

Model Fit - Monte Carlo simulations of Negative Binomial Distribution

October 5, 2020

Contents

1	Description	1
2	Bias of $\hat{\lambda}$	3
2.1	Different Levels of Dispersion α	3
2.2	Different Sample Sizes N	20
3	CV	37
3.1	Different levels of Dispersion α	37
3.2	Different Sample Sizes N	53

1 Description

Although, the Poisson distribution for MOI is justified, still it can fail in practice. An alternative for the Poisson distribution is the negative binomial distribution. Unlike the Poisson distribution, variance of the negative binomial distribution differs from its mean. It is especially useful for data whose sample variance exceeds the sample mean. In the model violation we also condition on infected samples. Therefore, we considered the conditional negative binomial distribution (CNB). Since the negative binomial distribution has one more parameter than the Poisson, the second parameter can be used to adjust the variance independently of the mean. In order to derive the range of values for parameters of CNB, we did as follows: while the mean of the distributions agree, we considered various levels of dispersion from the mean for CNB, i.e.,

$$\mu^{(CNB)} = \mu^{(CPD)} \quad (1)$$

$$\sigma^{2(CNB)} = \alpha \sigma^{2(CPD)} \quad (2)$$

where α is the level of dispersion. We solved 1 for (r, q) (the CNB parameters) for different values of λ (the CPD parameter) and α , namely $\lambda = 0.1, 0.15, \dots, 1.95, 2$ and $\alpha = 1.05, 1.1, \dots, 1.95, 2$. We generated $S = 10,000$ datasets under alternative model for each set of values, and repeated the procedure described above.

2 Bias of $\hat{\lambda}$

2.1 Different Levels of Dispersion α

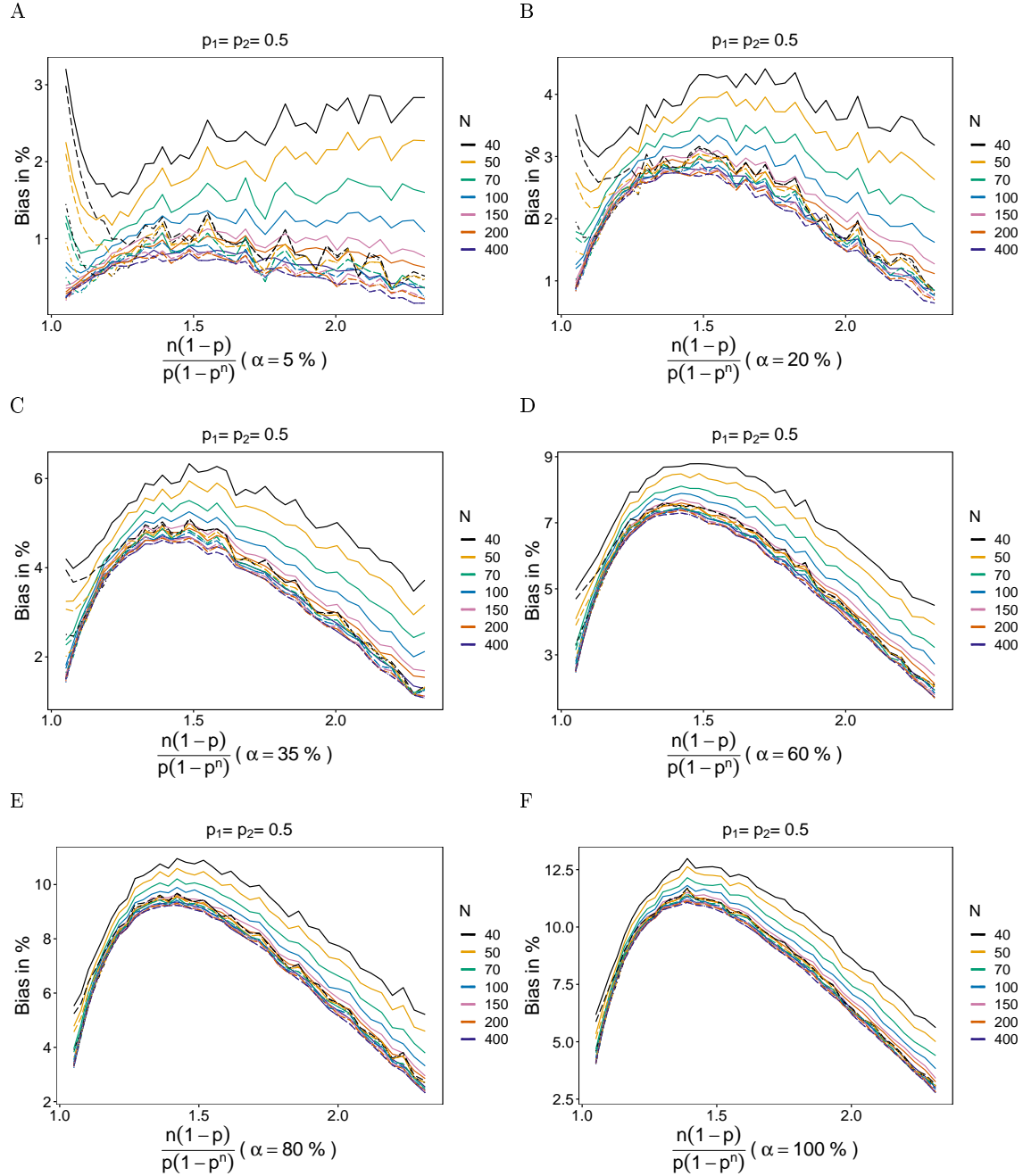


Figure 1: **Bias of bias-corrected MLE under conditional negative binomial model.** The figure shows the bias in % of different versions of bias-corrected MLE of mean MOI parameter along with MLE of $\psi = \frac{\lambda}{1-e^{-\lambda}}$ for different lineage frequency distributions under conditional negative binomial model. The simulated data is from conditional negative binomial model whereas the estimations are from conditional Poisson model. Each plot carries different levels of dispersion determined by α , i.e., the data is generated with $\sigma^{2(CNB)} = (1 + \alpha/100)\mu^{(CNB)}$. Different colors correspond to different sample sizes N . The solid, long-dashed and dot-dashed lines correspond to MLE, TBCMLE (theoretical bias-corrected MLE) and HBCMLE (heuristic bias-corrected MLE), respectively.

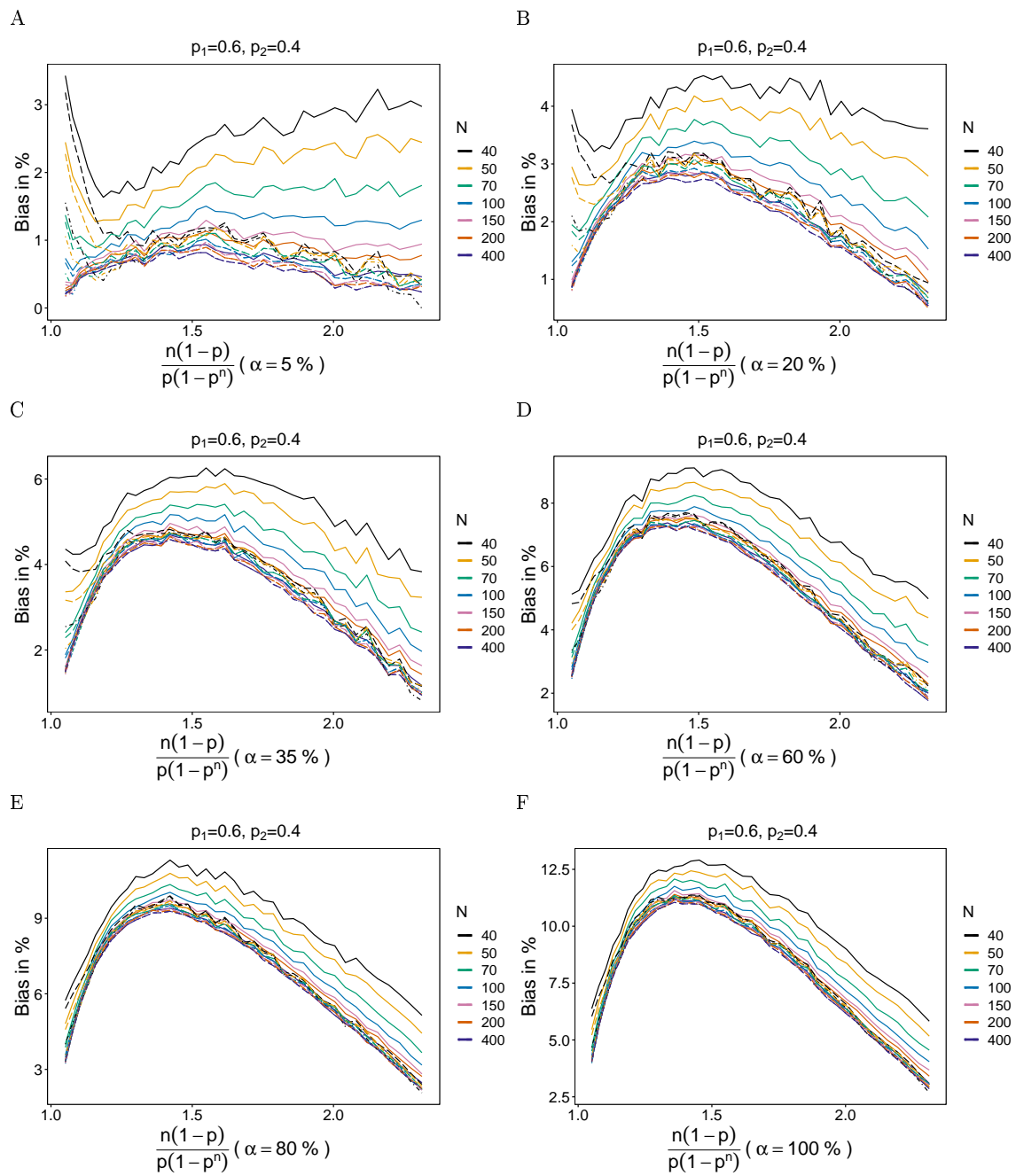


Figure 2: Same as Figure 1

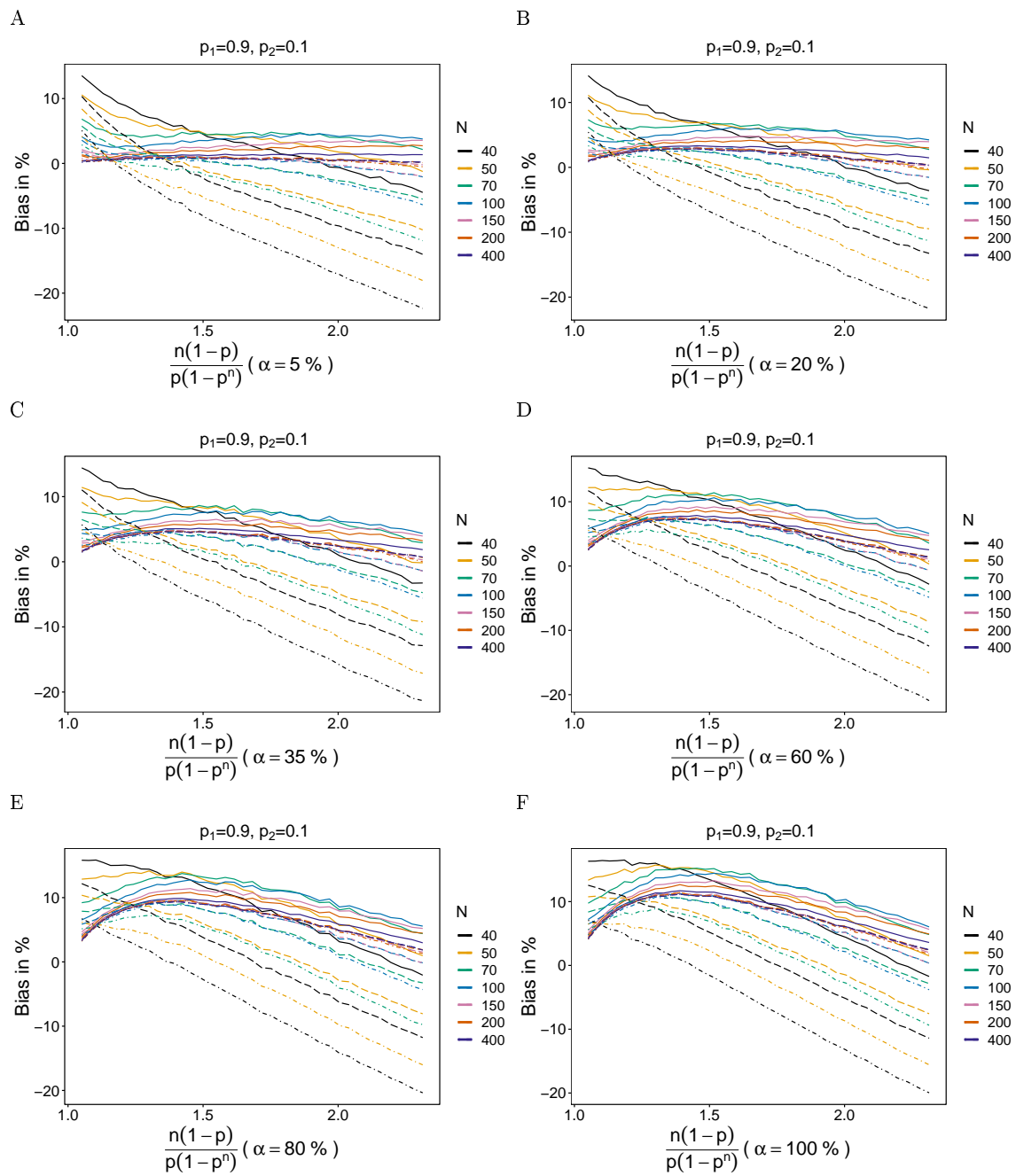


Figure 3: Same as Figure 1

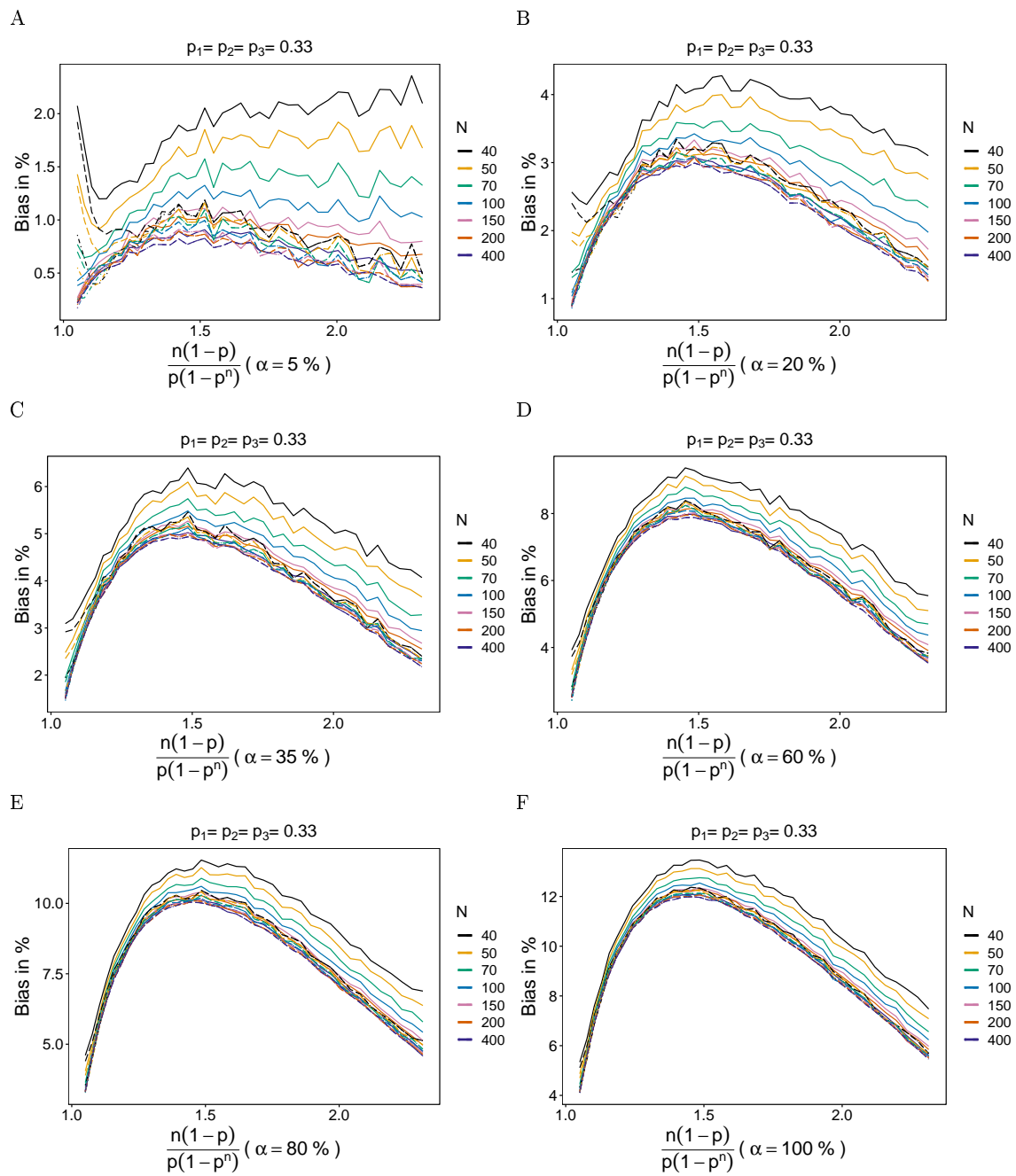


Figure 4: Same as Figure 1

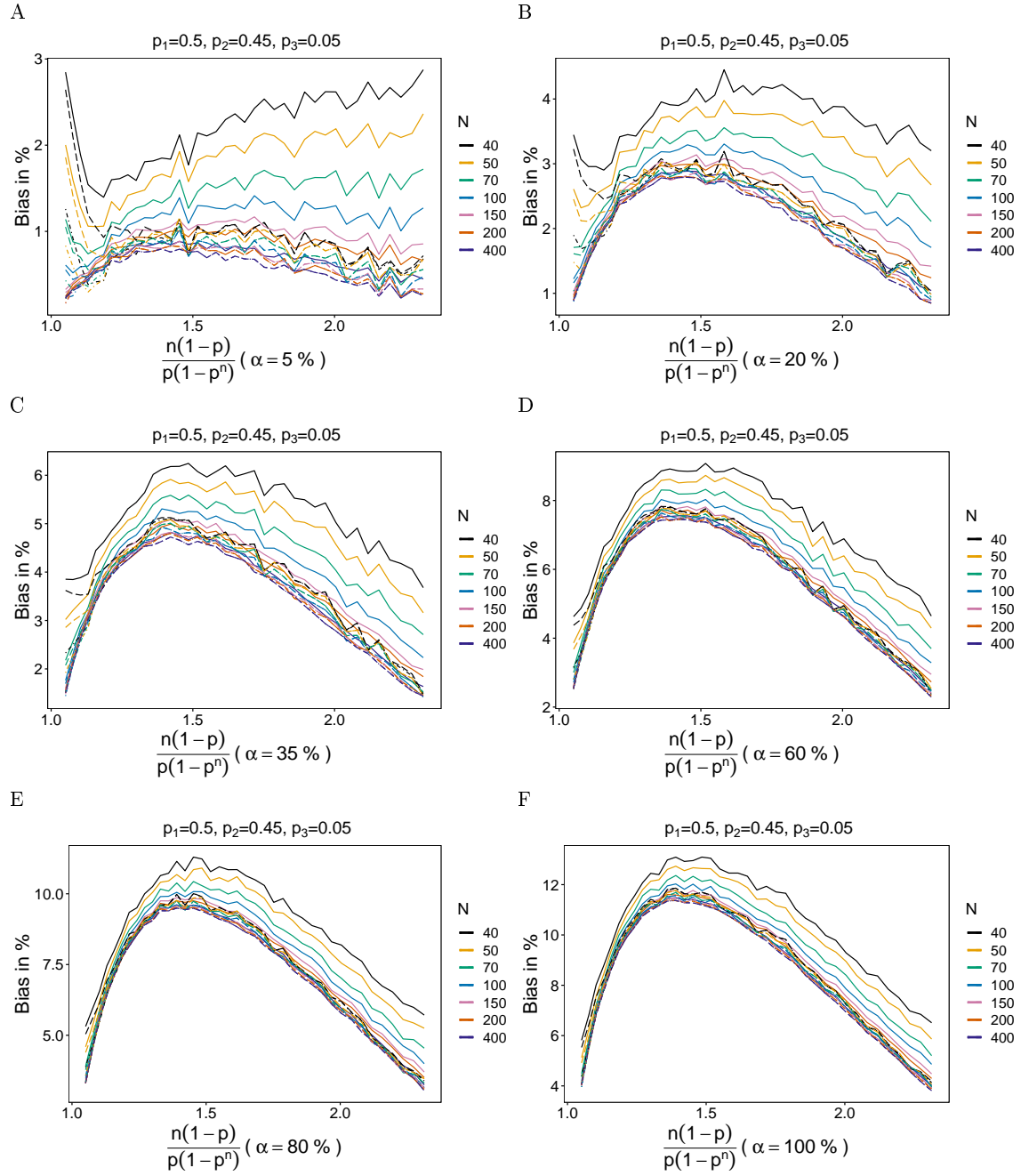


Figure 5: Same as Figure 1.

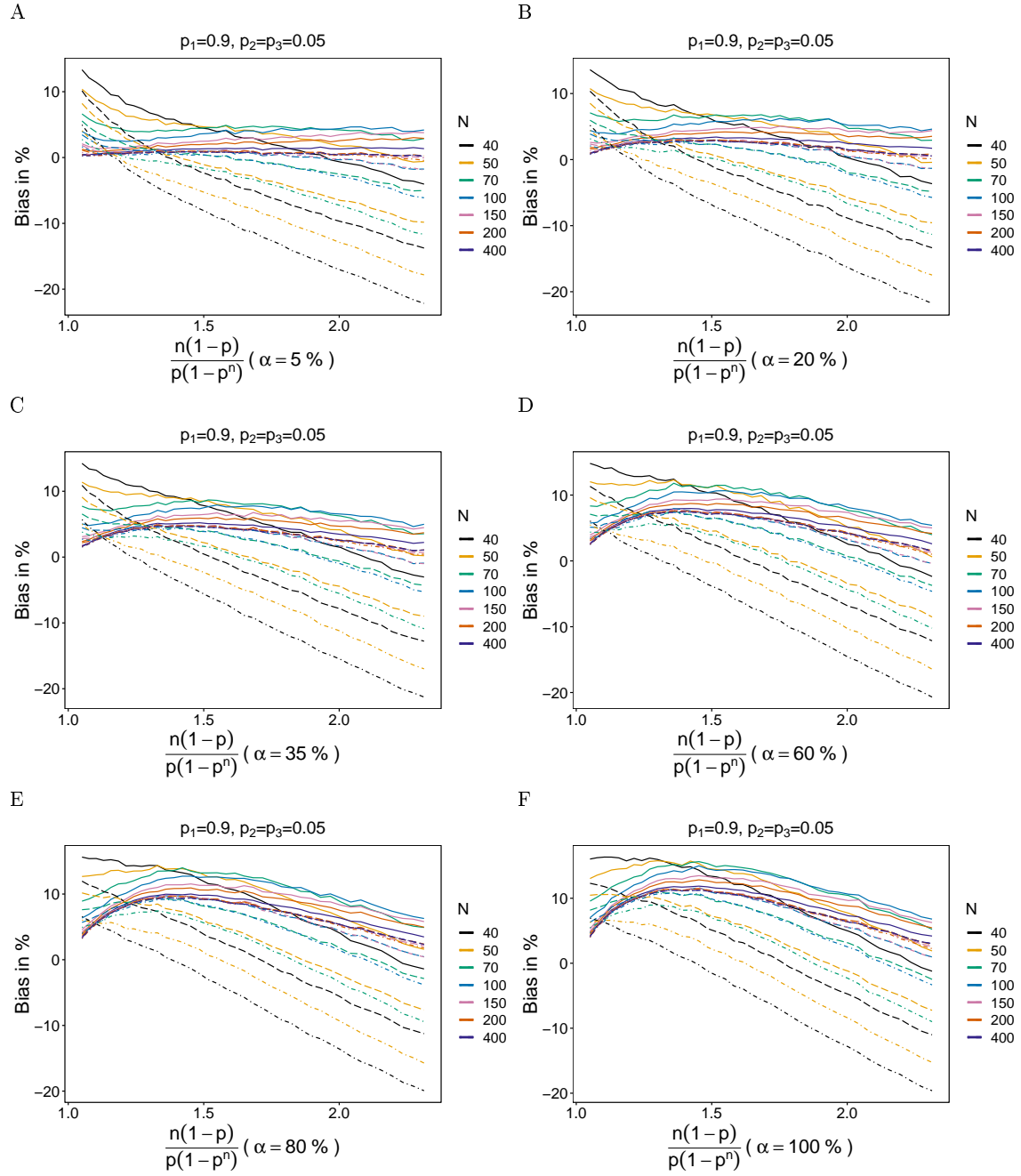


Figure 6: Same as Figure 1.

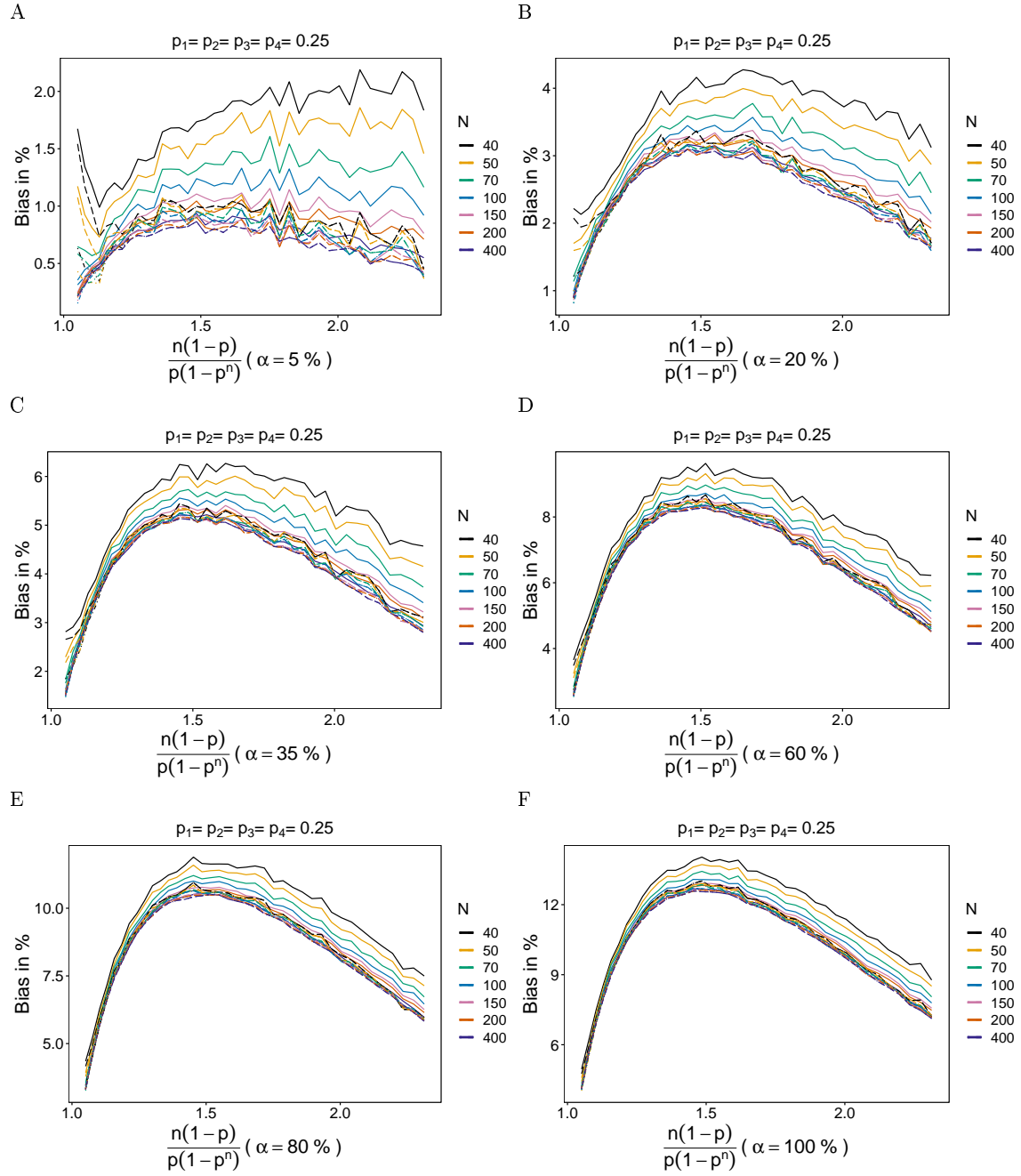


Figure 7: Same as Figure 1.

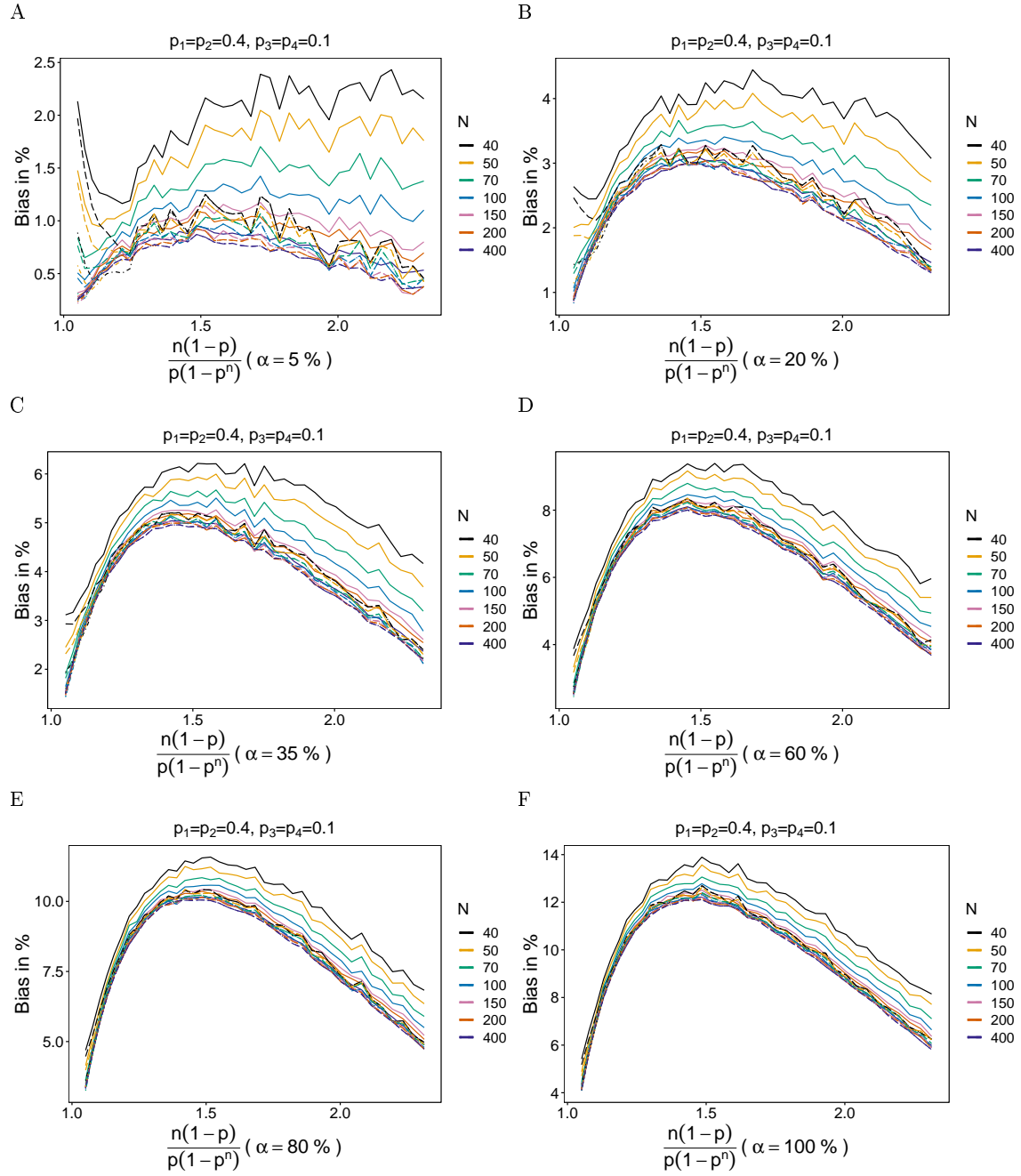


Figure 8: Same as Figure 1.

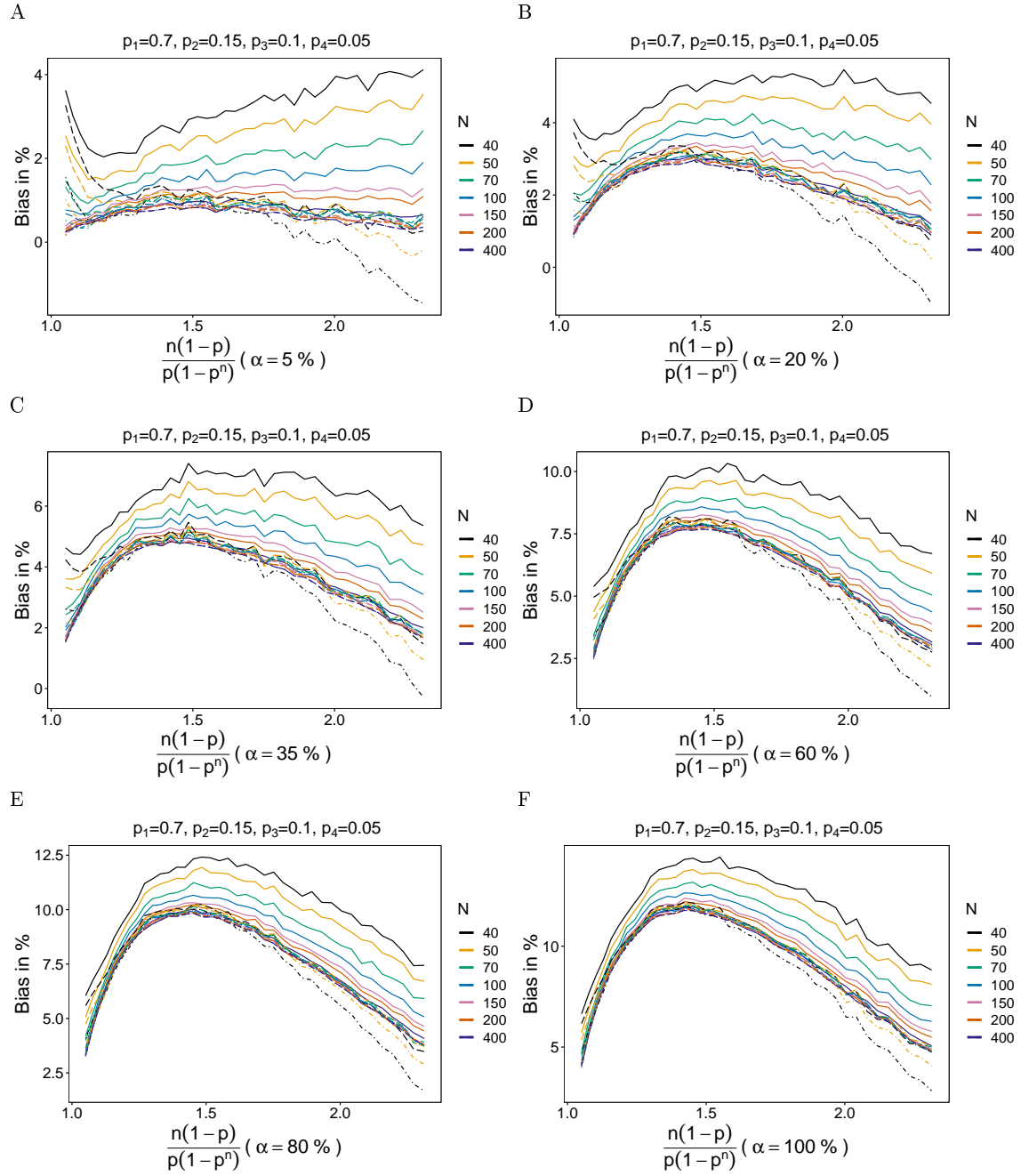


Figure 9: Same as Figure 1.

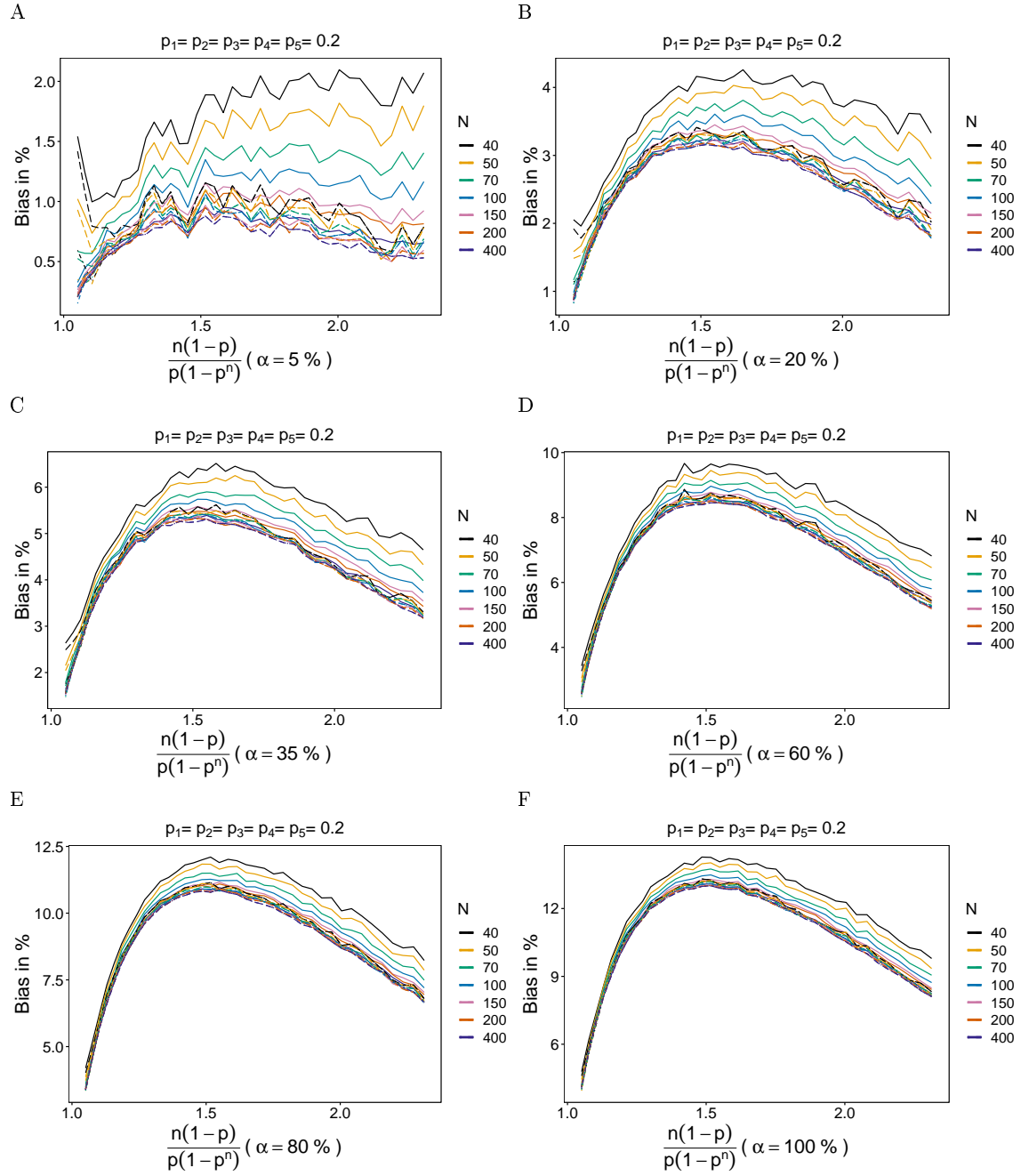


Figure 10: Same as Figure 1.

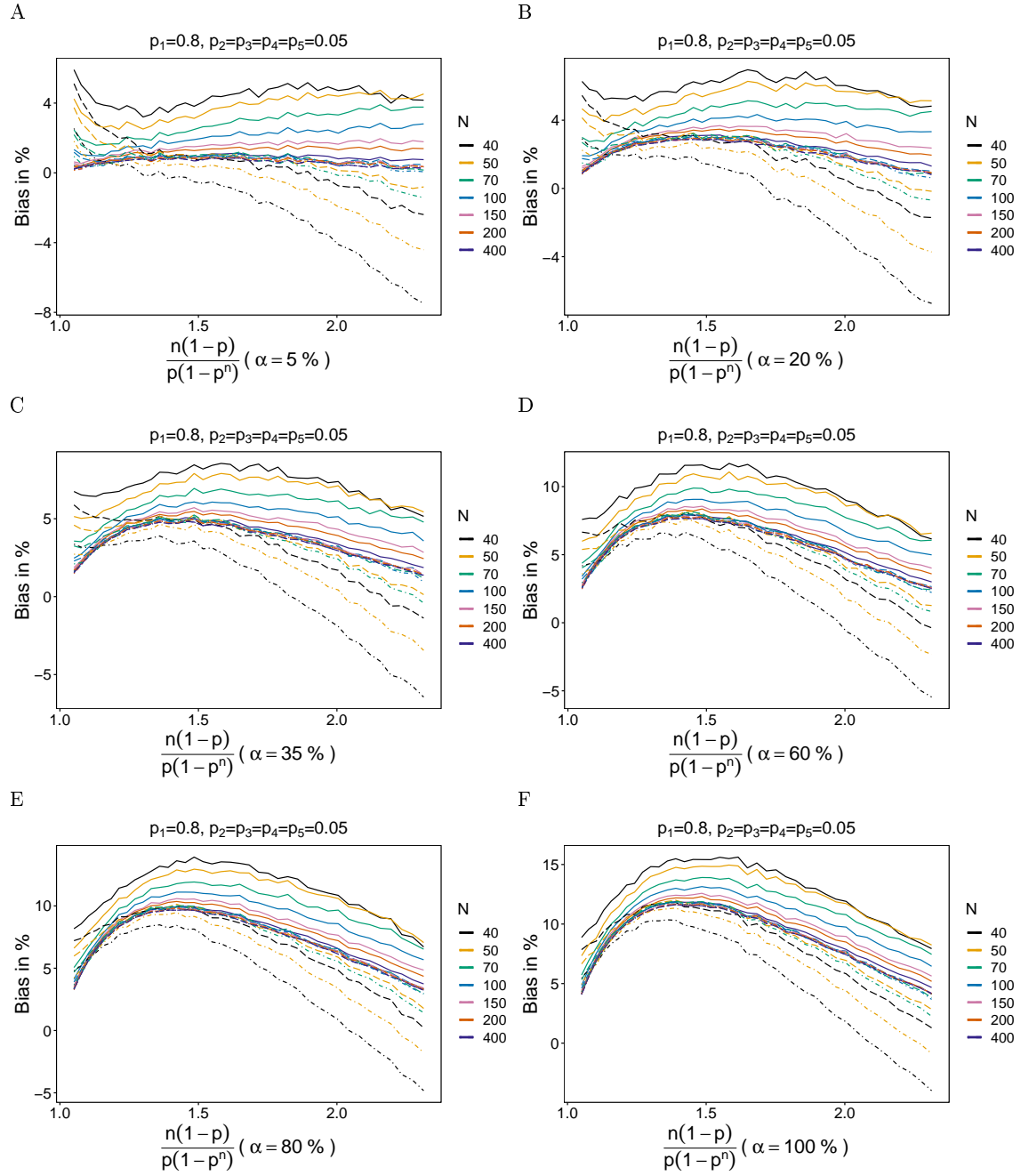


Figure 11: Same as Figure 1.

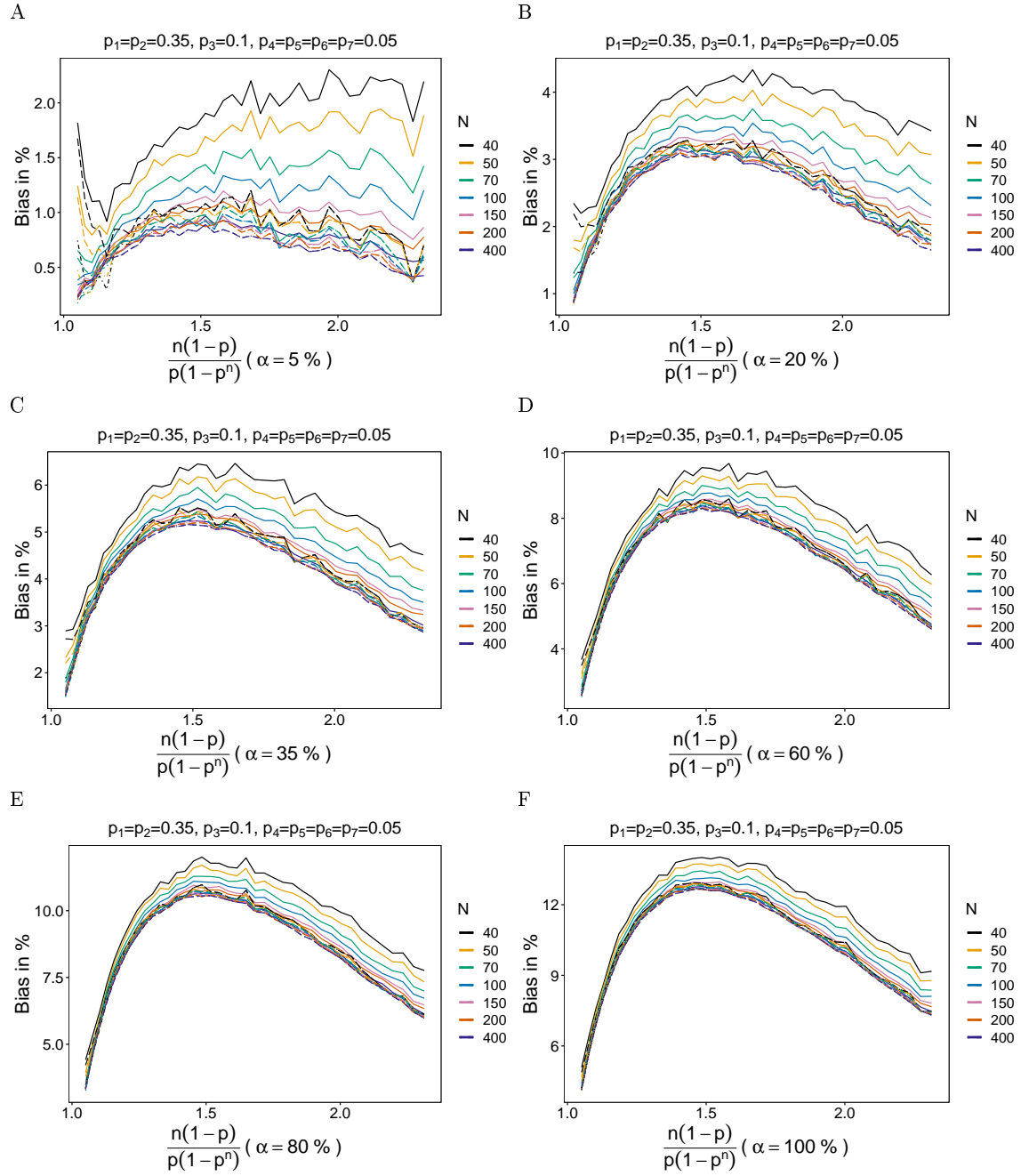


Figure 12: Same as Figure 1.

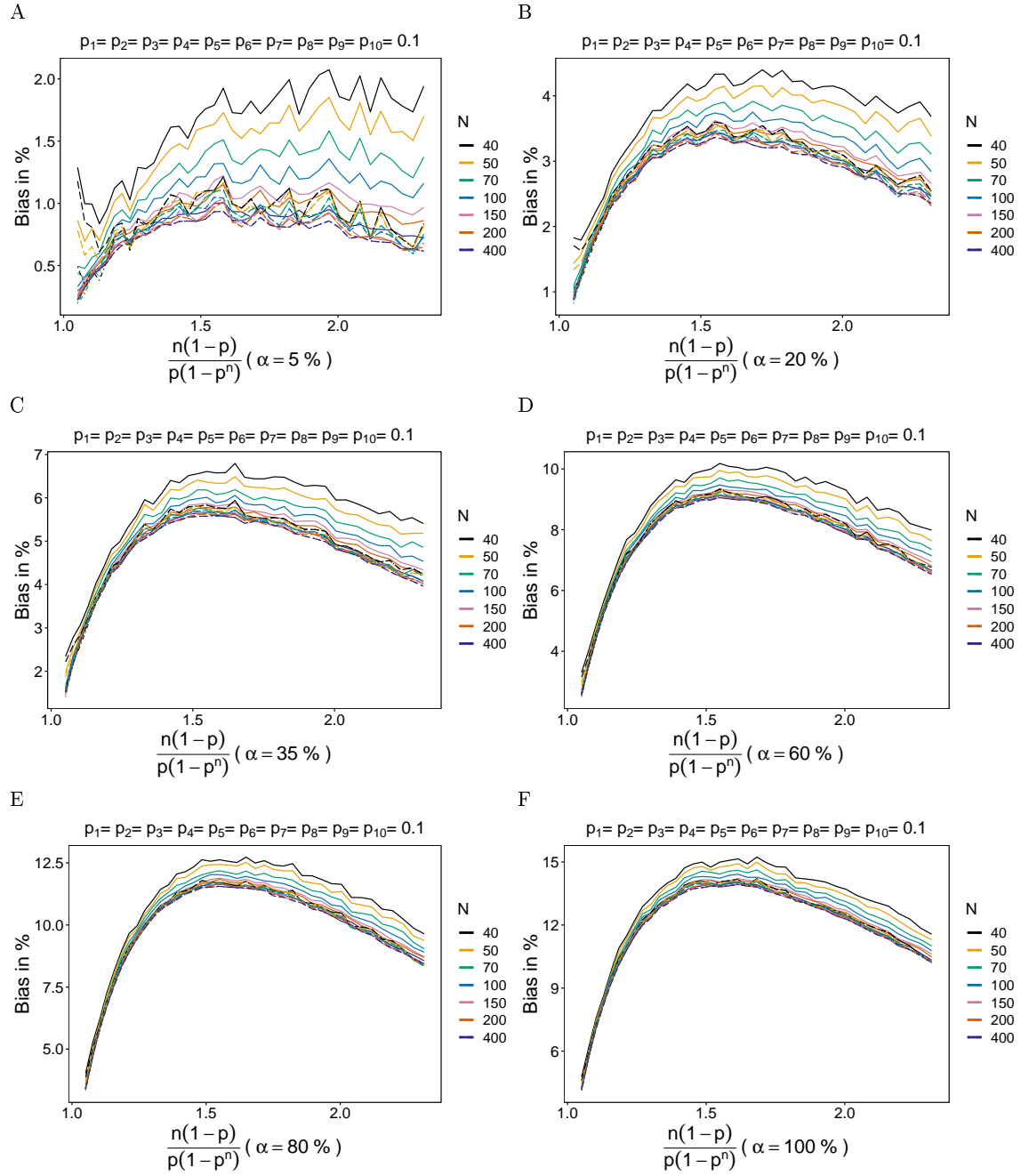


Figure 13: Same as Figure 1.

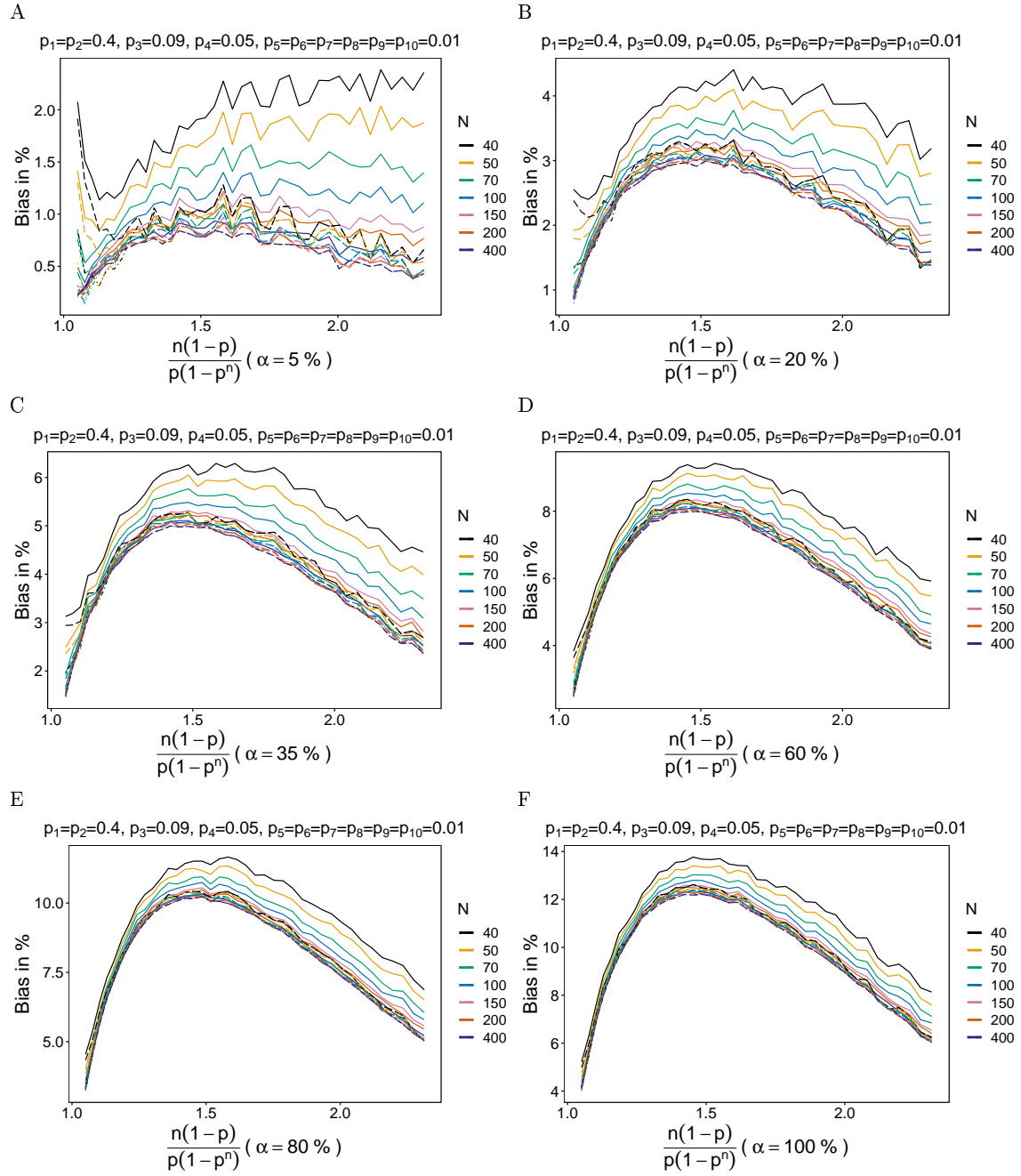


Figure 14: Same as Figure 1.

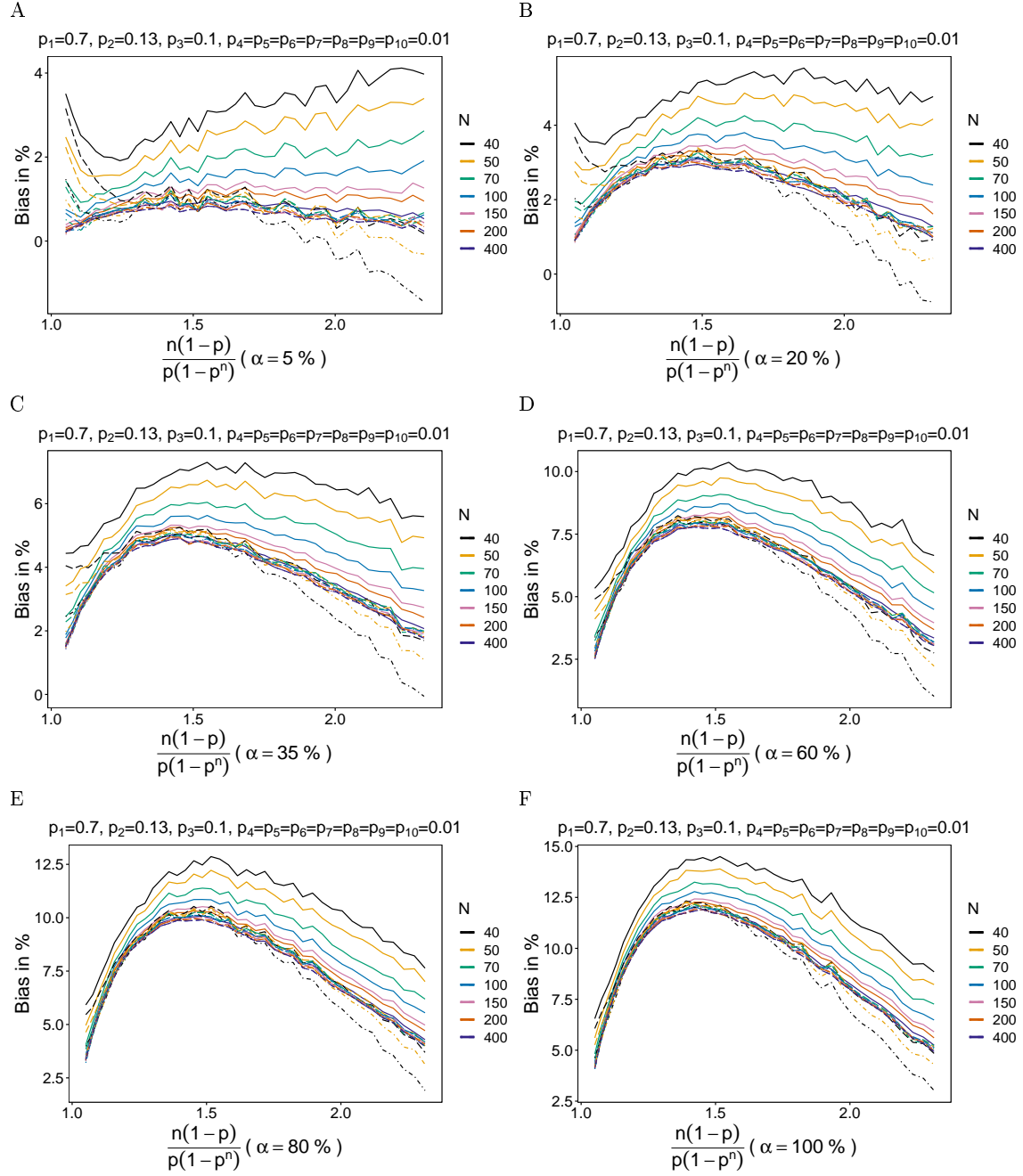


Figure 15: Same as Figure 1.

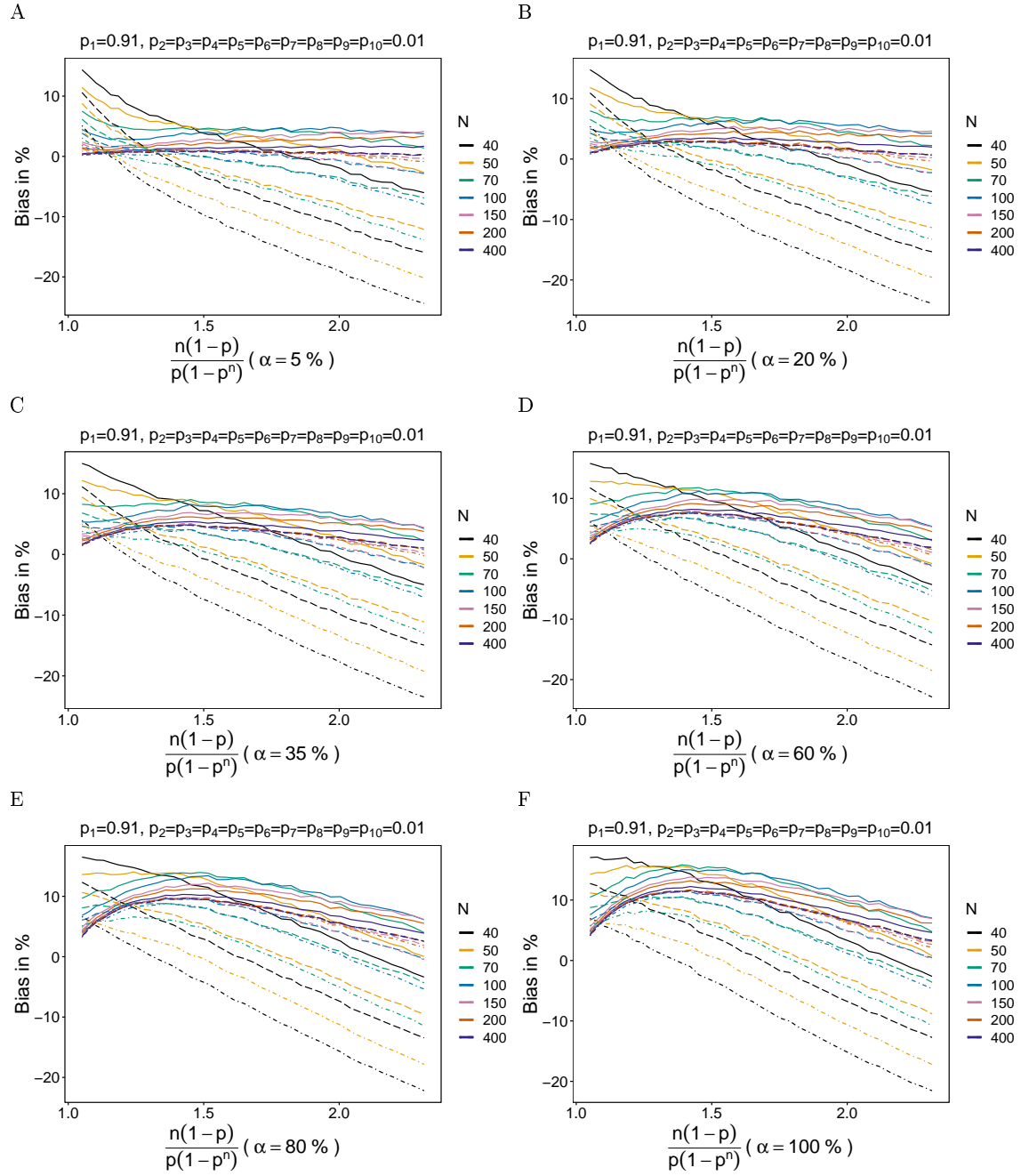


Figure 16: Same as Figure 1.

2.2 Different Sample Sizes N

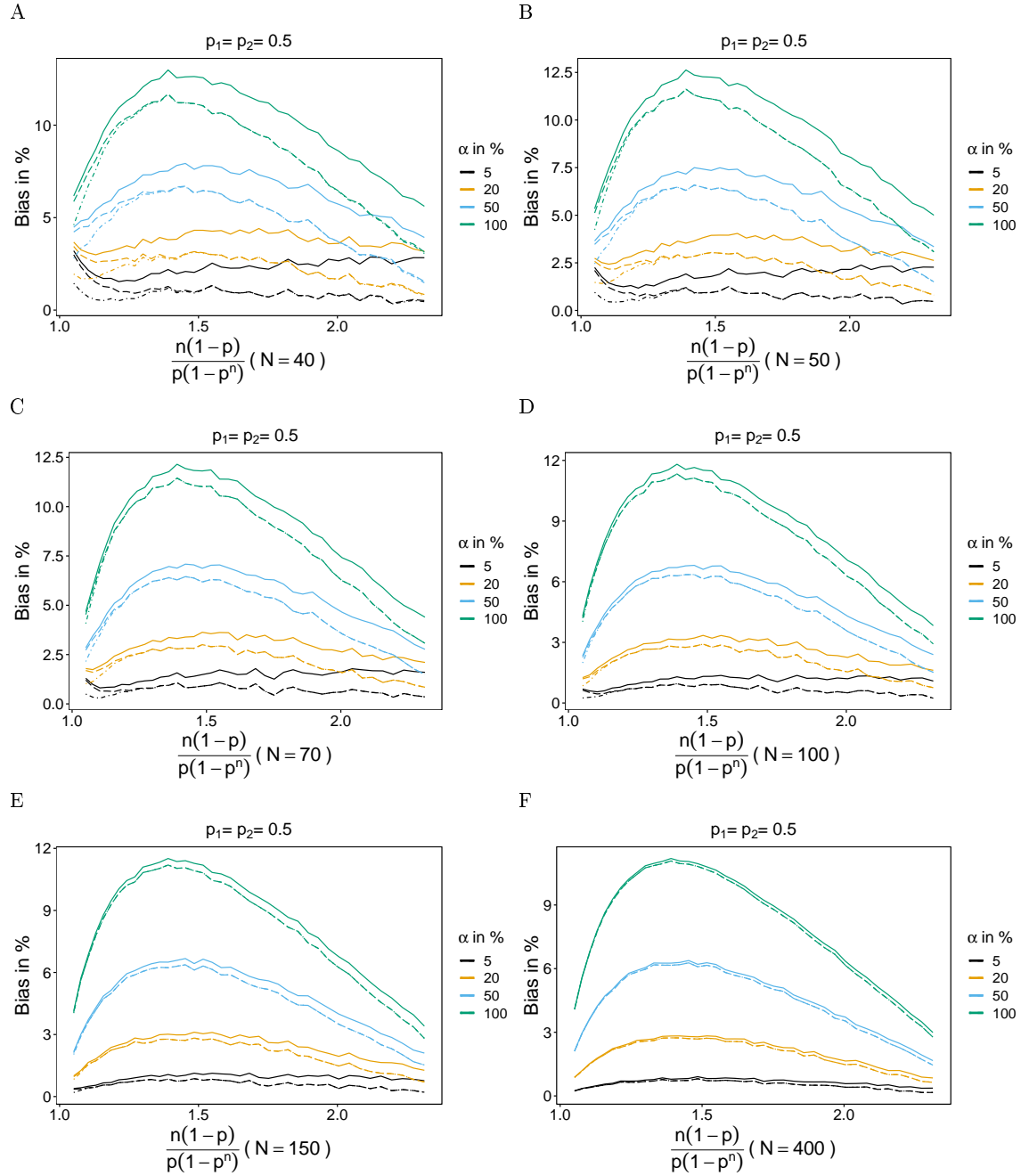


Figure 17: **Bias of bias-corrected MLE under conditional negative binomial model.** The figure shows the bias in % of different versions of bias-corrected MLE along with MLE of mean MOI parameter $\psi = \frac{\lambda}{1-e^{-\lambda}}$ for different lineage frequency distributions under conditional negative binomial model. The simulated data is from conditional negative binomial model whereas the estimations are from conditional Poisson model. Different colors correspond to different dispersion levels and every plot is considered for a fixed sample size N . The solid, long-dashed, dot-dashed and dashed lines correspond to MLE, TBCMLE (theoretical bias-corrected MLE), HBCMLE (heuristic bias-corrected MLE) and PBBCMLE (parametric bootstrap bias-corrected MLE), respectively.

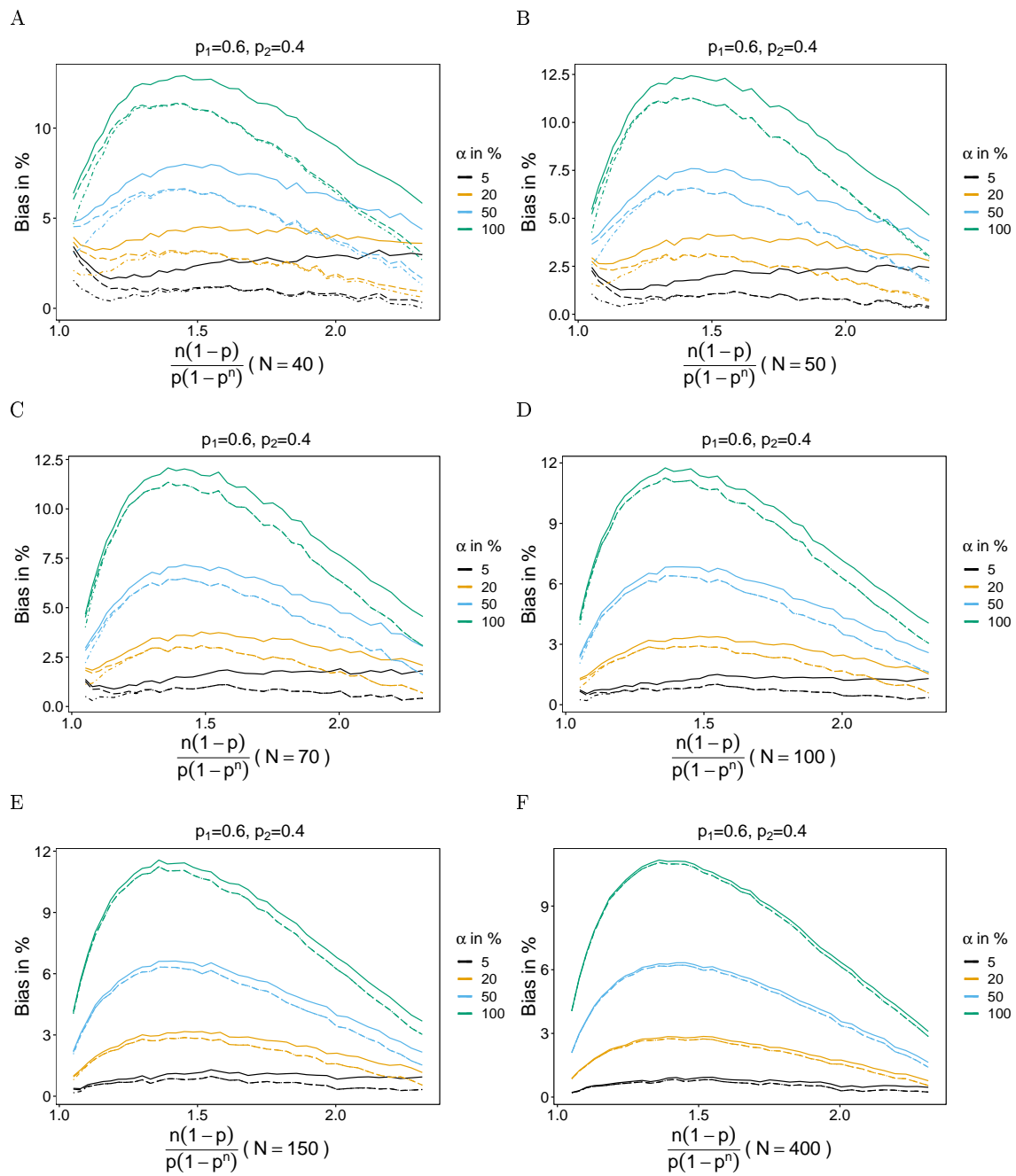


Figure 18: Same as Figure 17.

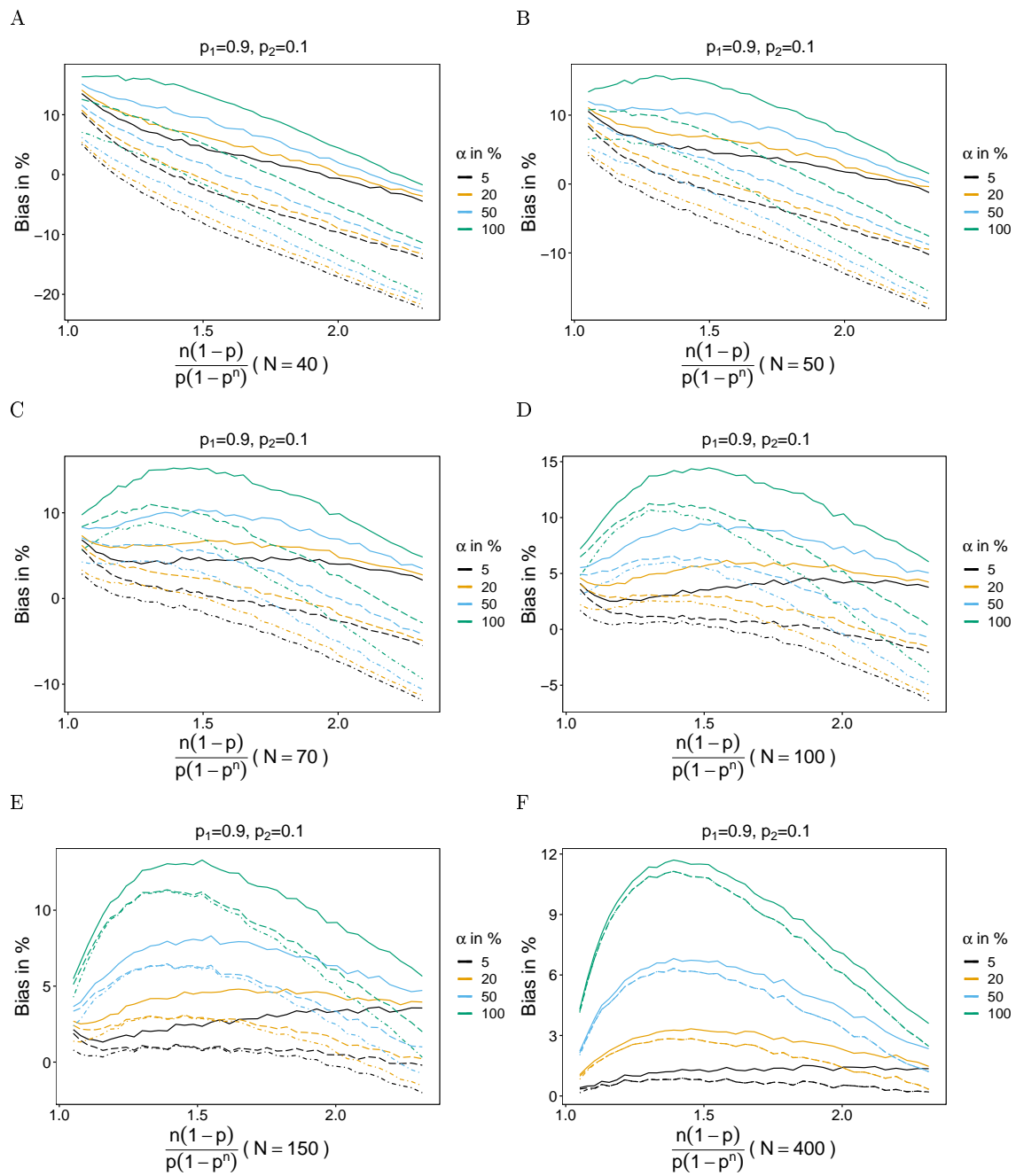


Figure 19: Same as Figure 17.

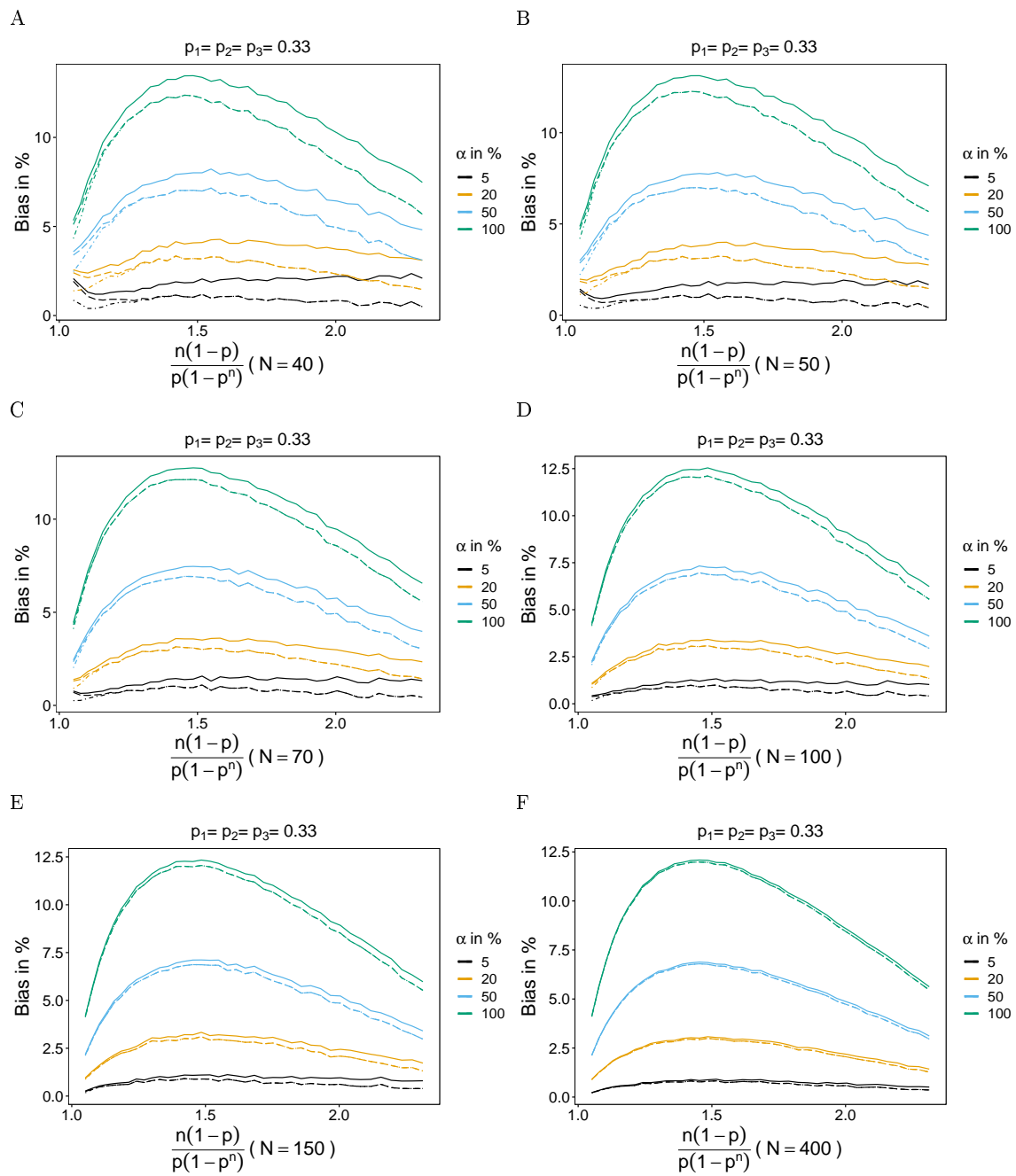


Figure 20: Same as Figure 17.

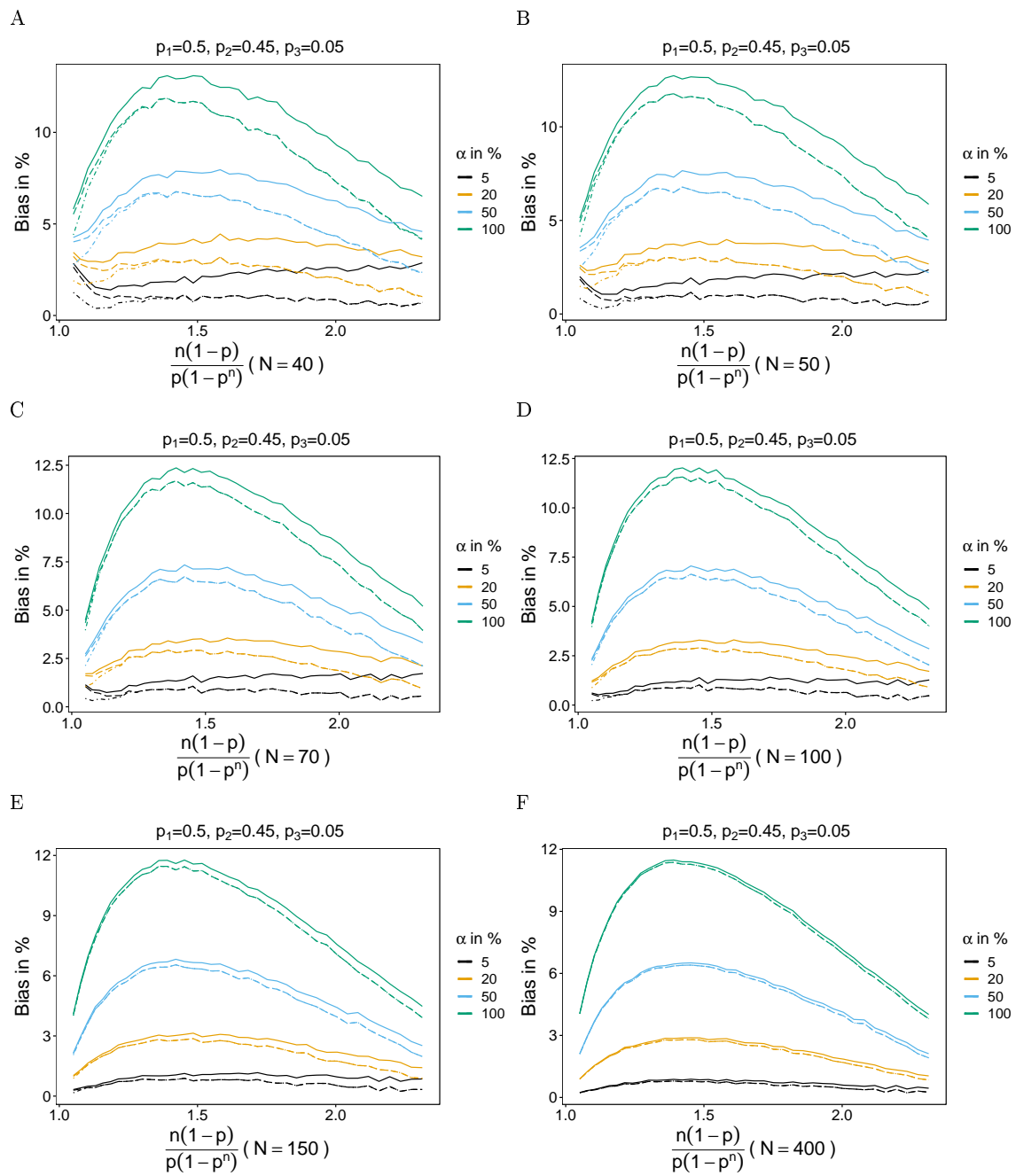


Figure 21: Same as Figure 17.

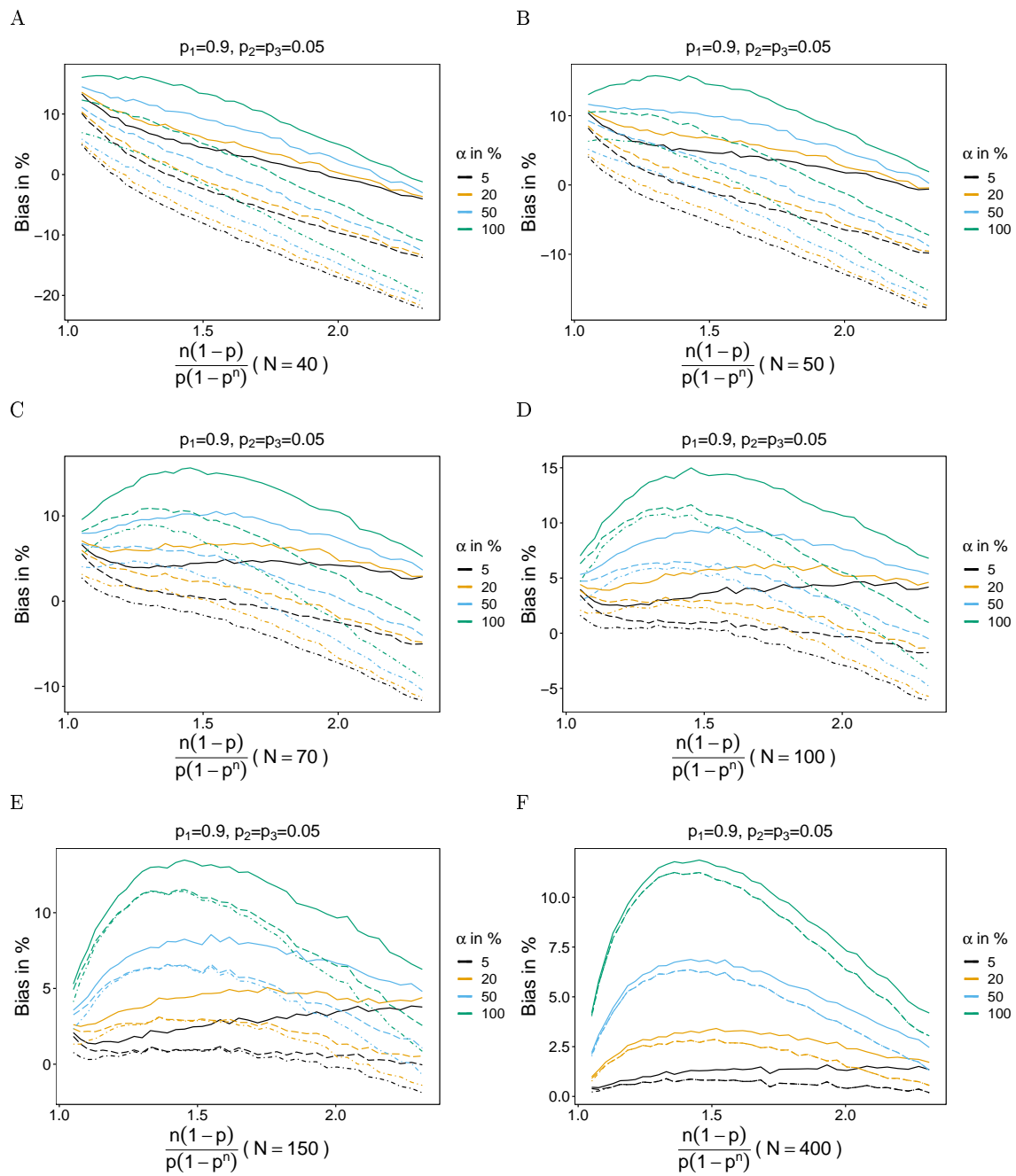


Figure 22: Same as Figure 17.

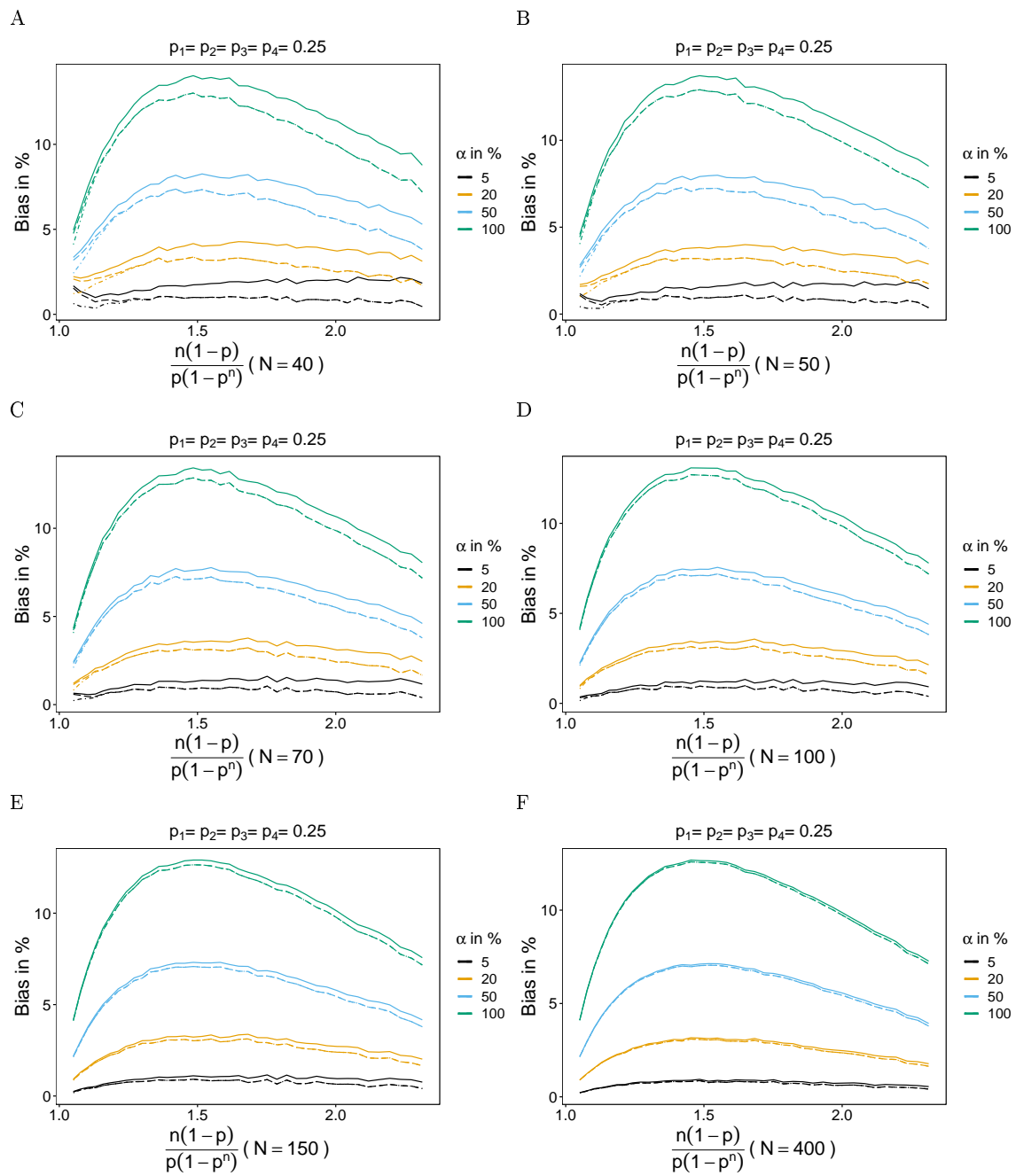


Figure 23: Same as Figure 17.

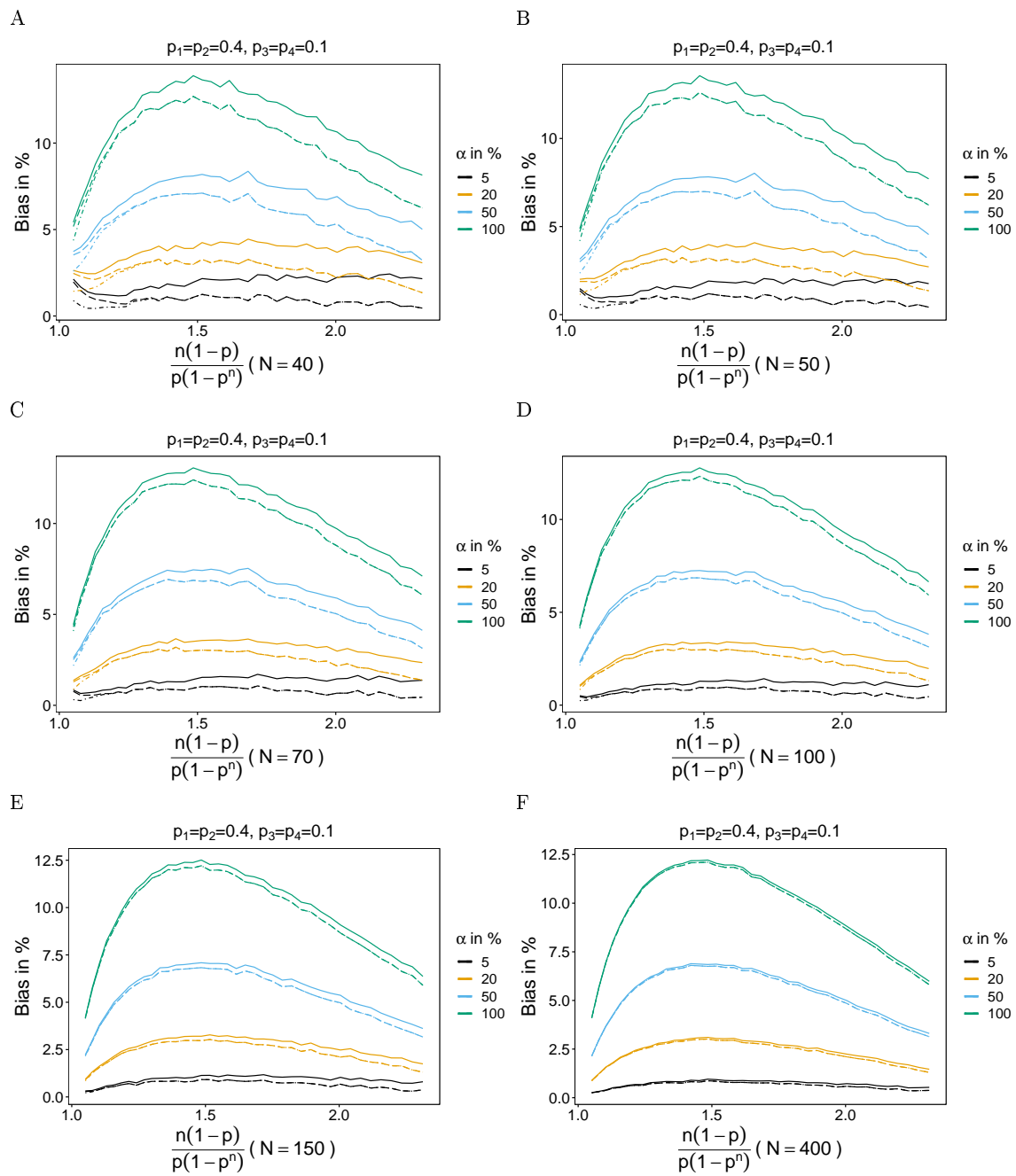


Figure 24: Same as Figure 17.

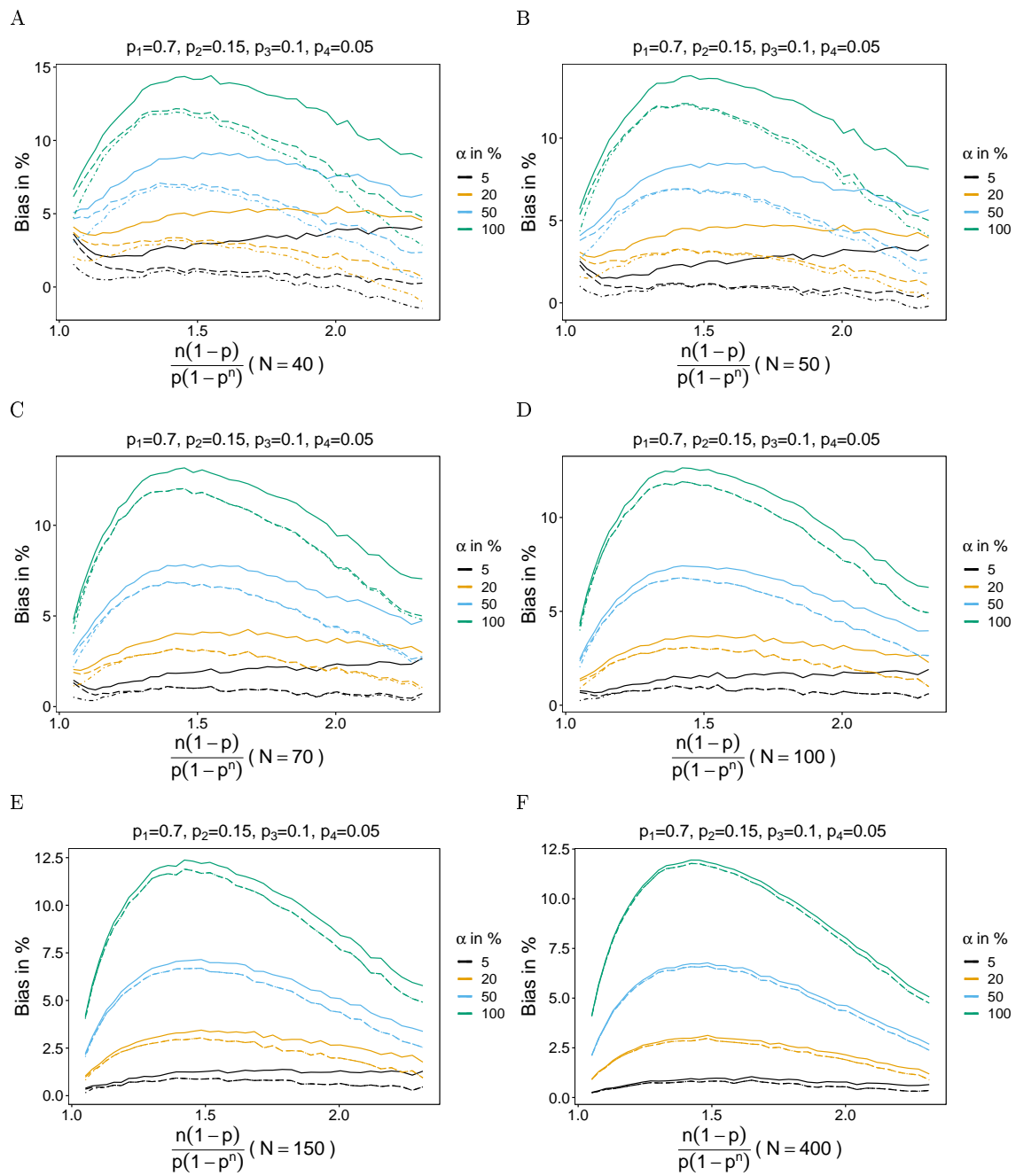


Figure 25: Same as Figure 17.

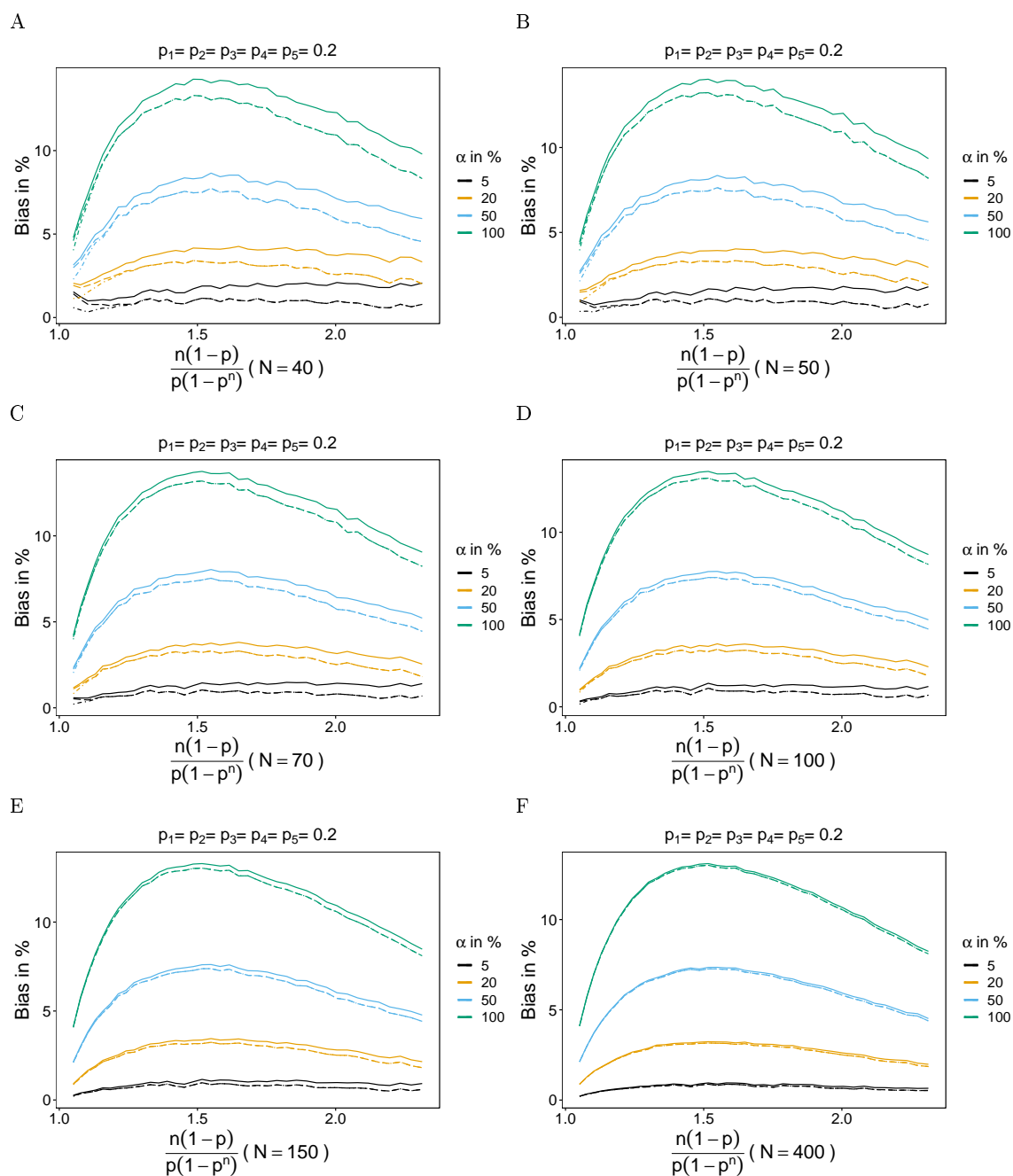


Figure 26: Same as Figure 17.

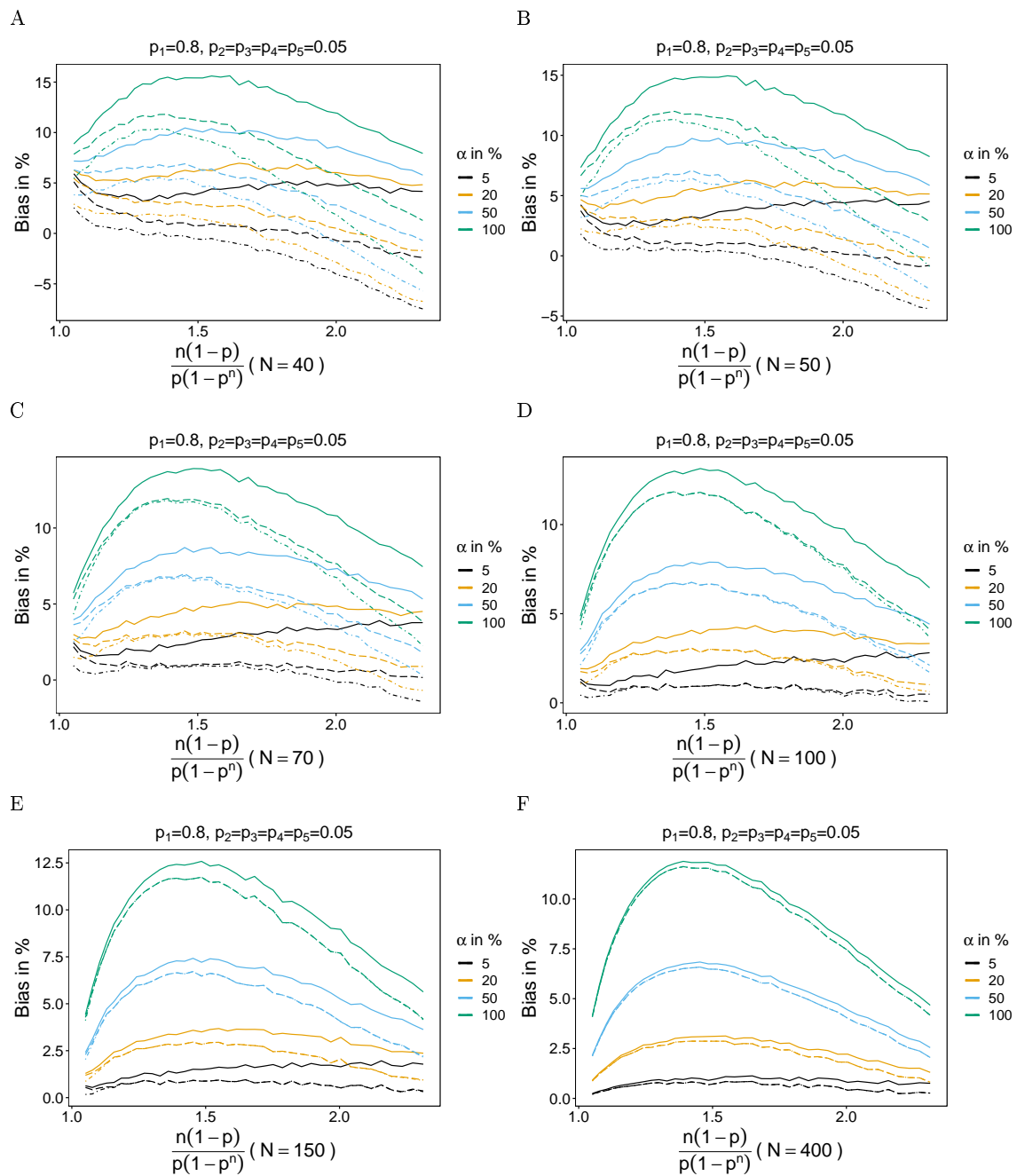


Figure 27: Same as Figure 17.

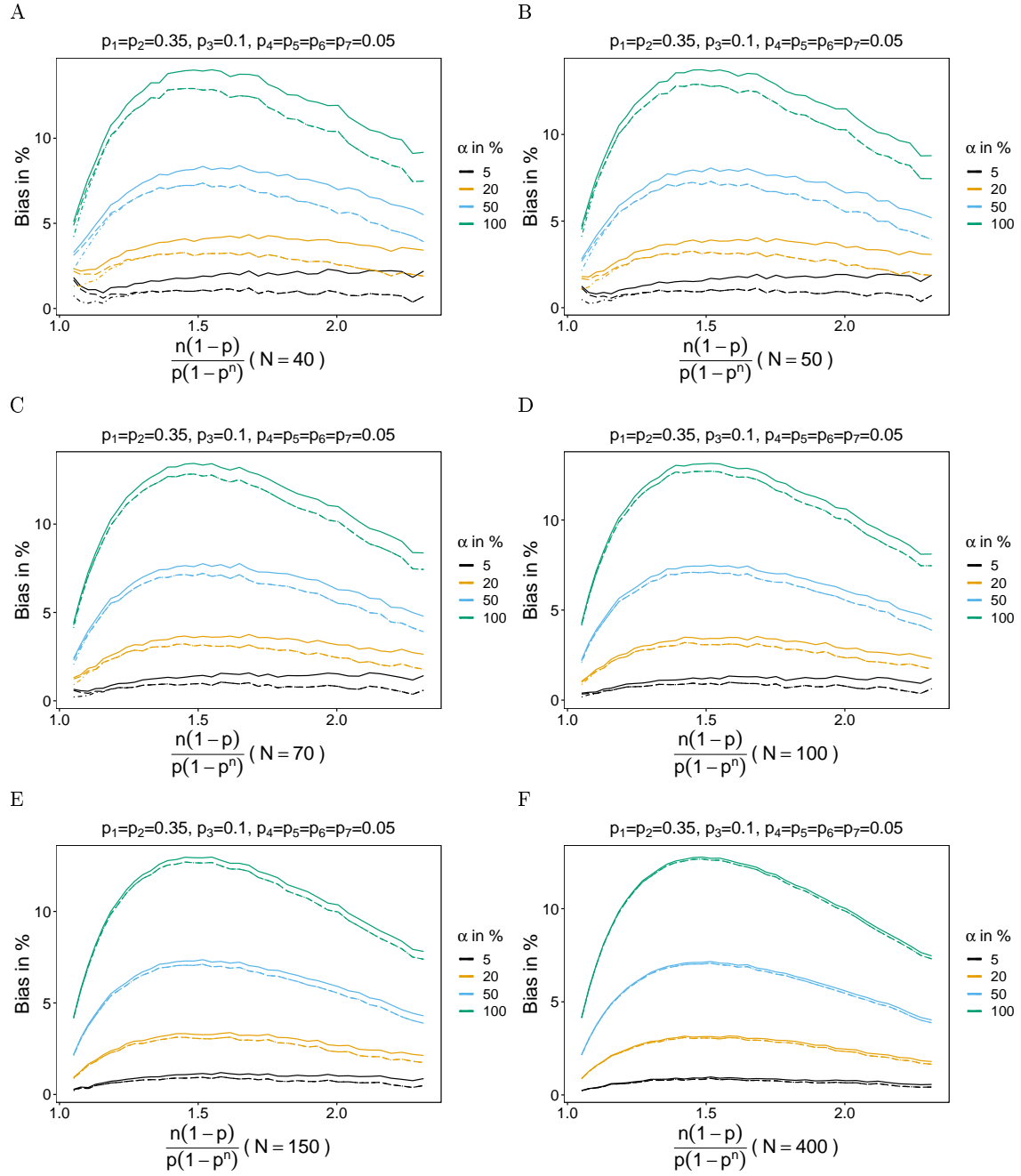


Figure 28: Same as Figure 17.

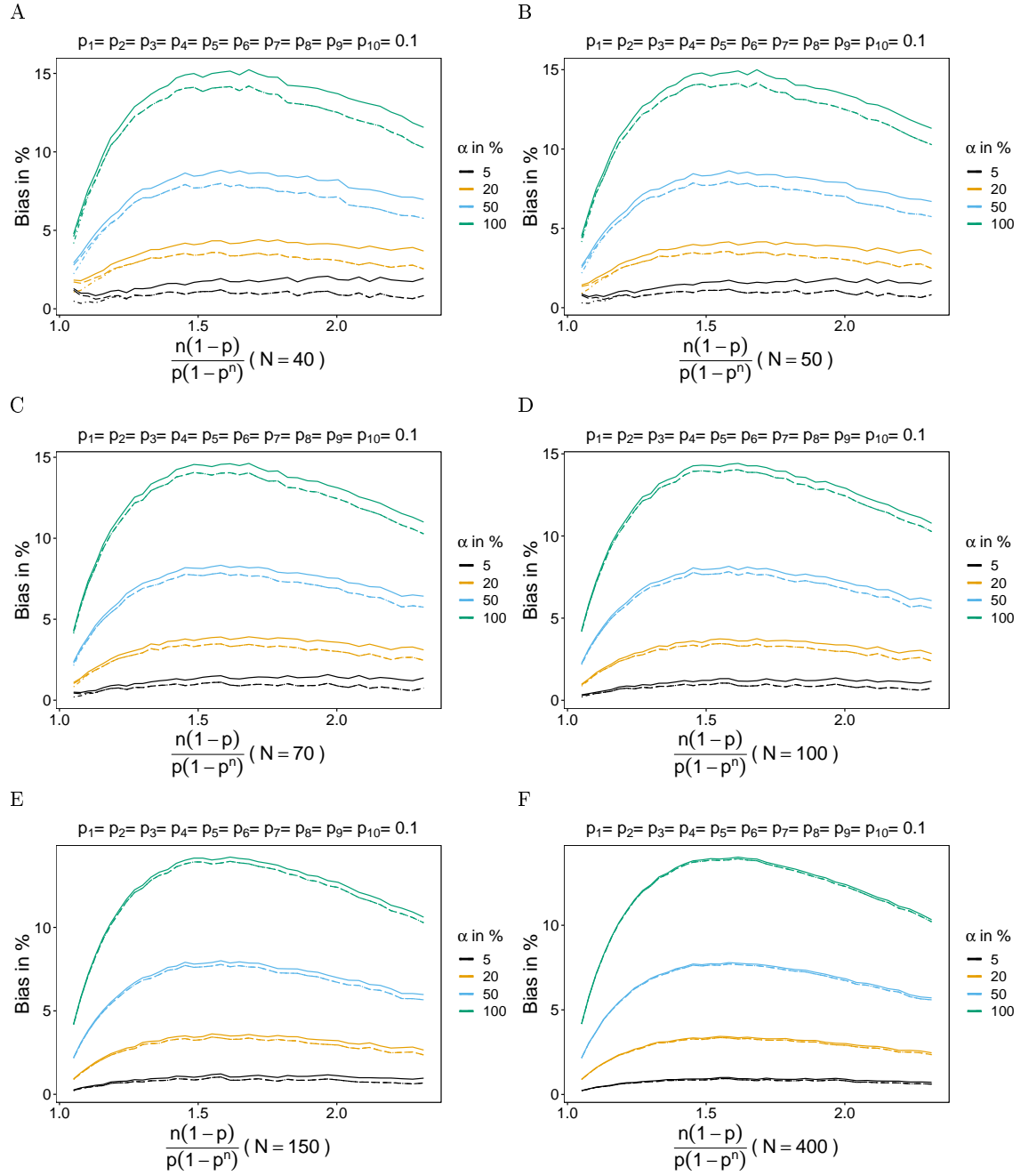


Figure 29: Same as Figure 17.

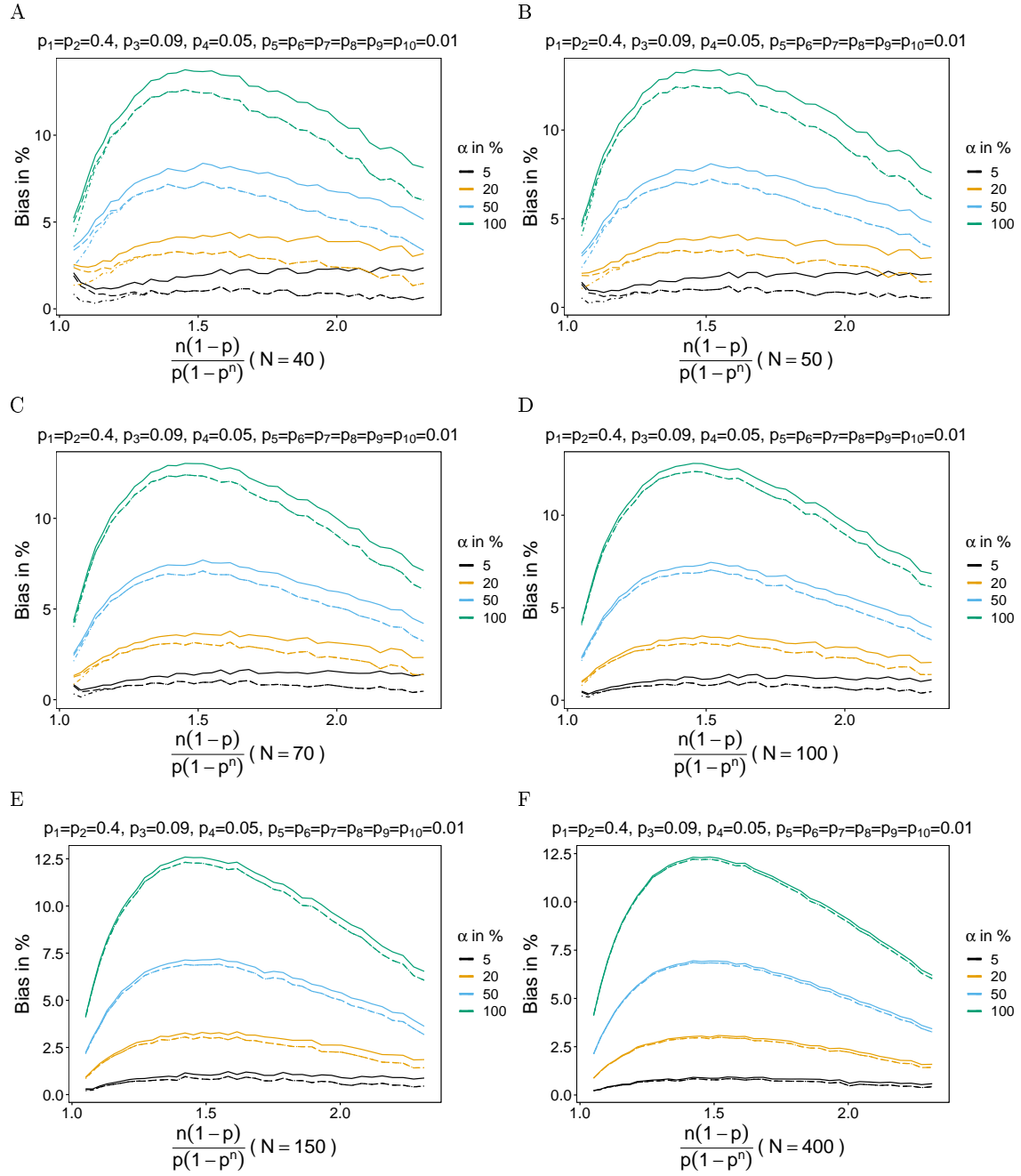


Figure 30: Same as Figure 17.

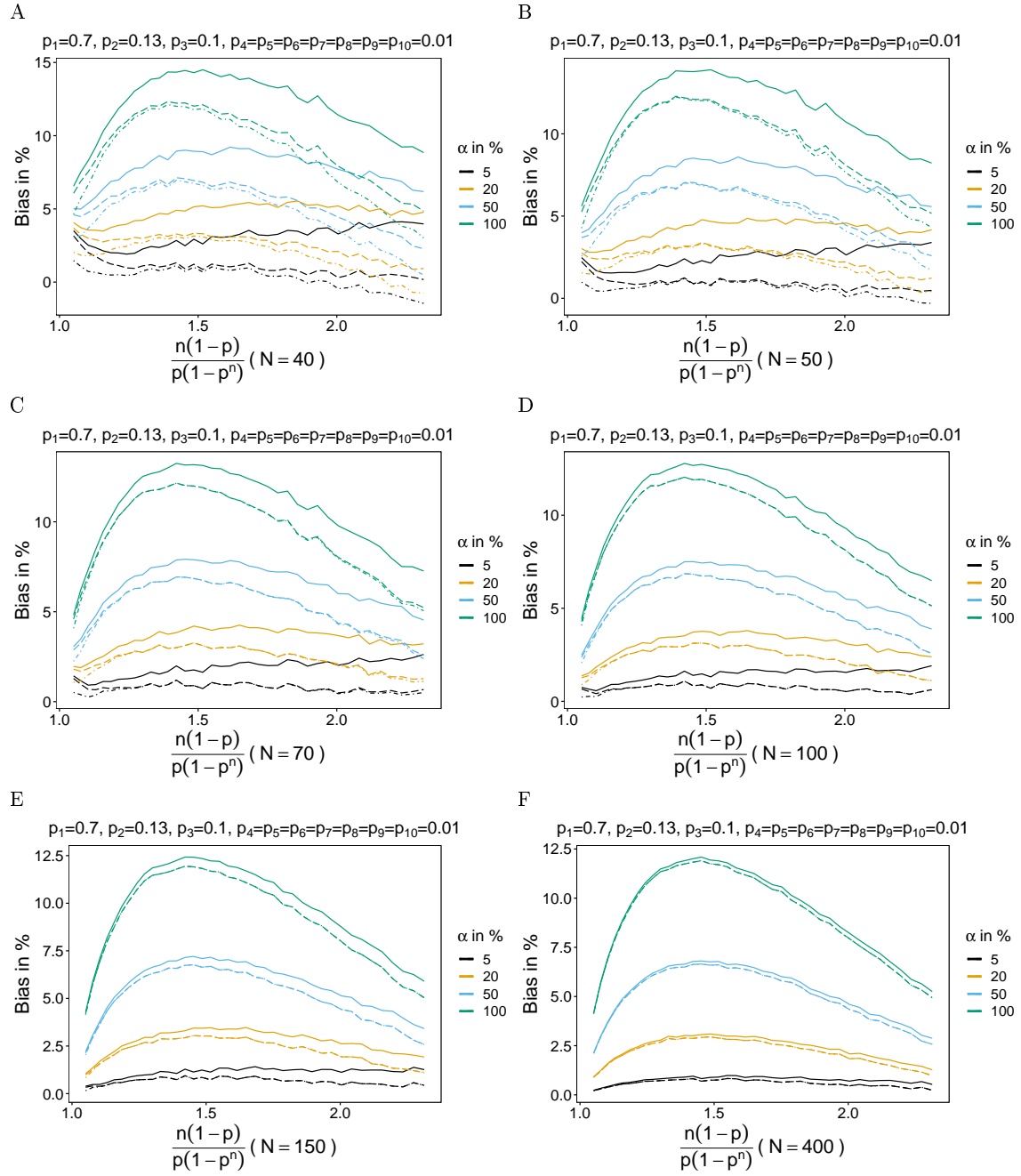


Figure 31: Same as Figure 17.

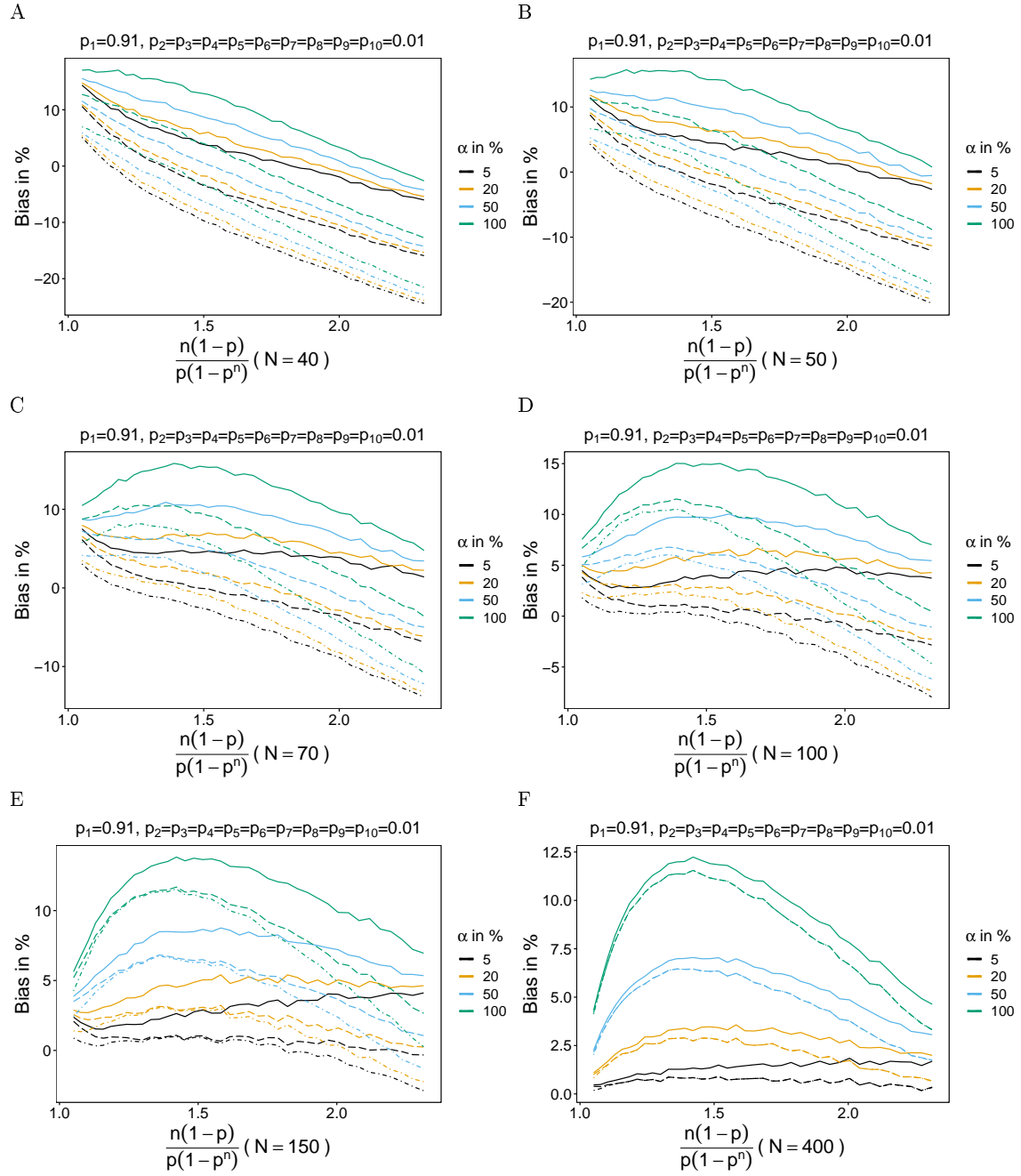


Figure 32: Same as Figure 17.

3 CV

3.1 Different levels of Dispersion α

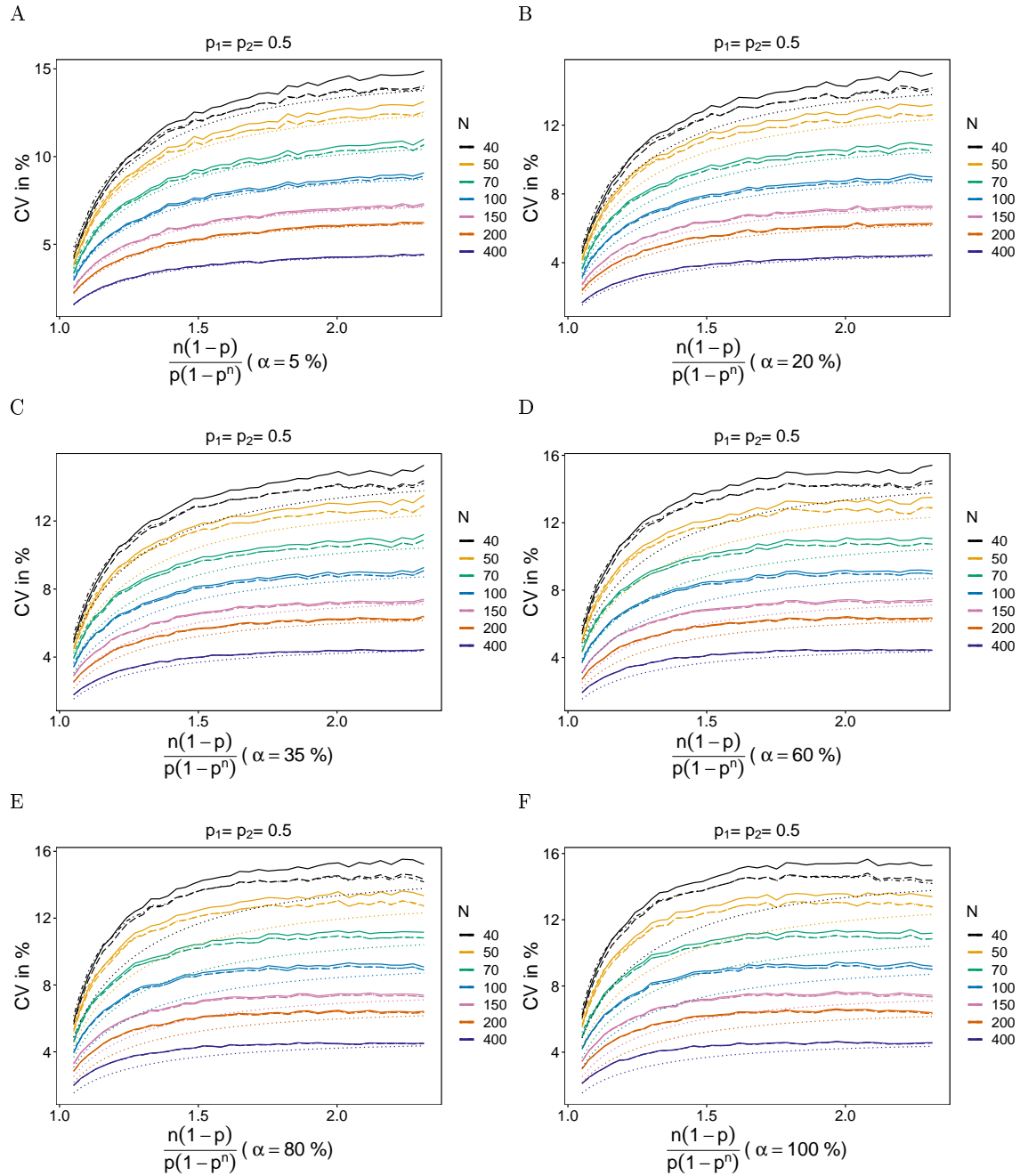


Figure 33: Coefficient of variation in % of bias-corrected MLE under conditional negative binomial model. Same as Figure 1 but for CV. The dotted lines are the Cramér Rao lower bounds.

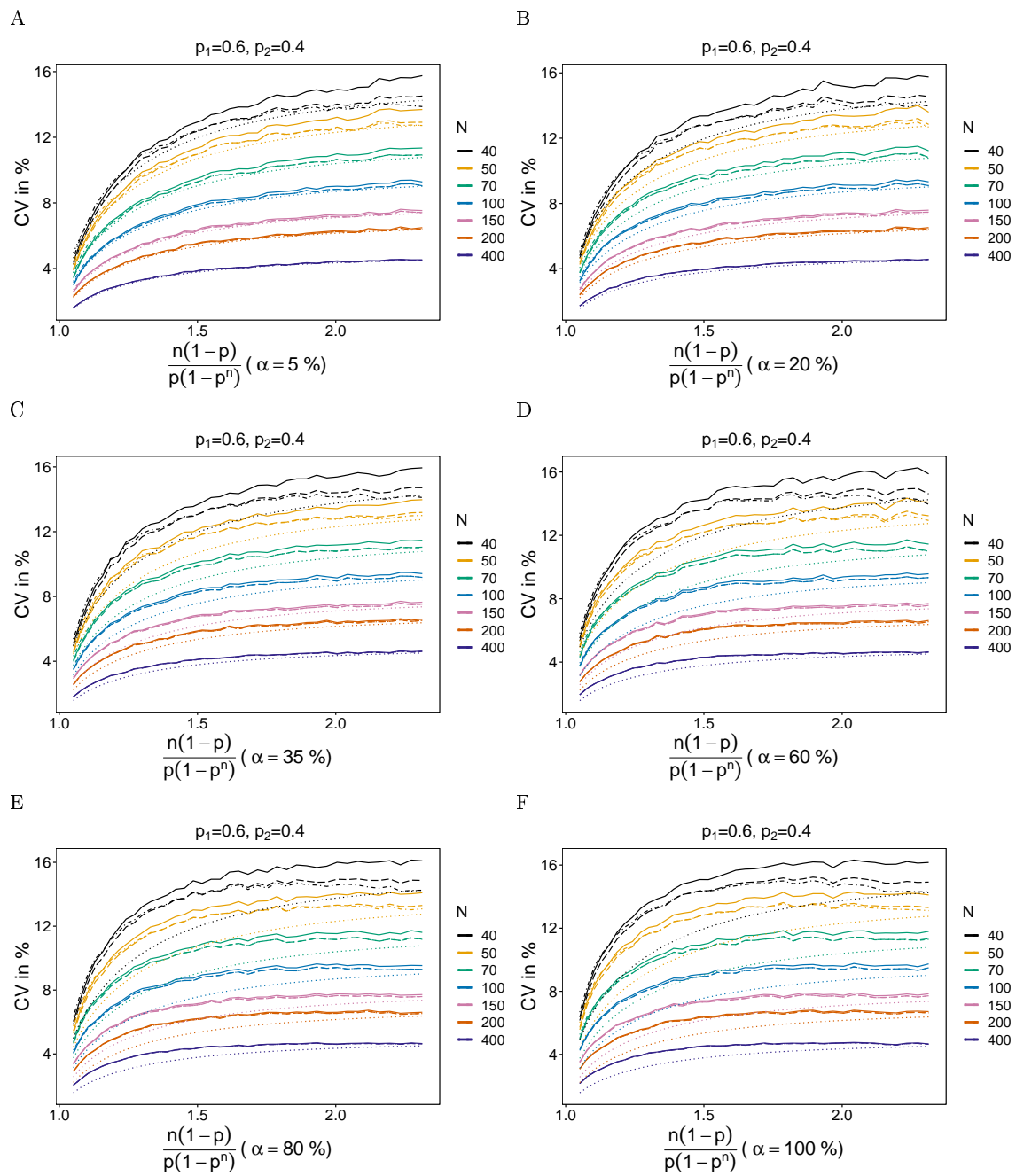


Figure 34: Same as Figure 33.

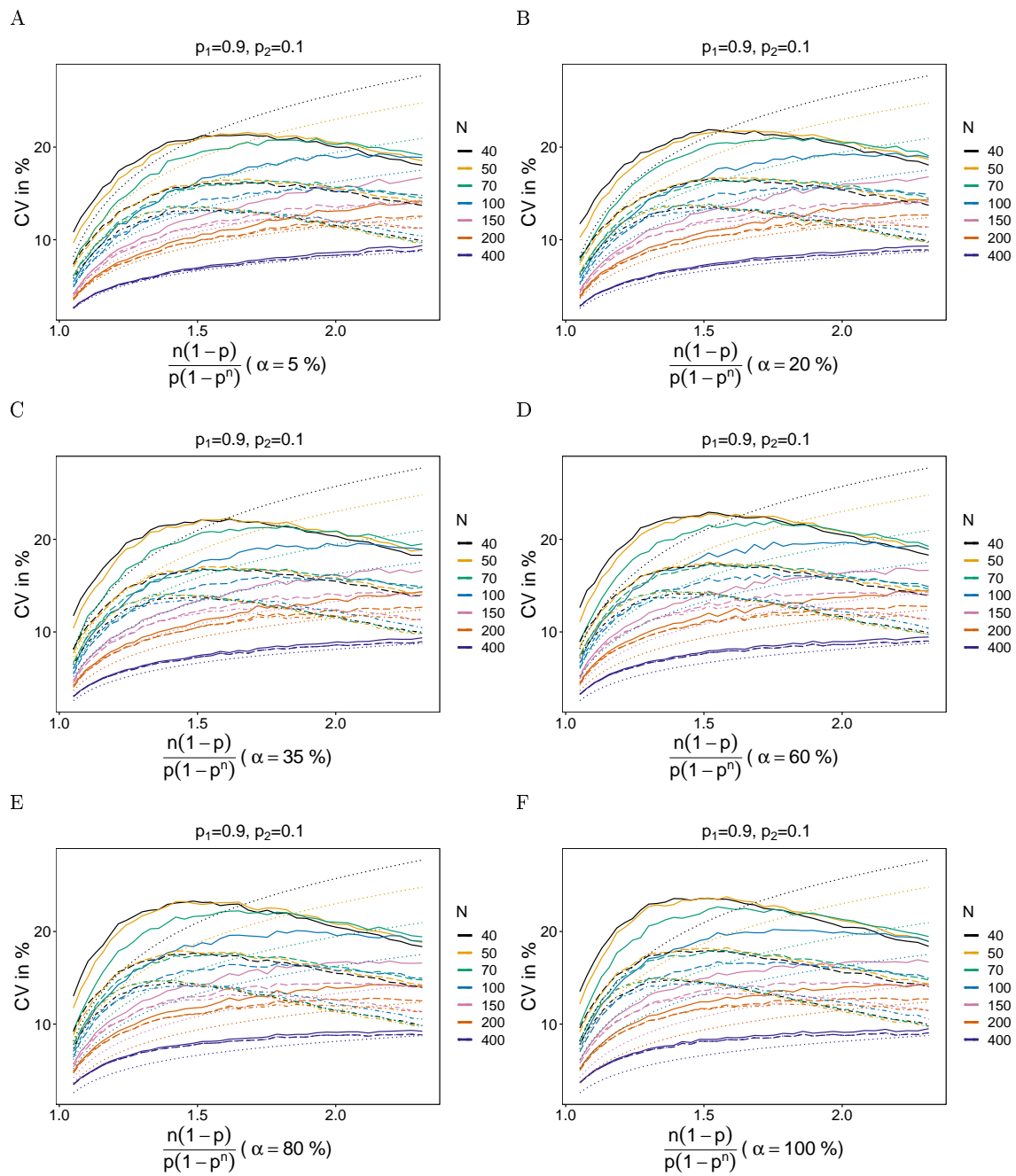


Figure 35: Same as Figure 33.

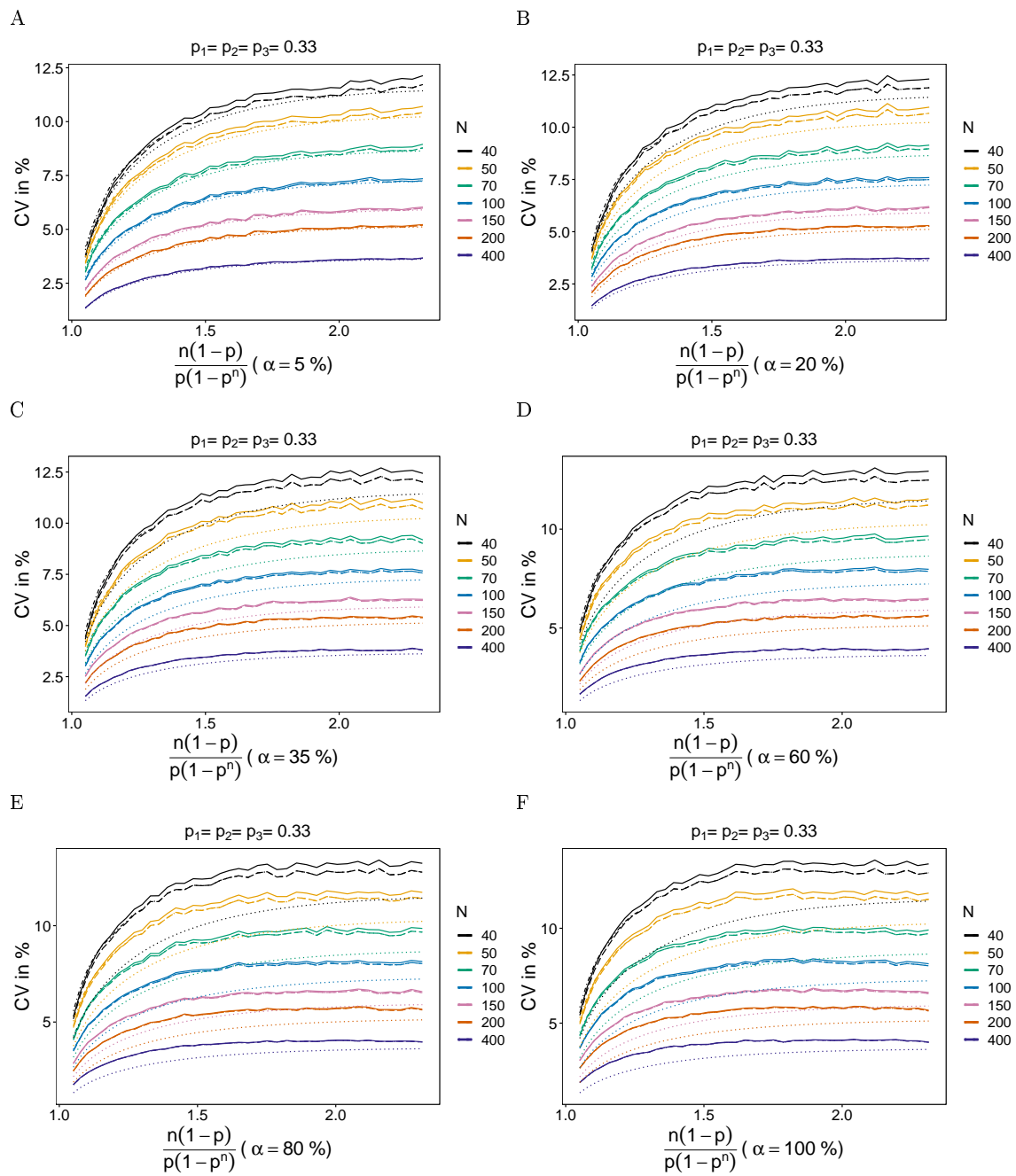


Figure 36: Same as Figure 33.

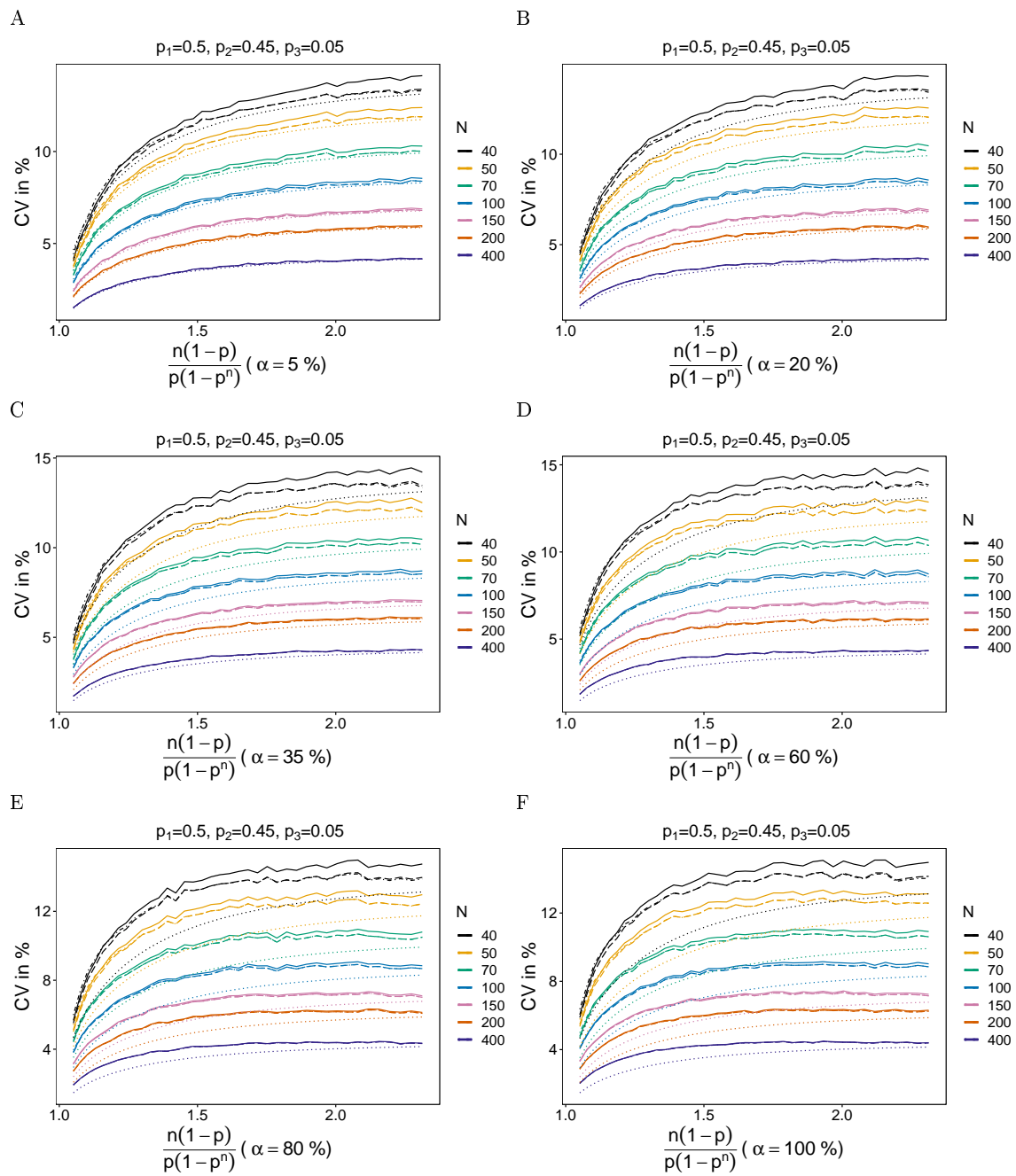


Figure 37: Same as Figure 33.

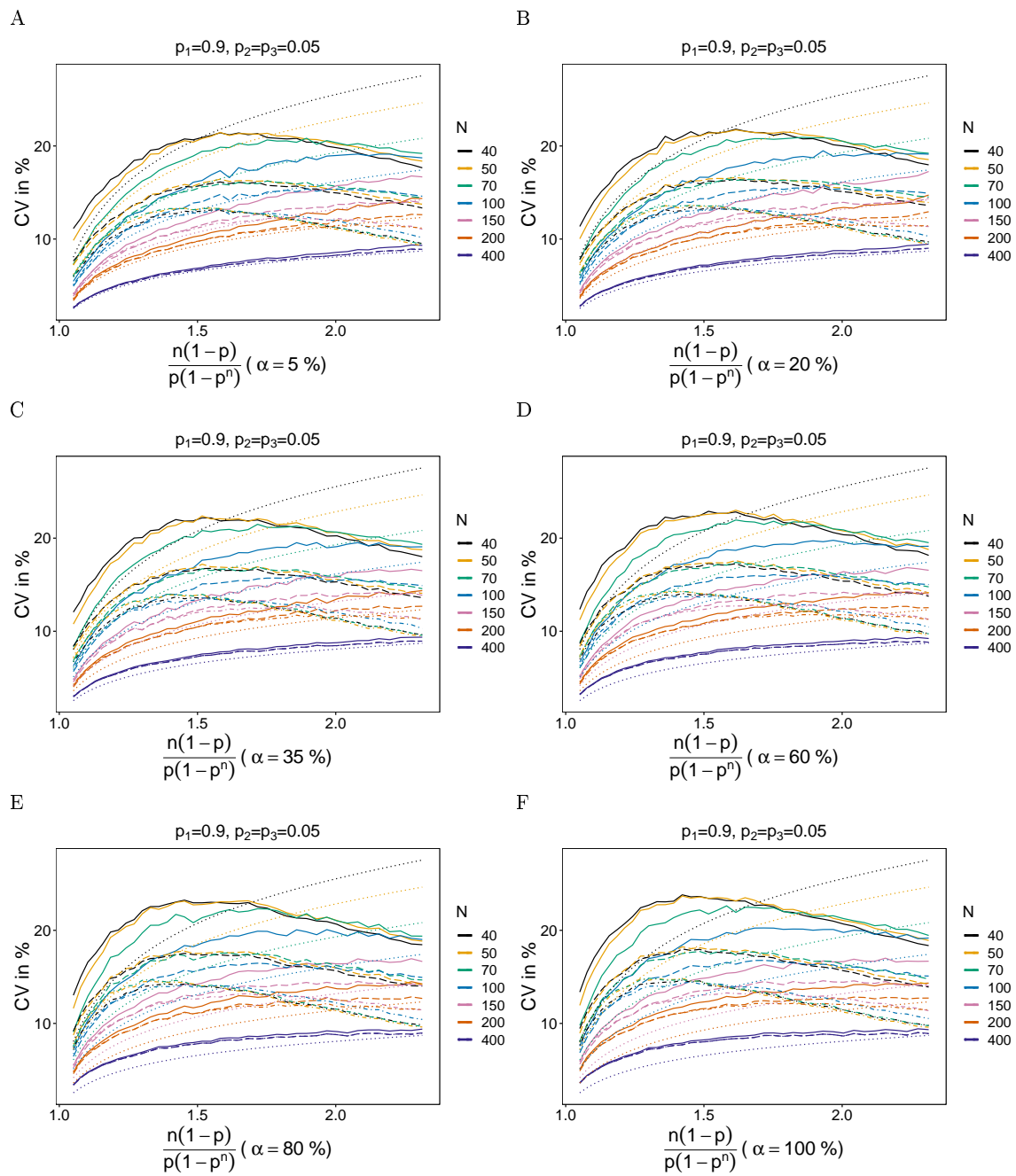


Figure 38: Same as Figure 33.

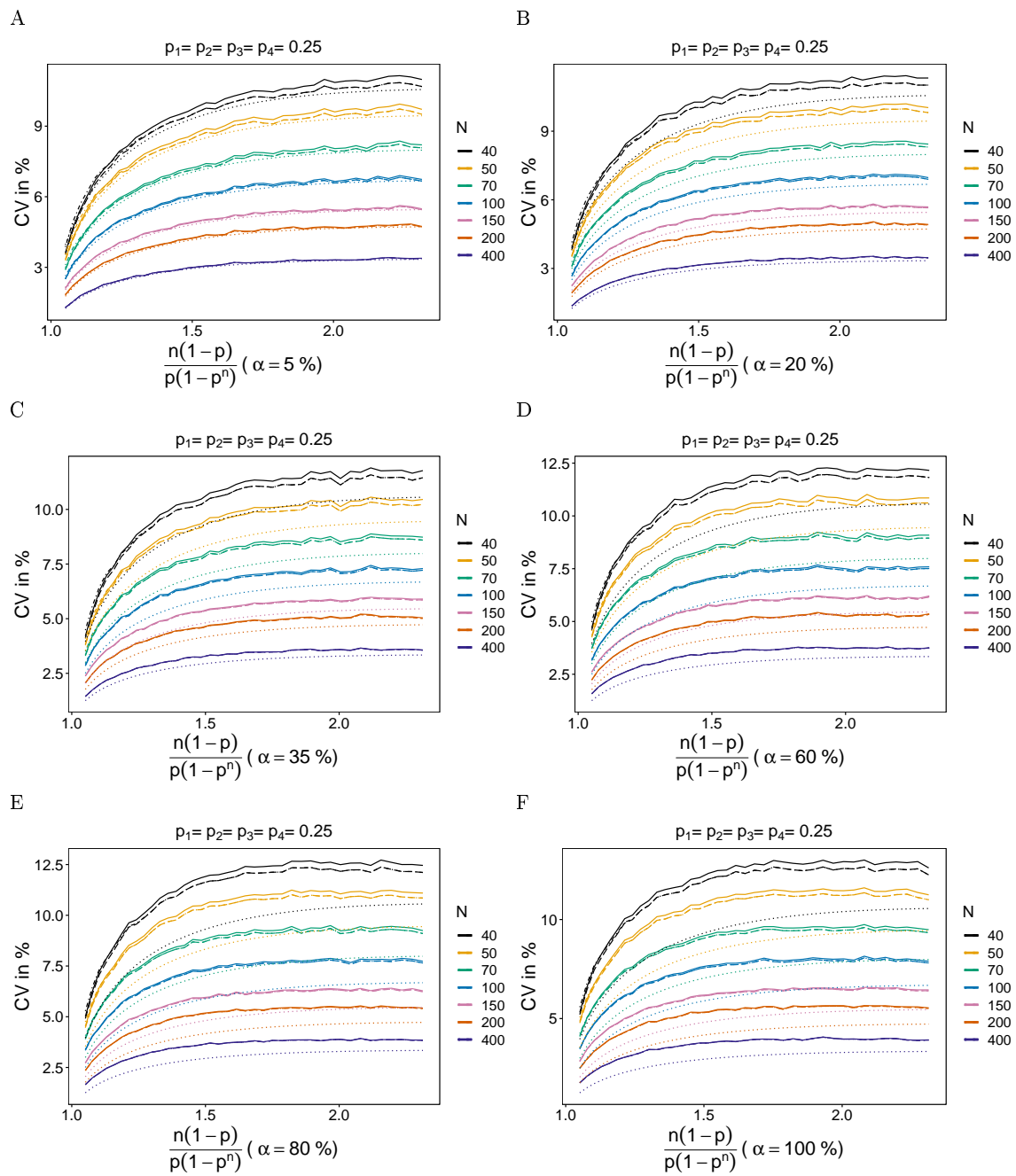


Figure 39: Same as Figure 33.

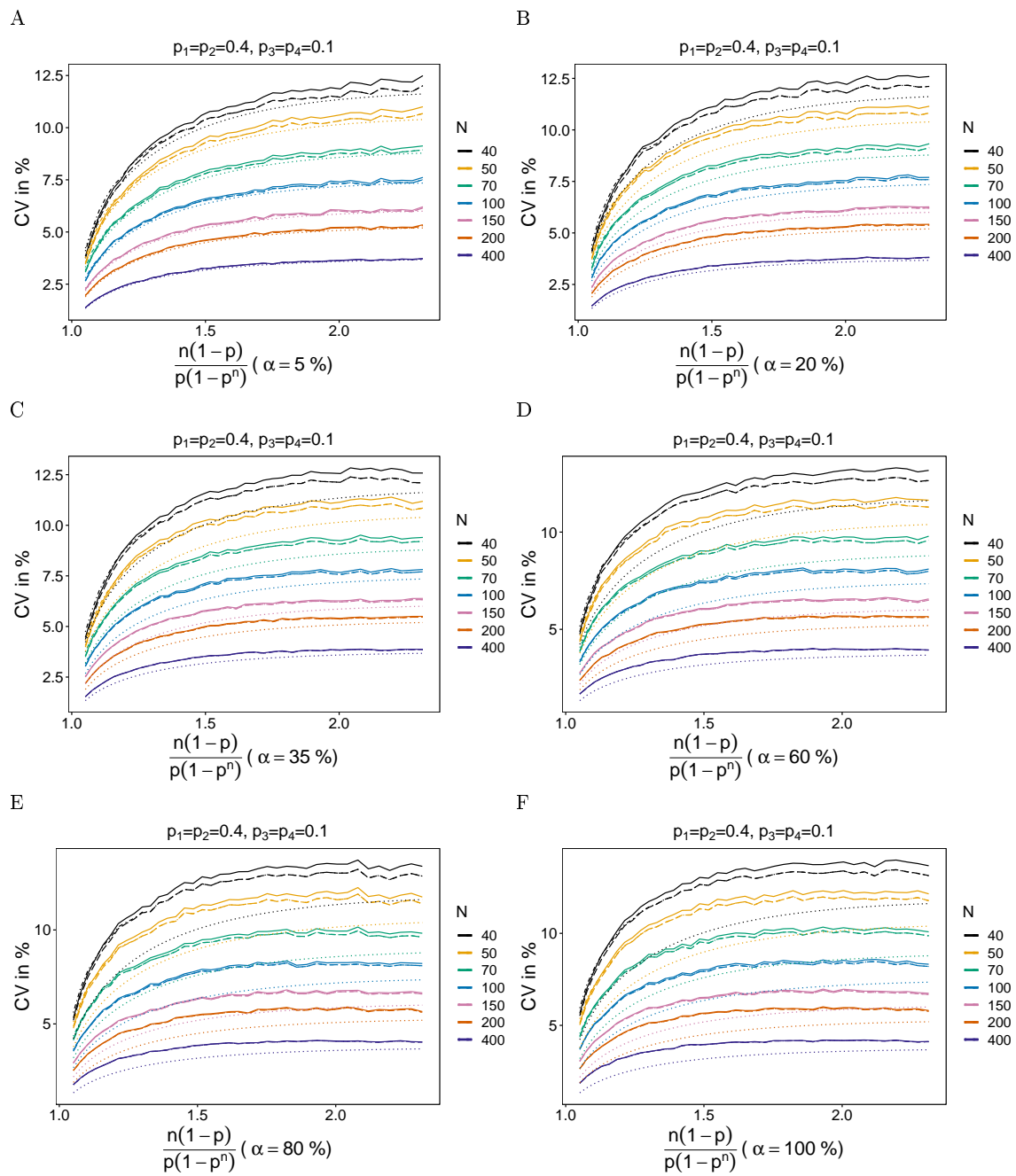


Figure 40: Same as Figure 33.

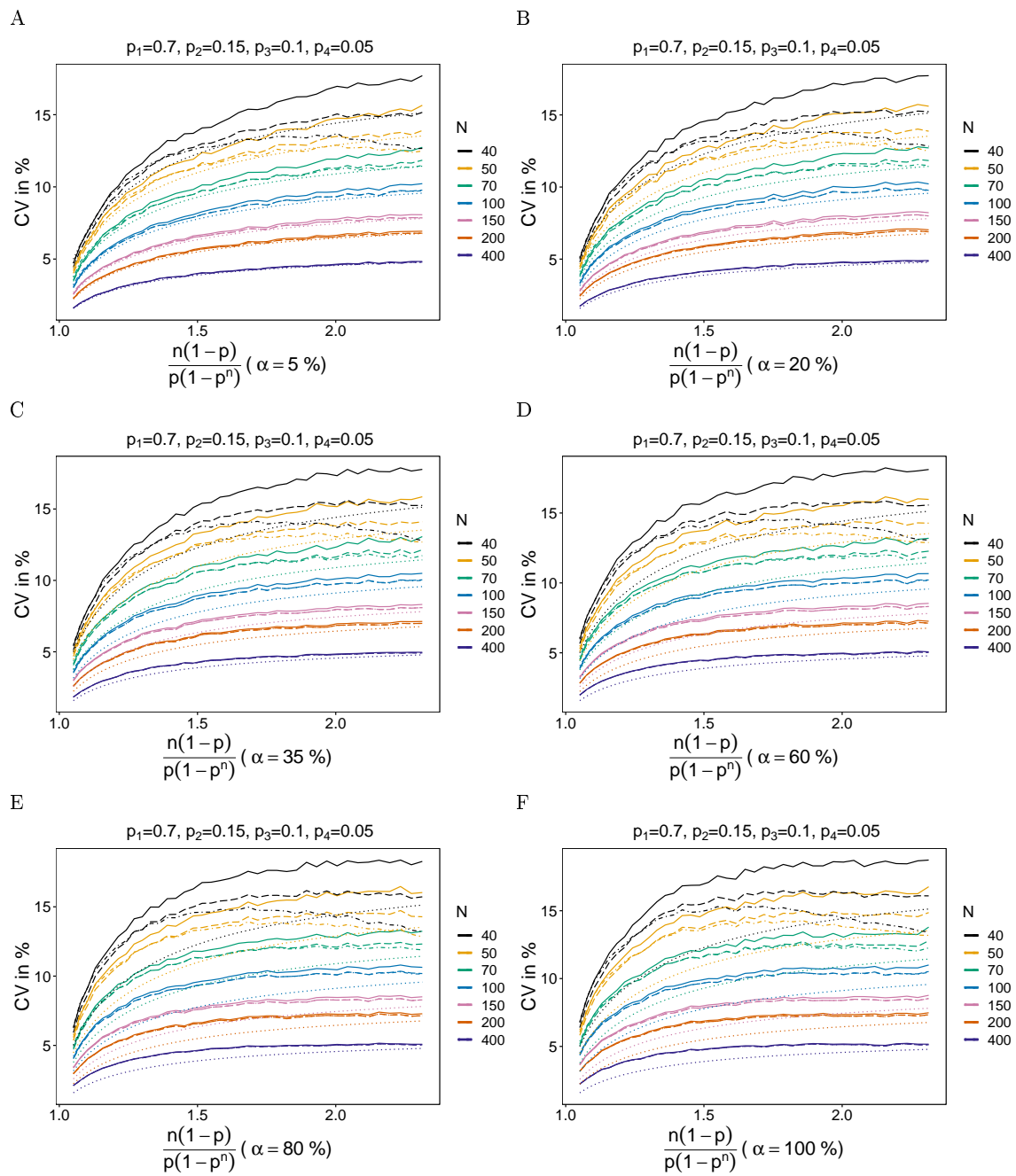


Figure 41: Same as Figure 33.

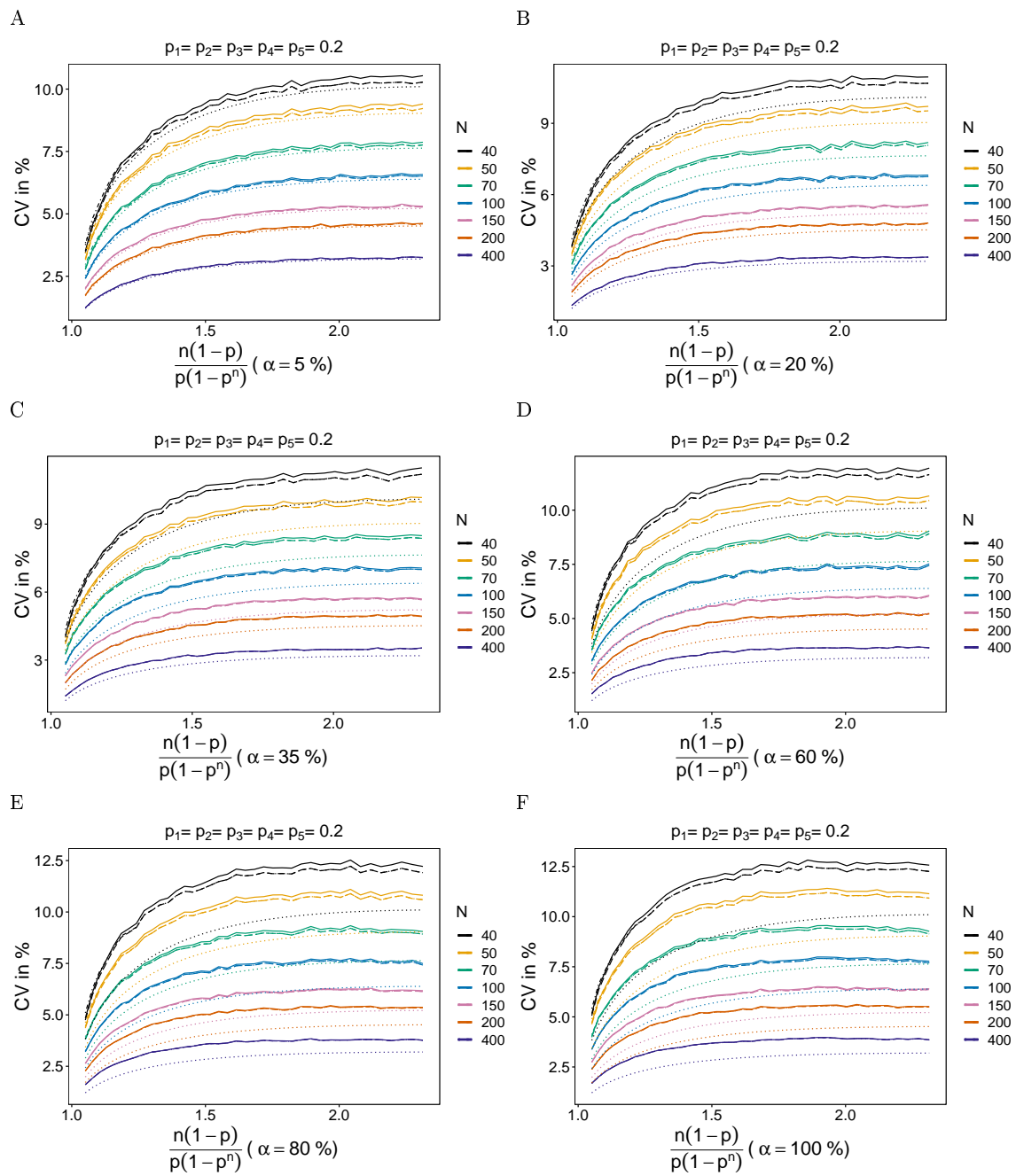


Figure 42: Same as Figure 33.

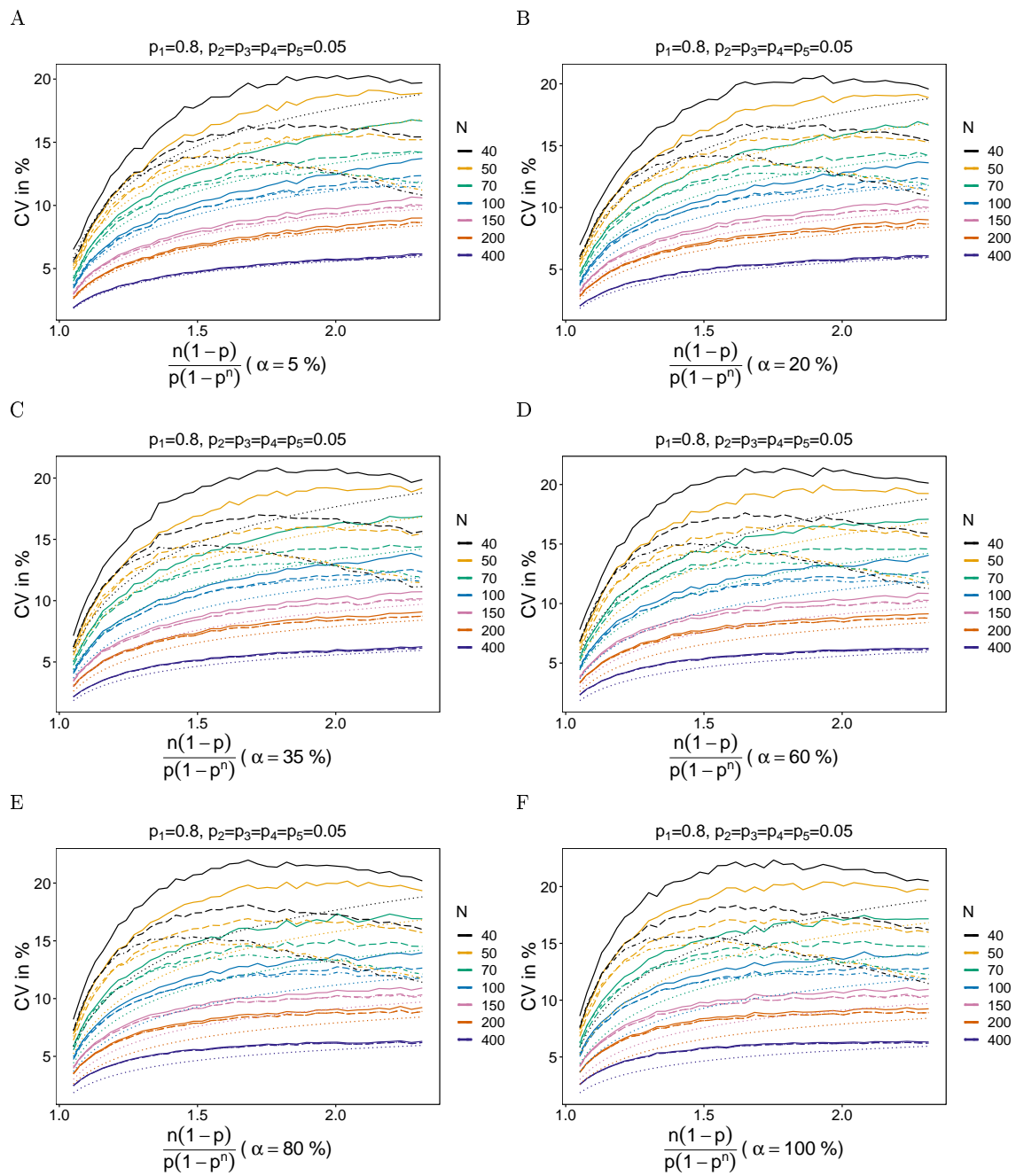


Figure 43: Same as Figure 33.

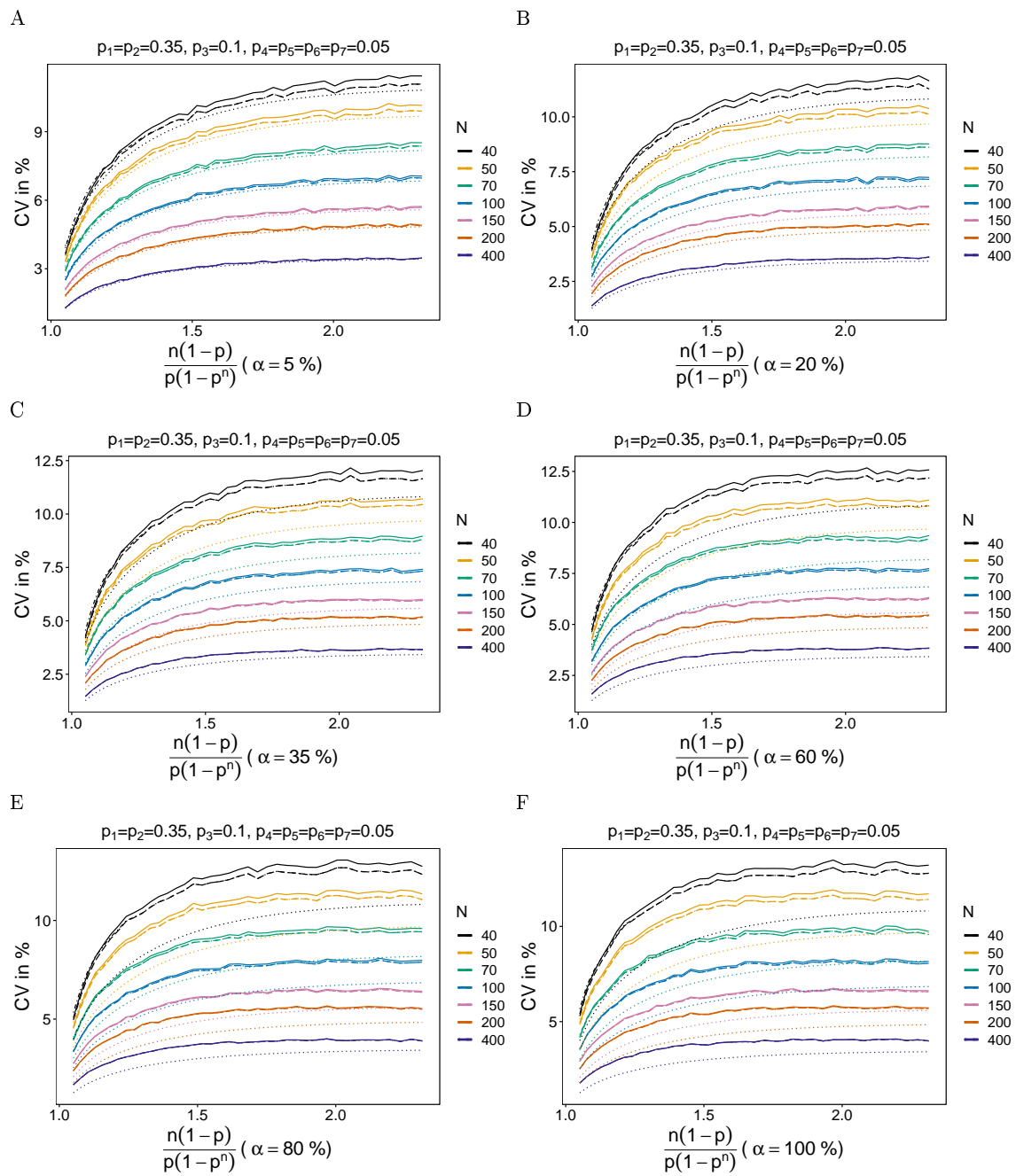


Figure 44: Same as Figure 33.

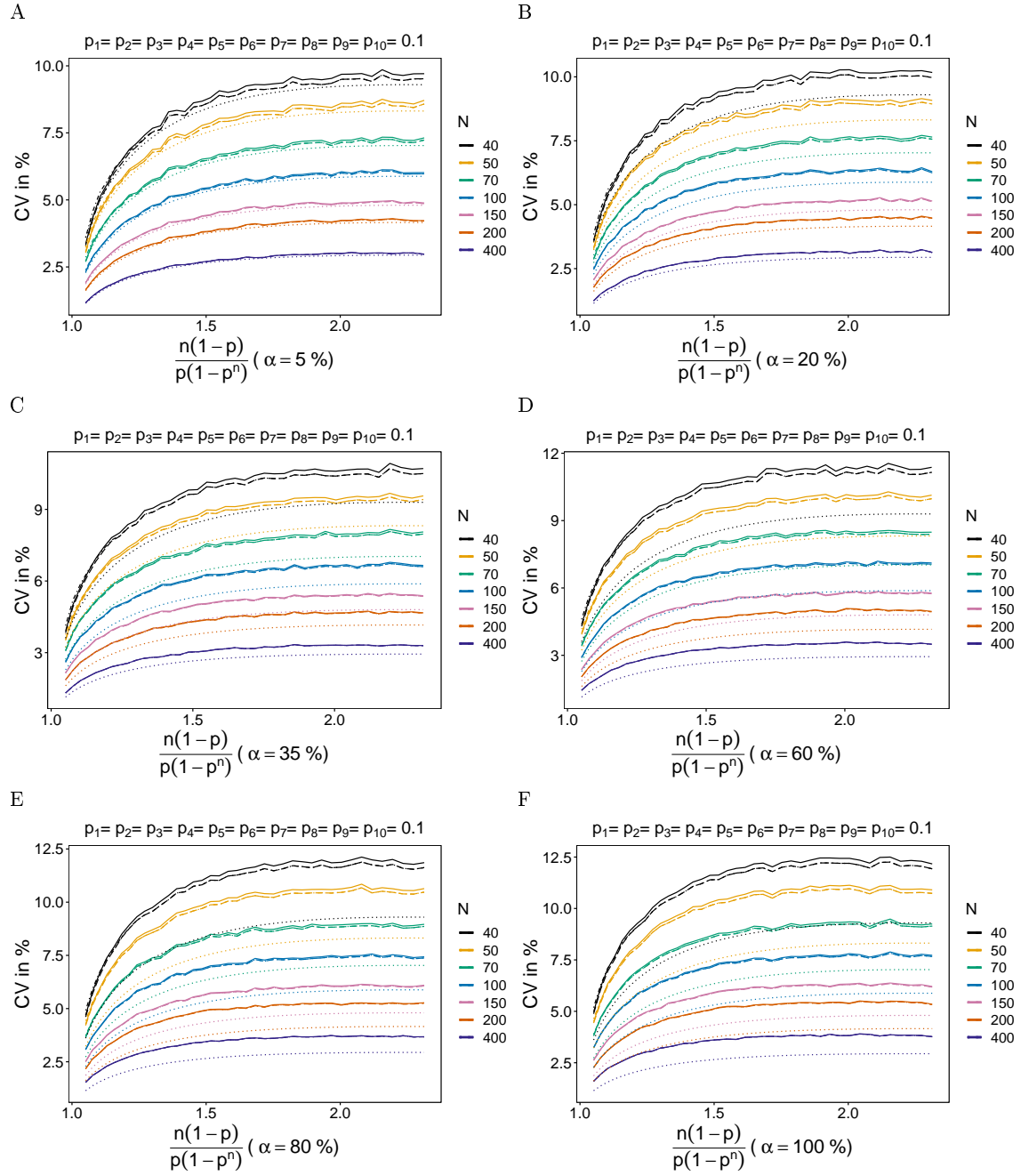


Figure 45: Same as Figure 33.

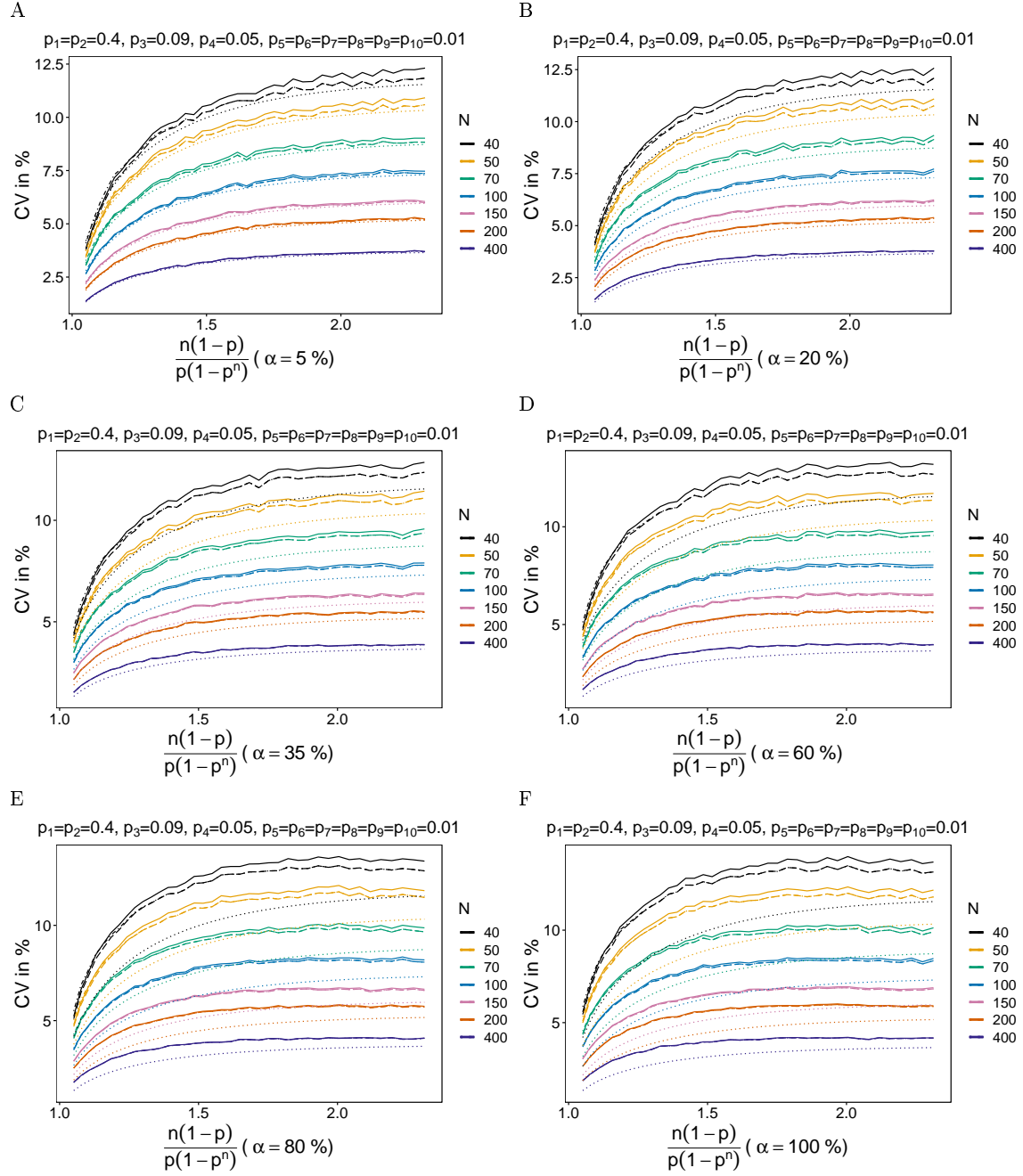


Figure 46: Same as Figure 33.

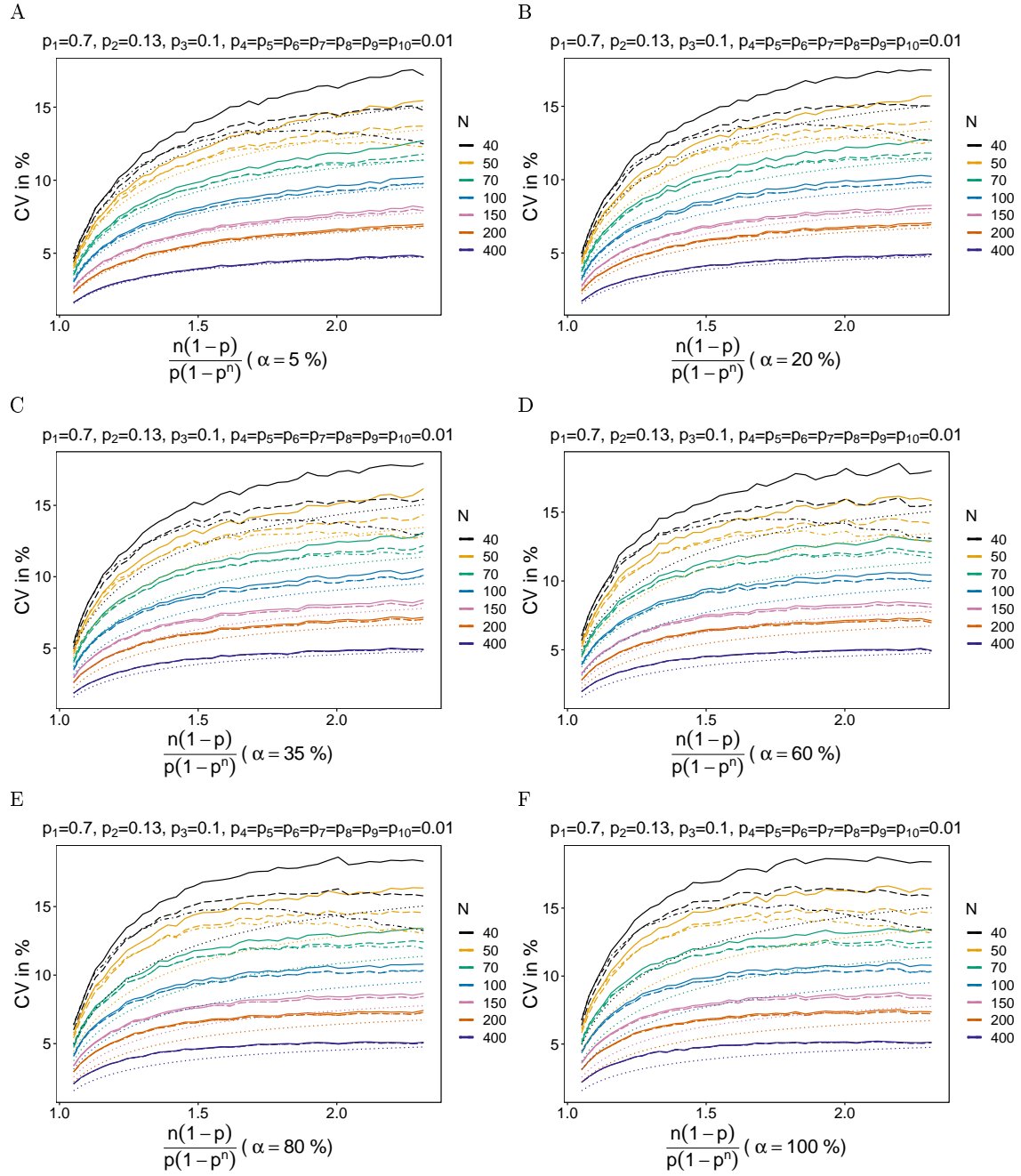


Figure 47: Same as Figure 33.

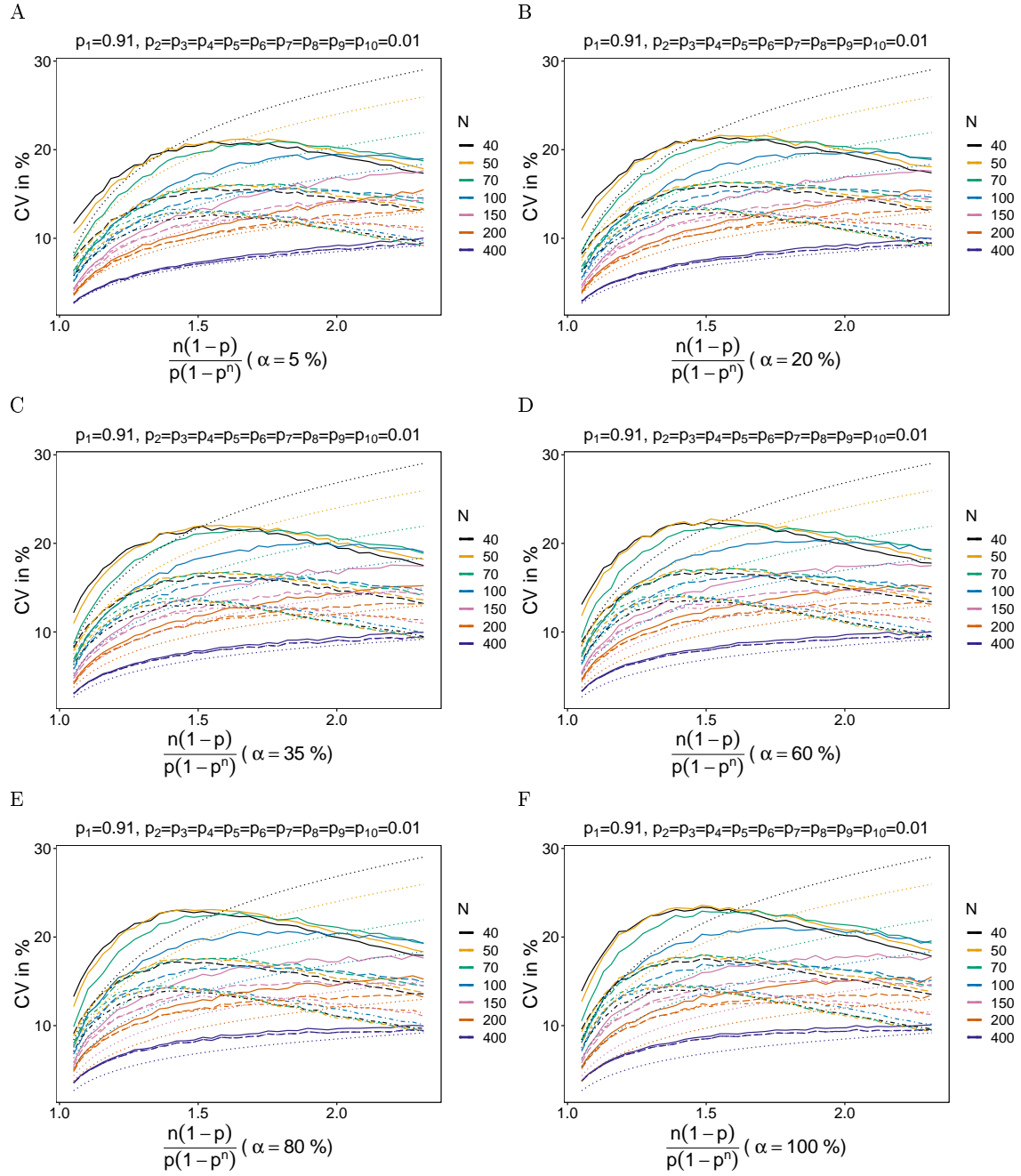


Figure 48: Same as Figure 33.

3.2 Different Sample Sizes N

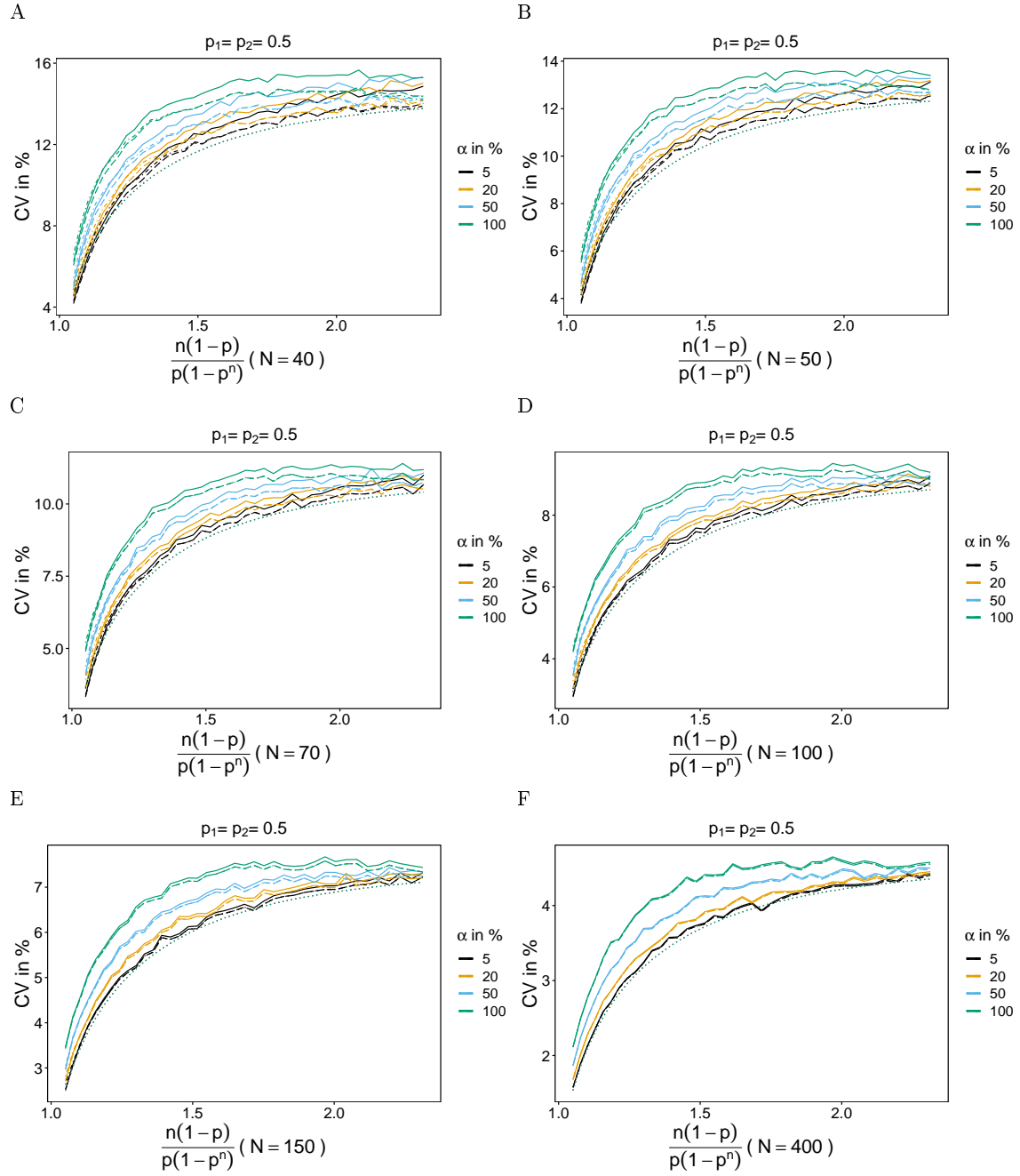


Figure 49: Same as Figure 17 but for CV. The dotted line is the Cramér Rao lower bound.

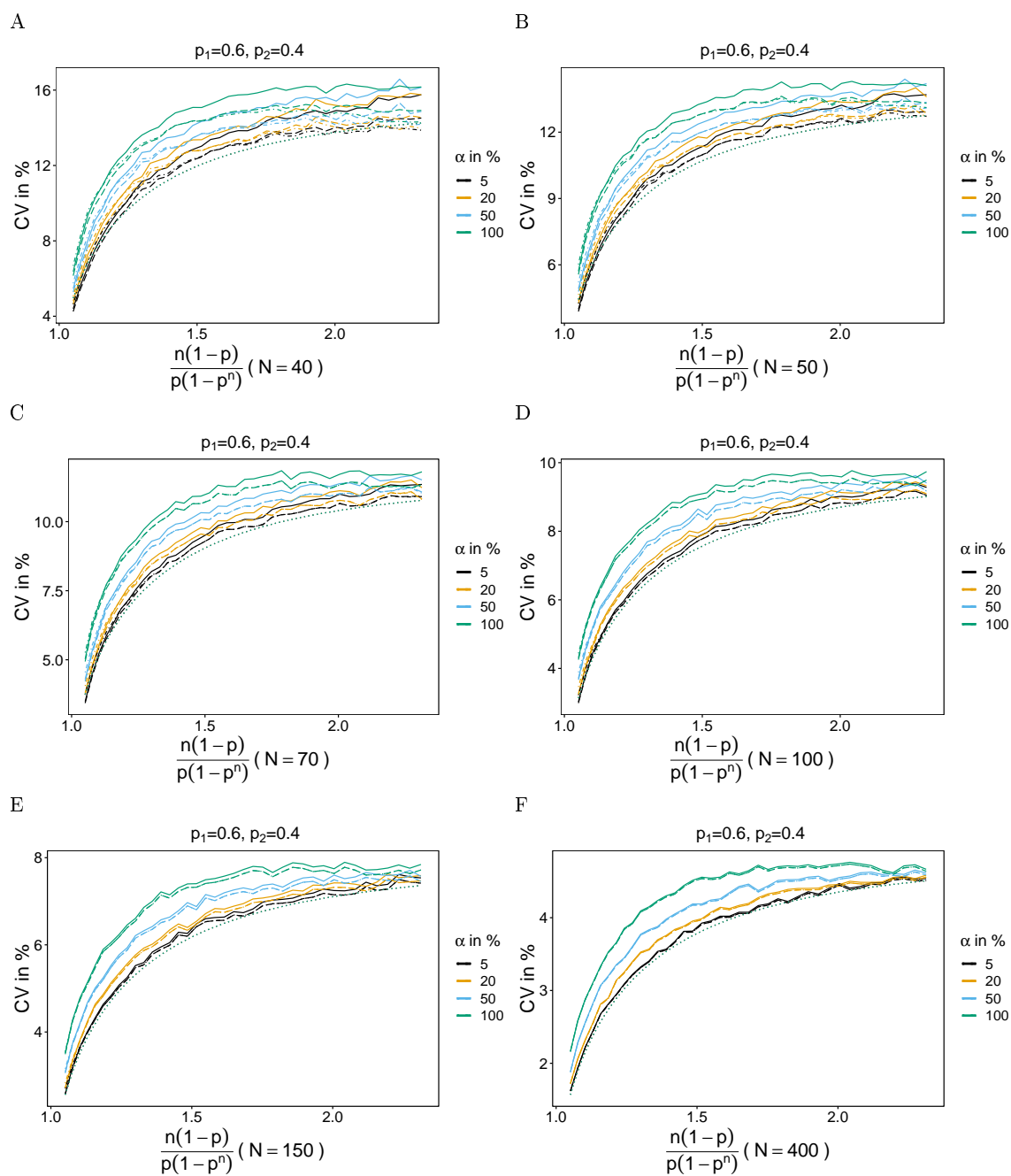


Figure 50: Same as Figure 49.

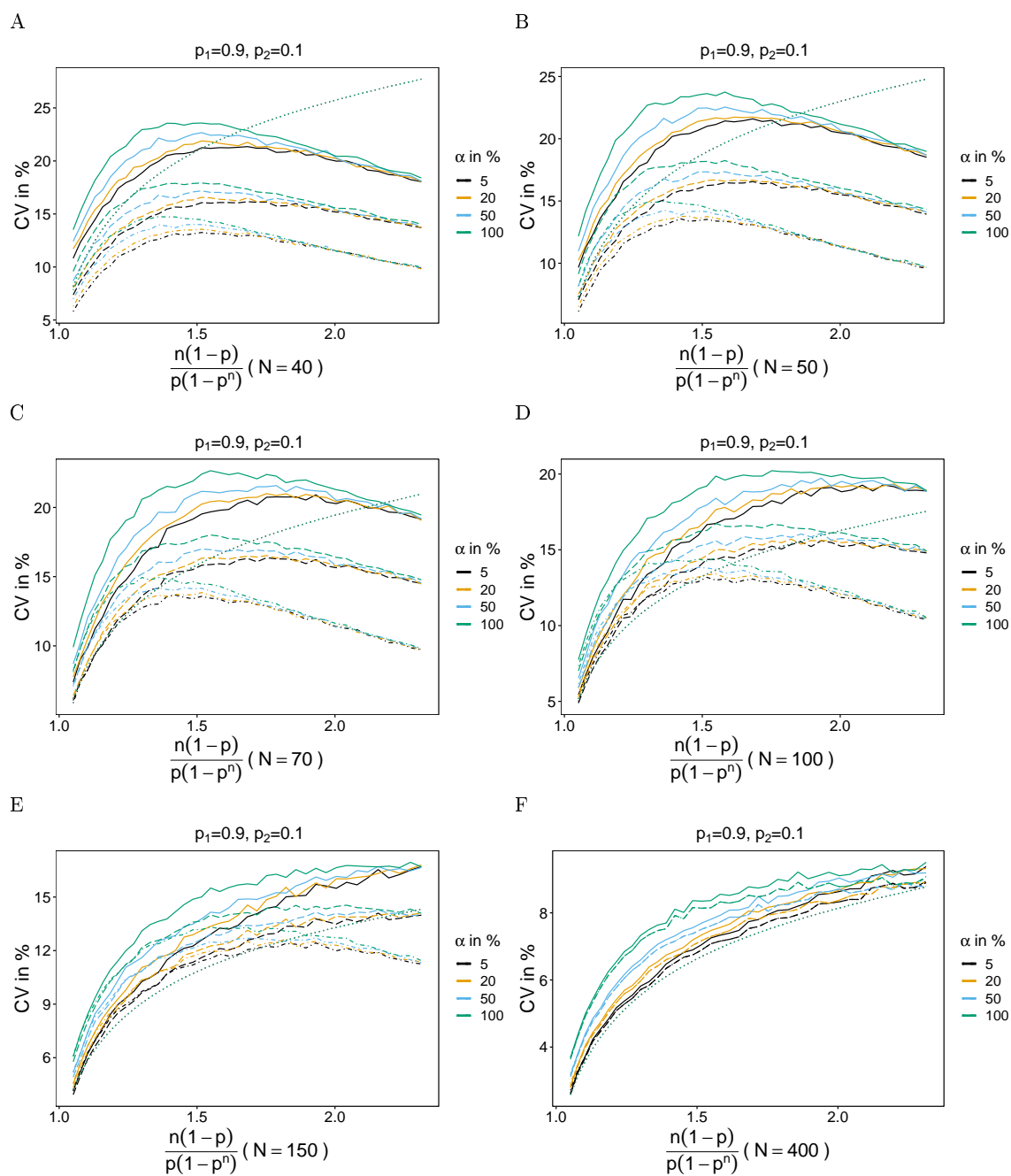


Figure 51: Same as Figure 49.

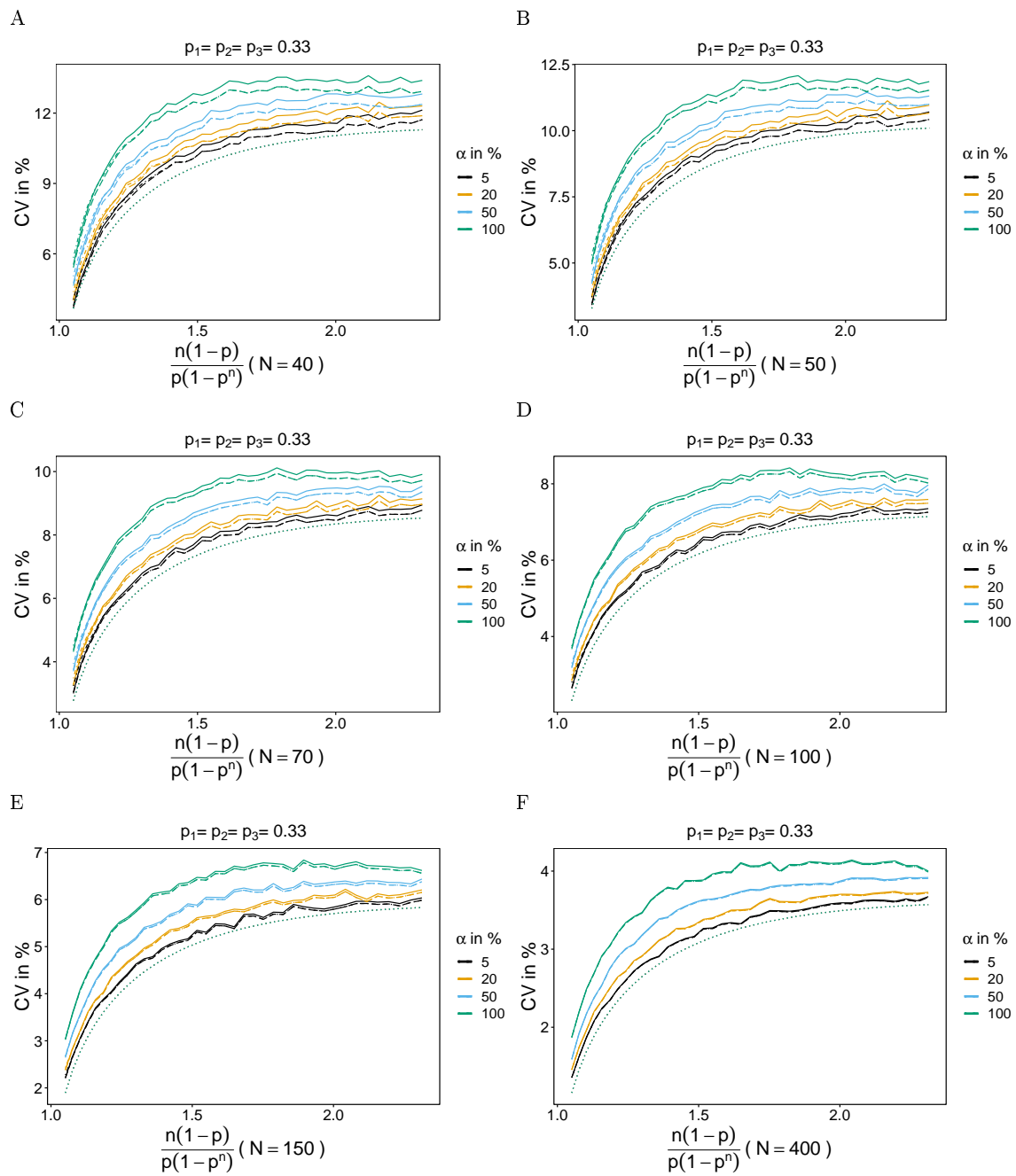


Figure 52: Same as Figure 49.

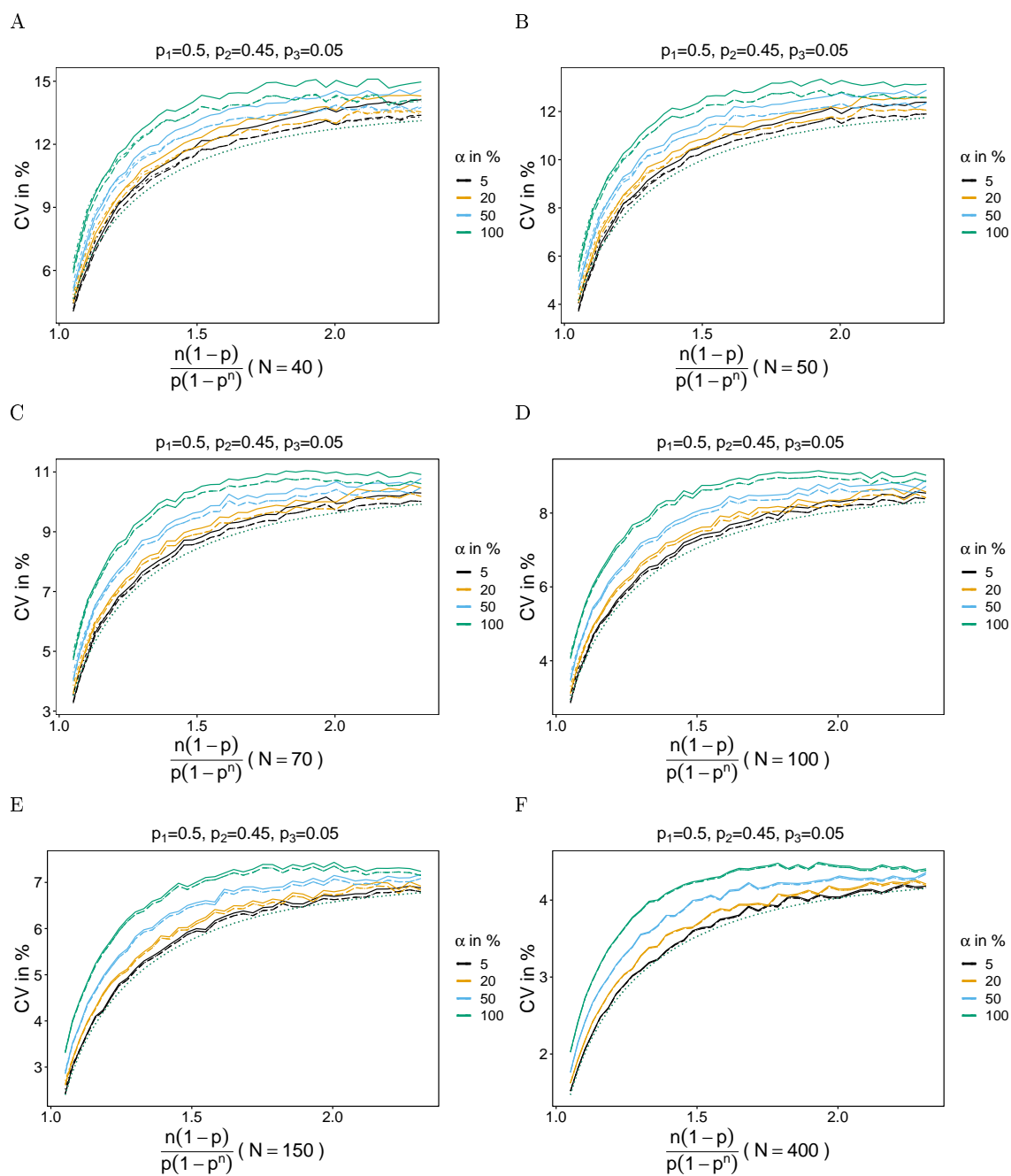


Figure 53: Same as Figure 49.

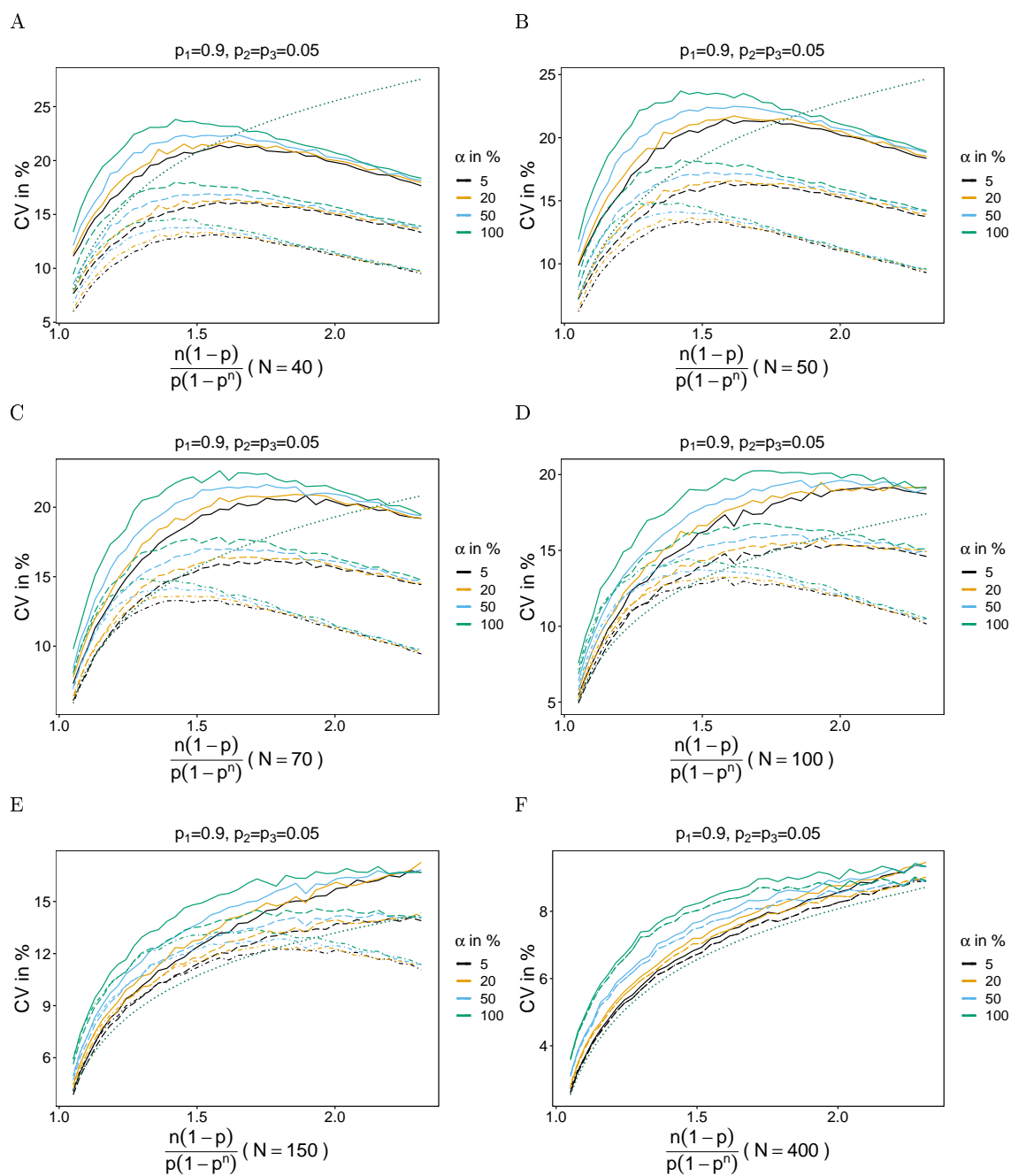


Figure 54: Same as Figure 49.

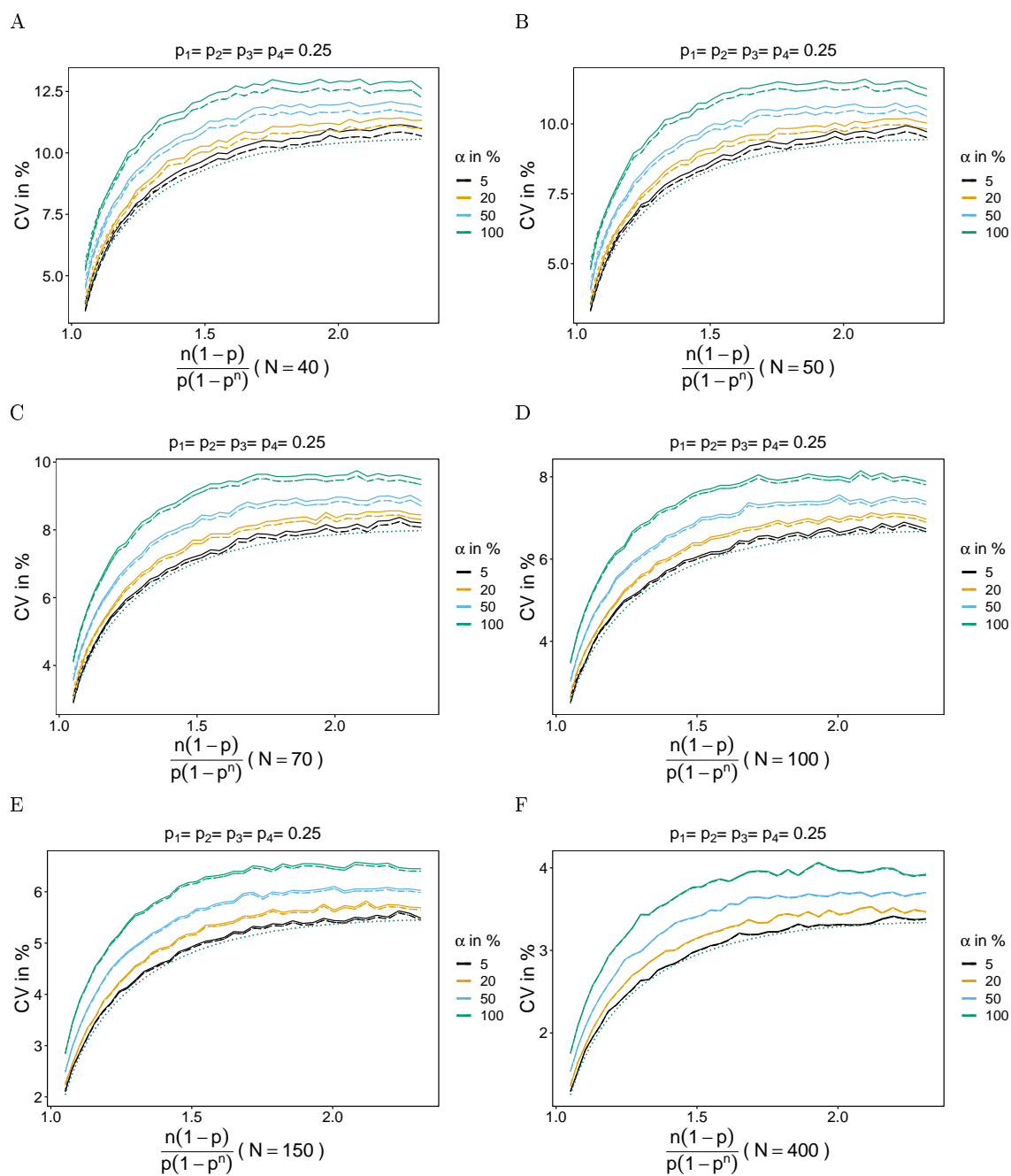


Figure 55: Same as Figure 49.

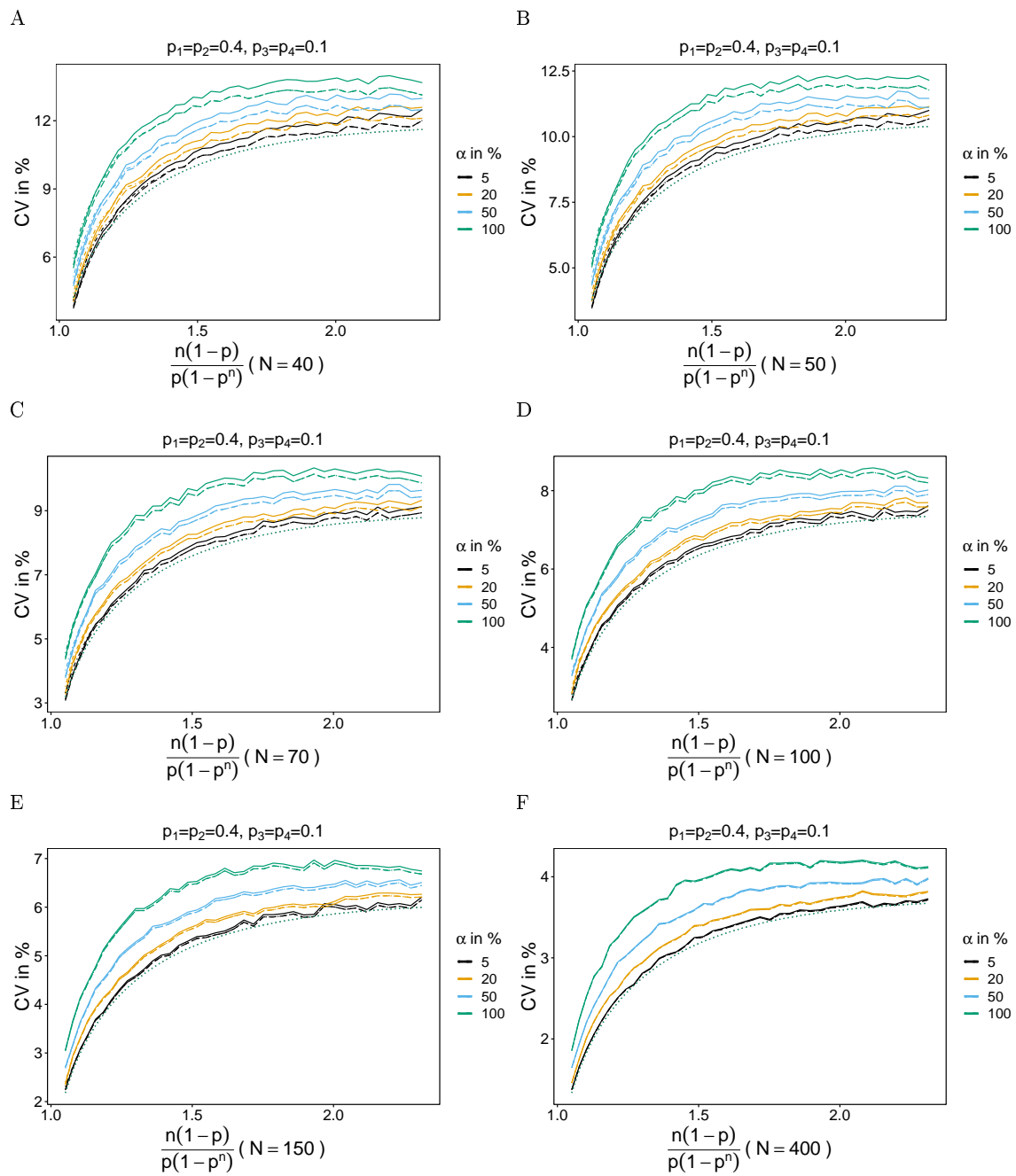


Figure 56: Same as Figure 49.

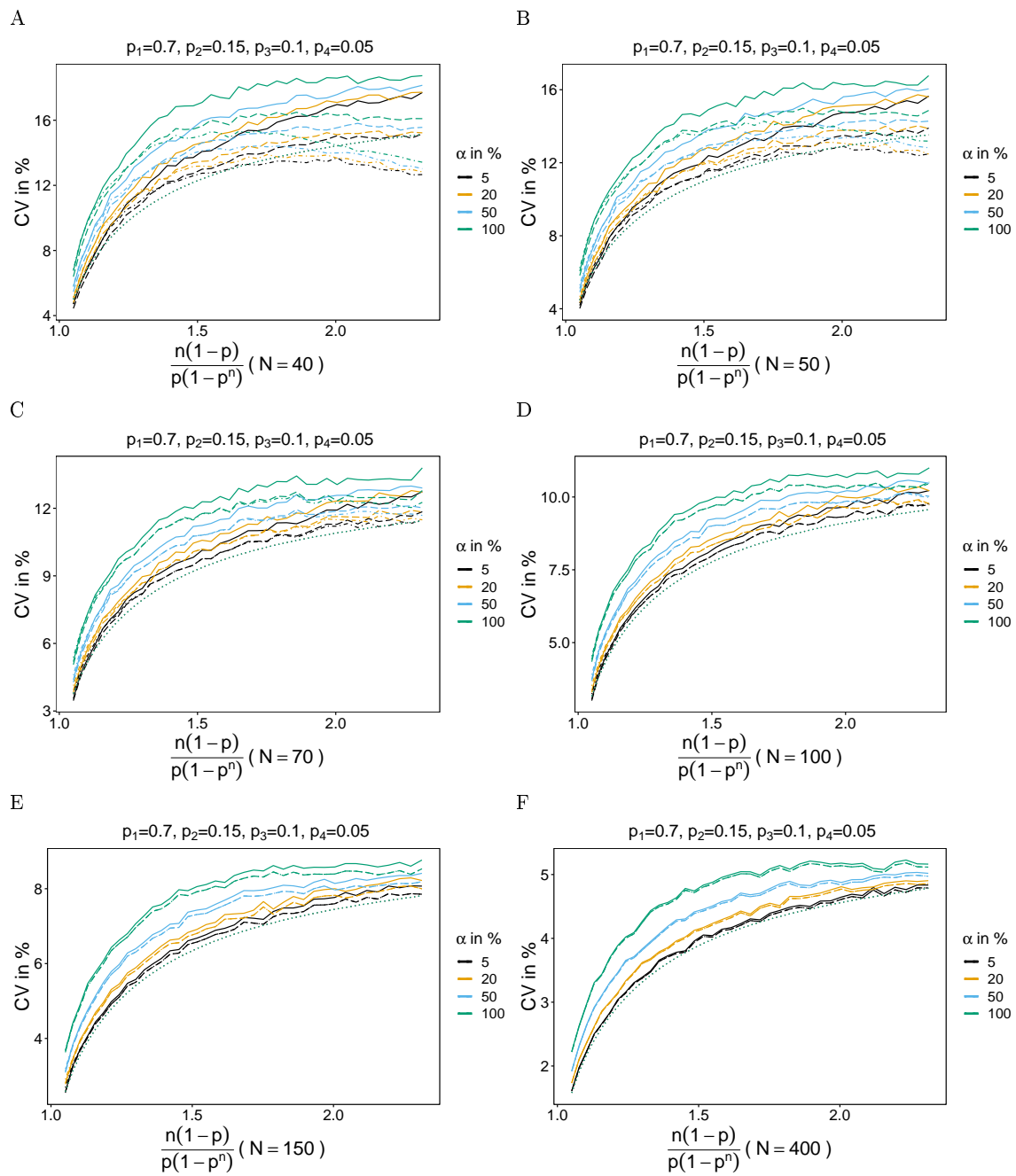


Figure 57: Same as Figure 49.

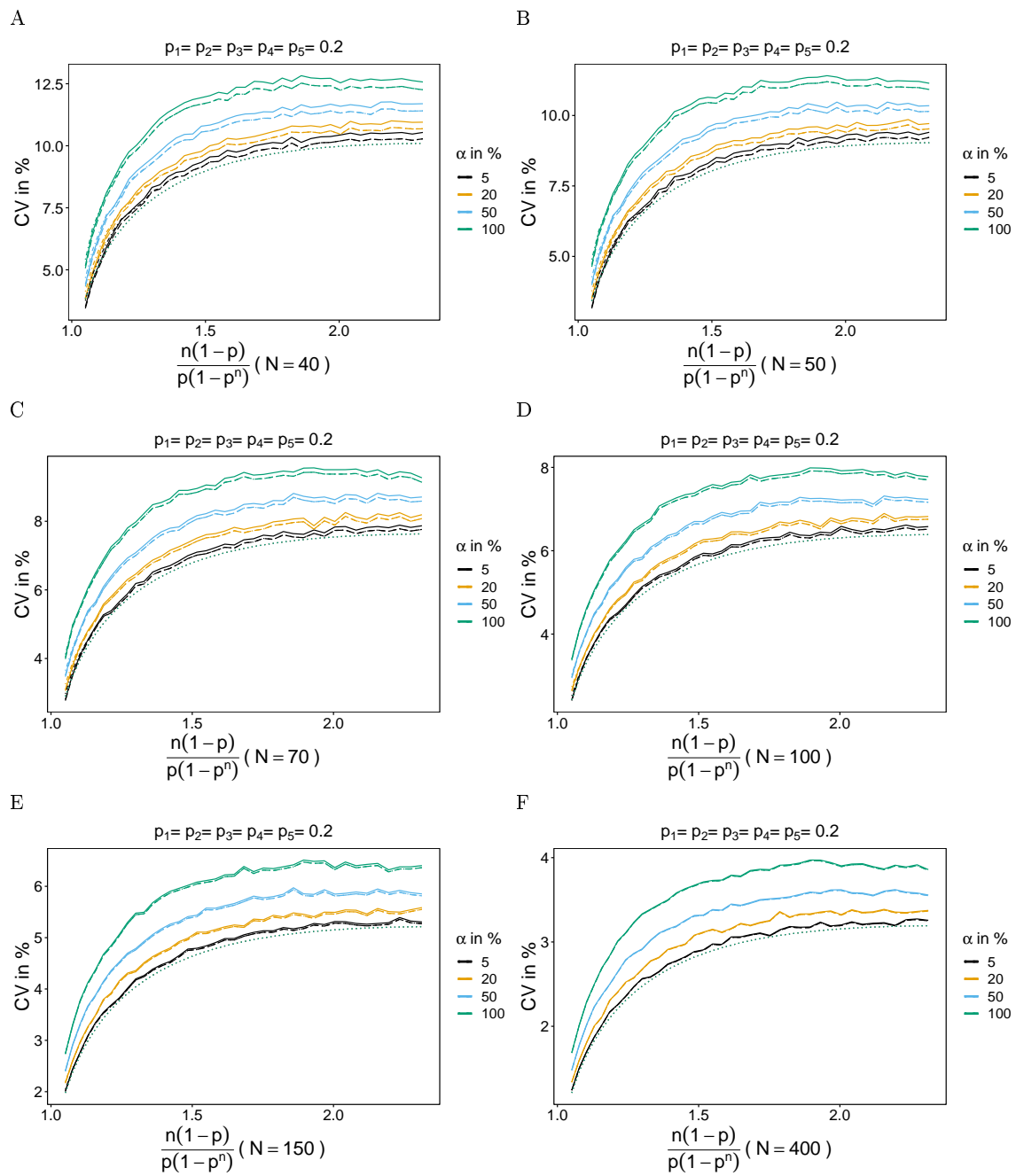


Figure 58: Same as Figure 49.

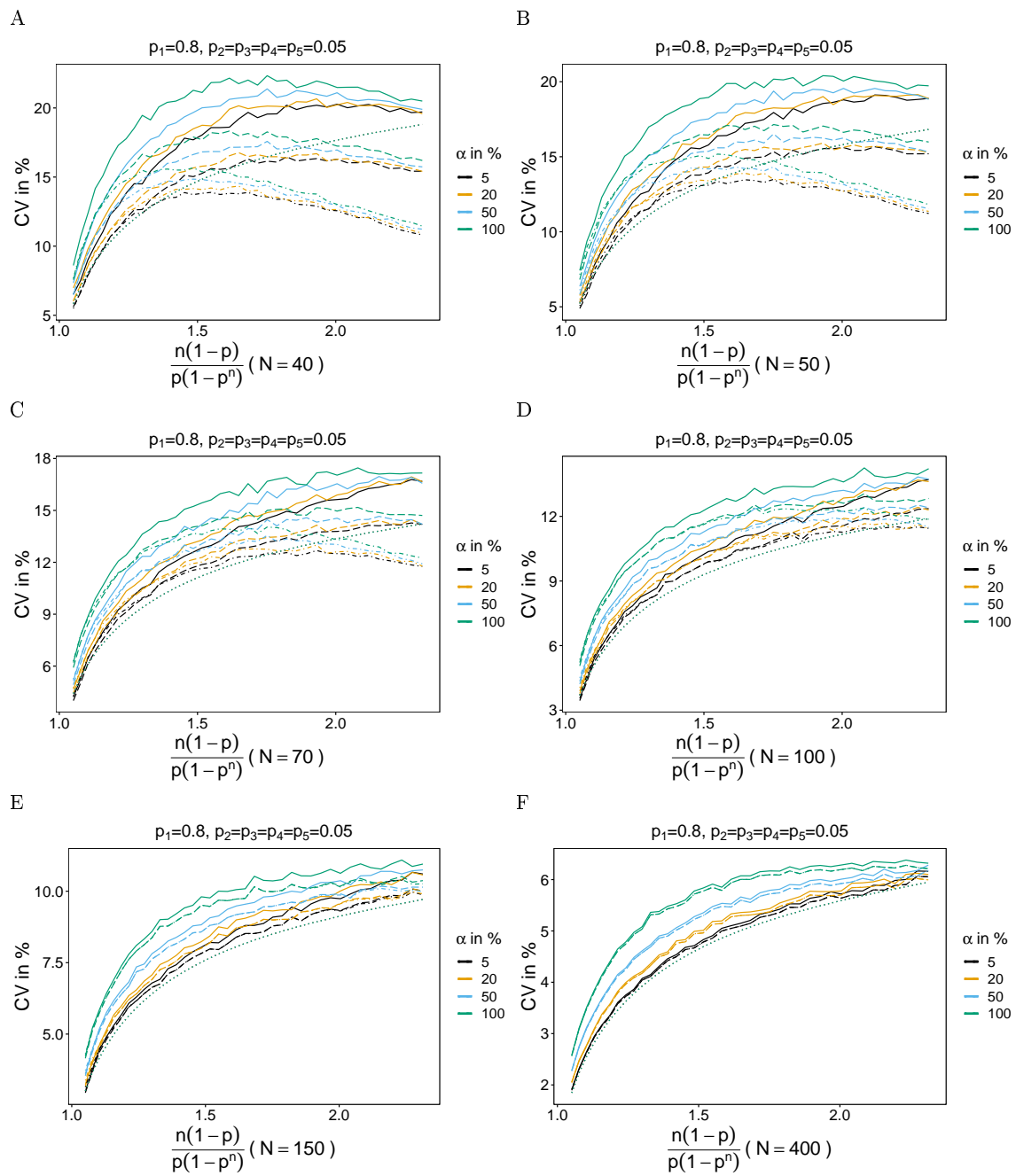


Figure 59: Same as Figure 49.

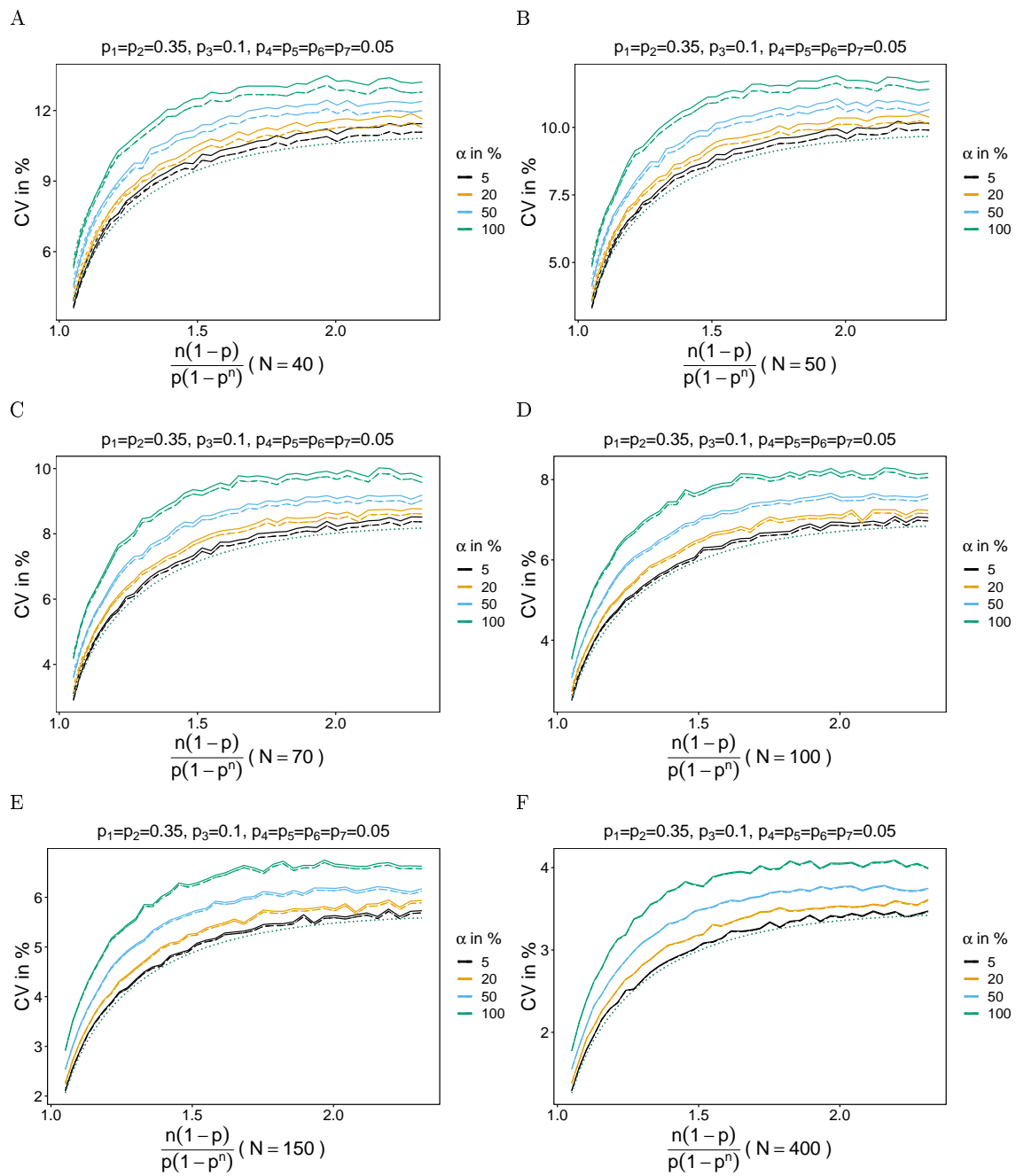


Figure 60: Same as Figure 49.

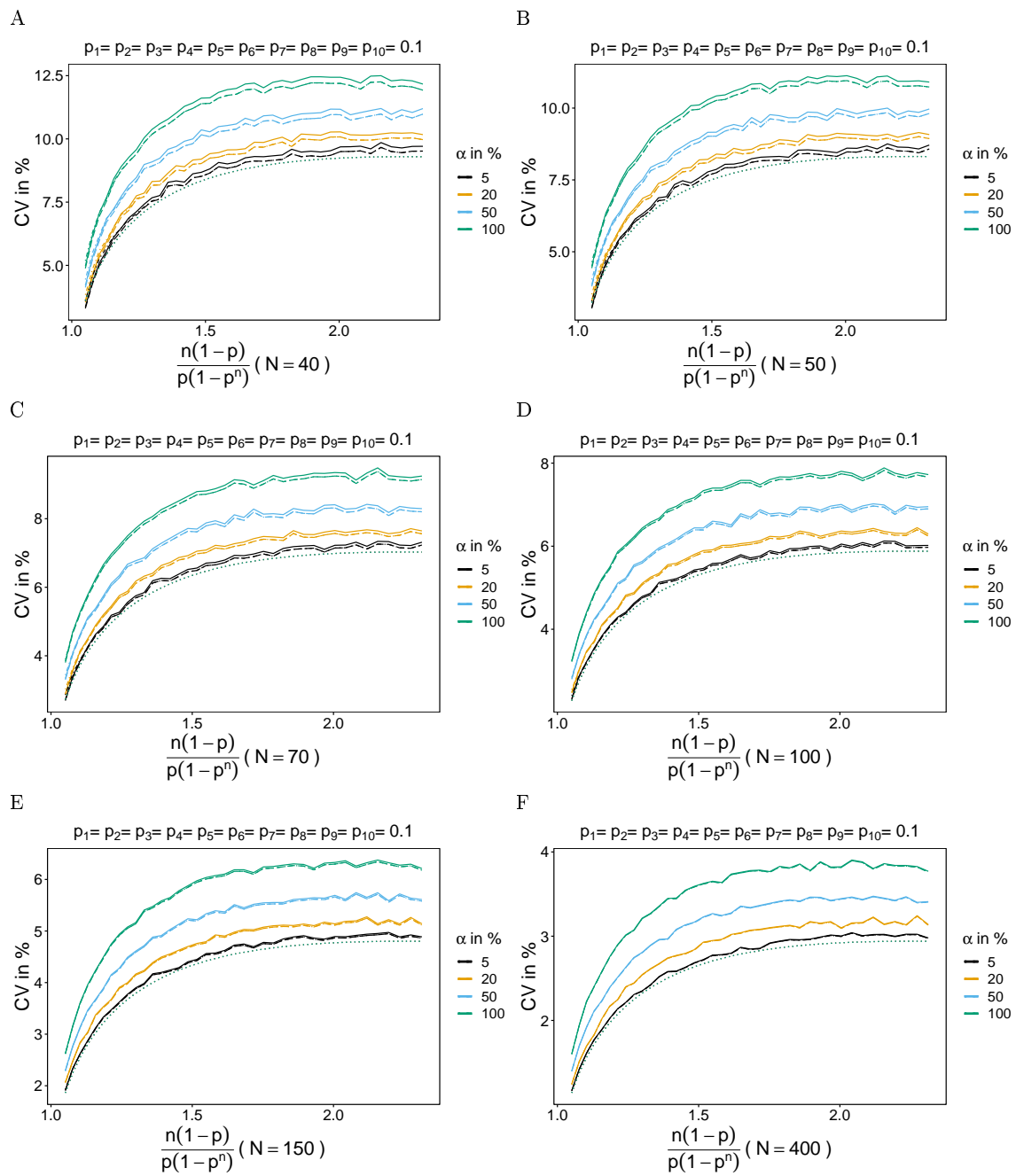


Figure 61: Same as Figure 49.

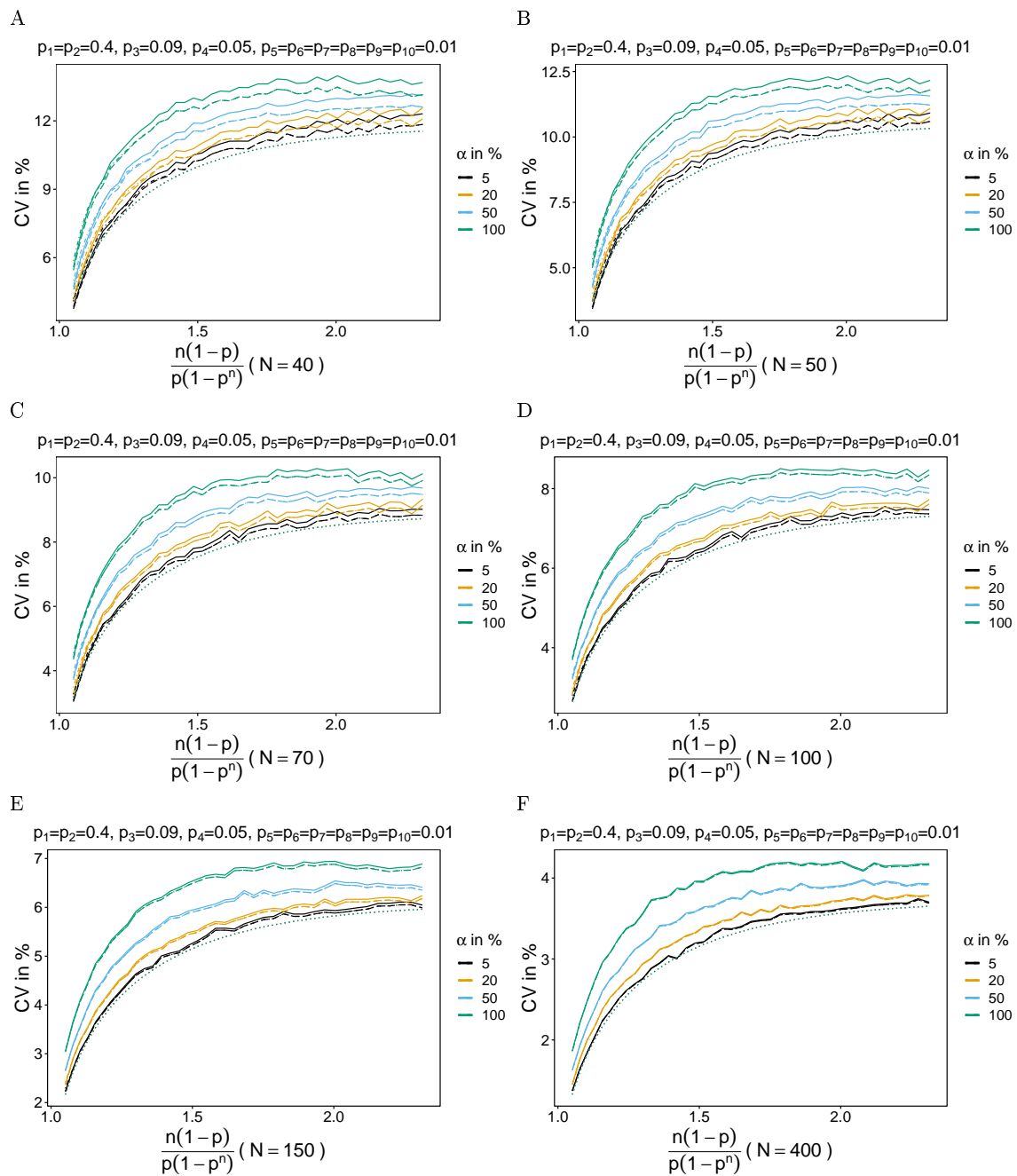


Figure 62: Same as Figure 49.

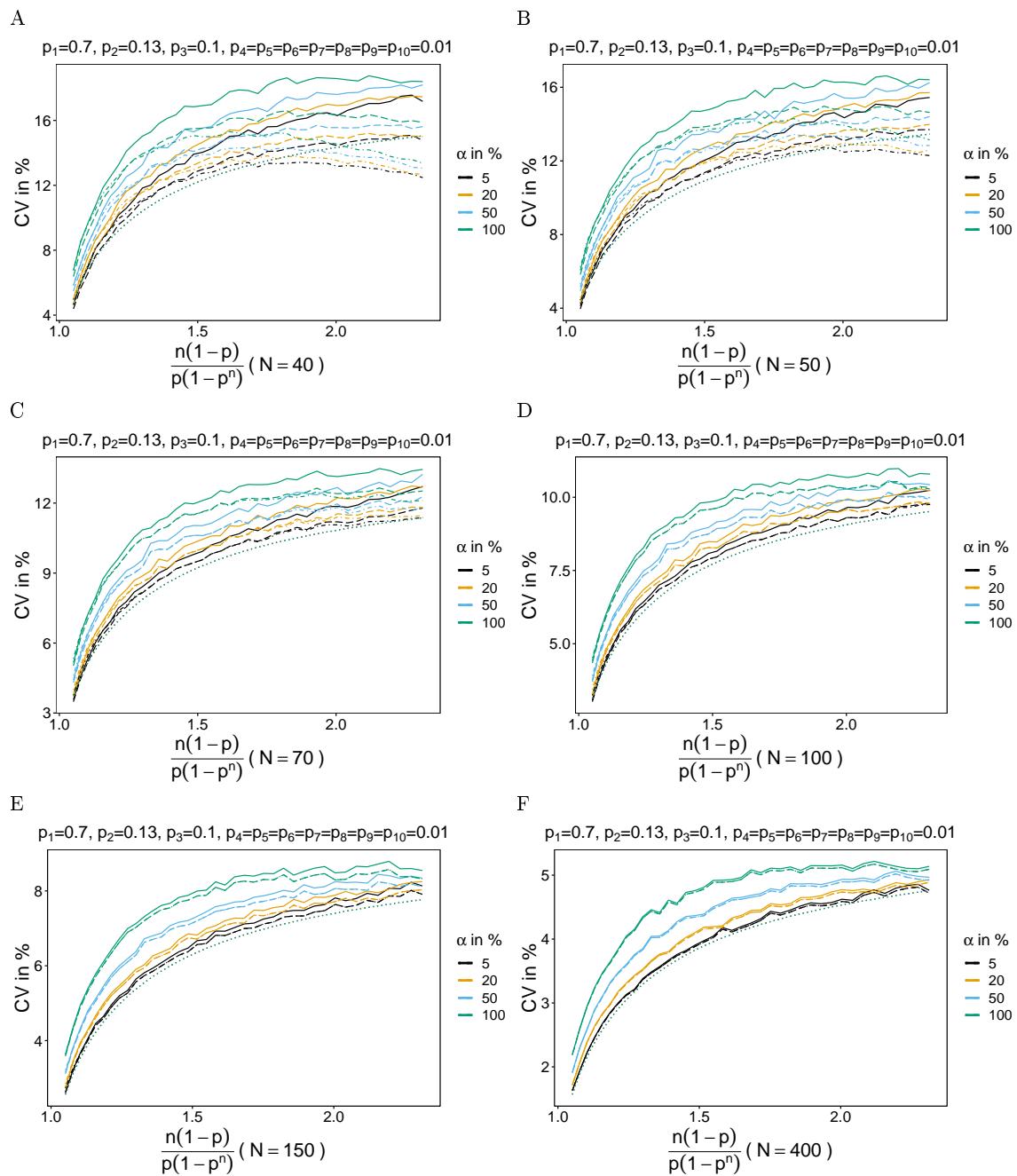


Figure 63: Same as Figure 49.

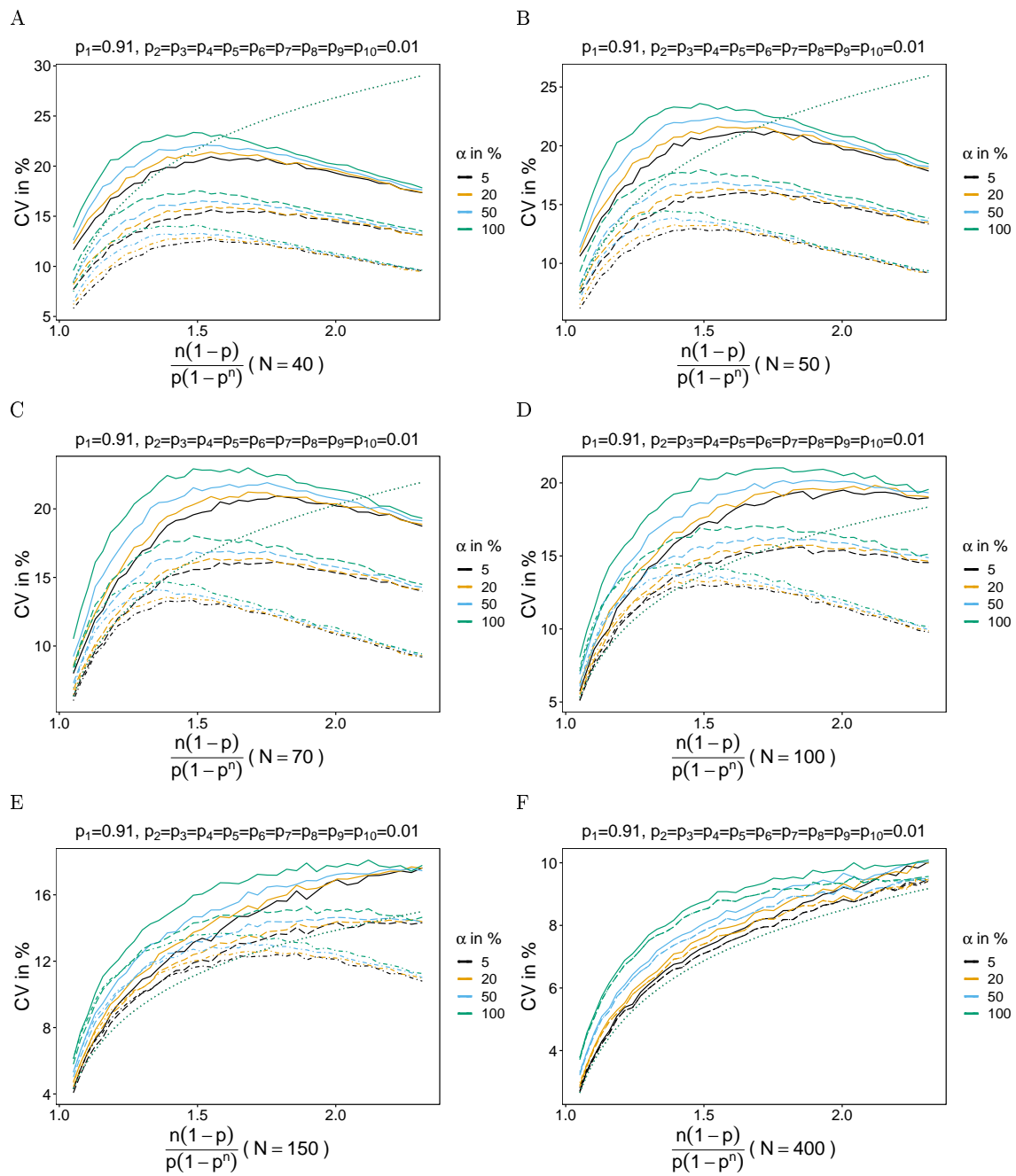


Figure 64: Same as Figure 49.