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Industrial Engineering™

Kidney Pairing Donations – Solutions for modern problems

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Introduction

The processes of pairing up organ donations both for the donor and the recipient side is a critical part of maximizing a finite supply of organs to limit mortality. Within organ transplants there exists the specific linkages of blood type and its compatibility with other blood types. More importantly there exists an incompatibility of blood/organ types that must be adhered to strictly to eliminate organ rejection.

The Kidney Pairing Donation (KPD) program and allocation system exists to maximize donor-recipient combinations based on blood type compatibility and eliminate incompatibility. Within the finite pool of donors, we need to ensure that each donor is paired with the correct recipient and not needlessly pair a more compatible donor with a more restrictive recipient. If a universal donor is used to donate to a universal recipient, then the more restrictive recipients/blood type might not have the chance to find a match.

Within the dataset/field provided there are recipients provided with their respective blood type. Additionally, the dataset there exists anywhere from 1 to 3 donors per recipient. A recipient can only receive a kidney from one donor, and one donor can only donate one kidney. Thus exists the need to maximize the compatibility and pairing of donors and recipients. Within this study the focus is primarily on blood compatibility, but there exist several other parameters such as age, relative health and probability of organ acceptance that drive the optimal pairing of donors to recipients.

The transplant medical system refers to two types of scenarios for a donor-recipient combination: cycles and chains. Cycles involve compatible pairings that result in a successful transplant of an organ. John Hopkins Hospital was a pioneer in the early days of organ transplantation by facilitating the framework needed to assemble donor-recipient chains: the mapping of willing incompatible donors with their respective incompatible recipients (*Kidney paired donation* 2024).

Kidney Donor Pairing Criteria

Humans have 35 major blood types, all subsisting of their own red blood cell types. Depending on a person's blood type the blood will display chemical characteristics that define its' antigen production. The human body's immune system creates antibodies whose purpose is to protect the body from foreign borne illness/disease. If, for instance, a blood type was infused into an incompatible pairing of blood the recipient's immune system would release antibodies against the antigens present and attack/reject the incoming blood. The same exists for organ transplants, as the recipient's body interprets an incompatible organ as a hostile foreign body.

The first group definition of blood, the ABO group, contains A, B, AB and O blood types. Defining and building cycles, then building donation pairings or chains, relies on the definition parameters of compatibility (and incompatibility, by extension).

Donor A => A & AB; Donor B => B & AB; Donor AB => AB; Donor O => A, B, AB & O

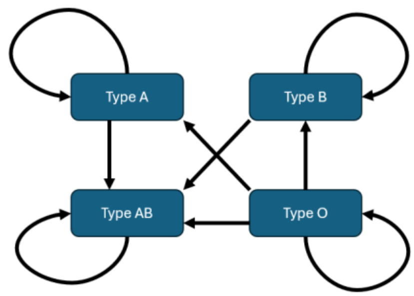

Figure 1: Donor – Recipient compatibility (*Programs for donor/recipient pairs with incompatible blood types* 2024)

Data is provided in the following format: Recipient "A"; Donor "A,B,O". There are 746 potential pairings within the provided dataset, each pairing/node representing a recipient and potential donor. If, for instance, a donor is not compatible with its recipient, then the donor and recipient can be paired with a compatible donor-recipient pair of another pairing/node.

Problem Statement

As Operations Researchers for the Organ Procurement and Transplantation Network (OPTN) we have been given the task of maximizing the efficiency of donors to recipients to ensure that organs are being maximally donated while strictly adhering to compatibility guidelines. The Kidney Paired Donation (KPD) system offers a solution by facilitating exchanges between compatible pairings while maximizing subsequent matchings of incompatible donor-recipient pairs.

In this system, pairs who cannot donate directly to their intended recipients can swap donors with other pairs to create mutually beneficial matches. However, optimizing the KPD system poses significant challenges. The process involves identifying and matching compatible exchanges among numerous donor-recipient pairs while considering complex medical compatibility requirements and specific constraints. Moreover, to increase the likelihood of successful transplants, it is essential to limit the exchange cycles to manageable sizes of 2 and 3 pairs. The goal of this project is to develop an Operations Research (OR) model to maximize the number of successful kidney transplants in the KPD system.

By using mathematical optimization techniques, we aim to create an efficient Operations Research (OR) model that maximizes the number of successful kidney transplants in the KPD system. By using mathematical optimization techniques, we aim to create an efficient matching plan that selects the maximum number of compatible exchanges while ensuring that each donor can only donate once, each recipient can only receive one kidney, and exchanges are restricted to cycles of 2 or 3 pairs. This approach not only optimizes the allocation of scarce donor kidneys but also enhances the overall effectiveness of the KPD program.

<u>OR Model (in math, then words)</u>

$$Maximize\ Z = \sum_{(i,j)\in E} x_{ij}$$

$$Subject\ to:$$

$$x_{ij} = 0, \quad if\ donor\ i\ is\ not\ compatible\ with\ recipient\ j$$

$$\sum_{i\in P} x_{ij} \leq 1, \quad \forall j \in P$$

$$\sum_{j\in P} x_{ij} \leq 1, \quad \forall i \in P$$

$$\sum_{(i,j)\in C_k} x_{ij} \leq k, \quad for\ k = 2\ or\ 3$$

$$Where:$$

$$x_{ij} = Binary\ decision\ variable\ (1\ if\ donor\ i\ donates\ to\ recipient\ j,\ and\ 0\ otherwise)$$

$$E = Set\ of\ all\ possible\ exchanges.$$

$$P = Set\ of\ all\ donors\ and\ recipients.$$

$$C_k = Set\ of\ cycles\ of\ size\ k.$$

The objective of this model is to optimize the Kidney Paired Donation (KPD) process by maximizing the number of successful kidney transplants between incompatible patient-donor pairs. The model uses binary decision variables x_{ij}, where $x_{ij} = 1$ if a donor from pair i donates to a recipient from pair j , and $x_{ij} = 0$ otherwise. The primary goal is to select as many exchanges as possible to maximize the total number of transplants. The objective function achieves this by summing all valid x_{ij} values to obtain the highest number of matches.

The model also consists of constraints. First, a compatibility constraint ensures that a donor can only donate to a recipient if they are medically compatible. Second, each recipient can receive at most one kidney, ensuring that no recipient is assigned multiple donors. Third, each donor is restricted to donating only once. Lastly, the model includes a cycle length constraint that restricts exchange cycles to a maximum size of 2 or 3 pairs. By balancing the goal of maximizing successful transplants with realistic constraints, the model efficiently supports the KPD program, optimizing the allocation of kidneys while addressing practical and medical limitations.

Python/Gurobi Code

Networkx and Gurobipy were the two core python functions that aided in maximizing/solving the Kidney pairing dataset, and their functions were used throughout the code.

Our Python/Gurobi code and data can be found at the following GitHub repository: https://github.com/mharvey2696/IE5318_Red_Raider_Engineers.

Experiment Discussion

This optimization model was written and solved using Gurobi Optimizer version 11.0.3 in Jupyter Notebook. The coding language used was Python. The model ran on a Windows 11 desktop computer with 32GB of RAM and an AMD Ryzen 9 5900X 12-Core @ 3.7GHz Processor. The LP model was solved in **94.57 seconds**. The Objective Value was found to be **356**.

An Optimal Pairing and Plan (Results)

To begin developing the model, a directed graph was required in order to find cycles of size 2 and 3. The graph contained nodes and edges created with the data set of pairs and was organized via the donor-recipient compatibility stated earlier in the report.

Once the graph was created, cycles of size 2 and 3 could be developed. To find cycles of size 3, an undirected graph was first created to show all potential cycles of size 3. Using both the undirected and directed graphs, cycles of size 3 could be found in the project data set by comparing pairs to their neighbors and looking for compatibility. Per our code, there were no cycles of size 3 found. To find cycles of size 2, the same method was used as in cycles of size 3. There were 37,202 cycles of size 2 found.

Running the maximization model using our constraints reduced the total number of cycles into the optimal amount of pairs based on our data set, 356. This value, while being the objective value, is not actually the final number of matched donor-recipient pairs. Each pair in the 356 are one half of a cycle of size 2. Therefore, the total combined matched pairs is half of that, 178.

Evaluation of the Plan

The results of our model can be found on the GitHub in Total Combined Matched Pairs.txt. The proposed plan met the following criteria:

- Only compatible blood-type matchings were paired
- No more than one donor was matched with no more than one recipient
- Recipients can only receive one kidney
- Cycle size constraint of at least 2, but no more than 3

The output results conform to the requirements of the donation program due to the restrictions placed on pairings. Since the constraints eliminate incompatible blood types as well as removing donors/recipients after pairing from the available pool there are no errors in matching. The goal of the program is to maximize the amount of successful recipient pairings with the finite number of donors in the available pool. Through computational analysis from Gurobi optimizer the simulated pairings maximize matching.

This model does not guarantee that all 746 pairs will be matched together (only 356 pairs for 178 combined matches).

Conclusion

In conclusion, this project presents a Kidney Paired Donation (KPD) model designed to maximize optimal matchings of two and three pair groups for incompatible donor-recipient pairs. The model ensures that each pair is matched with only one other compatible pair, provided the donors and recipients meet medical compatibility criteria.

<u>**References**</u>

Programs for donor/recipient pairs with incompatible blood types. National Kidney Foundation. (2024, August 28). https://www.kidney.org/transplantation/programs-donor-recipient-pairs-incompatible-blood-types

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