Pairwise Approximations Fail to Capture Cross Feeding interactions in Microbial Communities

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Non-specific cross feeding in microbial communities

- Microbes secrete various forms of metabolites into their environment (overflow metabolism, extracellular enzymes, etc.).
- Metabolite secretion permits interactions beyond competition for resources.
- Lotka-Volterra (henceforth pairwise) models don't account for the mechanisms of cooperative interactions.

The MiCRM

The microbial consumer resource model **Goldford** modifies MacArthur's classical model by including resource leakage.

$$\frac{1}{C_i}\frac{dC_i}{dt} = \sum_{\alpha=1}^{M}(1-l_\alpha^i)u_{i\alpha}R_\alpha - m_i \qquad (1)$$

$$\frac{dR_\beta}{dt} = \rho_\beta - \sum_{i=1}^{N} u_{i\beta}R_\beta C_i + \sum_{\alpha=1}^{M}\sum_{i=1}^{N} l_{\alpha\beta}^i u_{i\alpha}C_i R_\alpha \qquad (2)$$

Effective pairwise model

Assuming a regime where resources reach equilibrium faster than consumers ($\frac{dR_\alpha}{dt} = 0$), we can approximate the MiCRM with a pairwise Lotka-Volterra model.

$$\frac{1}{C_i}\frac{dC_i}{dt} = r_i + \sum_{j=1}^{N}\alpha_{ij}C_j \qquad (3)$$

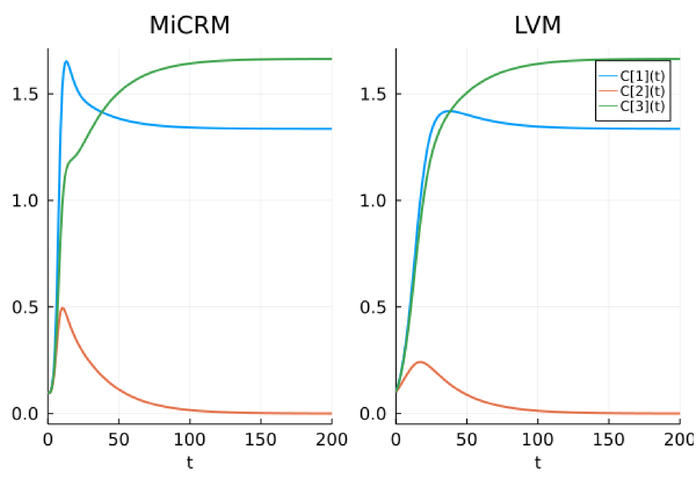

Figure 1:Comparison between MiCRM solution (left) and a successful LVM approximation (right).

The leakage parameter

The Leakage parameter can create strong feedback between consumers and resources, violating the assumptions necessary for the pairwise approximation.

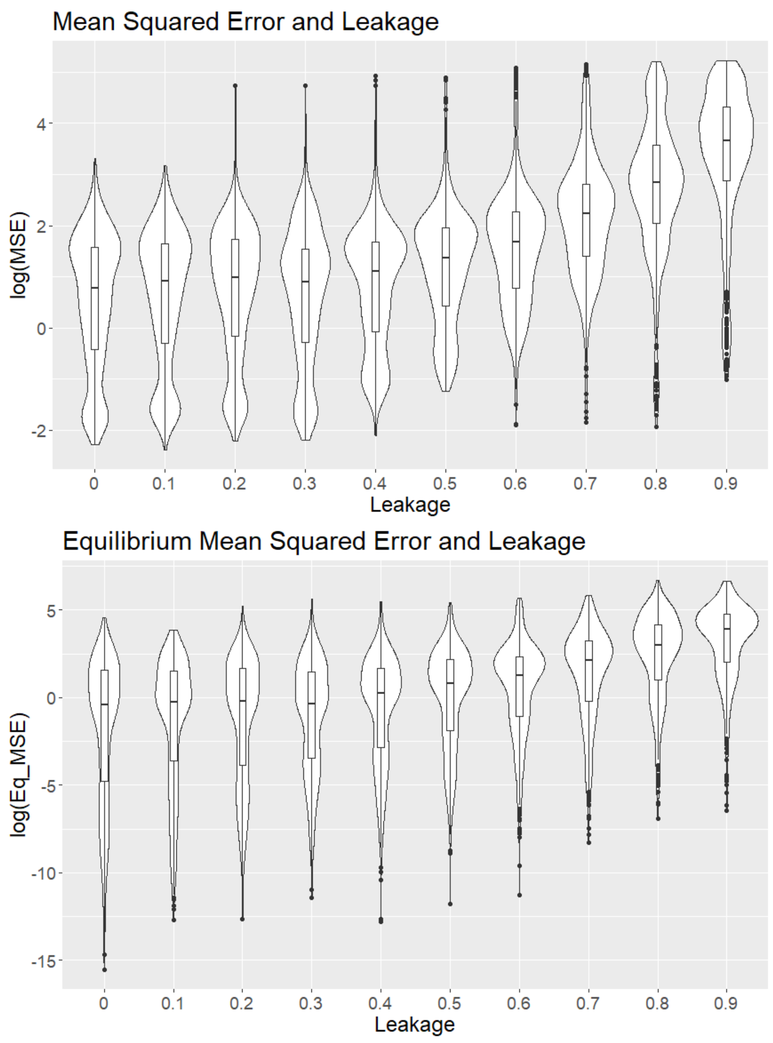

Figure 2:Distributions of log mean squared error (MSE) between MiCRM and LV model for different values of leakage. Top figure is MSE across solution trajectories and bottom is MSE between equilibrium solutions.

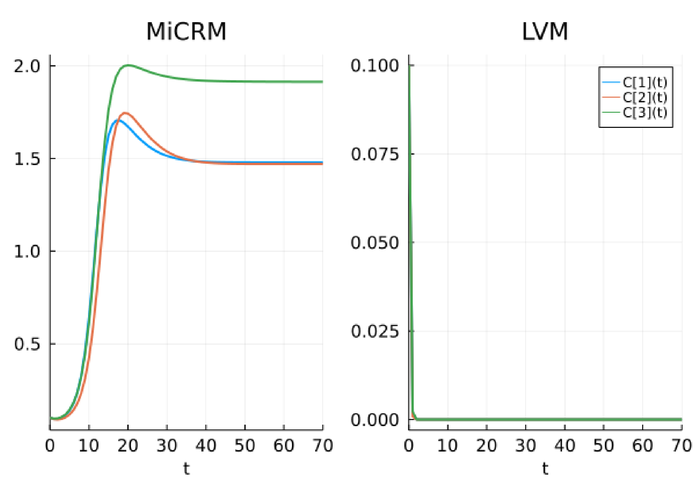

Figure 3:Comparison between MiCRM solution (left) and a failed LVM approximation (right).

Niche overlap modulates the importance of leakage

Niche overlap can decrease the leakage values for which significant discrepancies in the approximation are likely to occur.

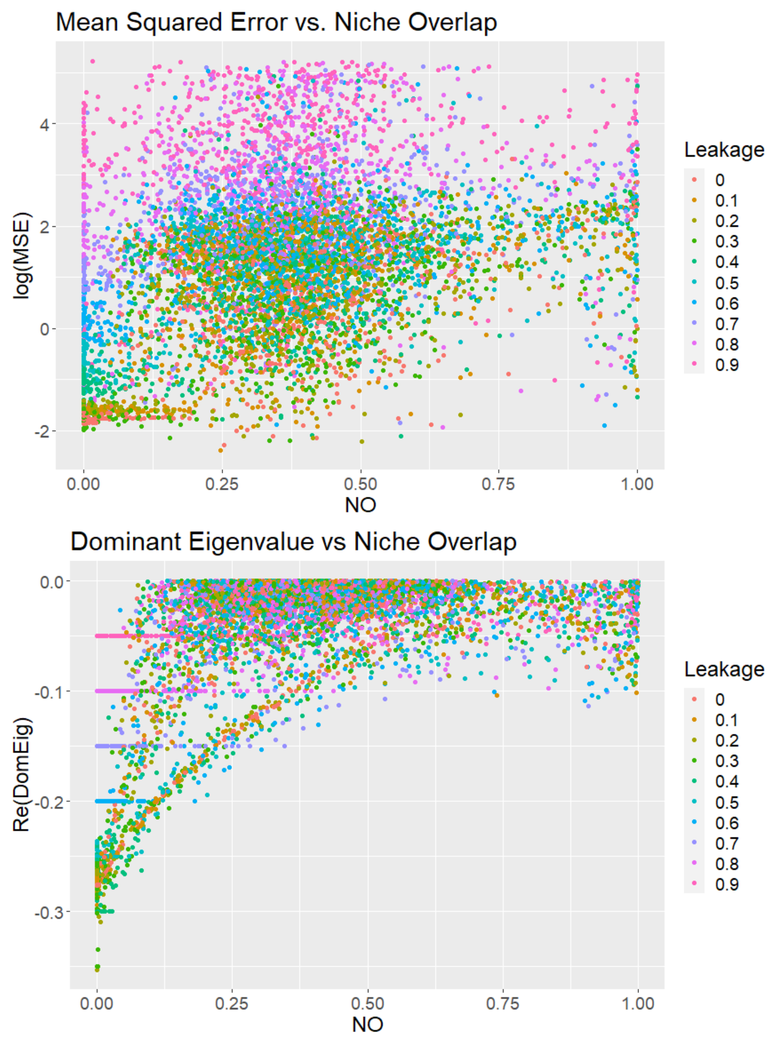

Figure 4:Top figure: scatter plot of log mean squared error vs. niche overlap. Bottom figure: scatter plot of the real parts of the MiCRM's dominant eigenvalues vs. niche overlap.

Conclusions

- Leakage largely determines when pairwise approximations will work.
- Structural characteristic of a model can change the degree of influence leakage has.

References

Marsland *et al.*, (2019)
Goldoford *et al.*, (2018)