Nuclear Single-Particle Energy Levels Using the Woods–Saxon Potential with Spin–Orbit Interaction

Mhamad Cheikh

Abstract

In this work, we study single-particle bound states in atomic nuclei using a realistic Woods–Saxon mean-field potential including the spin–orbit interaction. The radial Schrödinger equation is solved numerically using the Numerov method. Bound energy levels are obtained automatically via a shooting method, and the resulting shell structure exhibits the characteristic spin–orbit splitting observed in nuclear systems.

1 Introduction

The nuclear shell model provides a powerful framework to describe the structure of atomic nuclei. A crucial ingredient for reproducing experimental magic numbers is the inclusion of a strong spin–orbit interaction in the nuclear mean-field potential. The Woods–Saxon potential offers a realistic description of the finite depth and diffuse surface of the nucleus, in contrast to the harmonic oscillator model.

In this paper, we compute nuclear single-particle energy levels using a Woods–Saxon potential augmented by a spin–orbit term. The radial Schrödinger equation is solved numerically using the Numerov algorithm.

2 Woods–Saxon Potential

The central nuclear potential is modeled by the Woods–Saxon form

$$V_{\mathrm{WS}}(r) = -\frac{V_0}{1 + \exp\left(\frac{r-R}{a}\right)}, \tag{1}$$

where:

- V_0 is the potential depth,
- R is the nuclear radius,
- a is the surface diffuseness.

This potential accounts for the finite size of the nucleus and the smooth transition at its surface.

3 Spin–Orbit Interaction

The spin–orbit interaction plays a fundamental role in nuclear structure. It is given by

$$V_{\mathrm{SO}}(r) = \lambda \frac{1}{r} \frac{dV_{\mathrm{WS}}}{dr} \left[j(j+1) - l(l+1) - \frac{3}{4} \right], \tag{2}$$

where l is the orbital angular momentum and $j = l \pm \frac{1}{2}$ is the total angular momentum.

This interaction lifts the degeneracy between states with the same l but different j, leading to the experimentally observed shell structure.

4 Radial Schrödinger Equation

The radial Schrödinger equation for a nucleon of mass m reads

$$\frac{d^2u(r)}{dr^2} = \left[\frac{2m}{\hbar^2}(V(r) - E) + \frac{l(l+1)}{r^2}\right] u(r), \tag{3}$$

where $u(r) = rR(r)$ is the reduced radial wavefunction.

The centrifugal term acts as a repulsive barrier for $l > 0$.

5 Numerov Method

The Numerov method is a sixth-order accurate algorithm for solving second-order differential equations of the form

$$\frac{d^2u}{dr^2} = f(r)u(r). \tag{4}$$

It is particularly well suited for quantum mechanical bound-state problems due to its numerical stability and efficiency. Boundary conditions are chosen such that

$$u(0) = 0, \tag{5}$$

ensuring regularity at the origin.

6 Bound-State Search

Bound states correspond to negative energy solutions whose wavefunctions vanish at large distances. A shooting method is employed by scanning the energy range $E < 0$ and detecting sign changes of the wavefunction at the outer boundary. Each sign change indicates the presence of an eigenvalue.

The degeneracy of each level is given by

$$g = 2j + 1. \tag{6}$$

7 Results and Discussion

The computed energy spectrum exhibits clear spin–orbit splitting. For example, states such as $p_{3/2}$ and $p_{1/2}$ are separated in energy, with the $j = l + 1/2$ states lying lower, in agreement with experimental observations.

The resulting shell structure qualitatively reproduces the nuclear magic numbers and confirms the essential role of the spin–orbit interaction.

8 Conclusion

We have numerically computed nuclear single-particle bound states using a Woods–Saxon potential including a spin–orbit interaction. The Numerov method provides an accurate and efficient solution of the radial Schrödinger equation. The results demonstrate the emergence of realistic nuclear shell structure and highlight the importance of the spin–orbit coupling in nuclear physics.

References

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