499 cshs logit model

$$\log(\frac{Y}{1-Y}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

X_1=0 represents the aa group and X_1=1 represents the non-aa group.

X_2 represents age.

log-odds case:

It is the same as the OLS regression. Since there is no interaction between X_1 and X_2, the difference in the response variable $\log(\frac{Y}{1-Y})$ for one unit change in X_1 is constant across all levels of X_2.

after exponentiating,

$$\begin{aligned}
\frac{Y_1}{1-Y_1} &= e^{\beta_0+\beta_1 X_1+\beta_2 X_2} \\
Y_1 &= (1-Y_1) \times e^{\beta_0+\beta_1 X_1+\beta_2 X_2} \\
Y_1 &= \frac{e^{\beta_0+\beta_1 X_1+\beta_2 X_2}}{1+e^{\beta_0+\beta_1 X_1+\beta_2 X_2}} \\
Y_1 &= \frac{e^{\beta_0+\beta_2 X_2}}{1+e^{\beta_0+\beta_2 X_2}},\ with\ X_1 = 0 \\
Y_2 &= \frac{e^{\beta_0+\beta_1+\beta_2 X_2}}{1+e^{\beta_0+\beta_1+\beta_2 X_2}},\ with\ X_1 = 1 \\
Y_2 - Y_1 &= \frac{e^{\beta_0+\beta_1+\beta_2 X_2}}{1+e^{\beta_0+\beta_1+\beta_2 X_2}} - \frac{e^{\beta_0+\beta_2 X_2}}{1+e^{\beta_0+\beta_2 X_2}} \\
&= \frac{e^{\beta_0+\beta_1+\beta_2 X_2} - e^{\beta_0+\beta_2 X_2}}{(1+e^{\beta_0+\beta_1+\beta_2 X_2}) \times (1+e^{\beta_0+\beta_2 X_2})}
\end{aligned}$$

As we can see, $Y_2 - Y_1$ is dependent on X_2 and therefore the difference in Y with one unit change in X_1 is not constant across all age levels.

flipping the covariates,

log-odds case: same with the previous one

after exponentiating:

$$\begin{aligned}
Y_1 &= \frac{e^{\beta_0+\beta_1 X_1+\beta_2 X_2}}{1+e^{\beta_0+\beta_1 X_1+\beta_2 X_2}} \\
Y_2 &= \frac{e^{\beta_0+\beta_1 X_1+\beta_2 (X_2+1)}}{1+e^{\beta_0+\beta_1 X_1+\beta_2 (X_2+1)}} \\
Y_2 &= \frac{e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\beta_2}}{1+e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\beta_2}},\ as\ X_2\ increases\ by\ one\ level \\
Y_2 - Y_1 &= \frac{e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\beta_2} - e^{\beta_0+\beta_1 X_1+\beta_2 X_2}}{(1+e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\beta_2}) \times (1+e^{\beta_0+\beta_1 X_1+\beta_2 X_2})}
\end{aligned}$$

Since $Y_2 - Y_1$ depends on X_1, the difference in Y with one unit change in X_2 is not constant across all levels of X_1.

the odds-ratio case: (but I think what you did in the test code is not for this case)

$$\begin{aligned} odds\ for\ aa &= \frac{Pr(Y|aa)}{1 - Pr(Y|aa)} = e^{\beta_0 + \beta_2 X_2} \\ odds\ for\ non - aa &= \frac{Pr(Y|non - aa)}{1 - Pr(Y|non - aa)} = e^{\beta_0 + \beta_1 + \beta_2 X_2} \\ odds\ ratio &= \frac{odds\ for\ aa}{odds\ for\ non - aa} = \frac{1}{e^{\beta_1}} \end{aligned}$$