an ESG variable should help or hurt returns. For the full picture, one also needs to analyze investor demand for ESG. In this section, we consider institutional ownership, trading activity, and signed order flow to capture investors' interest in owning or purchasing a given stock.

Table 2, Panel A, uses a similar setup as Table 1 to predict institutional holdings (in percent, using 13f data) based on ESG metrics three months earlier (where the lag chosen to ensure that the ESG variables are known before we observe institutional holdings) and our usual controls.

Table 2
Does environmental, social, and governance (ESG) score predict investor demand?
This table reports the regression of investor demand on measures of ESG. Investor demand is measured as institutional ownership (obtained from 13f reports, led by three months) in Panel A, trading activity (log number of trades in the next month) in Panel B, and signed order flow (dollar buy volume over total dollar volume) in Panel C. The ESG proxies and control variables are as in Table 1. The estimation method is either a pooled regression with month fixed effects (pooled) or Fama-MacBeth (FM). Robust *t*-statistics are in parentheses and are clustered at the stock level in pooled regressions or adjusted using a Newey-West weighting scheme in Fama-MacBeth regressions.

Panel A: Predicting institutional ownership

Dependent variable	*Institutional holdings* ($t+3$)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
E (low CO2)	2.206***	2.284***						
	(3.37)	(14.65)						
S (non-sin)			6.128**	7.037***				
			(2.43)	(11.50)				
G (low accruals)					1.060	3.208***		
					(0.74)	(2.98)		
ESG (MSCI)							0.343**	0.420***
							(2.55)	(6.98)
Beta	5.774***	5.912***	5.698***	6.905***	1.610***	3.038***	6.371***	5.512***
	(8.50)	(21.96)	(14.13)	(20.76)	(3.37)	(11.91)	(7.05)	(11.27)
Ln market cap	10.079***	10.057***	9.662***	9.691***	9.599***	9.650***	0.846***	−1.265***
	(50.48)	(108.99)	(62.30)	(64.95)	(53.67)	(85.18)	(3.32)	(−2.67)
Ln(P/B)	−0.321	−0.354***	−1.759***	−1.264***	−2.282***	−1.931***	1.136***	1.642***
	(−1.20)	(−5.08)	(−11.05)	(−8.39)	(−13.90)	(−13.83)	(3.86)	(9.22)
Constant	−10.649***	−10.400***	−17.176***	−19.342***	−3.402***	−5.076***	62.372***	82.049***
	(−6.77)	(−17.28)	(−6.40)	(−18.11)	(−3.00)	(−9.55)	(24.56)	(18.45)
Number of observations	378,623	378,623	962,867	962,867	737,865	737,865	180,326	180,326
R-squared	0.454	0.450	0.470	0.424	0.475	0.422	0.033	0.083
Estimation method	Pooled	FM	Pooled	FM	Pooled	FM	Pooled	FM

Panel B: Predicting number of trades

Dependent variable	*ln #trades* ($t+1$)			
	(1)	(2)	(3)	(4)
E (low CO2)	−0.063***			
	(−3.46)			
S (non-sin)		−0.061		
		(−0.97)		
G (low accruals)			0.282***	
			(3.44)	
ESG (MSCI)				0.004
				(0.61)
Beta	1.382***	0.936***	0.940***	0.989***
	(29.97)	(43.48)	(43.56)	(21.81)
Ln market cap	0.898***	0.709***	0.724***	0.642***
	(67.04)	(111.50)	(108.31)	(37.60)
Ln(P/B)	−0.003	−0.062***	−0.085***	−0.075***
	(−0.16)	(−7.13)	(−9.80)	(−4.12)
Constant	−0.415***	−0.071	−0.178***	2.519***
	(−2.95)	(−0.85)	(−3.05)	(13.37)
Number of observations	49,264	312,487	263,217	28,703
R-squared	0.737	0.886	0.892	0.647
Estimation method	Pooled	Pooled	Pooled	Pooled

(continued on next page)

Table 2
Continued.

Panel C: Predicting signed order flow

Dependent variable	*Buy volume/total volume* ($t+1$)			
	(1)	(2)	(3)	(4)
E (low CO2)	−0.069***			
	(−4.07)			
S (non-sin)		0.321		
		(1.27)		
G (low accruals)			0.767*	
			(1.95)	
ESG (MSCI)				−0.015*
				(−1.67)
Beta	0.271***	1.593***	1.588***	0.097*
	(4.63)	(19.20)	(17.47)	(1.75)
Ln market cap	0.079***	0.740***	0.769***	−0.106***
	(4.11)	(30.86)	(28.03)	(−4.55)
Ln(P/B)	0.019	0.280***	0.249***	−0.023
	(0.79)	(8.20)	(6.62)	(−0.87)
Constant	48.874***	44.105***	44.206***	51.086***
	(238.71)	(139.66)	(207.70)	(225.45)
Number of observations	49,318	313,711	264,242	28,736
R-squared	0.011	0.122	0.121	0.166
Estimation method	Pooled	Pooled	Pooled	Pooled

Institutional investors (whose interest we measure using 13f filings) seem to incorporate ESG when forming their portfolios. All four ESG proxies correlate positively with institutional holdings. The economic effect of these variables is noticeable. For example, a one standard deviation increase in E (negated CO_2 intensity) is associated with increased institutional ownership of 1.3% in favor of greener firms. The corresponding number is 0.3%–1.3% for G and 0.6%–0.8% for overall ESG. As for our binary S proxy, a move from a sin stock to a non-sin stock implies an increase in holdings of 6%–7%.

Panels B and C in Table 2 consider measures of trading activity (logarithm of the number of trades) and signed order flow (the fraction of dollar volume that is attributable to buys). For brevity, we report only pooled regressions with date fixed effects. The results are perhaps most intuitive for accruals, where both the number of trades and the fraction of buys increase when this ESG proxy improves. For the other three metrics, evidence is not as straightforward. The number of trades seems to decrease for stocks with low carbon intensity and for non-sin stocks. For the former proxy, we also see a decrease in the fraction of buys.

Table 3
Environmental, social, and governance (ESG) score and valuation.

We regress each firm's valuation ratio (the logarithm of price-to-book) on the contemporaneous ESG score, controlling for the market beta. The ESG proxies are as in Table 1. Robust t-statistics are in parentheses and are clustered at the stock level in these pooled regressions.

Dependent variable	*Ln(P/B)*			
	(1)	(2)	(3)	(4)
E (low CO2)	0.086***			
	(7.25)			
S (non-sin)		0.020		
		(0.30)		
G (low accruals)			−0.470***	
			(−11.59)	
ESG (MSCI)				0.058***
				(8.25)
Beta	−0.449***	0.402***	0.338***	−0.348***
	(−16.39)	(28.48)	(21.13)	(−8.56)
Constant	1.391***	0.366***	0.514***	1.245***
	(38.32)	(5.48)	(27.37)	(21.81)
Number of observations	427,857	2120,679	1708,222	203,502
R-squared	0.050	0.073	0.077	0.046
Estimation method	Pooled	Pooled	Pooled	Pooled

4.6. Does ESG predict valuation and future returns?

The findings so far suggest that at least some ESG proxies (e.g., G) robustly correlate with future fundamentals. At the same time, some evidence exists that investors tilt their portfolios toward stocks with more attractive G. As we show in the theory section, the interplay between the two effects could lead to a return premium or discount, depending on which effect is stronger. The prediction is perhaps easier to make relative to the proxies for E, S, and overall ESG, for which we find less correlation to future fundamentals and stronger investor demand. Hence, the theory suggests that stocks with good E, S, or ESG should be more expensive and have lower future returns than stocks with good G. To assess these predictions, we consider valuations (Tobin's q) and risk-adjusted returns in Tables 3 and 4.

Table 3 shows how the ESG proxies correlate with the logarithm of the price-to-book ratio. Because our interest here is how much the market is willing to pay for ESG characteristics, we analyze the relation between contemporaneous valuation and ESG proxies. We control for market beta, but we naturally omit the previously used control variables that are related to valuation by construction (i.e., size and book-to-market).

Table 4
Does environmental, social, and governance (ESG) score predict returns?
This table reports the performance of high-ESG minus low-ESG portfolios. For each month, we sort stocks into portfolios based on quintiles of their ESG scores (defined as in Table 1). We then compute the return over the following month of the quintile with the best ESG scores minus that with the lowest scores. Stocks are equal weighted in Panel A and value weighted in Panel B. We report the portfolios' excess return, one-factor capital asset pricing model (CAPM) alpha, three-factor alpha that also controls for the Fama-French (FF) factors related to size and value, five-factor alpha that further controls for the FF factors related to profitability and investment, and six-factor alpha that also controls for momentum (Mom), annualized and in percentages. *t*-statistics are reported in parentheses.

	E (*low* CO_2)	*S* (*non-sin*)	*G* (*low accruals*)	*ESG* (*MSCI*)
Panel A: Equal-weighted returns				
Average excess return	5.15%	0.50%	7.84%***	0.38%
	(1.59)	(0.35)	(4.41)	(0.28)
CAPM alpha	7.02%**	−0.42%	7.87%***	1.29%
	(2.09)	(−0.30)	(4.39)	(1.00)
Three-factor (FF) alpha	5.03%	0.06%	7.30%***	0.74%
	(1.63)	(0.05)	(4.03)	(0.60)
Five-factor (FF) alpha	5.98%*	1.28%	8.85%***	0.28%
	(1.92)	(0.94)	(4.91)	(0.22)
Six-factor (FF + Mom) alpha	5.12%*	1.03%	8.71%***	0.27%
	(1.73)	(0.74)	(4.76)	(0.22)
Panel B: Value-weighted returns				
Average excess return	4.88%*	−3.04%**	3.01%**	0.02%
	(1.89)	(−2.07)	(2.30)	(0.01)
CAPM alpha	4.13%	−4.12%***	4.00%***	1.34%
	(1.52)	(−2.85)	(3.12)	(0.70)
Three-factor (FF) alpha	3.02%	−3.69%**	3.22%***	0.84%
	(1.14)	(−2.58)	(2.64)	(0.45)
Five-factor (FF) alpha	4.71%*	−0.20%	3.32%***	−0.58%
	(1.85)	(−0.15)	(2.76)	(−0.31)
Six-factor (FF + Mom) alpha	4.33%*	−0.36%	3.07%**	−0.59%
	(1.72)	(−0.26)	(2.52)	(−0.32)

Regression 1 in Table 3 suggests that prices of stocks with strong E scores (i.e., stocks with low carbon intensity, green stocks) are relatively higher than brown stocks' prices. This is consistent with the relatively higher demand from investors that we show earlier. A similar pattern is revealed when using the overall ESG metric (from MSCI) in Regression 4. In contrast, we find no significant difference in valuations between sin and non-sin stocks when using our S proxy in Regression 2.

Perhaps most interesting is Regression 4, suggesting that G (low accruals) is not priced by the market. In fact, we find low valuation ratios for stocks with high G scores despite the stronger forecasted profitability. This opens up the possibility that such stocks generate attractive returns, which is something we confirm below.

Table 4 shows the return predictability of the ESG proxies. Based on each of our four ESG proxies, we sort stocks into quintiles (in the case of the sin or non-sin indicator, into two portfolios) each month and then form a portfolio that goes long the best ESG stocks and short the worst ESG stocks. Table 4 presents the resulting performance for both the equal-weighted and value-weighted portfolios, controlling for a variety of asset pricing factors.

The portfolio based on G has highly significant returns. The economic magnitude of this effect is substantial: 7% a year for the equal-weighted and 3% a year for the value-weighted portfolio, even after controlling for the five Fama and French (2015) factors augmented with momentum. This finding reinforces our conclusion that G, or at least the particular aspect of governance we proxy for here over our sample, can be useful even for investors who already use multiple other investment factors in their portfolio decisions (in Section 4.2, our application was for simplicity, limited to the market factor and the value factor).

For the E and overall ESG proxies, we find little or weak evidence of abnormal returns. Over our sample period, less carbon intense companies seem relatively outperformed based on the point estimate, but this effect is significant only at the 10% level. Bolton and Kacperczyk (2019) find a carbon premium in returns but show that it disappears in richer specifications, for example, when they control for industry composition.

Finally, we find some evidence for the sin premium, as in Hong and Kacperczyk (2009). Because we consider a spread portfolio long in non-sin stocks (higher ESG) and short sin stocks (lower ESG), a sin return premium would be reflected as a negative alpha estimate. We find point estimates of a sin premium up to 4% a year with value-weighted returns, but the estimate is small and insignificant with equal-weighted returns and when we control for the five-factor or six-factor models (with both equal- and value-weighted returns), consistent with findings of Blitz and Fabozzi (2017). Our results are weaker than those of Hong and Kacperczyk (2009), possibly because of differences in methodology and in sample period.[20] We compare sin stocks with the whole universe of non-sin stocks, while Hong and Kacperczyk (2009) compare sin stocks with the closest peers that do not suffer from the sin stigma, that is, a different set of control stocks.

[20] The last years in our sample are particularly difficult for sin stocks. Tobacco companies posted historically weak results. For example, the MSCI World Tobacco index under-performed the cap-weighted benchmark in each of 2016, 2017, and 2018, by about 1%, 9%, and 28%, respectively.

5. Conclusion: ethical, saintly, and guiltless investing

Investors increasingly incorporate ESG views in their portfolios. Said simply and with a twist on the meaning of ESG, many investors want to own ethical companies in a saintly effort to promote good corporate behavior, while hoping to do so in a guiltless way that does not sacrifice returns.

Investors must realistically evaluate the costs and benefits of responsible investing, and we hope that our framework is a useful way to conceptualize and quantify these costs and benefits. We show that a responsible investor's decision can be conceptualized by the ESG-efficient frontier, a graphical illustration of the investment opportunity set. The benefit of ESG information can be quantified as the resulting increase in the maximum Sharpe ratio (relative to a frontier based on only non-ESG information). The cost of ESG preferences can be quantified as the drop in Sharpe ratio when choosing a portfolio with better ESG characteristics than those of the portfolio with maximum Sharpe.

In addition to its practical appeal, the ESG-efficient frontier is based on a rigorous theoretical framework. We explicitly derive the frontier and the corresponding set of optimal portfolios. The optimal portfolios are spanned by four funds, one of which captures stocks' ESG scores. This framework can be viewed as a theoretical foundation for what is called ESG integration, meaning that ESG characteristics are used directly in portfolio construction (not as screens).

Empirically, we find that when ESG is proxied for by a measure of governance based on accruals, the maximum SR is achieved for a relatively high level of ESG. Increasing the ESG level even further leads to only a small reduction in SR, implying that ethical goals can be achieved at a small cost. When we impose realistic constraints on the portfolio, we see a downward shift in the ESG-SR frontier. This is an expected outcome, because imposing constraints reduces the maximum Sharpe ratio that one can attain for any given ESG score. More surprisingly, screens that remove the lowest ESG assets from the investment universe can lead investors who maximize their Sharpe ratio to choose a portfolio with lower ESG scores than those chosen by unconstrained investors who allow investments in low-ESG assets. This result highlights nuances in optimally incorporating ESG into portfolio construction and suggests improvements to traditional approaches based on simple screening.

Turning to equilibrium asset prices, we derive an ESG-adjusted CAPM, which helps describe market environments that make ESG scores predict returns positively or negatively. To our knowledge, our model is the first to explicitly model heterogeneity in how investors use ESG information. We allow for investors to have preferences over ESG and for the possibility that investors find investment intelligence from ESG information. We argue that this feature is realistic, because not only do we observe large assets under management deployed with ESG in mind (e.g., the *2018 Global Sustainable Investment Review*), but ESG also is increasingly discussed as a potential alpha signal, both in academic outlets [going back to at least Sloan (1996) and Gompers et al. (2003)] and in practitioner journals (e.g., Nagy et al., 2015). This heterogeneity results in a range of possible equilibria depending on the relative importance of each investor type, leading to a relation between ESG and expected returns that is positive, negative, or neutral.

We test the empirical predictions of the theory using a range of ESG proxies that reflect different aspects of our model and that can represent different clienteles of investors. Our proxy G has historically offered ESG investors guiltless saintliness, perhaps because good G predicts strong future fundamentals, while attracting modest investor demand, leading to relatively cheap valuations and positive returns. Our proxies for E, S, and overall ESG are weaker predictors of future profits, and investor demand appears stronger for these proxies, which could help explain the higher valuations of stocks that score well on these metrics and the low or insignificant returns.

In conclusion, we think that our model provides a useful framework for responsible investment that we hope will be useful both for future research on the costs and benefits of ESG investing and for ESG applications in investments practice.

Appendix

A.1. Relation between the ESG-SR frontier and the mean-variance frontier

Fig. 1 shows how the ESG-SR frontier is related to the standard mean-variance frontier. What follows is a sketch of the math behind the graph. Consider first the frontier among portfolios with a certain ESG score. To see that this is a hyperbola, minimize the variance $x'\Sigma x$ for all portfolios with a given expected return, $x'\mu = \bar{\mu}$, portfolio weights that sum to one, $x'1 = 1$, and a given ESG score $x's = \bar{s}$. The solution to this minimization problem is linear in the expected return, $\bar{\mu}$, which means that the corresponding variance is quadratic in $\bar{\mu}$, showing that the frontier is a hyperbola when plotted in the usual way (mean, standard deviation).

The hyperbola for a given ESG score clearly lies inside the standard hyperbola, because minimizing the variance among all portfolios must provide a result that is at least as small as minimizing over the subset with a given ESG score. In fact, the two hyperbolas touch in a single point. To see why, recall that the standard mean-variance frontier is spanned by two portfolios. In other words, there exist portfolios x_1, x_2 such that the frontier consists of portfolios of the form $ax_1 + (1-a)x_2$, where a runs from $-\infty$ to ∞. Because x_1 and x_2 have different ESG scores generically, all frontier portfolios have different ESG scores. Further, for each ESG score, exactly one frontier portfolio has this score, so this is where the two hyperbolas touch each other.

Finally, given that the hyperbola for a given ESG scores lies inside the standard frontier, then the Sharpe ratio of its tangency portfolio must be lower than the overall tangency portfolio (except in the single case when they are the same).

A.2. Example in Section 3.2

With prices p, the demand of type-A investors is

$$x = \frac{1}{\gamma}\mathrm{diag}\left(p^i\right)\bar{\Sigma}^{-1}\mathrm{diag}\left(p^i\right)\left(\mathrm{diag}\left(\frac{1}{p^i}\right)\bar{\mu} - 1 - r^f\right). \quad (25)$$

The demand in dollars is $W_0^A x$. In equilibrium, this dollar demand must equal the supply in dollars, net of what the type-M investors buy, $\mathrm{diag}(p^i)\bar{z}$. Here, $\bar{z}^i = \frac{1}{n}$ for the brown asset and $\bar{z}^i = \frac{1}{n}(1-b)$ for the green assets (because M investors have bought the remaining $\frac{b}{n}$ shares outstanding for green assets). Hence, we have the equilibrium condition

$$\bar{z} = \frac{W_0^A}{\gamma}\bar{\Sigma}^{-1}\left(\bar{\mu} - \left(1 + r^f\right)p\right), \quad (26)$$

which implies that

$$p = \frac{\bar{\mu} - \frac{\gamma}{W_0^A}\bar{\Sigma}\bar{z}}{1 + r^f}. \quad (27)$$

Using that the variance-covariance matrix is $\bar{\Sigma} = \sigma_f^2 11' + \sigma_\varepsilon^2 I$,

$$p^{\text{green}} = \frac{\bar{\mu} - \frac{\gamma}{W_0^A}\left(\sigma_f^2(1 - b + b/n) + (1-b)\sigma_\varepsilon^2/n\right)}{1 + r^f} = 0.918 \quad (28)$$

and

$$p^{\text{brown}} = \frac{\bar{\mu} - \frac{\gamma}{W_0^A}\left(\sigma_f^2(1 - b + b/n) + \sigma_\varepsilon^2/n\right)}{1 + r^f} = 0.916. \quad (29)$$

The corresponding excess returns are $E(r^{\text{green}}) = \frac{\bar{\mu}}{p^{\text{green}}} - 1 - r^f = 5.88\%$ and $E(r^{\text{brown}}) = \frac{\bar{\mu}}{p^{\text{brown}}} - 1 - r^f = 6.11\%$. (Excluded stocks are often highly correlated because they tend to share similar characteristics, e.g., belong to the same industry. We capture this effect by considering a small number of assets, $n = 10$, each representing a sector. Alternatively, one can consider a large number of individual stocks and include industry factors in addition to the market-wide risk and idiosyncratic risk).

A.3. Estimating the empirical ESG-efficient frontier

As discussed in Section 4.2, we model expected returns as linear functions of factor exposures. For instance, investor U estimates expected returns as

$$E_t^{\mathrm{U}}(r_{i,t+1}) = \overline{MKT}_t + bm_{i,t}\,\overline{BM}_t, \quad (30)$$

where $\overline{MKT}_t$ is the equity risk premium, $bm_{i,t}$ is stock i's cross-sectional book-to-market z-score, and $\overline{BM}_t$ is the return premium of the factor-mimicking value factor, and similarly for investor A, who also includes an ESG factor. To show how we estimate these models, it is helpful to write them in a general way that captures either investor type. We first show how we model the vector of realized returns, r_{t+1}, and then later we turn to the expected returns, $E_t^{\mathrm{j}}(r_{t+1})$, for investor $j \in \{U, A\}$. Realized returns follow a standard factor model:

$$r_{t+1} = X_t F_{t+1} + \epsilon_{t+1}, \quad (31)$$

where X_t is a matrix of all securities' factor exposures, F_{t+1} is a vector of factor returns, and ϵ_{t+1} are the idiosyncratic shocks. For investor U, X_t is an $N \times 2$ matrix in which the first column is a vector of ones and the second contains the book-to-market z-scores. For investor A, X_t is an $N \times 3$ matrix in which the first two columns are the same and the third column is a vector of ESG z-scores. Even though investors U and A use different factor models (i.e., different X_t and F_{t+1}), we use the same notation for simplicity.

The factor returns F_{t+1} are unobserved, but they can be estimated as follows. Each time period, we run a cross-sectional regression of stock returns on their characteristics and note that the regression coefficients are the factor returns. Specifically, we run a generalized least squares regression each period of stock-level returns on stock-level characteristics, using the Barra risk model to obtain an estimate of the residual covariance matrix, $\Sigma_t = \mathrm{var}(\epsilon_{t+1})$, which yields the following estimated factor returns

$$\hat{F}_{t+1} = \left(X_t^T \Sigma_t^{-1} X_t\right)^{-1} X_t^T \Sigma_t^{-1} r_{t+1}. \quad (32)$$

Here, we can interpret $\theta_t := (X_t^T \Sigma_t^{-1} X_t)^{-1} X_t^T \Sigma_t^{-1}$ as the factor-mimicking portfolio weights, i.e., $\hat{F}_{t+1} = \theta_t r_{t+1}$.

Finally, we need to compute expected returns:

$$E_t^{\mathrm{j}}(r_{t+1}) = X_t E_t^{\mathrm{j}}(F_{t+1}), \quad (33)$$

which means that we need to estimate expected factor returns, $E_t^{\mathrm{j}}(F_{t+1})$. The simplest way to do this is to assume that $E_t^{\mathrm{j}}(F_{t+1})$ is constant over time and then estimate the factor premiums as the sample average of factor returns. This simple method does not work well empirically, however, because it leads, for example, to perceived and realized ESG-SR frontiers that differ significantly even for investor A. This problem arises because investors have an incentive to choose extreme portfolios when the perceived risk is time-varying (i.e., sometimes very low) while the perceived expected return is constant.

A more realistic specification is to assume that each factor k has a time-varying risk and a constant Sharpe ratio, $E_t^{\mathrm{j}}(F_{t+1}^k) = \sigma_t^{F,k} SR^{F,k}$. The volatility of each factor, $\sigma_t^{F,k}$, can be computed based on the factor-mimicking portfolio weights and the overall risk model, $\sigma_t^{F,k} = \sqrt{\theta_t^k \Sigma_t (\theta_t^k)^T}$. Finally, we estimate $SR^{F,k}$ as the realized full-sample Sharpe ratio of the volatility-scaled factor returns, $\hat{F}_{t+1}^k / \sigma_t^{F,k}$.

A.4. Proofs

Proof of Proposition 1. Consider the problem of maximizing the return given a level of risk σ and an ESG score $\bar{s}$:

$$\max_x \left(x'\mu - \frac{\gamma}{2}\sigma^2 + f(\bar{s})\right). \quad (34)$$
$$\text{s.t. } \bar{s} = \frac{x's}{x'1}$$
$$\sigma^2 = x'\Sigma x$$

Clearly, maximizing the expected return for given level of σ and $\bar{s}$ is achieved by maximizing the Sharpe ratio for that σ and $\bar{s}$. Further, the resulting Sharpe ratio is the same for all levels of σ. To see why, suppose that x_1 is the optimal solution for $(\sigma_1, \bar{s})$ and x_2 is the optimal solution for $(\sigma_2, \bar{s})$. We can scale x_2 as $\sigma_1/\sigma_2 x_2$ to have a volatility of σ_1, and this scaled portfolio still has the same average ESG score, $\bar{s}$. Given that x_1 has the highest expected return among such portfolios,

$$SR(x_1) = \frac{x_1'\mu}{\sigma_1} \geq \frac{\frac{\sigma_1}{\sigma_2}x_2{}'\mu}{\sigma_1} = \frac{x_2{}'\mu}{\sigma_2} = SR(x_2). \tag{35}$$

The symmetric argument shows that the opposite inequality also holds, so $SR(x_1) = SR(x_2) = SR(\bar{s})$.

Let us solve the problem explicitly. If we rewrite the first constraint as $x'\tilde{s} = 0$, where $\tilde{s} = s - 1\bar{s}$, and introduce Lagrange multipliers π and θ, then the solution is characterized by the first-order condition

$$0 = \mu + \pi\tilde{s} - \theta\Sigma x, \tag{36}$$

meaning that the optimal portfolio is given by

$$x = \frac{1}{\theta}\Sigma^{-1}(\mu + \pi\tilde{s}). \tag{37}$$

Both constraints clearly bind, and the first one yields

$$0 = \frac{1}{\theta}\tilde{s}'\Sigma^{-1}(\mu + \pi\tilde{s}). \tag{38}$$

So, the first Lagrange multiplier is

$$\begin{aligned}\pi &= -\frac{\tilde{s}'\Sigma^{-1}\mu}{\tilde{s}'\Sigma^{-1}\tilde{s}} = -\frac{(s-1\bar{s})'\Sigma^{-1}\mu}{(s-1\bar{s})'\Sigma^{-1}(s-1\bar{s})} \\ &= \frac{c_{1\mu}\bar{s} - c_{s\mu}}{c_{ss} - 2c_{1s}\bar{s} + c_{11}\bar{s}^2}.\end{aligned} \tag{39}$$

The second constraint yields

$$\sigma^2 = \frac{1}{\theta^2}(\mu + \pi\tilde{s})'\Sigma^{-1}(\mu + \pi\tilde{s}). \tag{40}$$

Using the first constraint, we can simplify as

$$\sigma^2 = \frac{1}{\theta^2}\mu'\Sigma^{-1}(\mu + \pi\tilde{s}), \tag{41}$$

implying that the second Lagrange multiplier is

$$\theta = \frac{1}{\sigma}\sqrt{c_{\mu\mu} - \frac{\left(c_{s\mu} - c_{1\mu}\bar{s}\right)^2}{c_{ss} - 2c_{1s}\bar{s} + c_{11}\bar{s}^2}}. \tag{42}$$

This shows explicitly that we can write the optimal portfolio as $x = \sigma v$, where the vector v depends only on the exogenous parameters and $\bar{s}$, that is, not on σ.

Finally, as seen in Eq. (7), the optimal level of risk is given by $\sigma = SR(\bar{s})/\gamma$. Inserting this risk level yields $\frac{(SR(\bar{s}))^2}{2\gamma} + f(\bar{s})$. Multiplying by 2γ gives the result [Eq. (8)] in the proposition. □

Proof of Proposition 2. The maximum Sharpe ratio for a given ESG score $\bar{s}$ is the Sharpe ratio of the optimal portfolio given in the proof of Proposition 1:

$$SR(\bar{s}) = \frac{\mu'x}{\sigma} = \frac{\mu'\Sigma^{-1}(\mu + \pi\tilde{s})}{\sigma\theta}. \tag{43}$$

Using the last two equations in the proof of Proposition 1,

$$\begin{aligned}SR(\bar{s}) &= \sigma\theta = \sqrt{\mu'\Sigma^{-1}(\mu + \pi\tilde{s})} \\ &= \sqrt{c_{\mu\mu} - \frac{\left(c_{s\mu} - c_{1\mu}\bar{s}\right)^2}{c_{ss} - 2c_{1s}\bar{s} + c_{11}\bar{s}^2}}.\end{aligned} \tag{44}$$

The maximum Sharpe ratio clearly is attained by the tangency portfolio, which is proportional to $\Sigma^{-1}\mu$. This portfolio has the ESG score and Sharpe stated in the proposition. This result can also be derived by differentiating the $SR(\bar{s})$ with respect to $\bar{s}$ and considering the first- and second-order conditions (there are two solutions to the first-order condition). □

Proof of Proposition 3. We have from the proof of Proposition 1 that $x = \frac{1}{\theta}\Sigma^{-1}(\mu + \pi\tilde{s})$. Further, from the proofs of Propositions 1–2, we know that $\theta = \frac{1}{\sigma}SR(\bar{s})$ and $\sigma = SR(\bar{s})/\gamma$. Combining these yields $x = \frac{1}{\gamma}\Sigma^{-1}(\mu + \pi\tilde{s})$, where we recall that $\tilde{s} = s - 1\bar{s}$. □

Proof of Propositions 4–5. These results follow based on arguments analogous to those in the first part of the proof of Proposition 1 using that, for any $x \in X$ and $a > 0$, we have $ax \in X$ and $e(ax, s) = e(x, s)$. The optimization problem can be written as

$$\begin{aligned}&\max_{x\in X}\left(x'\mu - \frac{\gamma}{2}x'\Sigma x + e(x,s)\right) \\ &= \max_{\bar{e}}\left[\max_{\sigma}\left\{\max_{\substack{x\in X \\ \text{s.t. } \bar{e} = e(x,s) \\ \sigma^2 = x'\Sigma x}}\left(x'\mu - \frac{\gamma}{2}\,\sigma^2 + \bar{e}\right)\right\}\right] \\ &= \max_{\bar{e}}\left[\max_{\sigma}\left\{\sigma SR(\bar{e}) - \frac{\gamma}{2}\,\sigma^2 + \bar{e}\right\}\right] \\ &= \max_{\bar{e}}\left[\frac{(SR(\bar{e}))^2}{2\gamma} + \bar{e}\right]\end{aligned} \tag{45}$$

□

Proof of Propositions 6–7. For Proposition 7, the equilibrium condition with all investors of type-M is

$$\begin{aligned}p = &\frac{W}{\gamma}\text{diag}\left(p^i\right)\bar{\Sigma}^{-1}\text{diag}\left(p^i\right) \\ &\times\left(\text{diag}\left(\frac{1}{p^i}\right)\bar{\mu} - r^f + \pi\left(s - 1s^m\right)\right).\end{aligned} \tag{46}$$

This condition can be simplified by multiplying both sides by $\text{diag}(\frac{1}{p^i})$:

$$\begin{aligned}1 &= \frac{W}{\gamma}\bar{\Sigma}^{-1}\left(\bar{\mu} - \text{diag}\left(p^i\right)(r^f - \pi\left(s - 1s^m\right))\right) \\ &= \frac{W}{\gamma}\bar{\Sigma}^{-1}\left(\bar{\mu} - \text{diag}(r^f - \pi\left(s^i - s^m\right)p\right).\end{aligned} \tag{47}$$

Solving this equation for the vector of prices p yields

$$p = \text{diag}\left(\frac{1}{r^f - \pi\left(s^i - s^m\right)}\right)\left(\bar{\mu} - \frac{\gamma}{W}\bar{\Sigma}1\right), \tag{48}$$

which proves Eq. (20) stated in the proposition. To translate this result to expected excess returns, we multiply both sides by $\text{diag}(\frac{1}{p^i})$, yielding

$$1 = \text{diag}\left(\frac{1}{p^i}\right)\text{diag}\left(\frac{1}{r^f - \pi\left(s^i - s^m\right)}\right)\left(\bar{\mu} - \frac{\gamma}{W}\bar{\Sigma}1\right), \tag{49}$$

and rearrange to obtain

$$\mathrm{diag}\left(r^f - \pi\left(s^i - s^m\right)\right) = \mathrm{diag}\left(\frac{1}{p^i}\right)\left(\bar{\mu} - \frac{\gamma}{W}\bar{\Sigma}1\right). \tag{50}$$

The vector of expected excess returns μ thus is given by

$$\mu = \mathrm{diag}\left(\frac{1}{p^i}\right)\bar{\mu} - r^f$$
$$= \frac{\gamma}{W}\mathrm{diag}\left(\frac{1}{p^i}\right)\bar{\Sigma}1 - \mathrm{diag}\left(\pi\left(s^i - s^m\right)\right). \tag{51}$$

The expected excess return of the market portfolio $(\frac{p^i}{1'p})$ is given by

$$\mu^m = 1'\mathrm{diag}\left(\frac{p^i}{1'p}\right)\mu$$
$$= \frac{\gamma}{W(1'p)}1'\bar{\Sigma}1 - 1'\mathrm{diag}\left(\frac{p^i}{1'p}\right)\mathrm{diag}\left(\pi\left(s^i - s^m\right)\right). \tag{52}$$

That is,

$$\mu^m = \frac{\gamma\left(1'p\right)}{W}\mathrm{var}(r^m|s) - \pi\left(s^m - s^m\right) = \frac{\gamma\left(1'p\right)}{W}\mathrm{var}(r^m|s), \tag{53}$$

where we use the definition of the ESG score of the market $s^m = \frac{1}{1'p}p's$. The expected excess return of security i can be written as $\mu^i = z_i{}'\mu$ using the i'th unit vector $z_i = (0, \dots, 0, 1, 0, \dots, 0)$, that is,

$$\mu^i = \frac{\gamma}{W}z_i{}'\mathrm{diag}\left(\frac{1}{p^i}\right)\bar{\Sigma}1 - z_i{}'\mathrm{diag}\left(\pi\left(s^i - s^m\right)\right)$$
$$= \frac{\gamma\left(1'p\right)}{W}\mathrm{cov}(r_i, r^m|s) - \pi\left(s^i - s^m\right). \tag{54}$$

Combining with the expression above for μ^m, we get

$$\mu^i = \bar{\beta}_i\mu^m - \pi\left(s^i - s^m\right). \tag{55}$$

Finally, when all investors are of type-A and choose the tangency portfolio, we have $\pi = 0$, which is seen from Proposition 3 and the expression for π.

For Proposition 6, similar calculations show that prices are given by

$$p = \frac{1}{r^f}\left(\hat{\mu} - \frac{\gamma}{W}\hat{\Sigma}1\right) \tag{56}$$

and returns by the unconditional CAPM. Conditional expected returns are given by

$$E\left(r^i|s\right) = \frac{E\left(v^i s\right)}{p^i} - \left(1 + r^f\right) = \frac{\hat{\mu}^i + \lambda\left(s^i - s^m\right)}{p^i} - \left(1 + r^f\right). \tag{57}$$

Using the expression for the price,

$$E\left(r^i|s\right) = \frac{\frac{\gamma}{W}\mathrm{cov}\left(v^i, v^m\right) + \lambda\left(s^i - s^m\right)}{p^i}$$
$$= \frac{\mathrm{cov}\left(r^i, r^m\right)}{\mathrm{var}(r^m)}E(r^m) + \frac{\lambda\left(s^i - s^m\right)}{p^i}, \tag{58}$$

where $\frac{\mathrm{cov}(v^i, v^m)}{p^i(1'p)} = \mathrm{cov}(r^i, r^m)$ and $\frac{\gamma(1'p)}{W} = \frac{E(r^m)}{\mathrm{var}(r^m)}$. □

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