Model Fusion for Inertial-based Personal Dead Reckoning Systems

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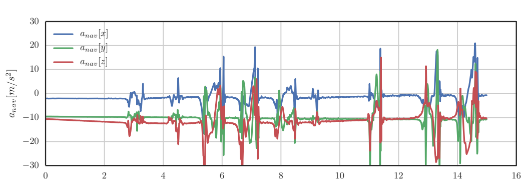
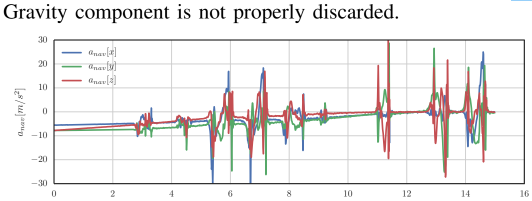
***Abstract*—This paper introduces a model fusion approach that improves the effectiveness of Personal Dead Reckoning Systems that exploits foot-mounted Inertial Measurement Units. Our solution estimates a sensor orientation by exploiting the Madgwick's algorithm integrated with popular Kalman-based solution. This way, attitude and heading correction is not based on the Zero-Velocity phase assumption which introduces significant error. The experiments conducted on ground-truth data shows, that the proposed approach outperforms state-of-the-art solution by reducing systematic and modelling errors and also provides better heading estimation.**

I. Introduction

Personal Dead Reckoning (PDR) based on foot-mounted Inertial Measurement Units is a basis in many infrastructure-free Indoor Navigation Systems [2]. Extensive research in this field formed a state-of-the-art technique that takes advantage of the so-called "Zero-Velocity Update" (ZUPT) approach [3], [4]. ZUPT-based systems assume that during each step a motionless (or zero-velocity) phase of the foot occurs, in which velocity is zero. However, direct integration of the accelerometers readings outputs incorrect, non-zero results in most cases. Information about this error can be usedduring second integration. Moreover, popular techniques for PDR apply Kalman Filter (KF) that corrects position, treating the foot orientation velocity in ZUPT as pseudo-observation.



(a) Acceleration in the navigational frame implemented in [1]. Gravity component is not properly discarded.



(b) Desired property of the system — acceleration converge to zero in ZUPT phase. Results from our implementation.

Fig. 1: Acceleration of the sensor in navigation frame computed by PDR with ZUPT technique with improperly set initial orientation. Gravity component is discarded. We refer to next sections for the explanation of both systems.

A few successful implementations and evaluations of the ZUPT-based system, as well as recent research in the field, ensure that the approach may be a promising solution to PDR. Moreover, deployed with cheap commercial-grade sensors, PDR may fill a niche where GPS signal or other radio-based or marker-based systems cannot be used. Such conditions are present, for instance, in places where fire-fighters or miners work [5].

The main factor that reduces performance of PDR system is an error in estimation of the sensor orientation. Especially, reducing the heading error is crucial: even if the error of position on the level of single step is very little, the improper heading can introduce great overall error (see Fig. 3 for example). This can be easily seen on turns or sudden movements of the foot. Moreover, extrinsic correction of the heading is problematic to perform since it can be determined only by magnetometers. In many cases, however, magnetic field introduces a significant level of disturbances in indoor environment.

Basic idea to overcome the mentioned problem is to use gravity component of the accelerations to correct the orientation during the ZUPT phase. Unfortunately, only attitude error can be corrected that way. Therefore, our approach uses a continuous error compensation (outside the ZUPT-phase) by fusing accelerometers with gyroscope readings. The idea is depicted on Fig. 1. Roughly speaking, rotation of acceleration vector by sensor orientation gives acceleration in navigational frame. If the orientation is estimated correctly, we would get only gravity component pointed downwards. By randomly choosing the initial orientation we see difference in both algorithms and the fact that our implementation converge to zero in ZUPT phase.

Contributions of this paper are as follows:

- identification of the ZUPT-based PDR limitations connected with orientation estimation,
- a novel model fusion approach for PDR, in which orientation is corrected without Zero-Velocity assumption.

Experiments were conducted using prototype PDR (see Fig. 2) and also using dataset with reference readings. Experimental results shows that the approach is simpler, outputs better results, makes KF parameter tuning less challenging and improves gyroscope drift compensation.

The rest of the papers is structured as follows: Section II outlines and reviews other solutions which face with the problem. Section III introduces and describes our approach. In Section IV we describe the experiment which were conducted to validate our model. The last section concludes the paper and presents our perspective on future work.

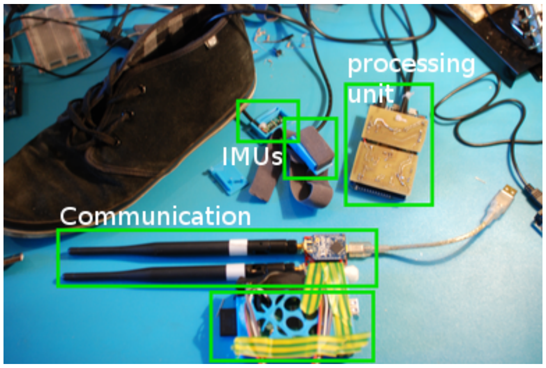


Fig. 2: Prototype of PDR system

II. Related Work

Initial approaches for PDR tried to face the problem by using the combination of pedometers and magnetometers. It is possible to get information about rough position, by multiplying average [6] or adaptively estimated [7] stride length by step count in the direction pointed by magnetometers. Performance of this "naïve implementation" is strongly based on sensor precision and bandwidth. ZUPT-based approach with KF, on the other hand, is relatively new solution to Personal Dead Reckoning in infrastructure-free environment. This solution is introduced in [3] and described in details in [4] and [1]. Kalman Filter with ZUPT forms a state-of-the-art method for non-GPS inertial navigation that can work using commercial-grade and small sensors.

Crucial aspect of the ZUPT-based system is the motionless (or zero-velocity) phase detection. Having walking patterns recognized, it is possible to construct model that describes it (e.g. HMM in [8]) and outputs zero-velocity-phase for different moving conditions. Other approaches detect zero-velocity by some statistical tests on sensor readings (see e.g. [9] for a generalization and evaluation model). Nevertheless, the authors in [10] claim that zero-velocity hypothesis in some cases can be incorrect due to the small movements in zero-velocity-phase, which in the end could lead to modelling error. This clearly lead us to the conclusion that correction in orientation should not depend on zero-velocity hypothesis.

Above-mentioned paper [3] introduces sensor orientation correction in terms of Kalman Filter Innovation (see Eq. 4) by exploiting a "transfer alignment" process. Moreover, the author of the aforementioned paper deliberate on heading drift compensation using magnetometer readings but (following other papers) there is considerable disturbance indoors that could introduce unstable results. Intuitively, compensation in orientation in zero-velocity phase can be easily introduced: in [11] authors corrects the attitude by introducing the *Angular Rate Update*. In the paper the authors add a correction during ZUPT basing on the gravity vector. Nevertheless, while there is a number of intuitive and straightforward methods of correction based on ZV-phase, it seems problematic to exploit it in a real-life application, since different gaits (or crawling) have measurable features which differ a lot. Hence, often ZV-phase can be ambiguously determined, which can cause a large mistake in the calculation of the vector acceleration of gravity. In this paper, therefore, the authors apply the amendment of the gravity vector to each step of the algorithm, and this lead to smaller error and to better solution.

The literature is lacking in systematic comparison of different approaches and examination on longer distances and genuine environment. It seems that much of the research is concerned to correct the PDR system on the patch level (e.g. by exploiting Wi-Fi fingerprinting), while our observations indicates that model itself still could be significantly improved.

III. Modified PDR Algorithm

The basis of our PDR system is the Kalman Filter which uses velocity pseudo-observation in the update phase. Under infrastructure-free assumption we cannot obtain any direct information about the feet movement. However, we can test a zero-velocity hypothesis by some statistical tests that use gyroscope or acceleration deviation. It was shown that errors in *a priori* state (defined later in this section) are correlated and it is possible to correct orientation error by correlating it with velocity. Number of researches addresses this approach (see [4], [9]).

However, according to [10], this technique has some theoretical and modelling drawbacks. It turns out that due to accumulation of errors, the feet touching the ground is usually not in the pure stationary state. Moreover, using precise sensors we can observe very small movements in ZUPT phase — that states whole zero-velocity assumption questionable. Furthermore, from the theoretical point of view we can say that the Kalman Filter is *not optimal* since the expectation of the measurement residual (the so-called *innovation*) is usually non zero.

Error in the entire PDR system that is based on Kalman Filtering is a combination of modelling and systematic (calibration/computation) errors. However, if we look closely on the model, then we will see that it can be substantially improved. For instance, Cayley-based KF can estimate efficiently only roll and pitch angles, while heading drift is observed in many researches — this is the main idea which leads us to conclusion that results still can be improved.

A. General setting

In this section, we present general algorithm used in PDR. In order to do so, we firstly introduce some necessary notation. It can be showed that considered system is driven by the following time-controlled linear equation

$$\hat{x}_k = A_k \hat{x}_{k-1} + B u_k, \tag{1}$$

where $\hat{x}$ is *a priori* state vector, A_k and B are linear transformations defined with respect to the method, which will be specified later. Vector u_k is the control factor. In Kalman Filter Algorithm error covariance matrix P_k is updated by

$$P_k := A_k P_{k-1} A_k^T + Q, \tag{2}$$

where $P_0 = \mathbb{O}$ and Q is the covariance matrix of noise of the Kalman Filter.

If ZUPT phase is triggered, we can perform update as follows. Assuming our observation state is velocity v_k, we define observation matrix $H := [\mathbb{O}\ \mathbb{I}]$, where $\mathbb{I}$ and $\mathbb{O}$ are the identity and zero matrices, respectively. Kalman Gain Matrix is then computed using error covariance matrix by

$$K_k := P_k H^T (H P_k H^T + R)^{-1}, \tag{3}$$

where R is the covariance matrix of the observation.

Afterwards, under assumption that the velocity is zero, we can compute *a posteriori* state by

$$x_k := K_k(\mathbb{O} - H\hat{x}_k), \tag{4}$$

where $\mathbb{O}$ is the zero vector (our initial hypothesis).

B. State-of-the-art orientation estimation

We describe below shortly original implementation of PDR. Following [1], during each step of the algorithm we will update, respectively, orientation matrix C_k, position vector p_k and velocity v_k according to the formula

$$\hat{x}_k := \begin{bmatrix} C_k \\ p_k \\ v_k \end{bmatrix} := \begin{bmatrix} C_{k-1} C_{k,\Omega_k} \\ p_{k-1} + v_{k-1}\Delta t \\ v_{k-1} + a_k^N \Delta t \end{bmatrix}, \tag{5}$$

where acceleration a_k^N expressed in the navigational frame is given by

$$a_k^N := C_k a_k^S - g_0. \tag{6}$$

In this equation accelerometer readings from sensor are rotated by the orientation matrix and then gravity component g_0 is discarded. Matrix C_{k,Ω_k}, which introduces a correction to the rotation matrix, is computed by integrating gyroscope readings. Details can be found in [1].

In the model (2) we set

$$B := \begin{bmatrix} \mathbb{O} & \Delta t & \Delta t & \Delta t \end{bmatrix}^T, \tag{7}$$

$$u_k := \begin{bmatrix} a_k^S & \mathbb{O} \end{bmatrix}. \tag{8}$$

Error covariance matrix P_k is computed according to Eq. (2) with the following (time-dependent) state transition matrix

$$A_k := \begin{bmatrix} \mathbb{I} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{I} & \Delta t \mathbb{I} \\ \Delta t S_k & \mathbb{O} & \mathbb{I} \end{bmatrix}, \tag{9}$$

where S_k is a skew-symmetric cross-product operator matrix given by

$$S_k := \begin{bmatrix} 0 & -a_{z,k}^N & a_{y,k}^N \\ a_{z,k}^N & 0 & -a_{x,k}^N \\ -a_{y,k}^N & a_{x,k}^N & 0 \end{bmatrix} \tag{10}$$

which allows us to relate errors manifested in the orientation matrix with velocity errors. This technique is called "alignment transfer" and intuition here lies in the fact that improperly estimated sensor orientation gives non-zero velocity when discarding the gravity component from acceleration in Eq. (6) during integration.

It was examined by experiment in many papers that the method work well for correcting the attitude of the sensor (roll and pitch angles) but the yaw angle still can remain incalculable [3], [4].

C. Continuous orientation estimation for PDR

In order to continuously estimate foot orientation we exploited the state-of-the-art orientation filter. The filter uses a quaternion representation allowing accelerometer data to be used in an analytically derived and optimised gradient descent algorithm which computes the direction of the gyroscope measurement error as a quaternion derivative. Moreover, gyroscope drift compensation can be performed using this filter through the integral feedback of the error in the rate of change of the orientation. For detailed description of the algorithm and the source code we refer to original paper of Madgwick [12].

The Madgwick's Algorithm calculates quaternion of orientation between the Earth frame and sensor frame by minimizing function

$$f(q, a^E, a^S) = q^{-1} \otimes a^E \otimes q - a^S \tag{11}$$

with respect to q under constrain $\|q\| = 1$, where q is an orientation quaternion of the sensor, a^E and a^S are accelerations in the Earth and sensor frame, respectively. Here $\otimes$ denotes multiplication in quaternion field. As we can see this is a quadratic functional on q which minimum in each coordinate determines quaternion orientation from the Earth frame to the sensor frame.

In the fusion step estimated sensor orientation $q_{est,m}$ is computed using previous estimate and quaternion calculated from gyroscope angular rates $\dot{q}_{\omega,m}$. Simultaneously, the direction of estimated error $\dot{q}_{\epsilon,m} := \frac{\nabla f}{\|\nabla f\|}$ is discarded, where $\nabla f := \nabla_q f(\dot{q}_{\omega,m}, a^E, a^S)$ is the objective function gradient with respect to q. Eventually, we obtain the following iterative schema

$$q_{est,m} := q_{est,m-1} + (\dot{q}_{\omega,m} - \beta \dot{q}_{\epsilon,m})\Delta t. \tag{12}$$

Parameter β is called *beta-gain* and indicates, roughly speaking, how much orientation estimated from functional f should influence $q_{est,m}$ over gyroscope angular rates.

Having estimated orientation quaternion $q_{est,m}$, we can substantially simplify Kalman Filter by taking into account only the *mechanization equations*. Therefore, the Kalman Predict and Update step will be as follows

$$\hat{x}_k = \begin{bmatrix} p_k \\ v_k \end{bmatrix} := \begin{bmatrix} p_{k-1} + v_{k-1}\Delta t_k \\ v_{k-1} + a_k^N \Delta t_k \end{bmatrix}. \tag{13}$$

Moreover, we simplified Eq. (12) by putting (time-independent) $A_k := A := \begin{bmatrix} \mathbb{I} & \Delta t \mathbb{I} \\ \mathbb{O} & \mathbb{I} \end{bmatrix}$, $B := \begin{bmatrix} 0 & 0 & 0 & \Delta t & \Delta t & \Delta t \end{bmatrix}^T$ and $u_k := \begin{bmatrix} 0 & 0 & 0 & q_{k-1} \otimes a_{k-1}^N \otimes q_{k-1}^{-1} - g_0 \end{bmatrix}$.

As was stated in Section II, in previous solutions authors did correction to the position only in stance phase. We, on the other hand, perform this correction continuously. Therefore, foot in move results in better orientation of the entire system.

The improvement in the method itself may not be a great, but it resulted in significant reduction in errors. We will show in the next section that the orientation error was one of the main factors which influences the quality of path estimation.

IV. Experimental results

We examined our algorithm in two experiments. For the first one we used the data published in [13] which provides high precision, ground-truth data obtained by optical systems and set of the visual markers mounted on the shoe along with inertial sensor. The dataset consist from files containing results from experiments in which subject were walking along a specified path. Only short files (with length of about $40\,\mathrm{s}$) with walking was chosen for the experiment. For the second experiment we used our prototype system and explicitly examined a heading drift over time.

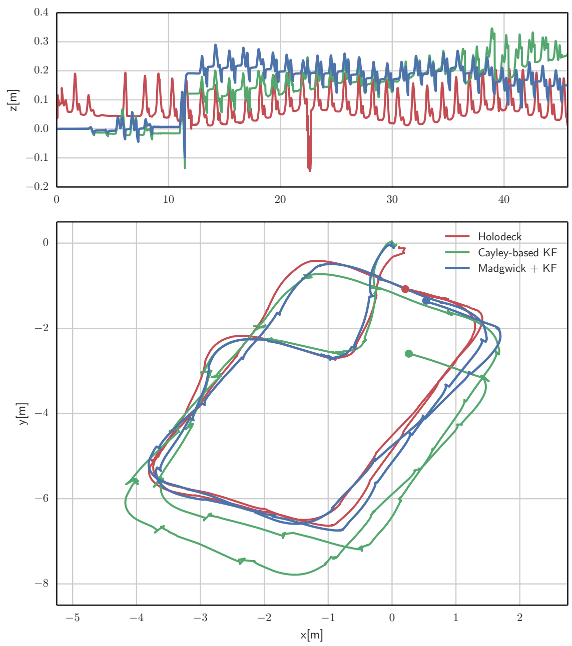


Fig. 3: Comparison of the path estimation with respect to ground-truth data (red line). Upper figure shows z-axis and lower figure depicts 2D path. Error in heading (easily observed on turns) in the state-of-the-art technique (green line) introduces significant error.

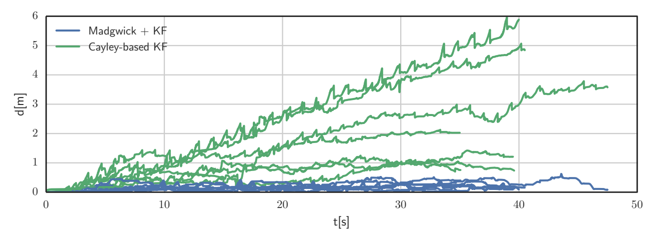
Two solutions were compared: (1) the state-of-the-art PDR implementation described in [1], which later will be referred as *Cayley-based KF*, and (2) our model-fusion implementation, which we will hereafter call *Madgwick + KF*. Note that implementation does not cover modelling sensor biases, because the first dataset, according to the authors of experiments, was prepared with properly tuned sensors with temperature compensation. Comparison in every experiment is done by running both algorithms on the same data with the same zero-velocity detection technique.

A. Path evaluation with reference data

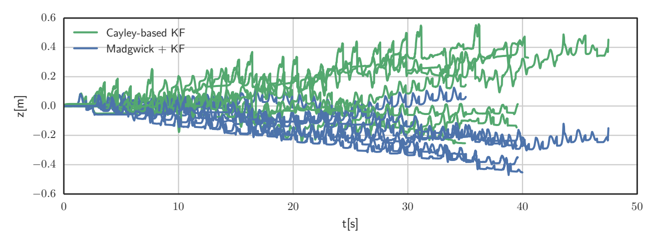
As an example we depicted the path estimated with both algorithms (see Fig. 3). The red colour denotes ground-truth data obtained by the optical system and blue and green colours stand for Cayley-based KF and Madgwick + KF ($\beta = 0.35$), respectively. Alignment of the reading was done using a stamp around $11.5\,\mathrm{s}$. We see that most of the errors are introduced at the turns, since side lengths seems to be improperly estimated. Our implementation at first glance is lacking this problem since end displacement is only $0.32\,\mathrm{m}$ (and most of the error is introduced in heading axis), comparing to $1.51\,\mathrm{m}$ in baseline implementation. It is worth to stress out that the readings was recorded with only $100\,\mathrm{Hz}$ — sudden turns are inherently miscalculated with such low sampling rate.

In order to examine rest of the files we measured for each of them distance d between estimation and ground-truth position at time t (see Fig. 4a). Average displacement in xOy-plane was $0.14\,\mathrm{m} \pm 0.11\,\mathrm{m}$, comparing to $2.70\,\mathrm{m} \pm 1.92\,\mathrm{m}$ in baseline implementation. Such a significant difference has two sources: the heading error and systematic error. After hand examinations of each file it turns out that heading displacement (similarly as in Fig. 3 is the main source of error — see next subsection).

The recording had taken place on flat surface so plain estimation is depicted on Fig. 4b. Both algorithm performed good introducing only few centimetres of error with exception that our algorithms tends to underestimate the height of the stride. We noted that adjusting the beta-gain coefficient minimizes the error and also adjusting zero-velocity threshold minimizes it as well. Having this in mind, we constructed a straightforward optimization technique of parameter adjustment.



(a)



(b)

Fig. 4: *Pictures show relative and absolute displacement from xOy-plane for two algorithms.*

B. Step evaluation with reference data

In order to look more closely on the nature of the error we estimated probability density function of error on single step. To prepare such estimation each step was extracted according to the zero-velocity intervals. PDR estimates and ground-truth data was aligned — beginning was moved to the centre of the frame and whole path was rotated around z-axis to match ground-truth with respect to the stamp (note that headings of individual steps was not aligned).

In Fig. 5 estimated distribution of errors for the two algorithms is depicted. Left graph represents standard KF, whereas right presents the results of our implementation. The smaller are the concentric distributions, the better. Moreover, smaller errors is noticed where peek of distribution is positioned in the origin. To give more insight into the magnitude of the error we also defined the integral of error by: $P_{a,b} := \iint_{[-a,a]\times[-b,b]} p(x,y)\,dx\,dy$, where $p(x,y)$ is an estimated probability of the error in direction (x,y).

Our solution turns out to better than baseline method: centre of a distribution of error probability in most cases is in the middle and dispersion is much smaller. Uneven shape arise from a specific walking path, which is chosen for the experiment. An interesting case is `3rdWalk_straight_020810_16_41` which is a walk recorded on one straight line performed in both directions — both algorithms did overestimation of step length. In our implementation, however, the probability of making an error of 1 cm or less has been reduced by 41 %. Unequal shapes suggest existence of a systematic error which can be modelled, e.g. by machine learning methods, as was suggested in the paper [13]. In the present form the error probability resemble a "white noise", but it can be still modelled to improve the estimation.

This evaluation allows us to determine main source of errors. In both implementations errors with respect to the ground-truth data are almost never higher than two centimetres. With a number of steps less than 40 (in most cases), we should get error less that 1 m. Results depicted on Fig. 4a reports much larger error. It clearly shows us that the greatest number of errors was introduced by improperly estimated direction of orientation.

C. Systematic heading drift

For experiment we have used gyroscope and accelerometer integrated circuit AltIMU-10 from Polulu. Data was gathered with sensor bandwidth of 1.344 kHz with scale of 2000 dps. Accelerometer was able to measure force of ±16 g. Sensor was mounted on foot and connected to Arduino DUE by I^{2}C interface.

Twenty trials was performed in which subject was walking over the straight line with moderate speed on the flat surface. The additional work was undertaken into precise foot positioning at the beginning in order to point it into the same direction. Gyroscope bias was measured at the beginning of each recording and initial calibration (sensor biases) was performed by a simple on-field hand-based process. As previously, Q and R matrices as well as beta-gain for Madgwick's Algorithm were tuned by trial and error.

The results are depicted in Fig. 6, although it may be misleading. Overall performance of our implementation was significantly better, taking into account y-axis displacement, which was 1.67 m comparing to 3.20 m. Nevertheless, standard deviation was only half meter better on a path with a length of 40 m. Explanation for this situation can lie in calibration process which was prone to error. Moreover, it seems that original implementation is more sensitive to noisy data and imprecise sensor biases. Taking this into account, we can

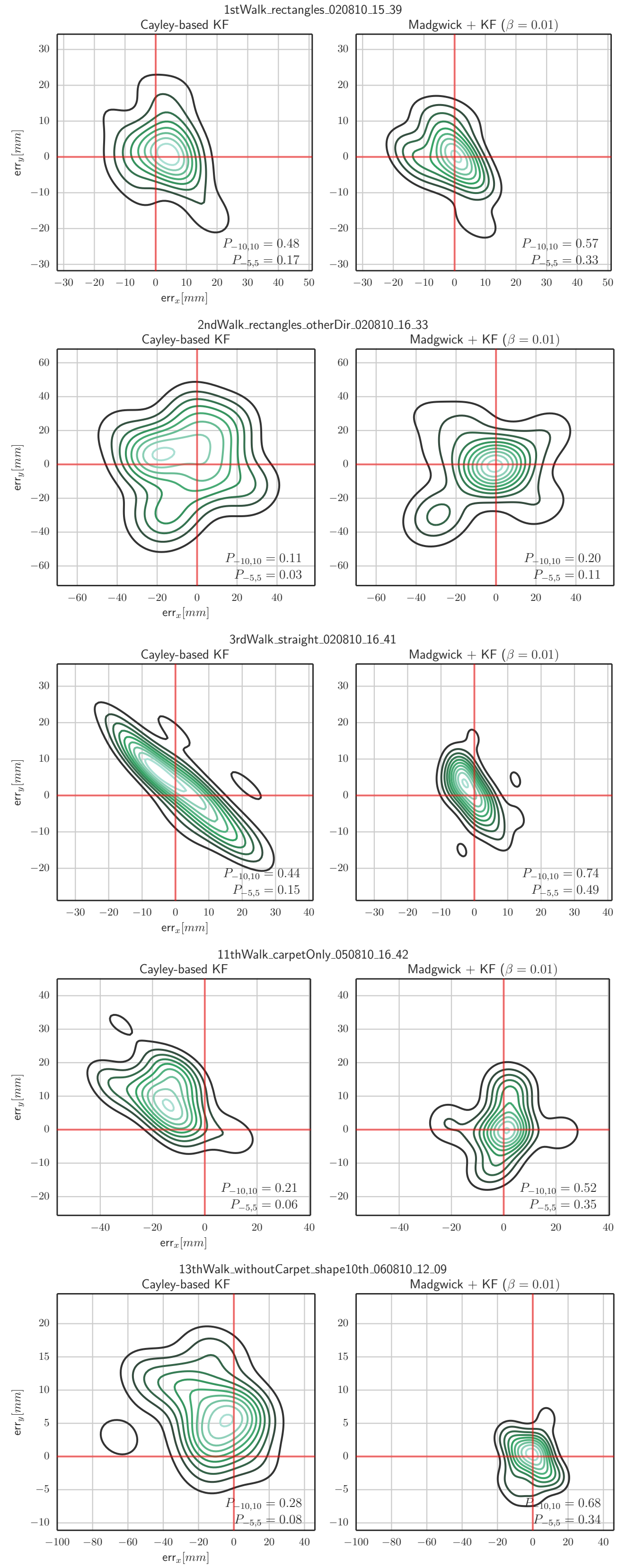


Fig. 5: Error probability of the individual steps for selected files. For more details see Section IV-B.

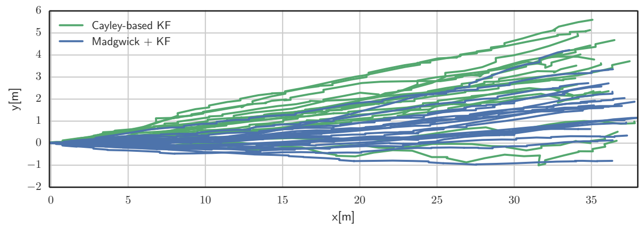


Fig. 6: Heading drift while walking over straight line. The y-axis displacement for: Madgwick + KF = $1.67\,\mathrm{m} \pm 1.04\,\mathrm{m}$ and Cayley-based KF = $3.20\,\mathrm{m} \pm 1.53\,\mathrm{m}$.

say that our technique is better by $32\,\%$ in heading drift compensation for PDR (because $1.04/1.53 \approx 68\,\%$).

We also examined gyroscope bias and acceleration in navigational frame. While gyroscope drift was not observed in such a small trial, the navigational force was displaced for almost $0.5\,\mathrm{m\,s^{-2}}$. This is due the errors in orientation matrix are propagating also and clearly states that some kind of means should be undertaken into correction of orientation. The technique for angular rate update introduced in [11] can be a solution, but still those updates are based on ZV-phase which can be difficult to employ in different gait styles or crawling. Our model fusion does not make any assumption on existence of stance phase for orientation estimation.

V. Conclusion

Despite that PDR systems was under research for some time, it seems that the original approach should be modified to take into account the latest improvements in IMU-related algorithms. In our experiment we showed that modified version of the Kalman Filter for PDR, where orientation estimation is corrected without taking into account the velocity error, can be a promising way to achieve better results.

Acknowledgement

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