Constructible Numbers

Name: _____ **Date: _____**

A number is said to be *constructible* if a line segment of that length can be constructed using a straightedge (a ruler with no markings), a compass, a pencil and two points given in the plane.

Let us begin by constructing the number 1.

Draw a line using your straightedge to connect these two points. We will assign this line segment a length of 1. Congratulations! You have constructed 1.

Let us now construct 2.

Place your compass on the second point and open it till your pencil touches the first point. Now, draw a circle. Next, draw a straight line through the two points using the straightedge and extend it until it meets the circle. The length of the line segment from the first point to where the line meets the circle is 2.

Q1. Construct 3 using these two points and the information given above.

Q2. Can you construct 152? What about 1947? There is no need to actually construct them. Just think about whether it is possible.

Now, let us try and construct fractions. Construct ½ by following the given instructions.

Draw a straight line between the two points using the straightedge. Open the compass to a random length and place it on the first point. Draw an arc above and an arc below the line. Without changing the length of the compass, place it on the second point and draw an arc above and below the line. These arcs should intersect the previous two arcs. Draw a straight line between the points of intersection of the two arcs. Congratulations! You have drawn a perpendicular bisector! The length of the two line segments thus formed is ½ each.

Q3. Construct ¼ using the previous instructions.

Look at the animation of the construction of $\frac{1}{5}$ shown in the slides.

Q4. Construct $\frac{1}{3}$ using these points and the discussion above.

Q5. Can you construct $\frac{2}{5}$? What about $\frac{3}{174}$? There is no need to actually construct them. Just think about whether it is possible.

Let us now construct $\sqrt{2}$.

First follow the previous steps to construct 2. Then, draw a perpendicular bisector through the line segment of length 2. Using the straightedge, connect the first dot to the point where the perpendicular bisector intersects the circle. This length is $\sqrt{2}$, as our discussion of the Pythagoras Theorem showed.

Q6. Is it possible to construct $\sqrt{3}$? What about $\sqrt{152}$? There is no need to actually construct them. Just think about whether it is possible.