The owners of Fish.net became aware of the depleting population when it fell to only 5,000 fish. In fear that the population would quickly be gone, but also not wanting to halt business, they decided to return to a harvesting rate of 12. After realizing this did not solve the issue they panicked further as the population fell to 4,000 fishes. It was at this point that Fish.net reached out to us for help in restoring the fish population. We assured them that there was an easy solution although it may damage their business's income temporarily.

Before we can determine a new harvesting rate that would allow the business to continue without depleting the hatchery, we must first understand why the population continued to decrease. Looking back at the model created with a harvesting rate of 12, recall that there were two points at which the population would not change. Those two values were when the population was equal to 10,000 and when the population was equal to 15,000. We followed up this discovery by determining the behavior around these population sizes. When the initial population was below 10,000, we saw a trend in population decease and when considering that our new initial condition for this harvesting rate is less than 10,000, it makes sense that our population would decrease further.

Now we are in the situation of determining what new harvesting rate can we apply to our initial population of 4,000, which would have a trend in increasing population. In order to answer this question, we can create a bifurcation graph to visualize every harvesting rate in comparison to the population.

In creating a bifurcation graph, we allow the x-axis to be the harvesting rate, k, and our y-axis to be the population, P. Recall in comparing harvesting values to the population we set our rate of change model equal to zero after plugging in our value for k. If we set our model to zero without plugging in any values for k, we are left with an equation, in terms of k and P, what the population will plateau at for different values of k.

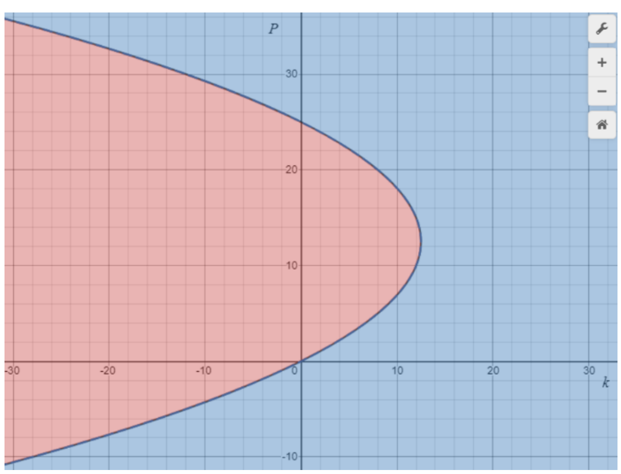
$$\frac{dP}{dt} = 2P(1 - \frac{P}{25}) - k$$

$$\frac{dP}{dt} = -\frac{2P^2}{25} + 2P - k = 0$$

$$a = -\frac{2}{25}, b = 2, c = -k$$

$$P = \frac{-(2) \pm \sqrt{(2)^2 - 4(-\frac{2}{25})(-k)}}{2(-\frac{2}{25})}$$

If we graph this solution, we are left with a bifurcation diagram. Using the behaviors we determined previously, we can represent the values in which population will decrease in blue and the values in which population will increase in red.




Population v. Harvesting Rate (Desmos.com)

Finally, in getting a new harvesting rate that would allow the population to restore, we graph the line P = 4 to represent the population of fish at the time. The intersection point will give us a coordinating value for k in which the population would remain 4,000 forever. Since we want to eventually restore the population, we will choose any k value less than that intersections k value. Since the k value at that intersection is 6.72, we now know that as long as less than 6,720 fish are harvested each year the population will experience a trend to increase. Additionally, by examining the upper plateau, we can see if they dropped the harvesting value below 6.72, the fish population has the potential raise to upwards of 21,000 or more, depending on the value for k. In returning to the original business model, once the population surpasses 10,000 fishes, they can return to a harvesting value of 12. The size of the population beyond 10,000 is irrelevant in the long run because after a long time the population will plateau at 15,000 fishes anyway. In concluding our work with Fish.net, it becomes clear that just because one harvesting rate may work efficiently for one population, does not mean it will work the same for others.