Discrete Signals & Systems
(Convolution & Correlation)
(DSP)
Part 3 (2)

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The Convolution Sum Graphically

- The process of computing the convolution between x(k) and h(k) involves the following 4 steps:
- 1- Folding : fold h(k) about k=0 to obtain h(-k) .
- 2- shifting, shift h(-k) by n0 to the right (left) if n0 is positive (negative) to obtain h(no-k).
- 3- multiplication. Multiply x(k) by h(n0-k) to obtain the product sequence $V_0(k) = x(k)h(n0-k)$.
- 4- Summation. Sum all the values of the product sequence $V_0(k)$ to obtain the value of the output at time n=n0.
- Step 2-4 repeated for all possible time shifts $-\infty < n < \infty$.

The Convolution Sum

Given the impulse response of LTI system is

$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$$

Determine the response of the system to the input signal

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

The Convolution Sum

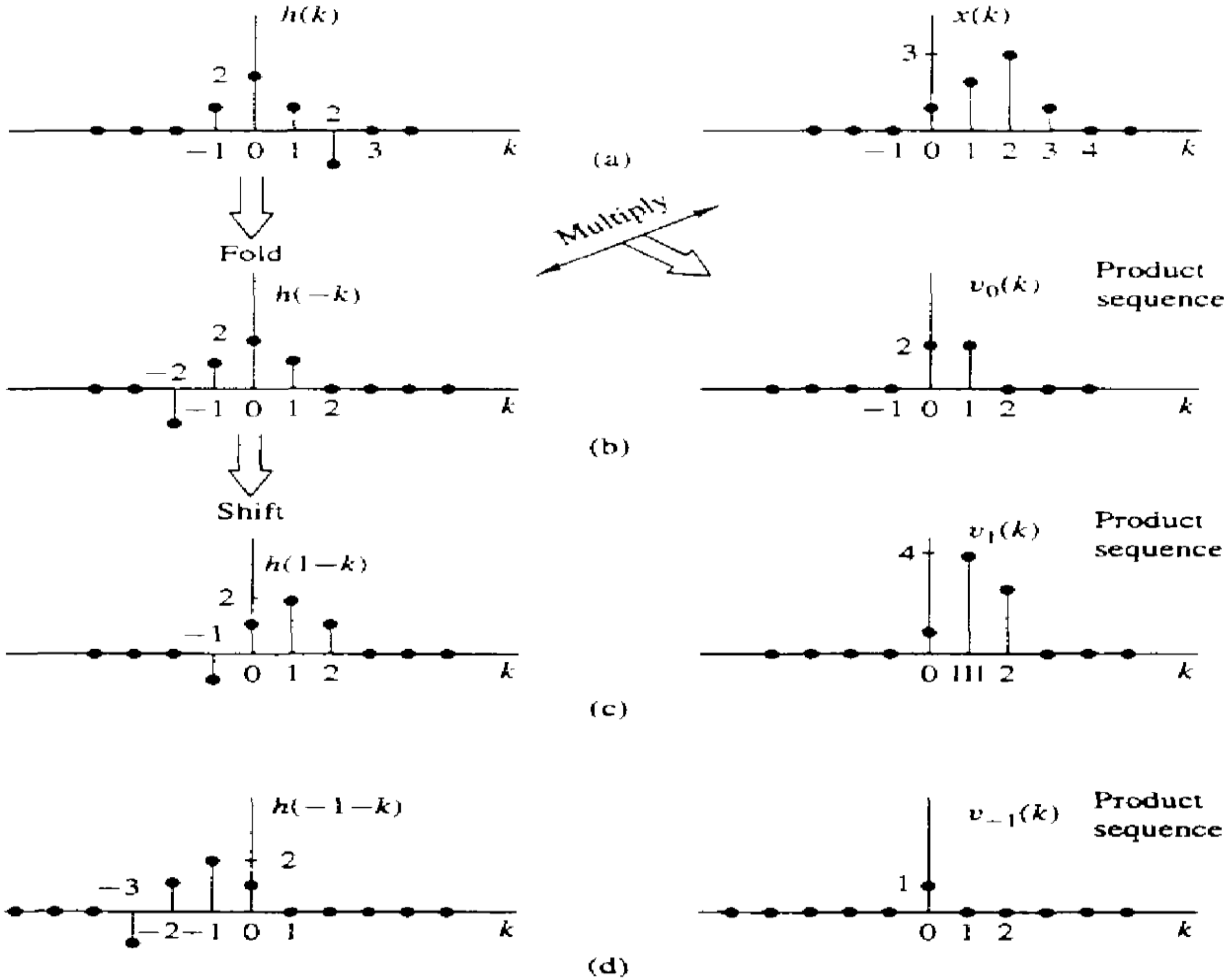

Figure 2.23 Graphical computation of convolution.

The Convolution Sum

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$

$$v_0(k) \equiv x(k)h(-k) \qquad y(0) = \sum_{k=-\infty}^{\infty} v_0(k) = 4$$

$$v_1(k) = x(k)h(1-k) \qquad y(1) = \sum_{k=-\infty}^{\infty} v_1(k) = 8$$

In a similar manner we obtain y(2) by shifting h(-k) two units to the right, forming the product sequence v2(k), and then summing all the terms in the product obtaining y(2)= 8. we obtain y(3) = 3. y(4) = - 2 , y (5) = - 1 . For n > 5, we find that v(n) = 0 because the product sequences contain all zeros. Thus, for n> 5 y(n)=0. Repeat for n=-1 (shift h(-k) to the left one unit) y(-1)=1. y(n)=0 for n≤- 2. Then we have:

$$y(n) = \{\ldots, 0, 0, 1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1, 0, 0, \ldots\}$$

Convolution Properties

$$y(n) = x(n) * h(n) \equiv \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = h(n) * x(n) \equiv \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

Commutative law

$$x(n) * h(n) = h(n) * x(n)$$

Associative law

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

Distributive law

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

Cross Correlation

- It is a process of measuring the degree of similarity (dependency) between two data sets (signals). Given two data sequences x1 and x2, the correlation between them is computed by taking the summation of the product of the corresponding pairs of points.

$$r_{12} = \sum_{n=0}^{N-1} x_1(n) x_2(n)$$

Applications of Correlation

- Robotic Vision
- Remote Sensing
- Radar and Sonar Systems
- Identification of Signals in Noise
- Control Engineering (observing effect of input on output)

Cross Correlation

- However, this definition produces a result which depends on the number of sampling points taken. This is corrected by normalizing the result to the number of points by dividing it by N.

$$r_{12} = \frac{1}{N}\sum_{n=0}^{N-1} x_1(n)x_2(n)$$

- Positive large results indicate strong correlation between signals, while, negative results indicate negative correlation which means an increase in one signal is associated with a decrease in the other signal and vice versa.
- Small results which tend toward zero indicate that the two signals are almost independent .

Cross Correlation Example

n	1	2	3	4	5	6	7	8	9
x_1	4	2	−1	3	−2	−6	−5	4	5
x_2	−4	1	3	7	4	−2	−8	−2	1

$$r_{12} = \frac{1}{9}(4 \times -4 + 2 \times 1 + -1 \times 3 + 3 \times 7 + -2 \times 4 + -6 \times -2 + -5 \times -8 + 4 \times -2 + 5 \times 1)$$

$$= 5$$

Thanks for Your Attention