

Stat414; Fall 2024; Worksheet 09; 20 Points;

Module09 deals with two important topics: obtaining prediction intervals for future observations from a process, and determining tolerance intervals that are expected to capture certain proportion of individuals from the population. These concepts are closely related to the concept of quantiles introduced earlier. Prediction intervals are used widely in many applications, especially in the area of statistical quality control. Tolerance intervals are used in biomedical fields, especially in determining acceptable ranges of biomarkers (such as level of sugar in blood samples, typical range of cholesterol levels in certain populations etc).

1. Suppose a population is well approximated by a Normal distribution with $\mu = 10$ and $\sigma = 2$.
 - (a) Obtain the quantiles for this population for $p = 0.025, 0.05, 0.95, 0.975$
 - (b) Determine a tolerance interval with 90% coverage. Determine a TI with a 95% coverage.
 - (c) For each coverage level, provide several other intervals of the form $[x_\delta, x_{\beta+\delta}]$ with the same coverage by taking different values of δ .
 - (d) Plot the length of these intervals vs δ . Comment.
 - (e) Obtain a random sample of size $n = 25$ from this population. Obtain 90% TI from your sample. Compare the TI based on exact TI obtained in part (a).
2. Suppose a population is well-modeled by a lognormal distribution with $\theta = 4$ and $\tau = 1$.
 - (a) Obtain the quantiles for this population for $p = 0.025, 0.05, 0.95, 0.975$
 - (b) Determine a tolerance interval with 90% coverage. Determine a TI with a 95% coverage.
 - (c) For each coverage level, provide several other intervals of the form $[x_\delta, x_{\beta+\delta}]$ with the same coverage by taking different values of δ .
 - (d) Plot the length of these intervals vs δ . Comment.
 - (e) Obtain a random sample of size $n = 25$ from this population. Obtain 90% TI from your sample. Compare the TI based on exact TI obtained in part (a).