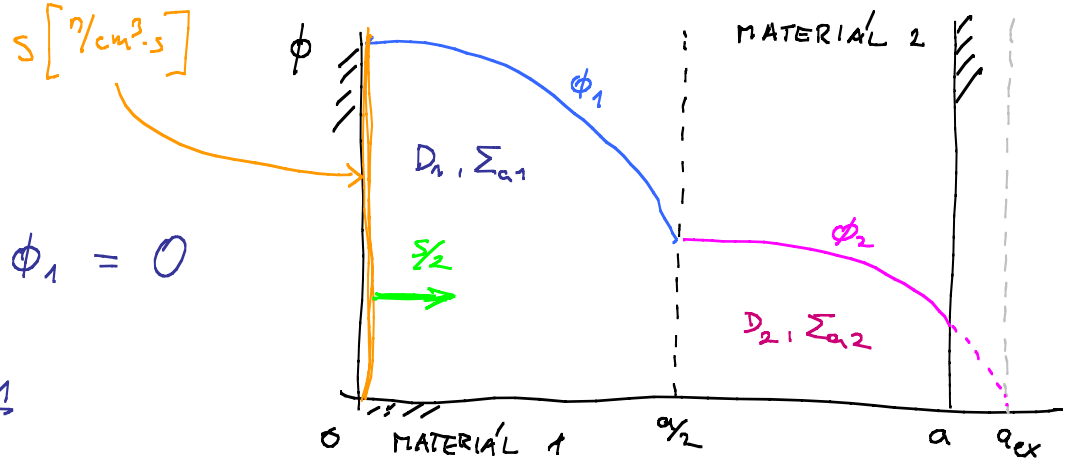
Prostředí s různými materiály



$$\frac{d^2\phi_1}{dx^2} - \frac{1}{L_1^2}\phi_1 = 0$$

$$J_1 = -D_1 \frac{d\phi_1}{dx}$$

$$\frac{d^2\phi_2}{dx^2} - \frac{1}{L_2^2}\phi_2 = 0$$

$$J_2 = -D_2 \frac{d\phi_2}{dx}$$

① $\lim_{x \to 0^+} J_1(x) = \frac{s}{2}$

② spojitost toků na rozhraní

$$\phi_1(a/2) = \phi_2(a/2)$$

③ spojitost proudů na rozhraní

$$J_1(a/2) = J_2(a/2)$$

④ $\phi_2(a + d_{ex}) = 0$, kde a_{ex}

$$\phi_1(x) = \boxed{1} A_1 \cosh\frac{x}{L_1} + \boxed{2} B_1 \sinh\frac{x}{L_1}$$

$$J_1(x) = -\frac{D_1 A_1}{L_1}\sinh\frac{x}{L_1} - \frac{D_1 B_1}{L_1}\cosh\frac{x}{L_1}$$

$$\phi_2(x) = \boxed{3} A_2 \cosh\frac{x}{L_2} + \boxed{4} B_2 \sinh\frac{x}{L_2}$$

$$J_2(x) = -\frac{D_2 A_2}{L_2}\sinh\frac{x}{L_2} - \frac{D_2 B_2}{L_2}\cosh\frac{x}{L_2}$$

4 neznámé
4 podmínky

① $\frac{S}{2} = \lim_{x \to 0+} \left(-\frac{D_1 A_1}{L_1} \sinh \frac{x}{L_1} - \frac{D_1 B_1}{L_1} \cosh \frac{x}{L_1} \right) = -\frac{D_1 B_1}{L_1}$

$\Rightarrow B_1 = -\frac{S L_1}{2 D_1}$

④ $0 = \Phi_2(a_{ex}) = A_2 \cosh \frac{a_{ex}}{L_2} + B_2 \sinh \frac{a_{ex}}{L_2}$

$\Rightarrow A_2 = -B_2 \tanh \frac{a_{ex}}{L_2}$

$\Phi_1(x) = A_1 \cosh \frac{x}{L_1} - \frac{S L_1}{2 D_1} \sinh \frac{x}{L_1}$

$J_1(x) = -\frac{D_1 A_1}{L_1} \sinh \frac{x}{L_1} + \frac{S}{2} \cosh \frac{x}{L_1}$

$\Phi_2(x) = B_2 \left(\sinh \frac{x}{L_2} - \tanh \frac{a_{ex}}{L_2} \cosh \frac{x}{L_2} \right)$

$J_2(x) = \frac{D_2 B_2}{L_2} \left(\tanh \frac{a_{ex}}{L_2} \sinh \frac{x}{L_2} - \cosh \frac{x}{L_2} \right)$

② $\Phi_1(a/2) - \Phi_2(a/2) = 0$ ③ $J_1(a/2) - J_2(a/2) = 0$

$$\begin{bmatrix} \cosh \frac{a}{2L_1} & \tanh \frac{a_{ex}}{L_2} \cosh \frac{a}{2L_2} - \sinh \frac{a}{2L_2} \\ -\frac{D_1}{L_1} \sinh \frac{a}{2L_1} & \frac{D_2}{L_2} \cosh \frac{a}{2L_2} - \frac{D_2}{L_2} \tanh \frac{a_{ex}}{L_2} \sinh \frac{a}{2L_2} \end{bmatrix} \begin{bmatrix} A_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \frac{S L_1}{2 D_1} \sinh \frac{a}{2L_1} \\ -\frac{S}{2} \cosh \frac{a}{2L_1} \end{bmatrix}$$

In[1]:=
$$M = \left(\begin{array}{c|c} \text{Cosh}\left[\frac{a}{2 L_1}\right] & \text{Tanh}\left[\frac{a_{ex}}{L_2}\right] \text{Cosh}\left[\frac{a}{2 L_2}\right] - \text{Sinh}\left[\frac{a}{2 L_2}\right] \\ \hline \frac{-D_1}{L_1} \text{Sinh}\left[\frac{a}{2 L_1}\right] & \frac{D_2}{L_2} \text{Cosh}\left[\frac{a}{2 L_2}\right] - \frac{D_2}{L_2} \text{Tanh}\left[\frac{a_{ex}}{L_2}\right] \text{Sinh}\left[\frac{a}{2 L_2}\right] \end{array}\right);$$

$$b = \left\{\frac{S L_1}{2 D_1} \text{Sinh}\left[\frac{a}{2 L_1}\right], -\frac{S}{2} \text{Cosh}\left[\frac{a}{2 L_1}\right]\right\};$$

In[3]:=
$$\{A_1, B_2\} = \text{LinearSolve}[M, b] \text{ // FullSimplify}$$

Out[3]=
$$\left\{\left(S L_1 \left(\text{Cosh}\left[\frac{a - 2 a_{ex}}{2 L_2}\right] \text{Sinh}\left[\frac{a}{2 L_1}\right] D_2 L_1 - \text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{ex}}{2 L_2}\right] D_1 L_2\right)\right) \Big/ \left(2 D_1 \left(\text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Cosh}\left[\frac{a - 2 a_{ex}}{2 L_2}\right] D_2 L_1 - \text{Sinh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{ex}}{2 L_2}\right] D_1 L_2\right)\right),$$
$$-\left(S \text{Cosh}\left[\frac{a_{ex}}{L_2}\right] L_1 L_2\right) \Big/ \left(2 \text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Cosh}\left[\frac{a - 2 a_{ex}}{2 L_2}\right] D_2 L_1 - 2 \text{Sinh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a - 2 a_{ex}}{2 L_2}\right] D_1 L_2\right)\right\}$$

In[4]:=
$$\{A_2, B_1\} = \left\{-B_2 \text{Tanh}\left[\frac{a_{ex}}{L_2}\right], -\frac{S L_1}{2 D_1}\right\} \text{ // FullSimplify}$$

Out[4]=
$$\left\{\frac{S \text{Sinh}\left[\frac{a_{ex}}{L_2}\right] L_1 L_2}{2 \text{Cosh}\left[\frac{a}{2 L_1}\right] \text{Cosh}\left[\frac{a-2 a_{ex}}{2 L_2}\right] D_2 L_1 - 2 \text{Sinh}\left[\frac{a}{2 L_1}\right] \text{Sinh}\left[\frac{a-2 a_{ex}}{2 L_2}\right] D_1 L_2}, -\frac{S L_1}{2 D_1}\right\}$$

- **Předpokládaný průběh neutronového toku**

In[5]:=
$$\phi_1[x\_] = A_1 \text{Cosh}\left[\frac{x}{L_1}\right] + B_1 \text{Sinh}\left[\frac{x}{L_1}\right];$$
$$\phi_2[x\_] = A_2 \text{Cosh}\left[\frac{x}{L_2}\right] + B_2 \text{Sinh}\left[\frac{x}{L_2}\right];$$

- **Ověření předepsaných podmínek**

In[7]:=
$$\phi_2[a_{ex}]$$

Out[7]= 0

In[8]:=
$$\text{Limit}[-D_1 \partial_x \phi_1[x], x \to 0]$$

Out[8]=
$$\frac{S}{2}$$

In[10]:=
$$\phi_1\left[\frac{a}{2}\right] - \phi_2\left[\frac{a}{2}\right] // \text{FullSimplify}$$

Out[10]= 0

In[12]:=
$$D_1 \partial_x \phi_1[x] - D_2 \partial_x \phi_2[x] /. x \to \frac{a}{2} // \text{FullSimplify}$$

Out[12]= 0

- **Ukázkový příklad**

In[62]:=
$$D_1 = 4; L_1 = 5; D_2 = 0.4; L_2 = 3; a = 20; a_{ex} = a + 2 D_2; S = 2;$$

In[63]:=
$$\text{Show}\left[\text{Plot}\left[\phi_1[x], \left\{x, 0, \frac{a}{2}\right\}\right], \text{Plot}\left[\phi_2[x], \left\{x, \frac{a}{2}, a_{ex}\right\}\right], \text{PlotRange} \to \text{Automatic}\right]$$

Out[63]=