Assessment 3 & 4 – (Problem solving & Programming assignment)

Instructions:

1. Don't change the question. Answer the question which is allotted to the respective roll no.
2. **Solve all the subdivisions in the allotted question. Program any two of the allotted questions (It should work for different test cases).**
3. Submit both i) the solved problems and ii) the code with output screen
4. Your document should contain Roll no., Name, Question no., Question, Solution, Program and screenshot of the output.
5. Submission date: 01.07.2022 before 5 pm

Allotted Question – (Refer appended Question)

Roll no.	Assignment question no.
2	1
4	2
6	3
8	4
10	5
12	6
14	7
16	8
18	9
20	10
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68	41
70	42
72	43
74	44
76	1
78	2
80	3
82	4
84	5
86	6
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138	32
140	40
142	41
144	42
146	43

Questions

UNIT-I

1\. a) How can the union & intersection of n sets that all are subsets of the universal set U be found using bit strings?

b) Show how bitwise operations on bit strings can be used to find these combinations of $A = \{a, b, c, d, e\}$ $B = \{b, c, d, g, p, t, v\}$ $C = \{c, e, i, o, u, x, y, z\}$ & $D = \{d, e, h, i, n, o, t, u, x, y\}$.

(i) $A \cup B$ (ii) $A \cap B$ (iii) $(A \cup D) \cap (B \cup C)$

(iv) $A \cup B \cup C \cup D$.

c) What is the bit string corresponding to the symmetric difference of 2 sets?

2\. a) Determine whether $f: Z \times Z \to Z$ is onto if

(i) $f(m,n) = 2m - n$ (ii) $f(m,n) = m^2 - 4$.

b) Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c & d are constants. Determine for which constants a, b, c & d it is true that $f \circ g = g \circ f$.

c) Let f & g be functions from $\{1, 2, 3, 4\}$ to $\{a, b, c, d\}$ and from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ respectively such that $f(1) = d$, $f(2) = c$, $f(3) = a$ and $f(4) = b$, and $g(a) = 2$, $g(b) = 1$, $g(c) = 3$ and $g(d) = 2$.

(i) Is f one-to-one? Is g one-to-one?

(ii) Is f onto? Is g onto?

(iii) Does either f or g have an inverse? If so, find this inverse.

3. a) Let R be the relation on the set of people such that $x R y$ if x and y are people and x is older than y. Show that R is not a partial ordering.

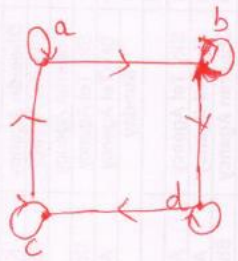
b) Let (S, R) be a poset. Show that (S, R^{-1}) is also a poset where R^{-1} is the inverse of R. 2

c) Which of these are posets?

a) $(R, =)$, b) $(R, <)$

c) $(Z, >)$, d) $(Z, \neq)$

4. a) Determine whether the relation with the directed graph shown is a partial order.

(i) 

(ii) 

b) Draw the Hasse diagram for the "greater than or equal to" relation on $\{0, 1, 2, 3, 4, 5\}$.

c) Determine whether the relation represented by these zero-one matrices is partial order.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

5. Show the following is tautology.

a) $(A \to B) \to (\neg(B \land C) \to \neg(C \land A))$

b) prove the equivalence

$$A \to B \equiv A \land \neg B \to \text{false}$$

c) Prove the following is a contingency (4)

i) $(A \vee B \to C) \wedge A \to (C \to B)$.

ii) $(A \to B) \wedge (B \to \neg A) \to A$.

6. a) Convert the following into DNF.

(i) $(A \vee B) \wedge (C \to D)$.

(ii) $(P \to Q) \to P$.

b) Transform the following into full DNF

(i) $(P \vee Q) \wedge (R \to Q)$

(ii) $Q \wedge \neg P \to P$

c) Show that the following set of operation is a complete set of connectives for the propositional calculus.

$\{\neg, \wedge\}$.

7. a) Convert the following into CNF

(i) $(A \wedge B) \vee (C \wedge D) \vee (E \to F)$

(ii) $P \to (Q \to P)$.

b) write full CNF for (5)

(i) $Q \wedge \neg P \to P$

(ii) $(P \vee Q) \wedge (R \to \neg P)$.

c) Show that each of the following sets of operation is a complete set of connective for the propositional calculus.

$\{\neg, \to\}$

8. a) Prove the following is tautology [using truth table & formal proof].

(i) $A \to (B \to (A \wedge B))$

(ii) $(A \to C) \to (A \to B \vee C)$

b) Consider the argument given by the following sentences.

"The team wins or I am sad. If the team wins, then I go to a movie. If I am sad, then my dog barks. My dog is quiet. therefore, I go to a movie."

Check the validity of the above.

9. Four men and four women were nominated for 2 positions on the school board. One man and one woman were elected to the positions. Suppose the men are named A, B, C and D and the women are named E, F, G and H. Further, suppose that the following 4 ~~sente~~ statements are true.

1. If neither A nor E won a position, then G won a position.

2. If neither A nor F won a position, then B won a position.

3. If neither B nor G won a position then C won a position.

4. If neither C nor F won a position then E won a position.

Who were the 2 people elected to the school board?
[Use logic].

(7)

10\. a) What are the contrapositive, converse & inverse of the conditional statement "The home team wins whenever it is raining"?

b) Determine whether the following is consistent

→ The diagnostic message is stored in the buffer or it is retransmitted.

→ The diagnostic message is not stored in the buffer.

→ If the diagnostic message is stored in the buffer, then it is retransmitted."

c) Construct truth table for

(i) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

(ii) $(p \rightarrow q) \wedge (\neg p \rightarrow r) \wedge (\neg q \rightarrow s)$

(8)

11. a) Use De Morgan's laws to find the negation of each of the following statements. [And prove them using truth table].

(i) Jan is rich & happy.

(ii) John walks or takes the bus to class.

b) Show that the following is tautology without using truth table.

(i) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

(ii) $\neg(p \to q) \to \neg q$.

c) Show that $(p \to q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.

12. a) Show that the propositions P_1, P_2, P_3, P_4 and P_5 can be shown to be equivalent by proving that the conditional statements $P_1 \to P_4$, $P_3 \to P_1$, $P_4 \to P_2$, $P_2 \to P_5$ and $P_5 \to P_3$ are true.

b) Determine whether $\forall x (P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are logically equivalent. Justify.

c) Use rules of inference to show that if $\forall x (P(x) \to (Q(x) \wedge S(x)))$ and $\forall x (P(x) \wedge R(x))$ are true, then $\forall x (R(x) \wedge S(x))$ is true.

13. a) Show that these statements are inconsistent: (9)

"If sergei takes the job offer then he will get a signing bonus."

"If sergei takes the job offer, then he will receive a higher salary."

"If sergei gets a signing bonus, then he will not receive a higher salary."

"Sergei takes the job offer."

b) Let $P(x,y)$ be a propositional functional functions. Show that $\exists_x \exists_y P(x,y) \to \forall_y \exists_x P(x,y)$ is a tautology.

c) Let $P(x)$ and $Q(x)$ be propositional functions. Show that $\exists_x (P(x) \to Q(x))$ and $\forall_x P(x) \to \exists_x Q(x)$ always have the same truth value.

14. a) Determine the validity of the following:

(i) $\forall_x (A(x) \to B(x)) \to (\forall_x A(x) \to \forall_x B(x))$

(ii) $\exists_x (A(x) \wedge B(x)) \to \exists_x A(x) \wedge \exists_x B(x)$.

b) Construct Prenex CNF.

- $\forall x \forall y (P(x,y) \to \exists z (P(x,z) \wedge P(y,z)))$.

c) Construct prenex DNF

$\exists_x P(x) \wedge \exists_x Q(x) \longrightarrow \exists_x (P(x) \wedge Q(x))$

15. Prove the validity of the following using direct (10)
a)
proof.

$\exists_x \forall_y P(x,y) \wedge \forall_x (P(x,x) \rightarrow \exists_y Q(y,x)) \rightarrow$
$\exists_y \exists_x Q(x,y).$

b) Use indirect proof to prove the validity.

$\exists_x \forall_y P(x,y) \wedge \forall_x (P(x,x) \rightarrow \exists_y Q(y,x)) \rightarrow$
$\exists_y \exists_x Q(x,y).$

c) Use partial truth table ~~and~~ and determine the validity of the following:-

$\exists_x (P(x) \wedge Q(x)) \rightarrow \exists_x P(x) \wedge \exists_x Q(x).$

UNIT-II

16.
Prove using mathematical induction

a) $1^2 + 3^2 + 5^2 + \cdots (2n+1)^2 = [(n+1)(2n+1) * (2n+3)]/3$

b) $1.2.3 + 2.3.4 + \cdots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4$

c) $3^n < n!$ if n is an integer greater than 6.

17 a). $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer

17. b) A license plate contains 2 letters followed by 3 digits with the first digit not zero. How many different license plate can be printed?

c b) A farmer buys 3 cows, 2 pigs and 4 hens from a man who has 6 cows, 5 pigs and 8 hens. How many choices does the farmer have?

18. a) Suppose a laundry bag contains many red, white & blue socks. Find the minimum number of socks that one needs to choose in order to get two pairs of the same color.

b) Find the minimum number of elements that one needs to take from the set = $\{1, 2 \cdots 9\}$ to be sure that 2 of the numbers add up to 10.

c) Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

(12)

19. a) How many strings of 4 decimal digits

(i) do not contain the same digit twice?

ii) end with an even digit?

iii) have exactly three digits that are 9s?

b) An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

C) How many positive integers not exceeding 100 are divisible either by 4 or by 6?

20. (a) How many ways are there for 10 women & 6 men to stand in a line so that no two men stand next to each other?

b) A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions 17 are ~~man~~ true. If the questions can be positioned in any order, how many different answer keys are possible?

C) How many ways are there to seat 6 people around a circular table, where seatings are considered to be the same, if they can be obtained from each other by rotating the table?

21.a) Draw Hasse diagram for the set {1,2,3,4,5,6} with divisibility relation

b) Draw Hasse diagram for the set {20,40,50,60, 100} with divisibility relation

22. a) In how many different ways can five elements be selected in order from a set with five elements when repetition is allowed?

b) A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels & plain bagels. How many ways are there to choose

i) six bagels

ii) a dozen bagels?

iii) a dozen bagels with at least one of each kind?

c) How many ways are there to assign 3 jobs to five employees if each employee can be given more than one job?

23. a) How many different strings can be made from the letters in MISSISSIPPI using all the letters?

b) How many different bit strings can be formed using six 1's & eight 0's?

c) How many ways are there to distribute hands of 5 cards to each of 4 players from the standard deck of 52 cards?

24. a) Of 32 people who save paper or bottles or both for recycling 30 save paper and 14 save bottles. Find the number of people who (i) save both, (ii) save only paper & (iii) save only bottles.

b) 12 people read Journal or Book or both. Given 3 people read only the Journal & six read both, find the number of people who read only Book.

c) There are 345 students at a college who have taken a course in Calculus, 212 who have taken a discrete maths course & 188 who have taken courses in both. How many students have taken a course in either Calculus or discrete?

25. a) Prove the principle of inclusion - exclusion using mathematical induction.

b) How many elements are in the union of 4 sets if each of the sets has 100 elements, each pair of the sets shares 50 elements, each three of the sets share 25 elements and there are 5 elements in all 4 sets?

c) write the explicit formula for Principle of Inclusion - exclusion for the no. of elements in the union of 5 sets.

Unit 2 &3

26.

a) Let N be the set of all natural numbers. For each of the following determine whether $*$ is an associative operation.

(i) $a * b = \max(a, b)$

(ii) $a * b = a + 2b$.

b) Consider the set N of positive integers, and let $*$ denote the operation of least common multiple on N.

(i) Is $(N, *)$ a semigroup?

(ii) Is it commutative?

(iii) Find identity element of $*$.

(iv) Which elements in N, if any, have inverse and what are they?

c) Let S be a semigroup with identity e, and let b and b' be inverse of a. Show that $b = b'$ (i.e.) that inverse are unique if they exist.

27.

a) State whether or not each of the following subsets of the +ve integers N is closed under the operation of multiplication.

i) $A = \{0, 1\}$

ii) $B = A\{1, 2\}$

iii) $C = \{x : x \text{ is Prime}\}$

iv) $D = \{1, 3, 5, \dots\} = \{x : x \text{ is odd}\}$

b) Consider the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7.

i) Find the multiplication table of G.

ii) Find 2^{-1}, 3^{-1}, 6^{-1}

iii) Find the orders & subgroups generated by 2 and 3.

c) Let σ & τ be the following elements of the symmetric group S_6.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix} \& \ \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$$

Find $\tau\sigma$, $\sigma\tau$, σ^2 & σ^{-1}.

28.

a) Show that the dimension of the vector space of all m by n real matrices over $\mathbb{R}$ is mn.

b) which of the following are vector spaces over $\mathbb{R}$?

i) $V_1 = \{x, y, z \in \mathbb{R}^3$ such that $y + z = 1\}$

ii) $V_1 = \{x, y, z \in \mathbb{R}^3$ such that $y \geq 0\}$.

c) Show that $\mathbb{Z}$ is not a vector space over $\mathbb{Q}$.

29.

a) Solve the recurrence relations for the given initial condition

i) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

b) Find the solution $a_n = 5a_{n-2} - 4a_{n-4}$ with $a_0 = 3$, $a_1 = 2$, $a_2 = 6$ & $a_3 = 8$.

c) what is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots $1, 1, 1, 1, -2, -2, -2, 3, 3, -4$?

30.

a) Find the first six term of the sequence defined by the recurrence relation & initial condition.

(i) $a_n = a_{n-1} - a_{n-2}$; $a_0 = 2$, $a_1 = -1$

(ii) $a_n = na_{n-1} + a^2_{n-2}$; $a_0 = -1$, $a_1 = 0$.

b) A person deposits \$1000 in an account that yields 9% interest compounded annually.

(i) set up a recurrence relation for the amount in the account at the end of 'n' years.

ii) Find an explicit formula for the amount in the account at the end of n years.

31.

a) (i) Find a recurrence relation for the number of bit strings of length n that do not contain three consecutive 0s.

(ii) what are the initial conditions?

(iii) How many bit strings of length seven do not contain 3 consecutive 0's?

b) (i) show that the Fibonacci numbers satisfy the recurrence relation $f_n = 5f_{n-4} + 3f_{n-5}$ for $n = 5, 6, 7, \cdots$ together with the initial conditions $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$.

(ii) use the above recurrence relation to show that f_{5n} is divisible by 5 for $n = 1, 2, 3, \ldots$

32.

a) Find the characteristic roots of the linear homogeneous recurrence relation $a_n = 2a_{n-1} - 2a_{n-2}$

b) & also find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

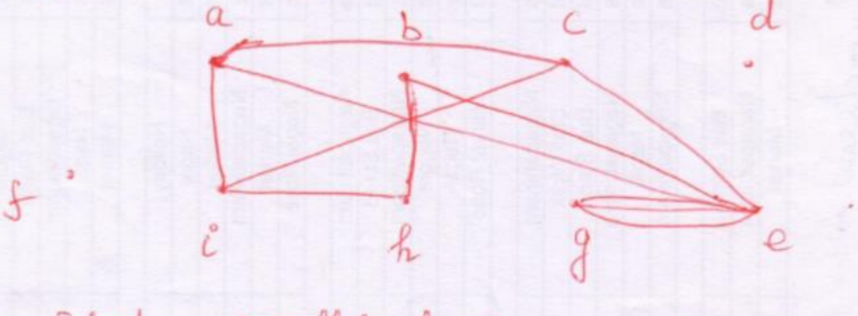
b) A new employee at an existing new software company starts with a salary of $50,000 and is promised that at the end of each year her salary will be double her salary of the previous year, with an extra increment of $10,000 for each year she has been with the company.

(i) construct a recurrence relation for her salary for her nth year of employment.

(ii) solve this recurrence relation to find her salary for her nth year of employment.

UNIT – V

40. a) Find the number of vertices, no. of edges and degree of each vertex in the given undirected graph. Identify all isolated, pendant vertices.



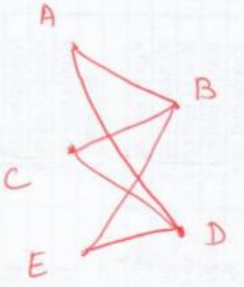
Find parallel loops, connected components

b) How many edges does a graph have if its degree sequence is 5, 2, 2, 2, 2, 1? Draw such a graph.

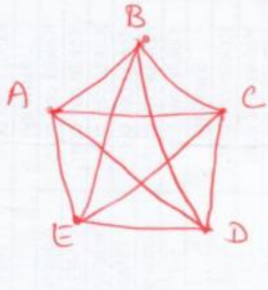
c) How many vertices does a regular graph of degree four with 10 edges have?

d) Suppose a graph G contains two distinct paths from a vertex u to a vertex v. Show that G has a cycle.

41. a) Consider Graph G_1 & G_2. (i) Find an Euler path or Euler Circuit. If it does not, why not?



G_1



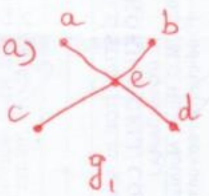
G_2

(ii) Find a Hamiltonian path or Hamiltonian Circuit. If not, justify the reason.

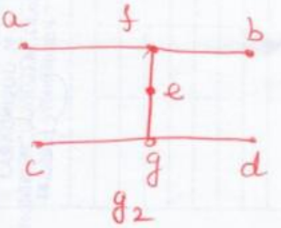
b) Draw two 3-regular graphs with eight vertices.

c) Show that in a simple graph with at least two vertics there must be two vertices that have the same degree.

42. a)



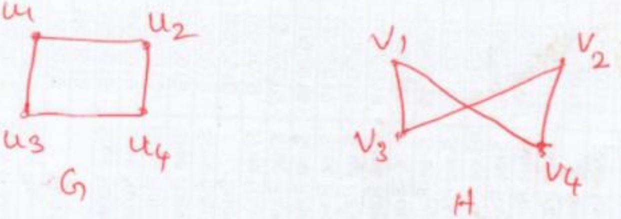
g_1



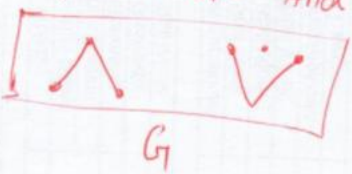
g_2

perform graph operations on the above graphs (union, intersection, deletion, complement, fusion & ring sum)

b) Show that graphs G and H are isomorphic (26)



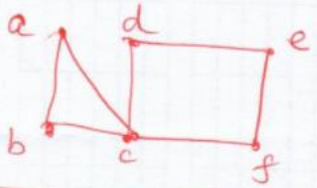
c) Find the no. of connected components in the graph G. And also represent them.



43. a) Show that every connected graph with n vertices has at least n-1 edges.

b) Draw 3-regular graphs with eight vertices.

c) Find all the cut edges in the graph.



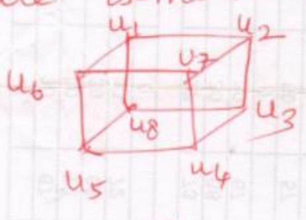
27

44. a) Show that if a connected simple graph G is the union of the graph G_1 & G_2, then G_1 & G_2 have at least one common vertex

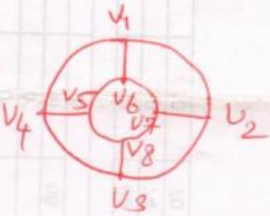
b) Find all the cut vertices of the graph G,

G.

c) ~~Suppose~~ Determine whether the 2 graphs (G_1 & G_2) are isomorphism.

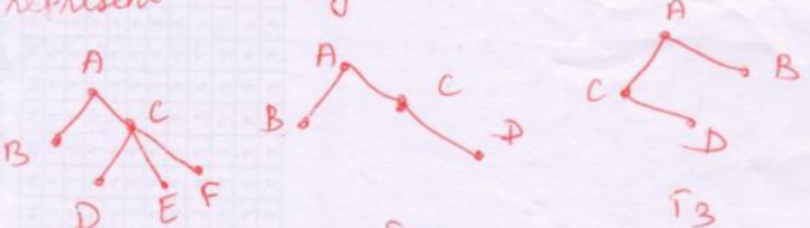


G_1



G_2.

d) consider 3 trees T_1, T_2, T_3. Identify those represent binary tree. Justify your answer



T_2 T_3