Digital Signal Processing (DSP)

Part 1-B

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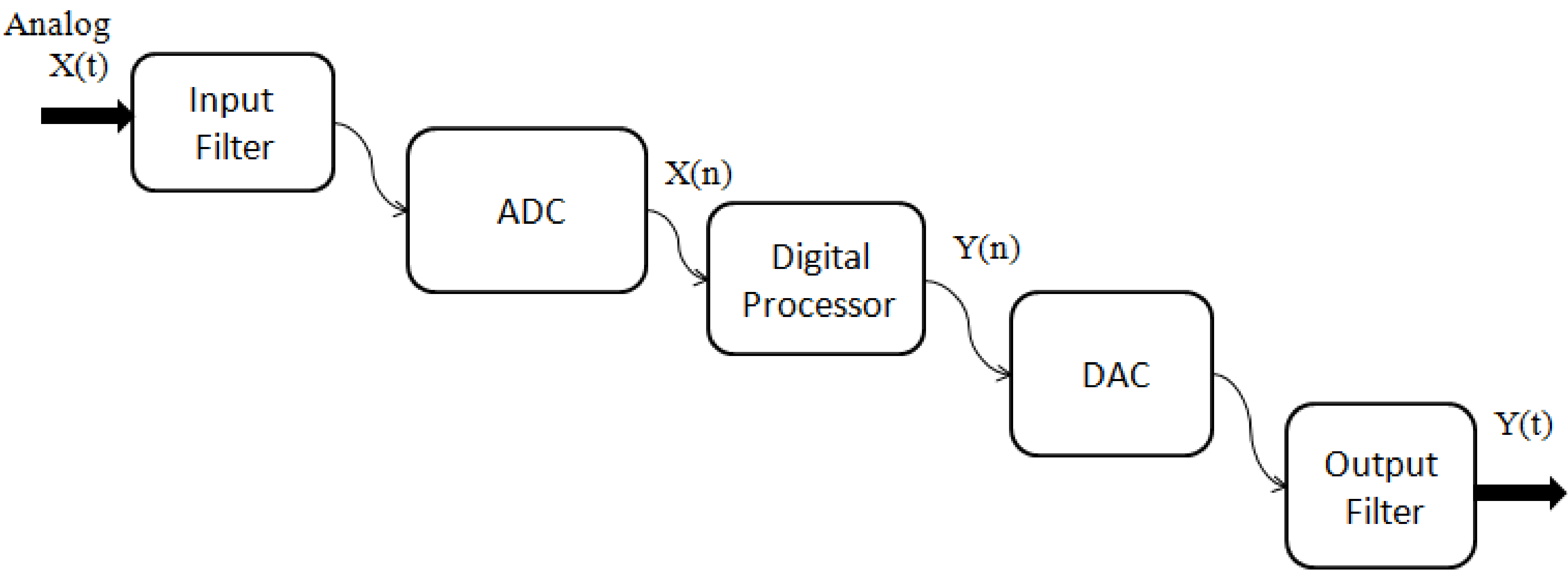
Typical Real-time DSP Systems



Typical Real-time DSP Systems

- *Input Filter* is used to band limit the analog input sign al prior to digitization to reduce aliasing.

- *ADC* most signals in real world are in analog form so an ADC is needed to convert input signal to digital form.

Typical Real-time DSP Systems

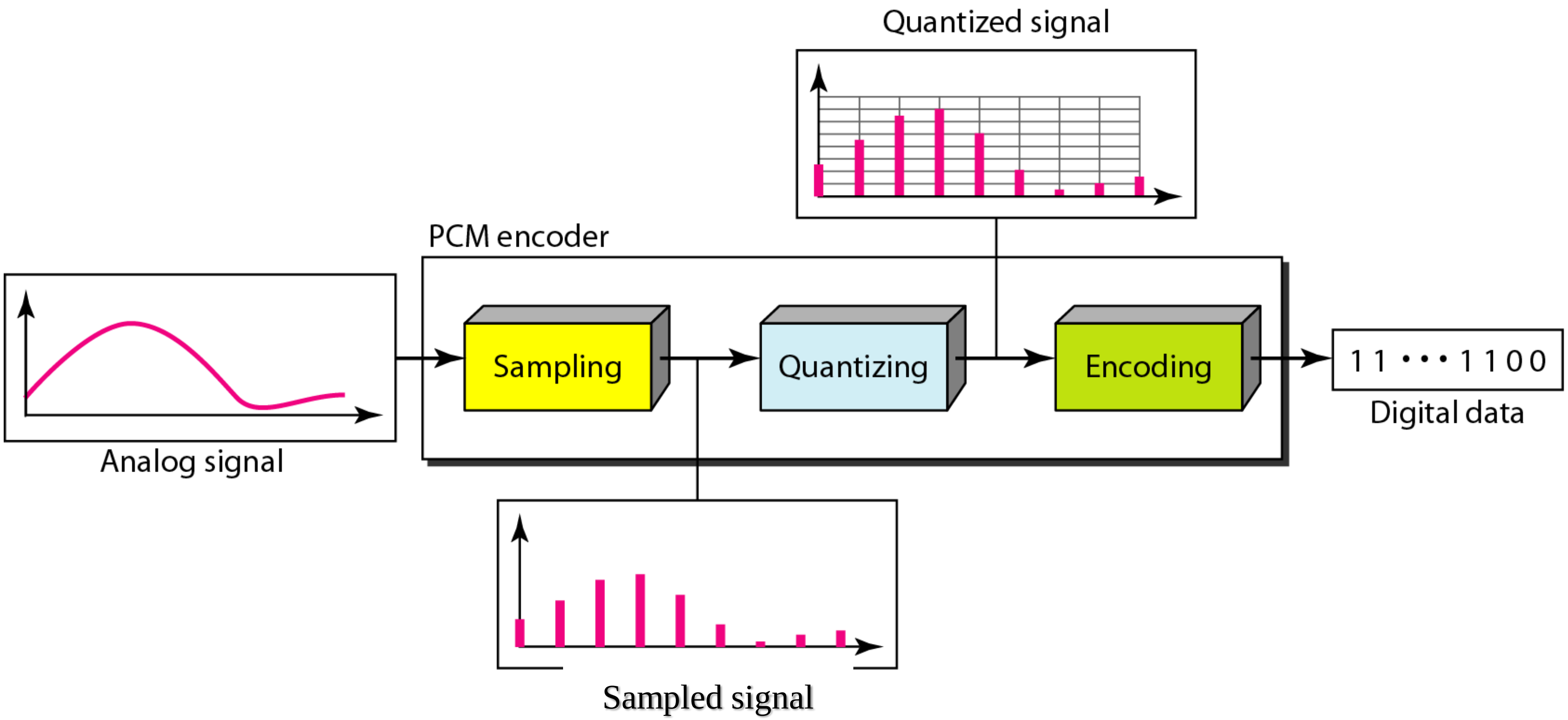
- *Digital Processor* is the core of the system, it applies one of the several DSP algorithms on the digital inpu t according to needed task.

- *DAC* used to return the processed signal back to its analog form.

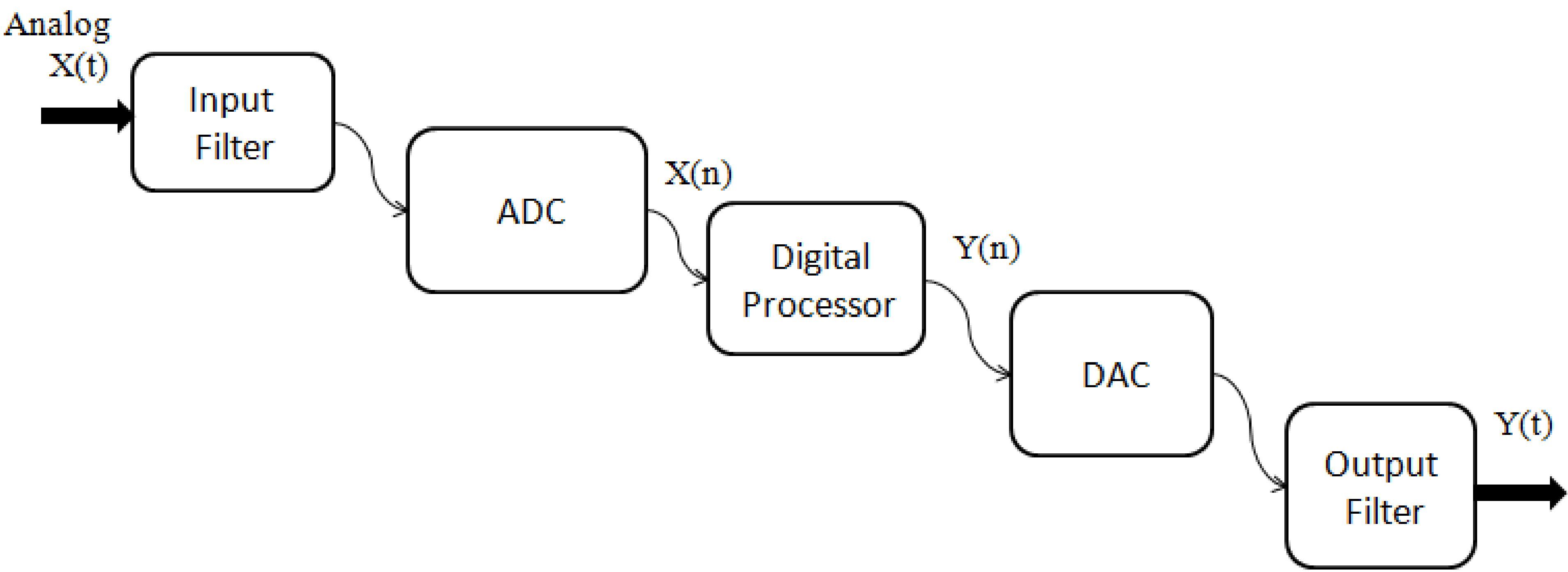
- *Output Filter* smoothes out the outputs of the DAC a nd removes unwanted high frequency components.

Analog-to-Digital Converter (ADC)

Pulse Code Modulation Technique (PCM)



Typical Real-time DSP Systems



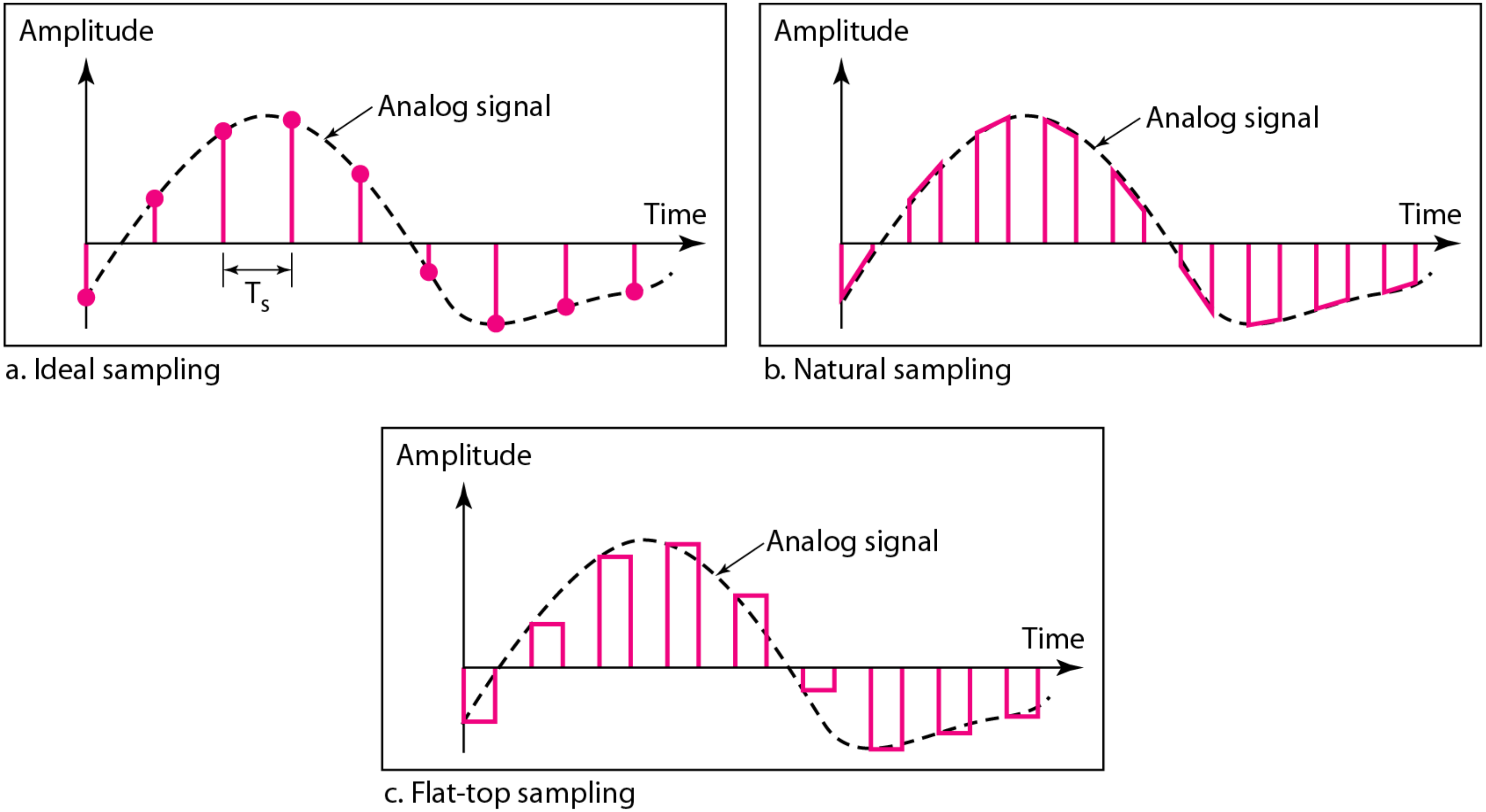
PCM Process

- It consists of three steps :
  1. Sampling
  2. Quantization
  3. Binary encoding
- Before we sample, we must filter the signal to limit the maximum frequency of the signal as it affects the sampling rate.
- Filtering should ensure that we do not distort the signal, ex: remove high frequency components that affect the signal shape.

Sampling

- There are 3 sampling methods:
  - Ideal - an impulse at each sampling instant
  - Natural - a pulse of short width with varying amplitude
  - Flattop - sample and hold, amplitude value

Sampling Methods



Uniform (Ideal) Sampling

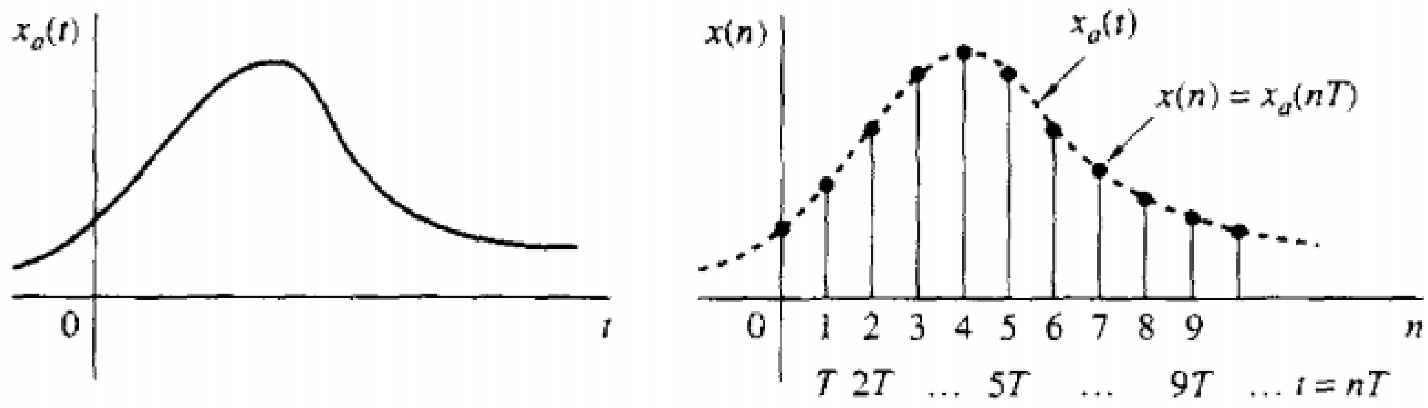
- Analog signal is sampled every T secs.

T is referred to as the sampling interval. $F_s = 1/T$ is called the sampling rate or sampling frequency

- The analog signal can be reconstructed from the samples with out any distortion provided that the sampling frequency is sufficiently high to avoid the aliasing problem.

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)



$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$

$$t = nT = \frac{n}{F_s}$$

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)

$$x_a(t) = A\cos(2\pi F t + \theta)$$

$$x_a(nT) \equiv x(n) = A\cos(2\pi F n T + \theta)$$

$$= A\cos\left(\frac{2\pi n F}{F_s} + \theta\right)$$

Relative or normalized frequency

$$f = \frac{F}{F_s}$$

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)

$$f = \frac{F}{F_s}$$

$-\infty < F < \infty \qquad -\infty < \Omega < \infty$

$-1/2 < f < 1/2 \qquad -\pi < w < \pi$

$$-\frac{1}{2T} = -\frac{F_s}{2} \le F \le \frac{F_s}{2} = \frac{1}{2T}$$

$$\frac{-\pi}{T} = -\pi F_s < w < \frac{\pi}{T} = \pi F_s$$

$$F_{\max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$\Omega_{\max} = \pi F_s = \frac{\pi}{T}$$

Uniform (Ideal) Sampling

- Sampling introduces an ambiguity , since the highest frequency in a continuous- time signal that can be uniquely distinguished when such a signal is sampled at a rate $F_s = 1/T$ is $F_{max} = F_s /2$

Ex: $X_1 = \cos 2\pi (10) t$, $X_2 = \cos 2\pi (50) t$
Both are sampled at $F_s = 40$.
Find $X_1 (n)$ and $X_2 (n)$?

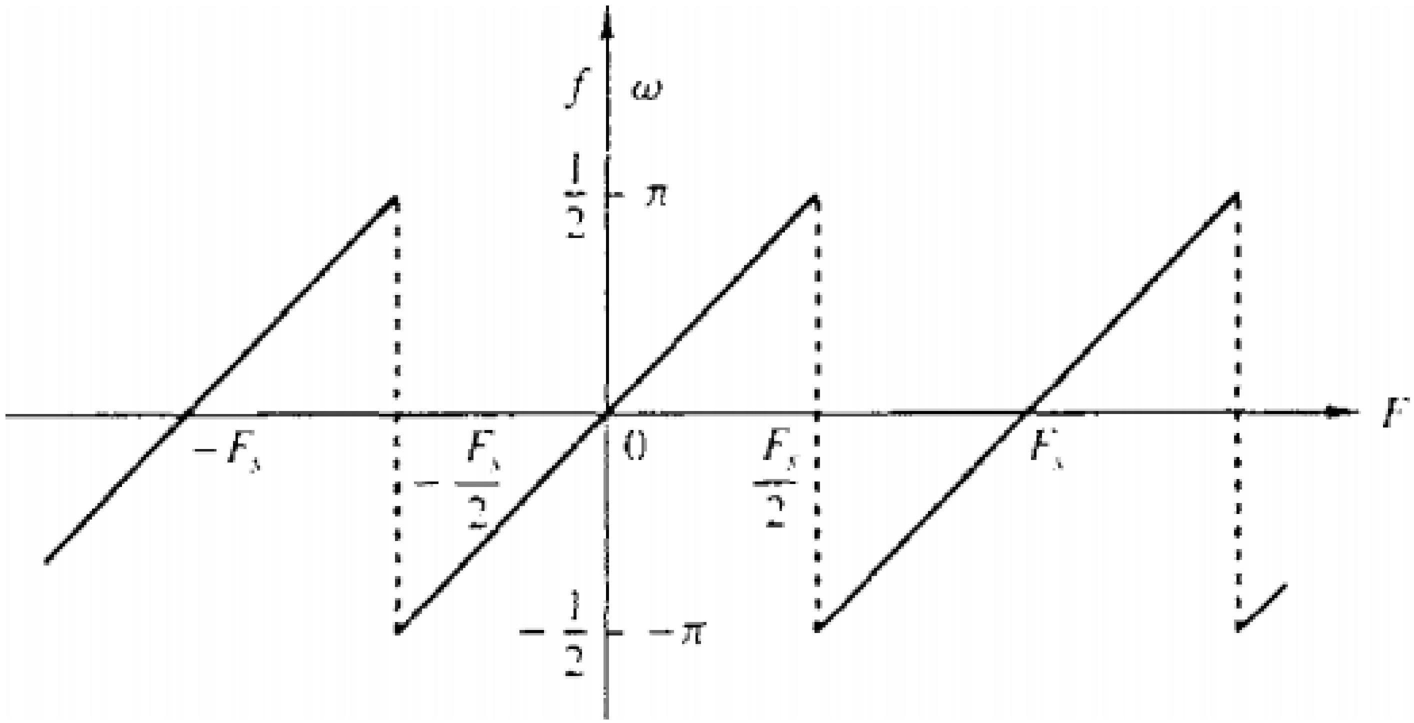
$$x_1(n) = \cos 2\pi \left(\frac{10}{40}\right) n = \cos \frac{\pi}{2} n$$

$$x_2(n) = \cos 2\pi \left(\frac{50}{40}\right) n = \cos \frac{5\pi}{2} n$$

$$= \cos\left(2\pi n + \frac{\pi}{2} n\right) = \cos\frac{\pi}{2} n = x_1(n)$$

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)



The frequency Fs/2 (or w= π) is called the ***folding frequency.***

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)
  Two sinusoids with frequencies $F_0 = 1/8$ Hz and $F_1 = -7/8$ Hz yield identical samples when a sampling rate of $F_s = 1$ Hz is used. it easily follows that for k = 1, $F_0 = F_1 + F_s = (-7/8+1)$ Hz = 1/8 Hz.

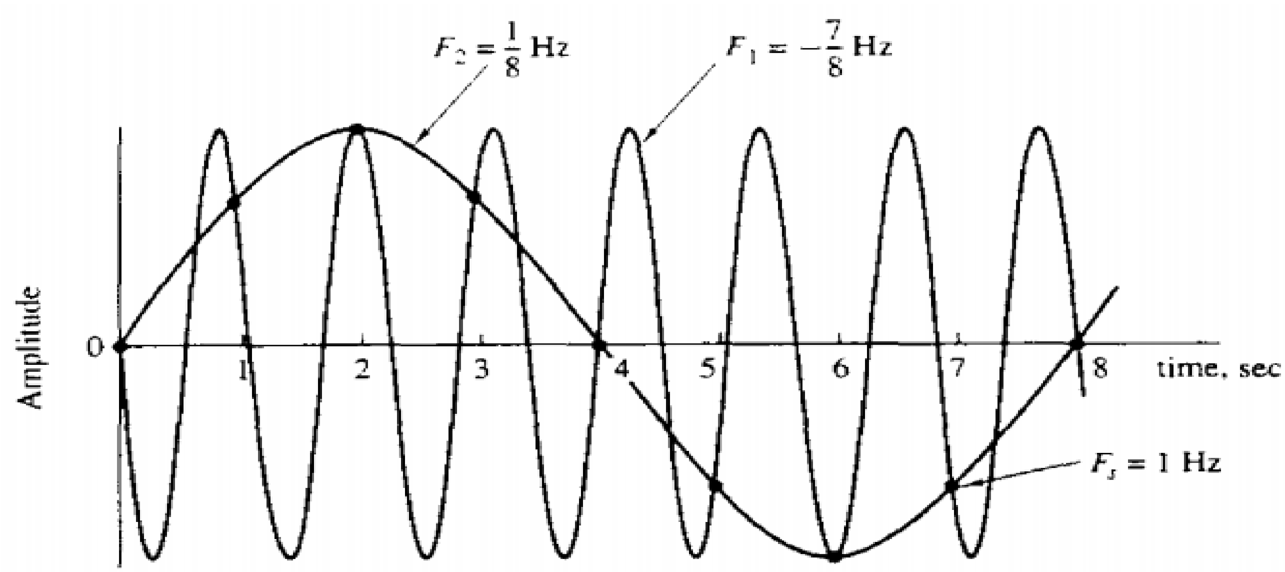


Illustration of aliasing.

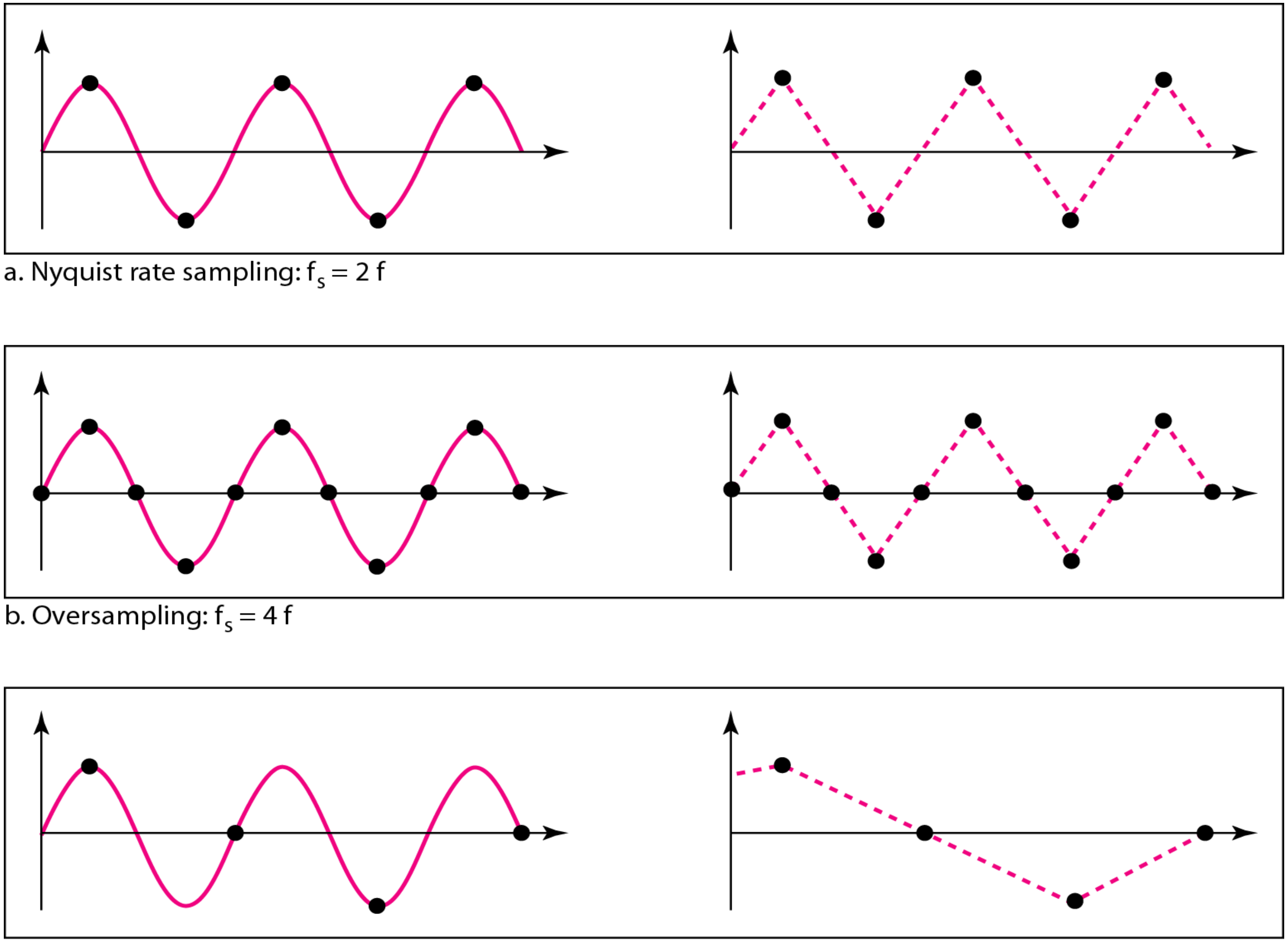
Sampling Theorem

- According to the Sampling theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

$$F_s > 2 F_{max}$$

The sampling rate F_s which is equals to 2 F_{max} is called ***Nyquist rate***.

Recovery of a Sampled Sine wave for different Sampling Rates



a. Nyquist rate sampling: $f_s = 2f$

b. Oversampling: $f_s = 4f$

c. Undersampling: $f_s = f$

Sampling Theorem Problems

- A signal has a bandwidth of 200 kHz, beginning from 0. What is the minimum sampling rate for this signal?

F_s = 400

- A signal has a bandwidth of 300 kHz. What is the minimum sampling rate for this signal?

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We do not know the maximum frequency in the signal.

Uniform (Ideal) Sampling Problems

Consider the analog signal

$$x_a(t) = 3\cos 100\pi t$$

(a) Determine the minimum sampling rate required to avoid aliasing.

(b) Suppose that the signal is sampled at the rate $F_s = 200$ Hz. What is the discrete-time signal obtained after sampling?

(c) Suppose that the signal is sampled at the rate $F_s = 75$ Hz. What is the discrete-time signal obtained after sampling?

(d) What is the frequency $0 < F < F_s/2$ of a sinusoid that yields samples identical to those obtained in part (c)?

Solution

(a) The frequency of the analog signal is $F = 50$ Hz. Hence the minimum sampling rate required to avoid aliasing is $F_s = 100$ Hz.

(b) If the signal is sampled at $F_s = 200$ Hz, the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$$

(c) If the signal is sampled at $F_s = 75$ Hz, the discrete-time signal is

$$\begin{aligned} x(n) &= 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n \\ &= 3\cos\left(2\pi - \frac{2\pi}{3}\right)n \\ &= 3\cos\frac{2\pi}{3}n \end{aligned}$$

(d) For the sampling rate of $F_s = 75$ Hz, we have

$$F = fF_s = 75f$$

The frequency of the sinusoid in part (c) is $f = \frac{1}{3}$. Hence

$$F = 25 \text{ Hz}$$

Clearly, the sinusoidal signal

$$\begin{aligned} y_a(t) &= 3\cos 2\pi Ft \\ &= 3\cos 50\pi t \end{aligned}$$

sampled at $F_s = 75$ samples/s yields identical samples. Hence $F = 50$ Hz is an alias of $F = 25$ Hz for the sampling rate $F_s = 75$ Hz.

Sampling Theorem Problems

Consider the analog signal

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$$

What is the Nyquist rate for this signal?

Solution The frequencies present in the signal above are

$$F_1 = 25 \text{ Hz}, \qquad F_2 = 150 \text{ Hz}, \qquad F_3 = 50 \text{ Hz}$$

Thus $F_{max} = 150$ Hz and according to (1.4.19),

$$F_s > 2F_{max} = 300 \text{ Hz}$$

The Nyquist rate is $F_N = 2F_{max}$. Hence

$$F_N = 300 \text{ Hz}$$

Discussion It should be observed that the signal component $10 \sin 300\pi t$, sampled at the Nyquist rate $F_N = 300$, results in the samples $10 \sin \pi n$, which are identically zero. In other words, we are sampling the analog sinusoid at its zero-crossing points, and hence we miss this signal component completely. This situation would not occur if the sinusoid is offset in phase by some amount θ. In such a case we have $10 \sin(300\pi t + \theta)$ sampled at the Nyquist rate $F_N = 300$ samples per second, which yields the samples

$$\begin{aligned} 10 \sin(\pi n + \theta) &= 10(\sin \pi n \cos \theta + \cos \pi n \sin \theta) \\ &= 10 \sin \theta \cos \pi n \\ &= (-1)^n 10 \sin \theta \end{aligned}$$

Thus if $\theta \neq 0$ or π, the samples of the sinusoid taken at the Nyquist rate are not all zero. However, we still cannot obtain the correct amplitude from the samples when the phase θ is unknown. A simple remedy that avoids this potentially troublesome situation is to sample the analog signal at a rate higher than the Nyquist rate.

Sampling Theorem Problems

- Consider the analog signal

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12{,}000\pi t$$

(a) What is the Nyquist rate for this signal?

(b) Assume now that we sample this signal using a sampling rate $F_s = 5000$ samples/s. What is the discrete-time signal obtained after sampling?

a) F1 = 1000 F2= 3000 F3= 6000

Nyquist frequency >= 2Fmax = 2(6000)=12000

b)

$$x(n) = 3\cos\frac{2000}{5000}\pi n + 5\sin\frac{6000}{5000}\pi n + 10\cos\frac{12000}{5000}\pi n$$

$$x(n) = 3\cos\frac{2}{5}\pi n + 5\sin\frac{6}{5}\pi n + 10\cos\frac{12}{5}\pi n$$

$$x(n) = 3\cos\frac{2}{5}\pi n + 5\sin(2\pi - \frac{4}{5}\pi)n + 10\cos(2\pi + \frac{2}{5}\pi)n$$

$$x(n) = 3\cos\frac{2}{5}\pi n - 5\sin(\frac{4}{5}\pi)n + 10\cos(\frac{2}{5})\pi n$$

$$x(n) = 13\cos\frac{2}{5}\pi n - 5\sin(\frac{4}{5}\pi)n$$

Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones (levels), each of height Δ.

$$\Delta = (\max - \min)/L$$

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Quantization Error

- Quantization is a nonreversible process that results in signal distortion.
- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and the quantized value is referred to as the quantization error.

$$e_q(n) = x_q(n) - x(n) \qquad -\frac{\Delta}{2} \le e_q(n) \le \frac{\Delta}{2}$$

the average quantized error power:

$$P_q = \frac{1}{N}\sum_{i=1}^{N} e_q^2$$

N is number of samples

- The more zones, the smaller Δ which results in smaller errors. BUT, the more zones the more bits required to encode the samples -> higher bit rate.

Quantization Error

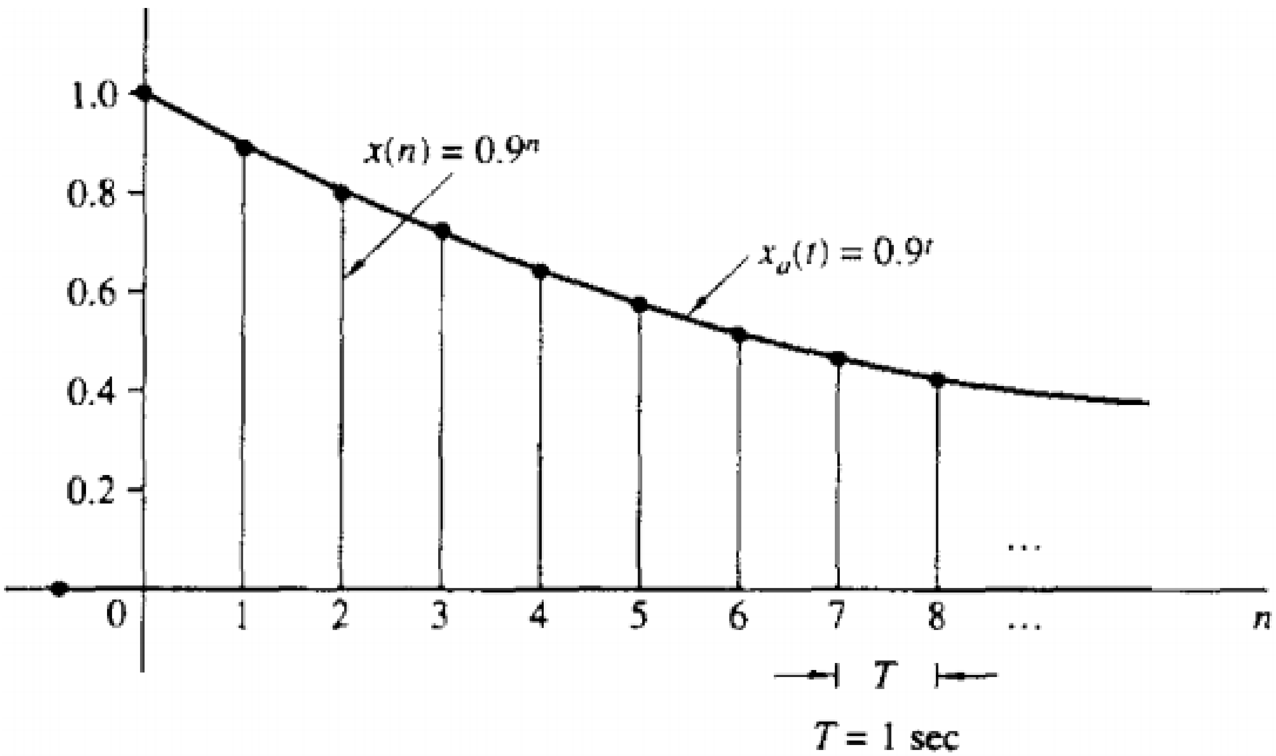
- Signals with lower amplitude values will suffer more from quantization error as the error range: Δ/2, is fixed for all signal levels.

- Non linear quantization is used to alleviate this problem. Goal is to keep quantization error fixed for all sample values.

- Ex: The quantization levels follow a logarithmic curve. Smaller Δ's at lower amplitudes and larger Δ's at higher amplitudes.

Quantization Error (Example)

$$x(n) = \begin{cases} 0.9^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Quantization Error (Example)

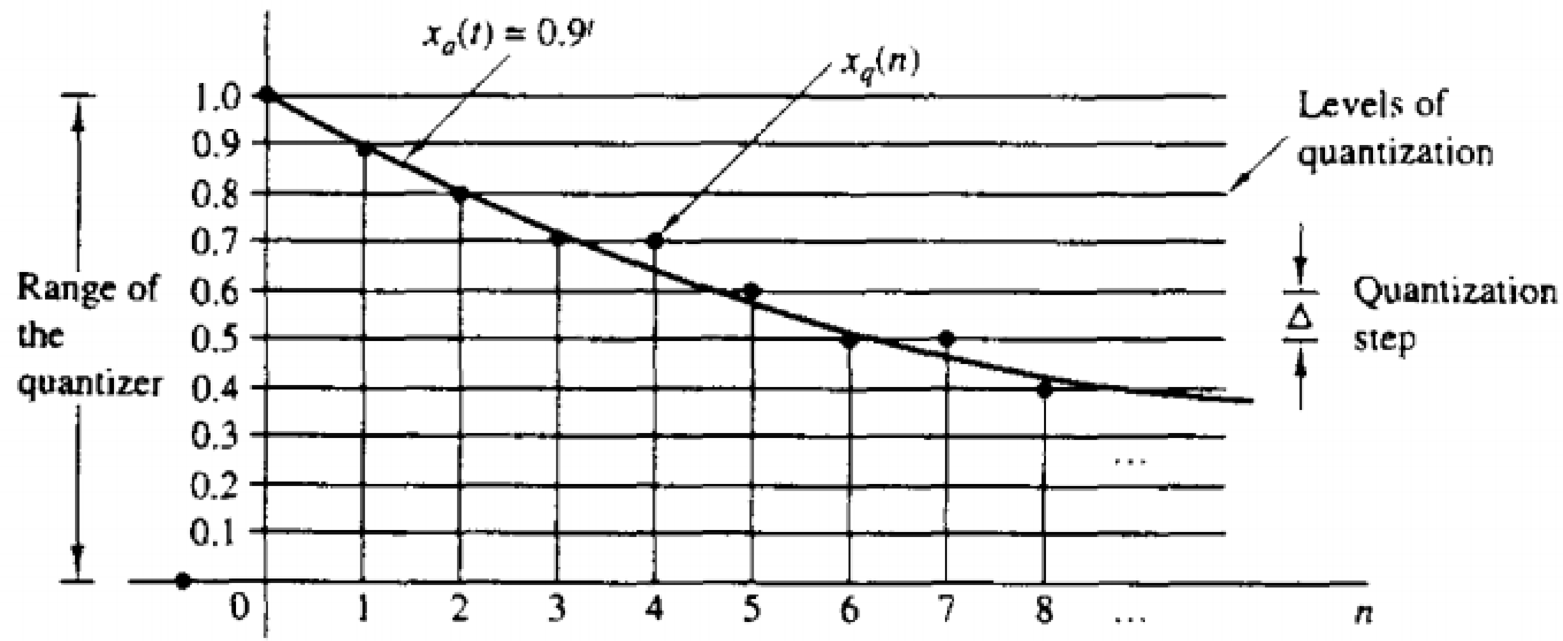


Illustration of quantization.

Encoding

- Each level is then assigned a binary code.
- The number of bits required to encode the levels, or the number of bits per sample as it is commonly referred to, is obtained as follows:

  $n_b = \log_2 L$
- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone 0 to 0.1
  - 001 to zone 0.1 to 0.2, etc.

Quantization & Encoding Problems

- Quantize and encode the following sampled signal $x(n)$ using 8 quantization levels and minimum number of bits. Compute the average error power.

  $x(n)= \{ 0.387, 0.430, 0.478, 0.531, 0.590, 0.6561, 0.729, 0.81, 0.9 , 1, 0.2\}.$

Quantization & Encoding Problems

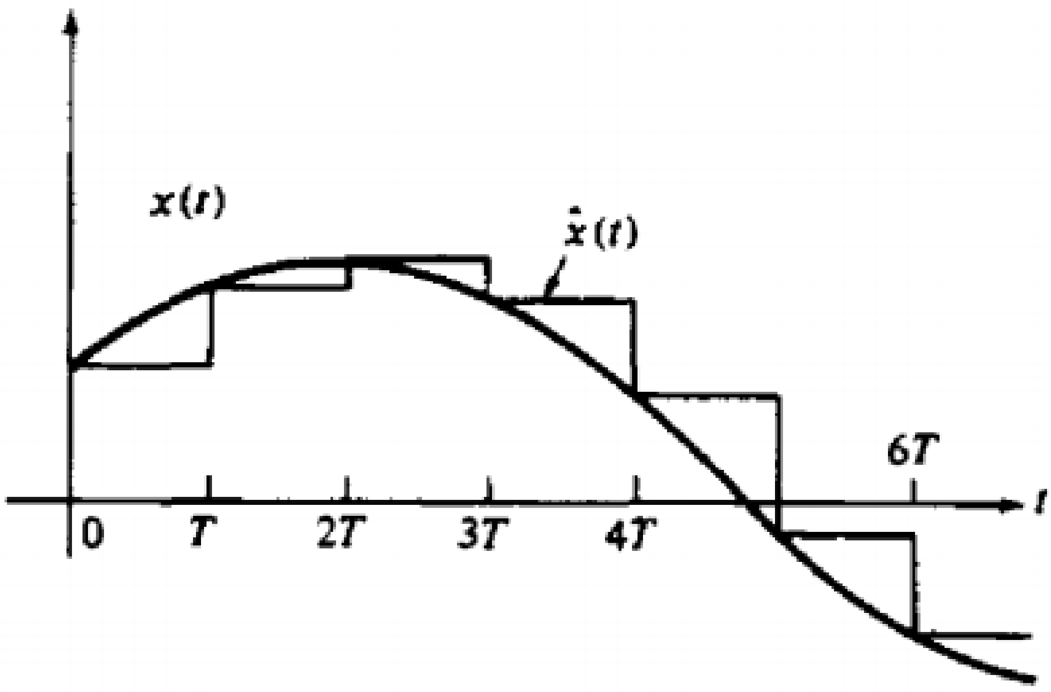
- Quantize and encode the following sampled signal x(n) using 4 quantization levels and minimum number of bits. Compute the average error power in both cases.

x(n)= {-1.22, 1.5, 3.24, 3.94, 2.20, -1.10, -2.26, -1.88, -1.2 }.

Digital to Analog Convertor

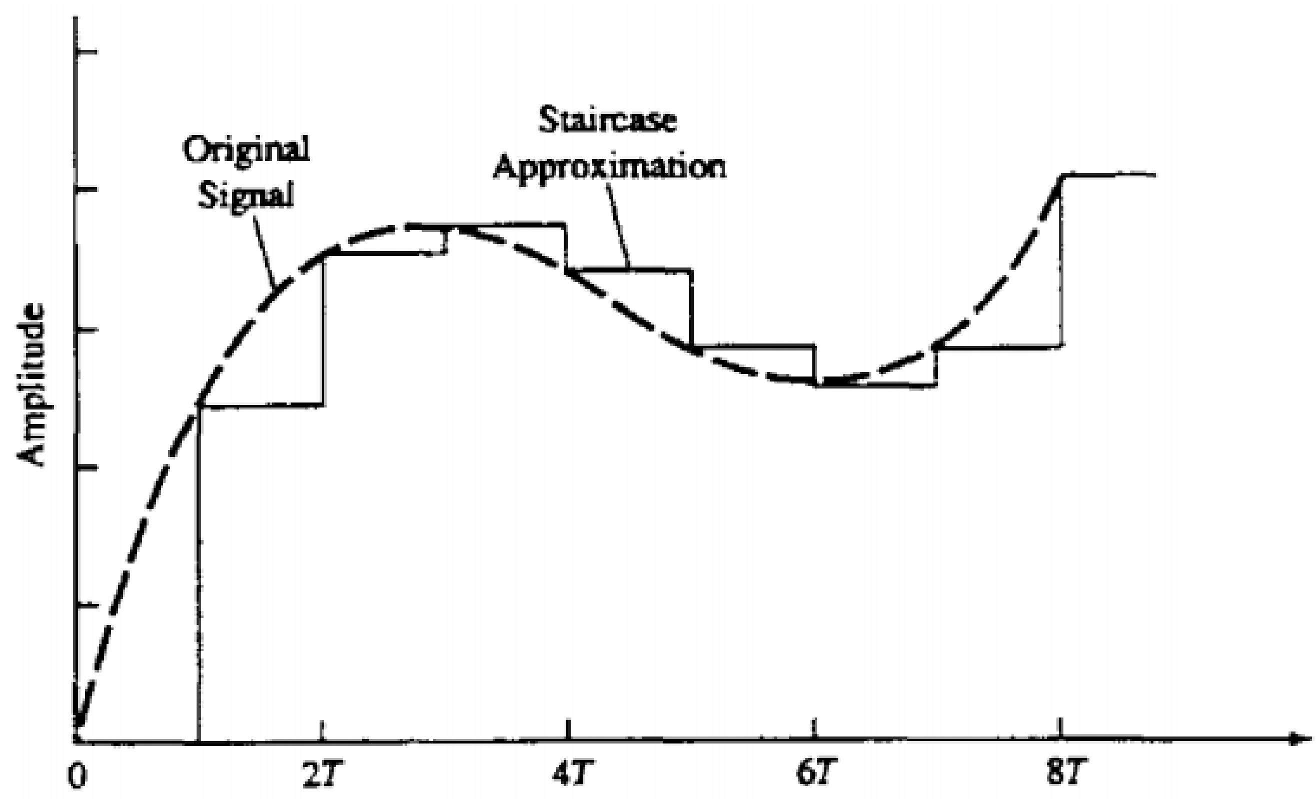
- The task of D/A converter is to interpolate between samples.
- The simplest D /A converter is the zero-order hold which simply holds constant the value of one sample until the next one is received .
- A first-order hold approximates x(t) by straight-line segments which have a slope that is determined by the current sample x(nT) and the previous sample x(nT-T).
- Additional improvement can be obtained by using linear interpolation to connect successive samples with straight-line segments.
- Better interpolation can be achieved by using more sophisticated higher-order interpolation techniques.

Digital to Analog Convertor



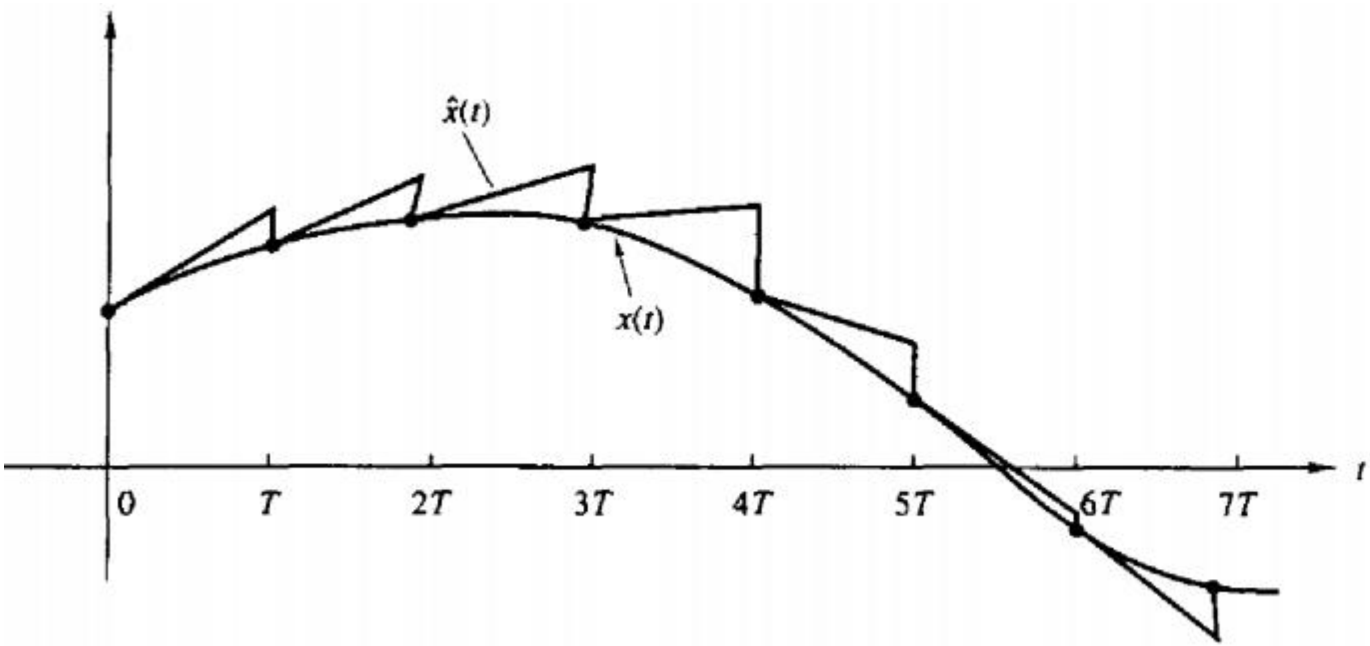
Zero-order hold digital-to-analog (D/A) conversion

Digital to Analog Convertor



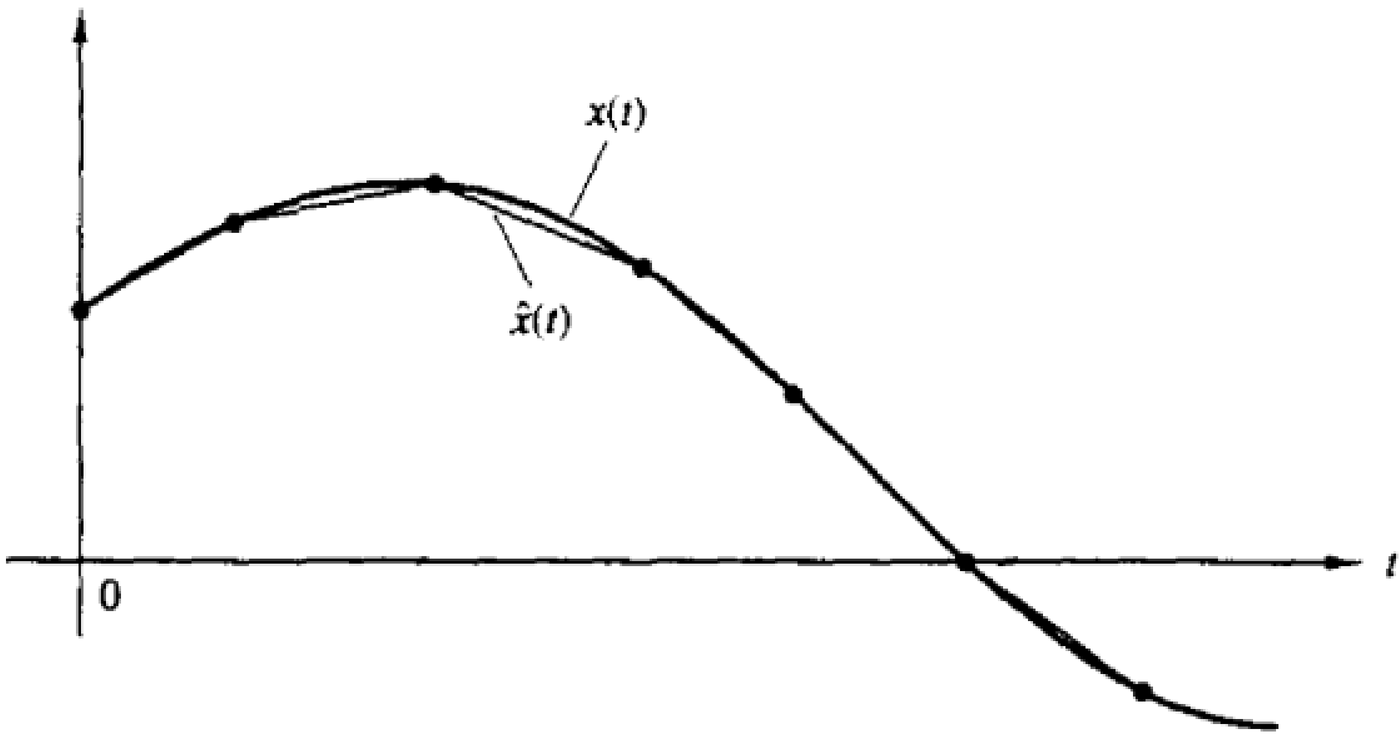
Zero-order hold (staircase) digital-to-analog (D/A) conversion

Digital to Analog Convertor



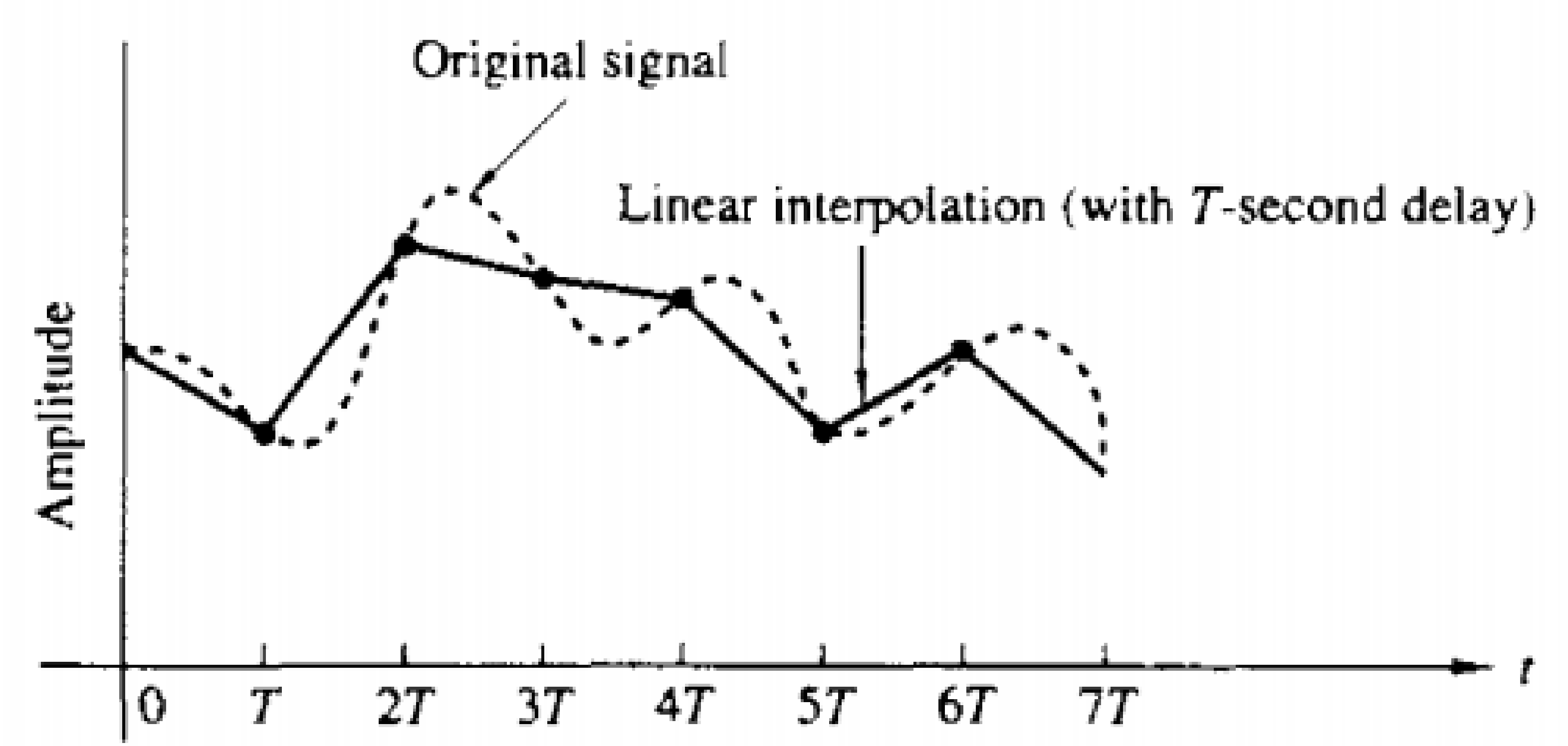
First-order hold digital-to-analog (D/A) conversion

Digital to Analog Convertor



Linear interpolator digital-to-analog (D/A) conversion

Digital to Analog Convertor



Digital to Analog Convertor

- In general, suboptimum interpolation techniques result in passing frequencies above the folding frequency. Such frequency components are undesirable and are usually removed by passing the output of the interpolator through a proper analog filter, which is called a postfilter or smoothing filter.

- Thus, D/A conversion usually involves a suboptimum interpolator followed by a postfilter.

Thanks for Your Attention