first difference poisson regression

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\begin{aligned}
log(Y) &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 \\
Y &= e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2} \\
x_1 = 0, Y_1 &= e^{\beta_0 + \beta_2 x_2} \\
x_1 = 1, Y_2 &= e^{\beta_0 + \beta_1 + \beta_2 x_2} \\
first\ difference &= Y_2 - Y_1 \\
&= e^{\beta_0 + \beta_1 + \beta_2 x_2} - e^{\beta_0 + \beta_2 x_2}
\end{aligned}
$$

Poisson regression assumes Y has poisson distribution. In this case, with x_1 changing from 0 to 1, the difference between Y's is dependent on x_2 as well even though there is no interaction term in the model.