Digital Signal Processing
(DSP)
Part 1
(Lecture 1)

Prepared by

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Course Outlines

- *Fundamentals of DSP*
- *Fourier Transform , its inverse and others*
- *Fast Fourier Transform (FFT)*
- *Discrete Systems (Convolution & Correlation)*
- *Finite Impulse Response (FIR) filters*
- *Infinite Impulse Response (IIR) filters*
- *Fundamentals of Wavelets*
- *Time domain & Frequency Domain Features*
- *Multi-rate Digital Signal Processing*

References

<u>Books</u>

- Digital Signal Processing
  Principles, Algorithms, and Applications
  Third Edition
  John G. Proakis, Dimitris G. Manolakis

- Digital Signal Processing
  A practical approach
  Third Edition
  Emmanuel C. Ifeathor, Barrie W. Jervis

Grading System

Final Exam : 50 marks
Midterm : 15 marks
Year work (Package) : 20 marks
Practical tasks + Oral : 15 marks

Part 1 Outlines

- *Signals, Systems, and Signal Processing*
- *Advantages of Digital over Analog Signal Processing*
- *Applications of DSP*
- *Classification of Signals*
- *The Concept of Frequency in Continuous -Time and Discrete-Time Signals*
- *Analog- to-Digital and Digital-to-A nalog Conversion*
  - *Analog to Digital Conversion (ADC)*
  - *Sampling Theorem*
  - *Quantization & Coding (Encoding)*
  - *Digital to Analog Conversion (DAC)*

Signals

- **Signal :** is defined as any physical quantity that varies with time, space, or any other independent variable or variables. Mathematically, we describe a signal as a function of one or more independent variables.

Ex: speech , biological signals, image, video, and radar signals

Mathematical representation:

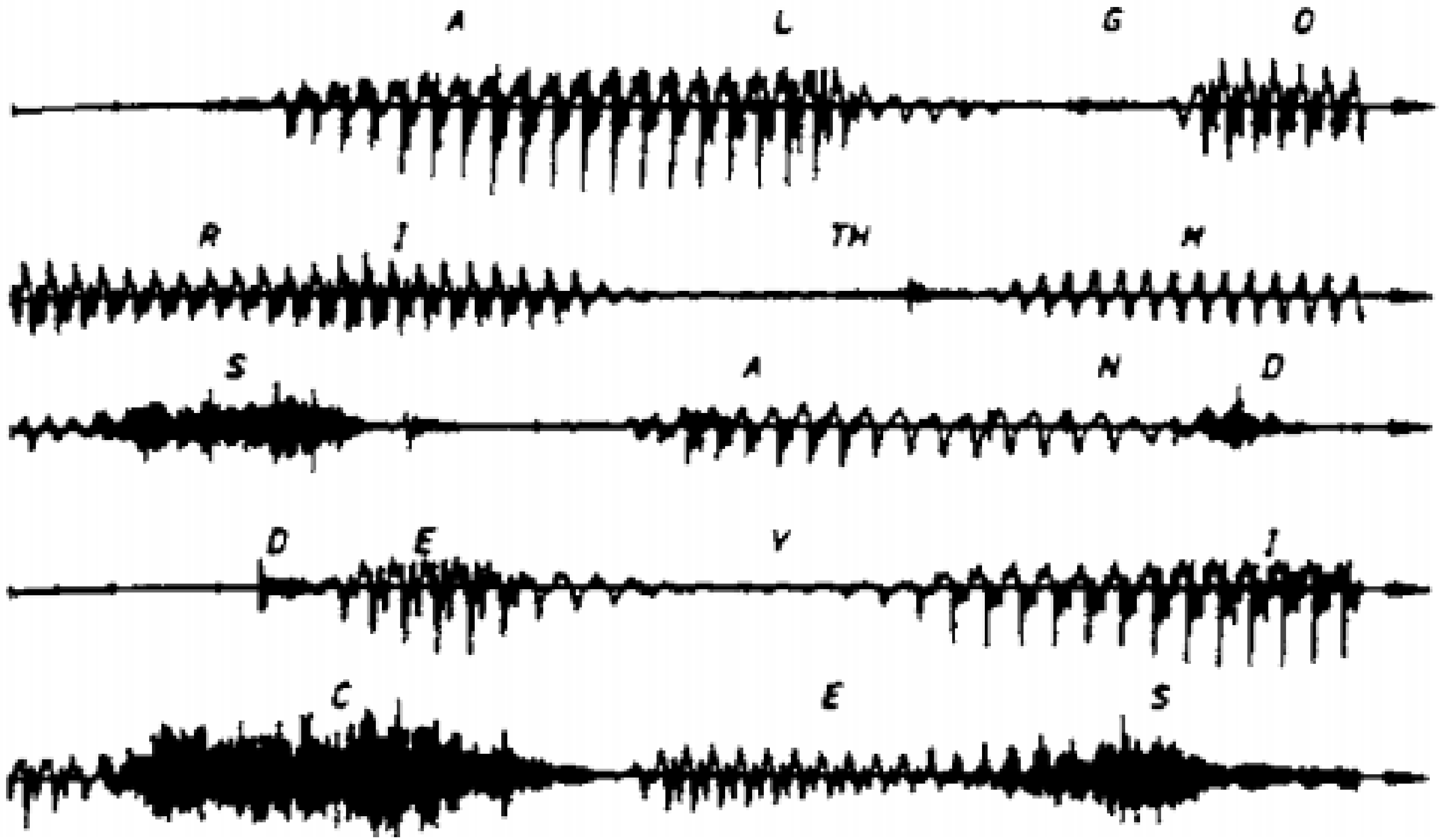
Ex: $A(t)=5t$

$$B(X, Y) = 5Y + 4XY + 10\,X^2$$

- ***All signals can be represented mathematically ?? No***

Ex: speech

Signals (Speech)



Signals (ECG)

Signals

- Associated with natural signals are the means by which such signals are generated . For example, speech signals are generated by forcing air through the vocal cords. Images are obtained by exposing a photographic film to a scene or an object. This stimulus is called the ***Signal Source***.

- A ***System*** is defined as the physical device (or software) that perform operation on the signal and it is characterized by this operation. Such operation are referred to as ***Signal Processing***

Ex: filtering system

The Filtering operation is signal processing

Signals

- Signal processing can be performed by a number of mathematical operations in ***software program*** or performed by ***digital hardware*** (logic circuits).

- General speaking, a system can be implemented as a combination of both hardware and software, each of which performs its own set of operations.

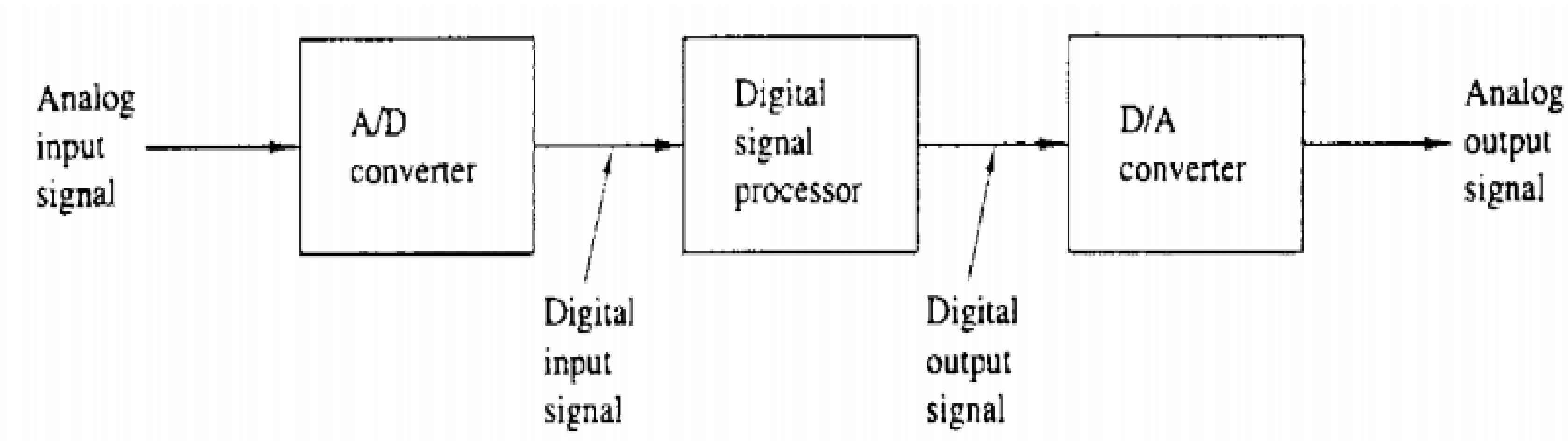
- The set of rules in the software implementation of the system that corresponds to the needed mathematical operations is called ***Algorithm***.

Signal processing System

- *Analog Signal Processing (ASP) System*

Signal Processing System

- ***Digital Signal Processing (DSP) System***



Advantages of DSP

- It allows the development of powerful, smaller, faster and cheaper digital computers and special-purpose digital hardware. Hence, DSP has made it possible to construct highly sophisticated digital systems capable of performing complex tasks.

- Digital processing hardware allows programmable operations. Through software, one can more easily modify functions to be performed by hardware. Thus, it provides a higher degree of flexibility .

Drawbacks of DSP

- Regarding signals with wide bandwidth (above 100 MHz), real-time processing is a requirement.

- Conversion of an analog signal to digital accomplished by sampling the signal and then quantizing the samples which results in distortion that prevents constructing the original signal back.

Can we control this amount of distortion ?? ***Yes***

Applications of DSP

- Biomedical applications
- Image processing
- Telecommunications
- Speech processing
- Military (radar or sonar processing)
- Data compression
- Instrumental/ control

ex: noise reduction and spectrum analysis

Classification of Signals

- ***Multi-channel and Multi-dimensional signal***

If the signal is generated by multiple sources or sensors, it is called multi-channel signal. On the other hand, if the signal is dependant on more than one independent variable it is called multi-dimensional signal.

Ex: S(X, Y) is a multi-dimension signal S (S can be a 2D image),

however,

$$\mathbf{S}_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

Classification of Signals

- ***Continuous-time Versus Discrete Signals***

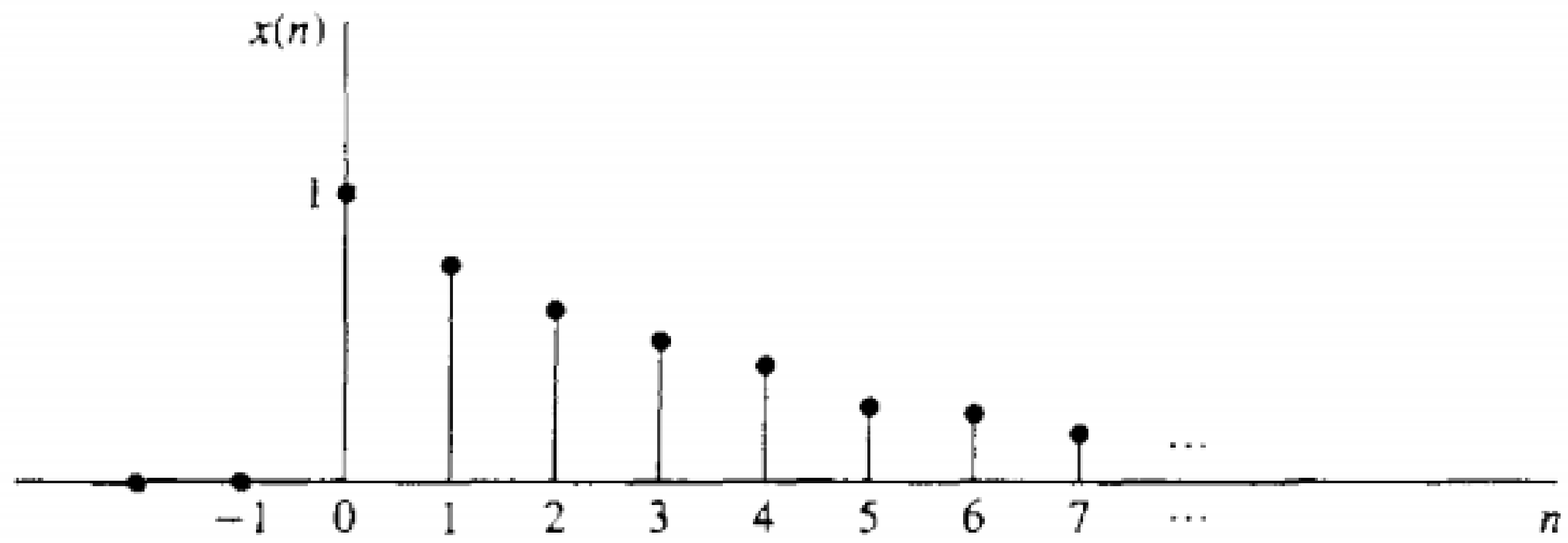
Continuous-Time (analog) signals are defined for every value of time from -∞ to ∞. On the other hand, Discrete-time signals are defined only at certain values of time (usually equally spaced).

Ex:

$$x(nT) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Classification of Signals

- ***Continuous-time Versus Discrete Signals***



Sampling process is the process of selecting values of an analog signal at discrete-time instants.

Classification of Signals

- ***Continuous-valued Versus Discrete-valued Signals***

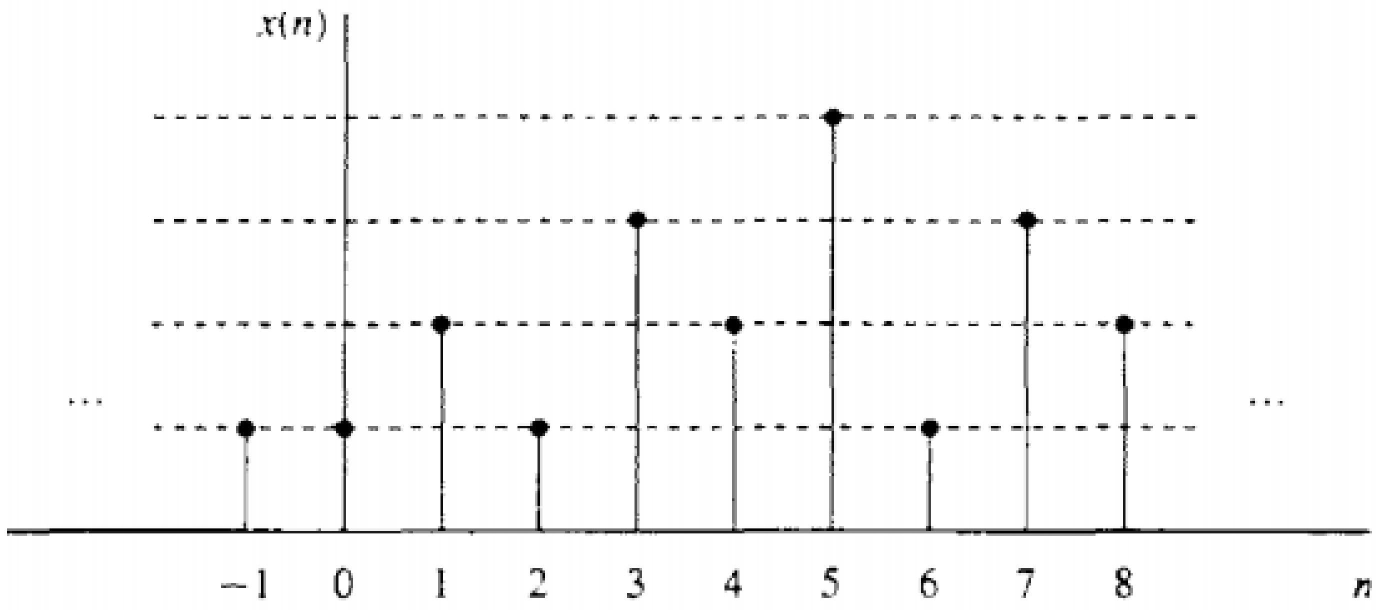
The signal is continuous valued, if it can take all possible values on a finite or infinite range. Alternatively, if the signal takes on values from a finite set of possible values, it is called discrete-valued signal.

A discrete-time signal having a set of discrete values is called a digital signal.

Quantization process is the process of converting continuous -valued signal into a discrete-valued signal. Is basically an approximation process.

Classification of Signals

- *discrete-valued signal*



Digital signal with four different amplitude values.

Classification of Signals

- ***Deterministic Versus Random Signals***

Any signal that can be uniquely described by an explicit mathe matical expression, table of data or a well defined is called deterministic. Alternatively, a random signal evolves in time in an unpredictable manner.

Is the classification of real world signals to deterministic or random a clear task ?? ***No***

Classification of Signals Problems

- Classify the following signals according to whether they are (1) one- or multidimensional; (2) single or multichannel, (3) continuous time or discrete time. (4) analog or digital (in amplitude).

  i) 12 lead ECG recording

  One dimension, multichannel, continuous and continuous-valued (analog)

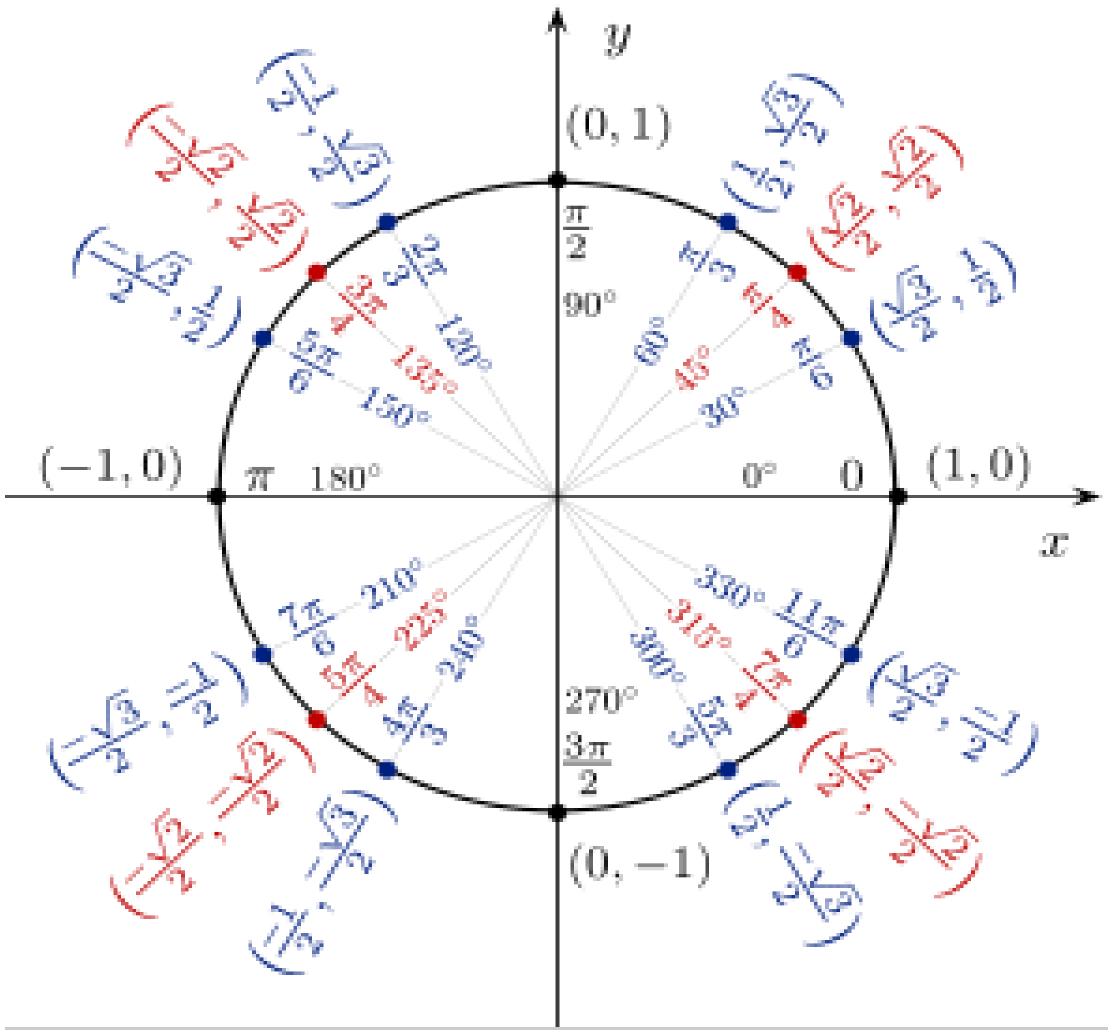
  ii) Blood pressure measured every hour for a patient

  One dimension, multichannel , discrete time and discrete-valued (digital)

  iii) Position of specific object captured every some instant of time in 3D space

  Multi-dimension, single channel discrete time and continuous-valued

Reminder to Trigonometric Rules



Reminder to Trigonometric Rules

Degrees	30°	60°	120°	150°	210°	240°	300°	330°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
Grads	33⅓ grad	66⅔ grad	133⅓ grad	166⅔ grad	233⅓ grad	266⅔ grad	333⅓ grad	366⅔ grad
Degrees	45°	90°	135°	180°	225°	270°	315°	360°
Radians	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
Grads	50 grad	100 grad	150 grad	200 grad	250 grad	300 grad	350 grad	400 grad

Reminder to Trigonometric Rules

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = +\cos\theta$$

$$\sin(\theta + 2\pi) = +\sin\theta$$
$$\cos(\theta + 2\pi) = +\cos\theta$$

$$e^{ix} = \cos x + i\sin x$$
$$e^{-ix} = \cos(-x) + i\sin(-x) = \cos x - i\sin x$$
$$\cos x = \mathrm{Re}\{e^{ix}\} = \frac{e^{ix} + e^{-ix}}{2}$$
$$\sin x = \mathrm{Im}\{e^{ix}\} = \frac{e^{ix} - e^{-ix}}{2i}$$

Frequency in Continuous and Discrete Time Signals

- ***Periodic Signals (Sinusoidal Signals)***

The concept of frequency is directly related to the concept of time. Actually , it has the dimension of inverse time. Thus, we should expect that the nature of time (continuous or discrete) would affect the nature of the frequency accordingly.

$$F = 1/T$$

T is the period of the signal

Frequency in Continuous and Discrete Time Signals

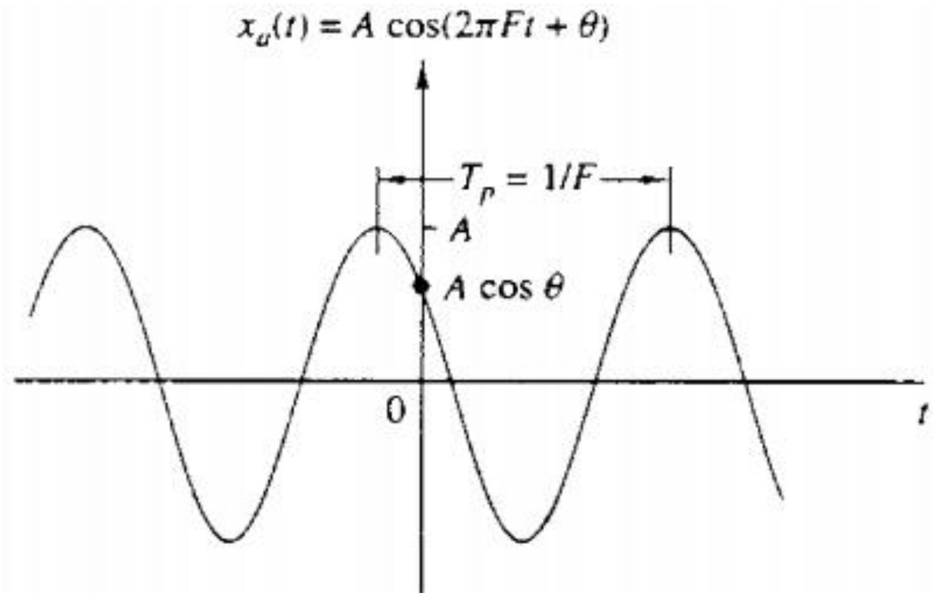
- ***Continuous Sinusoidal Signals***

$$x_a(t) = A\cos(\Omega t + \theta), -\infty < t < \infty$$

$$\Omega = 2\pi F$$

$$x_a(t) = A\cos(2\pi F t + \theta), -\infty < t < \infty$$

The subscript a used with x(t) denotes an analog, A is the amplitude of the sinusoid. Ω is the frequency in radians per second (rad/s), and θ is the phase in radians.



Frequency in Continuous and Discrete Time Signals

- ***Continuous Sinusoidal Signals***

❑ For every fixed value of the frequency F, $X_a(t)$ is periodic. Indeed, it can easily be shown using elementary trigonometry

$$X_a(t + T_p) = X_a(t)$$

where $T_p = 1/F$ is the fundamental period of the sinusoid al signal.

❑Continuous-time sinusoidal signals with distinct frequencies are themselves distinct.

❑ Increasing the frequency F results in an increase in the rate of oscillation of the signal, in the sense that more periods are included in a given time interval.

Negative Frequencies

- frequency is an inherently positive physical quantity . However, only for mathematical convenience, we need to introduce negative frequencies.

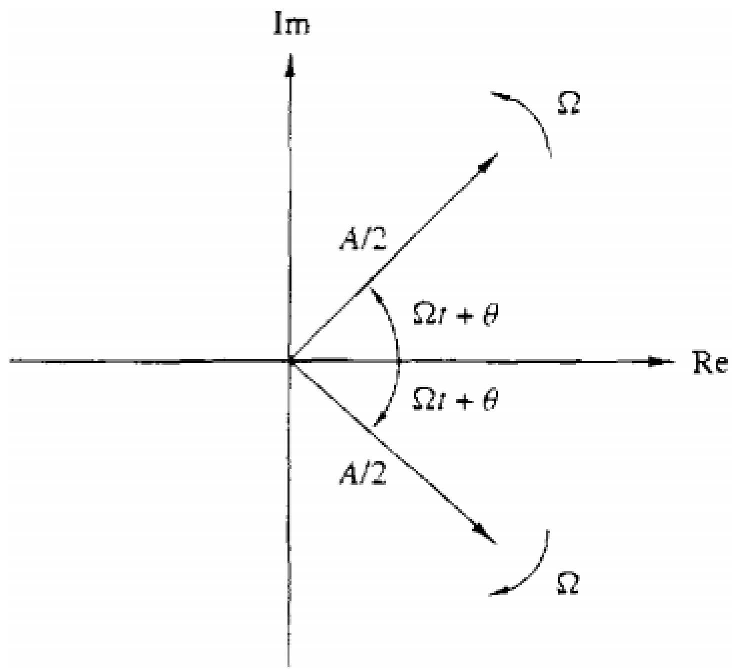
$$\cos x = \mathrm{Re}\{e^{ix}\} = \frac{e^{ix} + e^{-ix}}{2}$$

Euler Formula

$$x_a(t) = A\cos(\Omega t + \theta) = \frac{A}{2}\, e^{j(\Omega t+\theta)} + \frac{A}{2}\, e^{-j(\Omega t+\theta)}$$

Negative Frequencies

- As time progresses the phasors rotate in opposite directions with angular frequencies $\pm\Omega$ radians per second. Since a positive frequency corresponds to counterclockwise uniform angular motion, a negative frequency simply corresponds to clockwise angular motion.



- Hence the frequency range for analog sinusoids is $-\infty < F < \infty$.

Frequency in Continuous and Discrete Time Signals

- ***Discrete Sinusoidal Signals***

$$x(n) = A\cos(\omega n + \theta), \quad -\infty < n < \infty$$

$$\omega \equiv 2\pi f$$

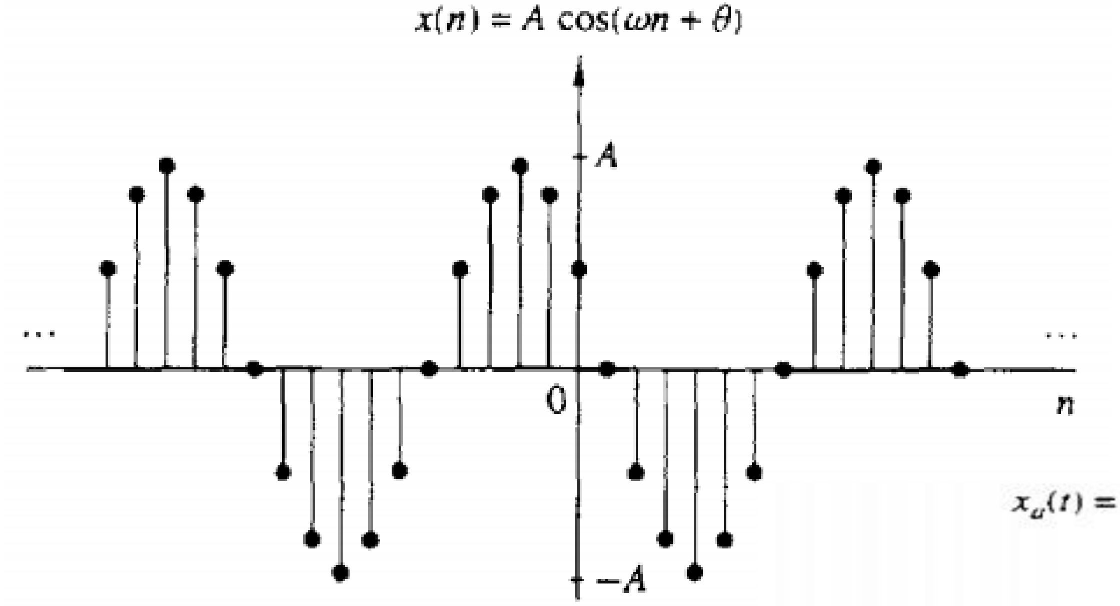
$$x(n) = A\cos(2\pi f n + \theta), \quad -\infty < n < \infty$$

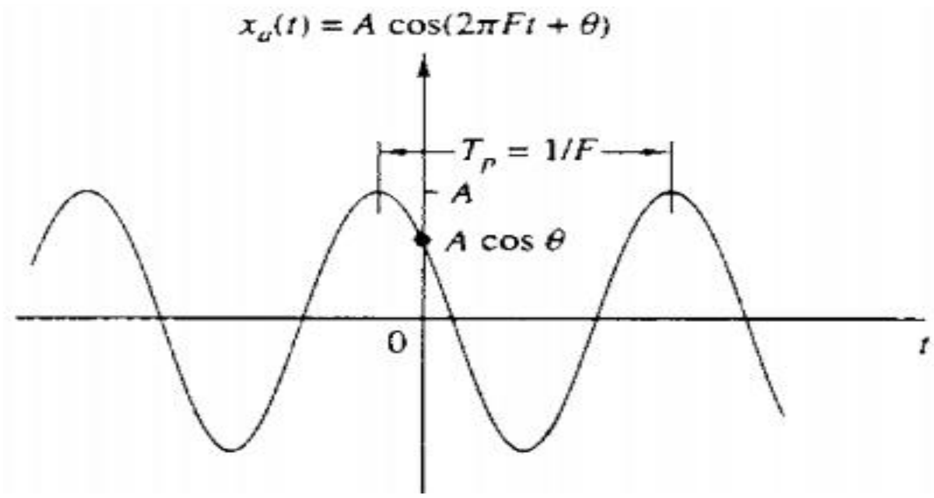
n is an integer variable called sample number; A is the amplitude of the discrete sinusoid. w is the frequency in radians per second (rad/s), and θ is the phase in radians.

- The frequency has dimensions of cycles per sample.

Frequency in Continuous and Discrete Time Signals

- Discrete Sinusoidal Signals (sampled analog signal)





Frequency in Continuous and Discrete Time Signals

- ***Discrete-time Sinusoidal Signals***

❑ A discrete-time sinusoid is periodic only if its frequency f is a rational number.

$$x(n+N) = x(n) \qquad \text{for all } n$$

The smallest value of N for which the equation is true is called the fundamental period.

$$\cos[2\pi f_0(N+n)+\theta] = \cos(2\pi f_0 n + \theta)$$

This relation is true if and only if there exists an integer k such that

$$2\pi f_0 N = 2k\pi$$

$$f_0 = \frac{k}{N}$$

Frequency in Continuous and Discrete Time Signals

- ***Discrete-time Sinusoidal Signals***

❑ Discrete-time sinusoids whose frequencies are separated by an integer multiple of 2π are identical.

For the sinusoidal cos (w_o n+ θ) . It easily follows that

$$\cos[(\omega_0 + 2\pi)n + \theta] = \cos(\omega_0 n + 2\pi n + \theta) = \cos(\omega_0 n + \theta)$$

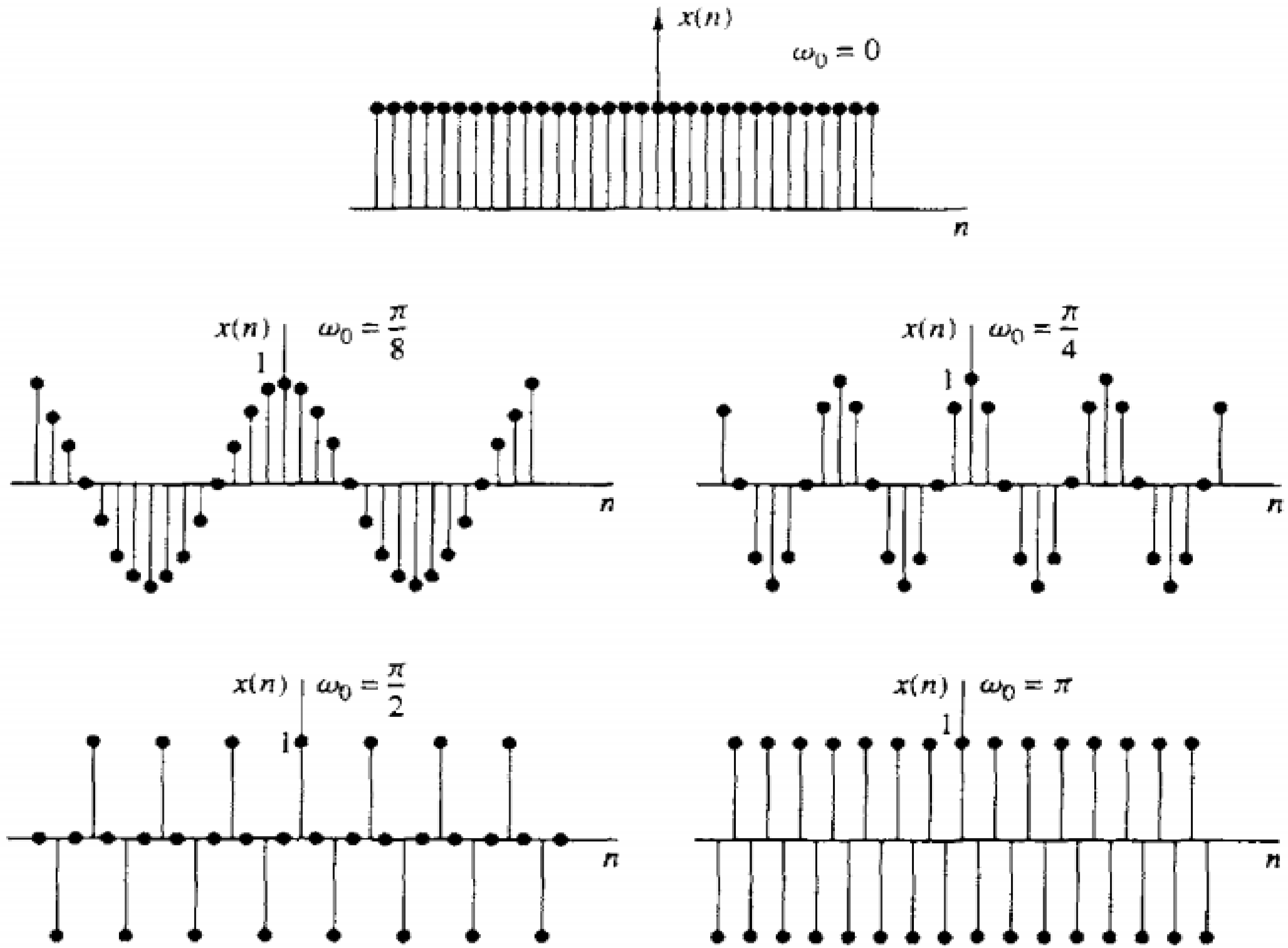
For all sinusoidals

$$x_k(n) = A\cos(\omega_k n + \theta), \qquad k = 0, 1, 2, \ldots$$

where $\omega_k = \omega_0 + 2k\pi, \qquad -\pi \le \omega_0 \le \pi$

are identical. Thus, we regard frequencies in the range 0 <= w <2π or 0 < f < 1 as unique and all frequencies outside this range are aliases.

Frequency in Continuous and Discrete Time Signals



Frequency in Continuous and Discrete Time Signals

- ***Discrete-time Sinusoidal Signals***

❑ The highest rate of oscillation in a discrete-time sinusoidal is attained when $w=\pi$ (or $w= -\pi$) or, equivalently , $f=1/2$ (or $f= -1/2$).

To see what happens for $\pi < w < 2\pi$. we consider the sinusoids with frequencies $w_1 = w_0$ and $w_2 = 2\pi - w_0$. Note that as w_1 varies from π to 2π. w_2 varies from π to 0. it can be easily seen that

$$x_1(n) = A\cos\omega_1 n = A\cos\omega_0 n$$

$$x_2(n) = A\cos\omega_2 n = A\cos(2\pi - \omega_0)n$$

$$= A\cos(-\omega_0 n) = x_1(n)$$

Frequency in Continuous and Discrete Time Signals

w_2 is an alias of w_1. If we had used a sine function instead of a cosine function , the result would basically be the same, except for a 180 phase difference between the sinusoids $x_1(n)$ and x_2 (n).

As we increase the relative frequency w_o of a discrete-time sinusoid from π to 2π. its rate of oscillation decreases. For $w_o = 2\pi$ the result is a constant signal, as in the case for $w_o = 0$. Obviously , for $w_o = \pi$ (or $f = 1/2$) we have the highest rate of oscillation.

Frequency in Continuous and Discrete Time Signals

Since discrete-time sinusoidal signals with frequencies that are separated by an integer multiple of 2π are identical, it follows that the frequencies in any interval $w_1 <= w <= w_1 + 2\pi$ constitute all the existing discrete-time sinusoids.

Hence, the frequency range for discrete-time sinusoids is finite with duration 2π . Usually, we choose the range $-\pi <= w <= \pi$ $(-1/2 <= f <= 1/2)$,which we call the funda mental range.

Uniform (Ideal) Sampling

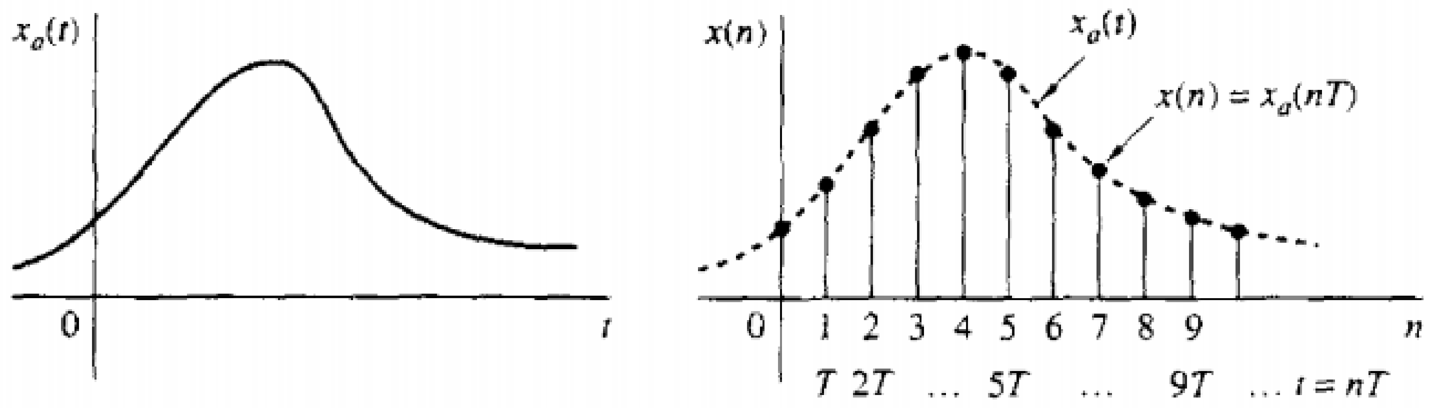
- Analog signal is sampled every T secs.

T is referred to as the sampling interval. $F_s = 1/T$ is called the sampling rate or sampling frequency

- The analog signal can be reconstructed from the samples with out any distortion provided that the sampling frequency is sufficiently high to avoid the aliasing problem.

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)



$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$

$$t = nT = \frac{n}{F_s}$$

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)

$$x_a(t) = A\cos(2\pi Ft + \theta)$$

$$x_a(nT) \equiv x(n) = A\cos(2\pi FnT + \theta)$$

$$= A\cos\left(\frac{2\pi nF}{F_s} + \theta\right)$$

Relative or normalized frequency

$$f = \frac{F}{F_s}$$

Uniform (Ideal) Sampling

- Relation between F (analog frequency) &f (discrete frequency)

$$f = \frac{F}{F_s}$$

$-\infty < F < \infty \qquad -\infty < \Omega < \infty$

$-1/2 < f < 1/2 \qquad -\pi < w < \pi$

$$-\frac{1}{2T} = -\frac{F_s}{2} \le F \le \frac{F_s}{2} = \frac{1}{2T}$$

$$\frac{-\pi}{T} = -\pi F_s < w < \frac{\pi}{T} = \pi F_s$$

$$F_{\max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$\Omega_{\max} = \pi F_s = \frac{\pi}{T}$$

Sampling Theorem

- According to the Sampling theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

$$F_s > 2\,F_{max}$$

The sampling rate F_s which is equals to 2 F_{max} is called ***Nyquist rate***.

Thanks for Your Attention