Main page
Contents
Featured content
Current events
Random article
Donate to Wikipedia
Wikipedia store

Interaction
Help
About Wikipedia
Community portal
Recent changes
Contact page

Tools
What links here
Related changes
Upload file
Special pages
Permanent link
Page information
Wikidata item
Cite this page

Print/export
Create a book
Download as PDF
Printable version

In other projects
Wikimedia Commons
Wikiquote

Languages
Afrikaans
????
العربية
Aragonés
Azərbaycanca
বাংলা
Башкортса
Беларуская
Български
Bosanski
Brezhoneg
Català
Čeština
Dansk
Deutsch
Eesti
Ελληνικά
Español

e (mathematical constant)

From Wikipedia, the free encyclopedia

"Euler's number" redirects here. For other uses, see List of things named after Leonhard Euler § Euler's numbers.

The number e is a mathematical constant that is the base of the natural logarithm: the unique number whose natural logarithm is equal to one. It is approximately equal to **2.71828**,[1] and is the limit of $(1 + 1/n)^n$ as n approaches infinity, an expression that arises in the study of compound interest. It can also be calculated as the sum of the infinite series[2]

$$e=\sum_{n=0}^{\infty}\frac{1}{n!}=\frac{1}{1}+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\cdots$$

The constant can be characterized in many different ways. For example, e can be defined as the unique positive number a such that the graph of the function $y = a^x$ has unit slope at $x = 0$.[3] The function $f(x) = e^x$ is called the (natural) exponential function. The natural logarithm, or logarithm to base e, is the inverse function to the natural exponential function. The natural logarithm of a positive number k can be defined directly as the area under the curve $y = 1/x$ between $x = 1$ and $x = k$, in which case e is the value of k for which this area equals one (see image). There are alternative characterizations.

Sometimes called **Euler's number** after the Swiss mathematician Leonhard Euler, e is not to be confused with γ, the Euler–Mascheroni constant, sometimes called simply *Euler's constant*. The number e is also known as **Napier's constant**, but Euler's choice of the symbol e is said to have been retained in his honor.[4] The constant was discovered by the Swiss mathematician Jacob Bernoulli while studying compound interest.[5]

The number e is of eminent importance in mathematics,[6] alongside 0, 1, π and i. All five of these numbers play important and recurring roles across mathematics, and are the five constants appearing in one formulation of Euler's identity. Like the constant π, e is irrational: it is not a ratio of integers. Also like π, e is transcendental: it is not a root of *any* non-zero polynomial with rational coefficients. The numerical value of e truncated to 50 decimal places is

2.71828 18284 59045 23536 02874 71352 66249 77572 47093 699
(sequence A001113 in the OEIS).

Part of a series of articles on the
mathematical constant e

Properties
Natural logarithm · Exponential function

Applications
compound interest · Euler's identity · Euler's formula · half-lives (exponential growth and decay)

Defining e
proof that e is irrational · representations of e · Lindemann–Weierstrass theorem

People
John Napier · Leonhard Euler

Related topics
Schanuel's conjecture

v · t · e

Contents

Esperanto
Euskara
فارسی
Français
Galego
贛語
한국어
Հայերեն
हिन्दी
Hrvatski
Bahasa Indonesia
Interlingua
Íslenska
Italiano
עברית
ಕನ್ನಡ
ქართული
Қазақша
Latina
Latviešu
Lietuvių
Lumbaart
Magyar
Македонски
मराठी
Bahasa Melayu
Монгол
Nederlands
日本語
Norsk bokmål
Norsk nynorsk
Polski
Português
Română
Русский
Shqip
සිංහල
Simple English
Slovenčina
Slovenščina
کوردیی ناوەندی
Српски / srpski
Srpskohrvatski / српскохрватски
Suomi
Svenska
தமிழ்
ไทย
Türkçe
Українська
اردو
Tiếng Việt
粵語
中文
Edit

History [edit]

The first references to the constant were published in 1618 in the table of an appendix of a work on logarithms by John Napier.[5] However, this did not contain the constant itself, but simply a list of logarithms calculated from the constant. It is assumed that the table was written by William Oughtred. The discovery of the constant itself is credited to Jacob Bernoulli in 1683,[7][8] who attempted to find the value of the following expression (which is in fact e):

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n.$$

The first known use of the constant, represented by the letter b, was in correspondence from Gottfried Leibniz to Christiaan Huygens in 1690 and 1691. Leonhard Euler introduced the letter e as the base for natural logarithms, writing in a letter to Christian Goldbach of 25 November 1731.[9][10] Euler started to use the letter e for the constant in 1727 or 1728, in an unpublished paper on explosive forces in cannons,[11] and the first appearance of e in a publication was Euler's *Mechanica* (1736).[12] While in the subsequent years some researchers used the letter c, e was more common and eventually became the standard.

Applications [edit]

Compound interest [edit]

Jacob Bernoulli discovered this constant in 1683 by studying a question about compound interest:[5]

> An account starts with $1.00 and pays 100 percent interest per year. If the interest is credited once, at the end of the year, the value of the account at year-end will be $2.00. What happens if the interest is computed and credited more frequently during the year?

If the interest is credited twice in the year, the interest rate for each 6 months will be 50%, so the initial $1 is multiplied by 1.5 twice, yielding $1.00×1.5^2 = $2.25 at the end of the year. Compounding quarterly yields $1.00×1.25^4 = $2.4414..., and compounding monthly yields $1.00×(1+1/12)^{12} = $2.613035... If there are n compounding intervals, the interest for each interval will be $100\%/n$ and the value at the end of the year will be $1.00×$(1+1/n)^n$.

Bernoulli noticed that this sequence approaches a limit (the force of interest) with larger n and, thus, smaller compounding intervals. Compounding weekly ($n = 52$) yields $2.692597..., while compounding daily ($n = 365$) yields $2.714567..., just two cents more. The limit as n grows large is the number that came to be known as e; with *continuous* compounding, the account value will reach $2.7182818.... More generally, an account that starts at $1 and offers an annual interest rate of R will, after t years, yield e^{Rt} dollars with continuous compounding. (Here R is the decimal equivalent of the rate of interest expressed as a percentage, so for 5% interest, $R = 5/100 = 0.05$)

Bernoulli trials [edit]

The number e itself also has applications to probability theory, where it arises in a way not obviously related to exponential growth. Suppose that a gambler plays a slot machine that pays out with a probability of one in n and plays it n times. Then, for large n (such as a million) the probability that the gambler will lose every bet is approximately $1/e$. For $n = 20$ it is already approximately 1/2.79.

This is an example of a Bernoulli trials process. Each time the gambler plays the slots, there is a one in one million chance of winning. Playing one million times is modelled by the binomial distribution, which is closely related to the binomial theorem. The probability of winning k times out of a million trials is:

$$\binom{10^6}{k}\left(10^{-6}\right)^k\left(1-10^{-6}\right)^{10^6-k}.$$

In particular, the probability of winning zero times ($k = 0$) is

$$\left(1-\frac{1}{10^6}\right)^{10^6}.$$

This is very close to the following limit for $1/e$:

$$\frac{1}{e} = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n.$$

Derangements [edit]

Another application of e, also discovered in part by Jacob Bernoulli along with Pierre Raymond de Montmort, is in the problem of derangements, also known as the *hat check problem*:[13] n guests are invited to a party, and at the door each guest checks his hat with the butler who then places them into n boxes, each labelled with the name of one guest. But the butler does not know the identities of the guests, and so he puts the hats into boxes selected at random. The problem of de Montmort is to find the probability that *none* of the hats gets put into the right box. The answer is:

$$p_n = 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+\frac{(-1)^n}{n!} = \sum_{k=0}^{n}\frac{(-1)^k}{k!}.$$

As the number n of guests tends to infinity, p_n approaches $1/e$. Furthermore, the number of ways the hats can be placed into the boxes so that none of the hats are in the right box is $n!/e$ rounded to the nearest integer, for every positive n.[14]

Asymptotics [edit]

The number e occurs naturally in connection with many problems involving asymptotics. A prominent example is Stirling's formula for the asymptotics of the factorial function, in which both the numbers e and π enter:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n.$$

A particular consequence of this is

$$e = \lim_{n\to\infty}\frac{n}{\sqrt[n]{n!}}.$$

Standard normal distribution [edit]

Main article: Standard normal distribution

The simplest case of a normal distribution is known as the *standard normal distribution*, described by this probability density function:

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}.$$

The factor $1/\sqrt{2\pi}$ in this expression ensures that the total area under the curve $\phi(x)$ is equal to one[proof]. The $\frac{1}{2}$ in the exponent ensures that the distribution has unit variance (and therefore also unit standard deviation). This function is symmetric around $x = 0$, where it attains its maximum value $1/\sqrt{2\pi}$; and has inflection points at +1 and −1.

e in calculus [edit]

The principal motivation for introducing the number e, particularly in calculus, is to perform differential and integral calculus with exponential functions and logarithms.[15] A general exponential function $y = a^x$ has derivative given as the limit:

$$\frac{d}{dx}a^x = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = \lim_{h\to 0}\frac{a^x a^h - a^x}{h} = a^x\left(\lim_{h\to 0}\frac{a^h-1}{h}\right).$$

The limit on the far right is independent of the variable x: it depends only on the base a. When the base is e, this limit is equal to 1, and so e is symbolically defined by the equation:

$$\frac{d}{dx}e^x = e^x.$$

Consequently, the exponential function with base e is particularly suited to doing calculus. Choosing e, as opposed to some other number, as the base of the exponential function makes calculations involving the derivative much simpler.

Another motivation comes from considering the base-a logarithm.[16] Considering the definition of the derivative of $\log_a x$ as the limit:

$$\frac{d}{dx}\log_a x = \lim_{h\to 0}\frac{\log_a(x+h)-\log_a(x)}{h} = \frac{1}{x}\left(\lim_{u\to 0}\frac{1}{u}\log_a(1+u)\right),$$

where the substitution $u = h/x$ was made in the last step. The last limit appearing in this calculation is again an undetermined limit that depends only on the base a, and if that base is e, the limit is equal to 1. So symbolically,

$$\frac{d}{dx}\log_e x = \frac{1}{x}.$$

The logarithm in this special base is called the natural logarithm and is represented as $\ln$; it behaves well under differentiation since there is no undetermined limit to carry through the calculations.

There are thus two ways in which to select a special number $a = e$. One way is to set the derivative of the exponential function a^x to a^x, and solve for a. The other way is to set the derivative of the base a logarithm to $1/x$ and solve for a. In each case, one arrives at a convenient choice of base for doing calculus. In fact, these two solutions for a are actually *the same*, the number e.

Alternative characterizations [edit]

See also: § Representations, and Characterizations of the exponential function

Other characterizations of e are also possible: one is as the limit of a sequence, another is as the sum of an infinite series, and still others rely on integral calculus. So far, the following two (equivalent) properties have been introduced:

1. The number e is the unique positive real number such that
$$\frac{d}{dt}e^t = e^t.$$
2. The number e is the unique positive real number such that
$$\frac{d}{dt}\log_e t = \frac{1}{t}.$$

The following three characterizations can be proven equivalent:

3. The number e is the limit
$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$
Similarly:
$$e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$
4. The number e is the sum of the infinite series
$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots,$$
where $n!$ is the factorial of n.
5. The number e is the unique positive real number such that
$$\int_1^e \frac{1}{t}\,dt = 1.$$
[17]

Properties [edit]

Calculus [edit]

As in the motivation, the exponential function e^x is important in part because it is the unique nontrivial function (up to multiplication by a constant) which is its own derivative

$$\frac{d}{dx}e^x = e^x$$

and therefore its own antiderivative as well:

$$\int e^x\,dx = e^x + C.$$

Inequalities [edit]

The number e is the unique real number such that

$$\left(1+\frac{1}{x}\right)^x < e < \left(1+\frac{1}{x}\right)^{x+1}$$

for all positive x.[18]

Also, we have the inequality

$$e^x \geq x+1$$

for all real x, with equality if and only if $x = 0$. Furthermore, e is the unique base of the exponential for which the inequality $a^x \geq x + 1$ holds for all x.

Exponential-like functions [edit]

Steiner's problem asks to find the global maximum for the function

$$f(x) = x^{1/x}.$$

This maximum occurs precisely at $x = e$. For proof, the inequality $e^y \geq y+1$, from above, evaluated at $y = (x-e)/e$ and simplifying gives $e^{x/e} \geq x$. So $e^{1/e} \geq x^{1/x}$ for all positive x.[19]

Similarly, $x = 1/e$ is where the global minimum occurs for the function

$$f(x) = x^x$$

defined for positive x. More generally, for the function

$$f(x) = x^{x^n}$$

the global maximum for positive x occurs at $x = 1/e$ for any $n < 0$; and the global minimum occurs at $x = e^{-1/n}$ for any $n > 0$.

The infinite tetration

$$x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$$ or ${}^{\infty}x$

converges if and only if $e^{-e} \leq x \leq e^{1/e}$ (or approximately between 0.0660 and 1.4447), due to a theorem of Leonhard Euler.[20]

Number theory [edit]

The real number e is irrational. Euler proved this by showing that its simple continued fraction expansion is infinite.[21] (See also Fourier's proof that e is irrational.)

Furthermore, by the Lindemann–Weierstrass theorem, e is transcendental, meaning that it is not a solution of any non-constant polynomial equation with rational coefficients. It was the first number to be proved transcendental without having been specifically constructed for this purpose (compare with Liouville number); the proof was given by Charles Hermite in 1873.

It is conjectured that e is normal, meaning that when e is expressed in any base the possible digits in that base are uniformly distributed (occur with equal probability in any sequence of given length).

Complex numbers [edit]

The exponential function e^x may be written as a Taylor series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Because this series keeps many important properties for e^x even when x is complex, it is commonly used to extend the definition of e^x to the complex numbers. This, with the Taylor series for sin and cos x, allows one to derive Euler's formula:

$$e^{ix} = \cos x + i \sin x,$$

which holds for all x. The special case with $x = \pi$ is Euler's identity:

$$e^{i\pi} + 1 = 0$$

from which it follows that, in the principal branch of the logarithm,

$$\ln(-1) = i\pi.$$

Furthermore, using the laws for exponentiation,

$$(\cos x + i\sin x)^n = \left(e^{ix}\right)^n = e^{inx} = \cos(nx) + i\sin(nx),$$

which is de Moivre's formula.

The expression

$$\cos x + i\sin x$$

is sometimes referred to as $\mathrm{cis}(x)$.

Differential equations [edit]

The general function

$$y(x) = Ce^x$$

is the solution to the differential equation:

$$y' = y.$$

Representations [edit]

Main article: List of representations of e

The number e can be represented as a real number in a variety of ways: as an infinite series, an infinite product, a continued fraction, or a limit of a sequence. The chief among these representations, particularly in introductory calculus courses is the limit

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n,$$

given above, as well as the series

$$e = \sum_{n=0}^{\infty}\frac{1}{n!}$$

given by evaluating the above power series for e^x at $x = 1$.

Less common is the continued fraction (sequence A003417 in the OEIS).

$$e = [2;1,\mathbf{2},1,1,\mathbf{4},1,1,\mathbf{6},1,1,\ldots,\mathbf{2n},1,1,\ldots] = [1;\mathbf{0},1,1,\mathbf{2},1,1,\mathbf{4},1,1,\ldots,\mathbf{2n},1,1,\ldots],$$ [22]

which written out looks like

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{\mathbf{2} + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\mathbf{4} + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}}}} = 1 + \cfrac{1}{\mathbf{0} + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\mathbf{2} + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\mathbf{4} + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}}}}}}.$$

This continued fraction for e converges three times as quickly:

$$e = [1;0.5,12,5,28,9,44,13,\ldots,4(4n-1),(4n+1),\ldots],$$

which written out looks like

$$e = 1 + \cfrac{2}{1 + \cfrac{1}{6 + \cfrac{1}{10 + \cfrac{1}{14 + \cfrac{1}{18 + \cfrac{1}{22 + \cfrac{1}{26 + \ddots}}}}}}}.$$

Many other series, sequence, continued fraction, and infinite product representations of e have been developed.

Stochastic representations [edit]

In addition to exact analytical expressions for representation of e, there are stochastic techniques for estimating e. One such approach begins with an infinite sequence of independent random variables X_1, X_2..., drawn from the uniform distribution on [0, 1]. Let V be the least number n such that the sum of the first n observations exceeds 1:

$$V = \min\{n \mid X_1 + X_2 + \cdots + X_n > 1\}.$$

Then the expected value of V is e: $\mathrm{E}(V) = e$.[23][24]

Known digits [edit]

The number of known digits of e has increased substantially during the last decades. This is due both to the increased performance of computers and to algorithmic improvements.[25][26]

Number of known decimal digits of *e*

Date	Decimal digits	Computation performed by
1690	1	Jacob Bernoulli[7]
1714	13	Roger Cotes[27]
1748	23	Leonhard Euler[28]
1853	137	William Shanks[29]
1871	205	William Shanks[30]
1884	346	J. Marcus Boorman[31]
1949	2,010	John von Neumann (on the ENIAC)
1961	100,265	Daniel Shanks and John Wrench[32]
1978	116,000	Steve Wozniak on the Apple II[33]

Since that time, the proliferation of modern high-speed desktop computers has made it possible for amateurs to compute billions of digits of *e*.[34]

In computer culture [edit]

In contemporary internet culture, individuals and organizations frequently pay homage to the number e.

For instance, in the IPO filing for Google in 2004, rather than a typical round-number amount of money, the company announced its intention to raise $2,718,281,828, which is e billion dollars rounded to the nearest dollar. Google was also responsible for a billboard[35] that appeared in the heart of Silicon Valley, and later in Cambridge, Massachusetts; Seattle, Washington; and Austin, Texas. It read "{first 10-digit prime found in consecutive digits of e}.com". Solving this problem and visiting the advertised (now defunct) web site led to an even more difficult problem to solve, which in turn led to Google Labs where the visitor was invited to submit a resume.[36] The first 10-digit prime in e is 7427466391, which starts at the 99th digit.[37]

In another instance, the computer scientist Donald Knuth let the version numbers of his program Metafont approach e. The versions are 2, 2.7, 2.71, 2.718, and so forth.[38]

Notes [edit]

1. ^ Oxford English Dictionary, 2nd ed.: natural logarithm
2. ^ Encyclopedic Dictionary of Mathematics 142.D
3. ^ Jerrold E. Marsden, Alan Weinstein (1985). *Calculus*. Springer. ISBN 0-387-90974-5.
4. ^ Sondow, Jonathan. "e". *Wolfram Mathworld*. Wolfram Research. Retrieved 10 May 2011.
5. ^ ***a b c*** O'Connor, J J; Robertson, E F. "The number *e*". MacTutor History of Mathematics.
6. ^ Howard Whitley Eves (1969). *An Introduction to the History of Mathematics*. Holt, Rinehart & Winston. ISBN 0-03-029558-0.
7. ^ ***a b*** Jacob Bernoulli considered the problem of continuous compounding of interest, which led to a series expression for *e*. See: Jacob Bernoulli (1690) "Quæstiones nonnullæ de usuris, cum solutione problematis de sorte alearum, propositi in Ephem. Gall. A. 1685" (Some questions about interest, with a solution of a problem about games of chance, proposed in the *Journal des Savants* (*Ephemerides Eruditorum Gallicanæ*), in the year (anno) 1685.**), *Acta eruditorum*, pp. 219-223. On page 222, Bernoulli poses the question: *"Alterius naturæ hoc Problema est: Quæritur, si creditor aliquis pecuniæ summam fænori exponat, ea lege, ut singulis momentis pars proportionalis usuræ annuæ sorti annumeretur; quantum ipsi finito anno debeatur?"* (This is a problem of another kind: The question is, if some lender were to invest [a] sum of money [at] interest, let it accumulate, so that [at] every moment [it] were to receive [a] proportional part of [its] annual interest; how much would he be owed [at the] end of [the] year?) Bernoulli constructs a power series to calculate the answer, and then writes: *" … quæ nostra serie [mathematical expression for a geometric series] &c. major est. … si* a=b*, debebitur plu quam 2½*a *& minus quam 3*a*."* (… which our series [a geometric series] is larger [than]. … if *a*=*b*, [the lender] will be owed more than 2½*a* and less than 3*a*.) If *a*=*b*, the geometric series reduces to the series for *a* × *e*, so 2.5 < *e* < 3. (** The reference is to a problem which Jacob Bernoulli posed and which appears in the *Journal des Sçavans* of 1685 at the bottom of page 314.)
8. ^ Carl Boyer; Uta Merzbach (1991). *A History of Mathematics* (2nd ed.). Wiley. p. 419.
9. ^ Lettre XV. Euler à Goldbach, dated November 25, 1731 in: P. H. Fuss, ed., *Correspondance Mathématique et Physique de Quelques Célèbres Géomètres du XVIIIeme Siècle* … (Mathematical and physical correspondence of some famous geometers of the 18th century), vol. 1, (St. Petersburg, Russia: 1843), pp. 56-60 ; see especially page 58. From page 58: *" … (e denotat hic numerum, cujus logarithmus hyperbolicus est = 1), … "* (… (e denotes that number whose hyperbolic [i.e., natural] logarithm is equal to 1) …)
10. ^ Remmert, Reinhold (1991). *Theory of Complex Functions*. Springer-Verlag. p. 136. ISBN 0-387-97195-5
11. ^ Euler, *Meditatio in experimenta explosione tormentorum nuper instituta*.
12. ^ Leonhard Euler, *Mechanica, sive Motus scientia analytice exposita* (St. Petersburg (Petropoli), Russia: Academy of Sciences, 1736), vol. 1, Chapter 2, Corollary 11, paragraph 171, p. 68. From page 68: *Erit enim* $\frac{dc}{c} = \frac{dyds}{rdx}$ *seu* $c = e^{\int \frac{dyds}{rdx}}$ *ubi* e *denotat numerum, cuius logarithmus hyperbolicus est 1.* (So it [i.e., *c*, the speed] will be $\frac{dc}{c} = \frac{dyds}{rdx}$ or $c = e^{\int \frac{dyds}{rdx}}$, where *e* denotes the number whose hyperbolic [i.e., natural] logarithm is 1.)
13. ^ Grinstead, C.M. and Snell, J.L.*Introduction to probability theory* (published online under the GFDL), p. 85.
14. ^ Knuth (1997) *The Art of Computer Programming* Volume I, Addison-Wesley, p. 183 ISBN 0-201-03801-3.
15. ^ Kline, M. (1998) *Calculus: An intuitive and physical approach*, section 12.3 "The Derived Functions of Logarithmic Functions.", pp. 337 ff, Courier Dover Publications, 1998, ISBN 0-486-40453-6
16. ^ This is the approach taken by Kline (1998).
17. ^ More generally, by the substitution $t = u^n$, $\int_1^{e^n} \frac{1}{t}\, dt = n \int_1^e \frac{du}{u} = n.$
18. ^ Dorrie, Heinrich (1965). *100 Great Problems of Elementary Mathematics*. Dover. pp. 44–48.
19. ^ Dorrie, Heinrich (1965). *100 Great Problems of Elementary Mathematics*. Dover. p. 359.
20. ^ Euler, L. "De serie Lambertina Plurimisque eius insignibus proprietatibus." *Acta Acad. Scient. Petropol. 2*, 29–51, 1783. Reprinted in Euler, L. *Opera Omnia, Series Prima, Vol. 6: Commentationes Algebraicae*. Leipzig, Germany: Teubner, pp. 350–369, 1921. (facsimile)
21. ^ Sandifer, Ed (Feb 2006). "How Euler Did It: Who proved e is Irrational?" (PDF). MAA Online. Retrieved 2010-06-18.
22. ^ Hofstadter, D. R., "Fluid Concepts and Creative Analogies: Computer Models of the Fundamental Mechanisms of Thought" Basic Books (1995) ISBN 0-7139-9155-0
23. ^ Russell, K. G. (1991) *Estimating the Value of e by Simulation* The American Statistician, Vol. 45, No. 1. (Feb., 1991), pp. 66–68.

1991), pp. 60–68.
24. ^ Dinov, ID (2007) *Estimating e using SOCR simulation*, SOCR Hands-on Activities (retrieved December 26, 2007).
25. ^ Sebah, P. and Gourdon, X.; The constant e and its computation
26. ^ Gourdon, X.; Reported large computations with PiFast
27. ^ Roger Cotes (1714) "Logometria," *Philosophical Transactions of the Royal Society of London*, **29** (338) : 5-45; see especially the bottom of page 10. From page 10: *"Porro eadem ratio est inter 2,718281828459 &c et 1, … "* (Furthermore, by the same means, the ratio is between 2.718281828459… and 1, …)
28. ^ Leonhard Euler, *Introductio in Analysin Infinitorum* (Lausanne, Switzerland: Marc Michel Bousquet & Co., 1748), volume 1, page 90.
29. ^ William Shanks, *Contributions to Mathematics*, … (London, England: G. Bell, 1853), page 89.
30. ^ William Shanks (1871) "On the numerical values of *e*, $\log_e 2$, $\log_e 3$, $\log_e 5$, and $\log_e 10$, also on the numerical value of M the modulus of the common system of logarithms, all to 205 decimals," *Proceedings of the Royal Society of London*, **20** : 27-29.
31. ^ J. Marcus Boorman (October 1884) "Computation of the Naperian base," *Mathematical Magazine*, **1** (12) : 204-205.
32. ^ Daniel Shanks and John W Wrench (1962). "Calculation of Pi to 100,000 Decimals" (PDF). *Mathematics of Computation*. **16** (77): 76–99 (78). doi:10.2307/2003813. "We have computed e on a 7090 to 100,265D by the obvious program"
33. ^ Wozniak, Steve (June 1981). "The Impossible Dream: Computing *e* to 116,000 Places with a Personal Computer". *BYTE*. p. 392. Retrieved 18 October 2013.
34. ^ Alexander Yee. "e".
35. ^ First 10-digit prime found in consecutive digits of e}. Brain Tags. Retrieved on 2012-02-24.
36. ^ Shea, Andrea. "Google Entices Job-Searchers with Math Puzzle". *NPR*. Retrieved 2007-06-09.

Further reading [edit]

Maor, Eli; *e: The Story of a Number*, ISBN 0-691-05854-7

Commentary on Endnote 10 of the book *Prime Obsession* for another stochastic representation

McCartin, Brian J. (2006). "e: The Master of All" (PDF). *The Mathematical Intelligencer*. **28** (2): 10–21. doi:10.1007/bf02987150.

External links [edit]

An Intuitive Guide To Exponential Functions & e for the non-mathematician

The number e to 1 million places and 2 and 5 million places (link obsolete)

e Approximations – Wolfram MathWorld

Earliest Uses of Symbols for Constants Jan. 13, 2008

"The story of e", by Robin Wilson at Gresham College, 28 February 2007 (available for audio and video download)

e Search Engine 2 billion searchable digits of e, π and $\sqrt{2}$

Wikimedia Commons has media related to ***E (mathematical constant)***.

Wikiquote has quotations related to: ***E (mathematical constant)***

v · t · e **Irrational numbers**

Prime (ρ) · Euler–Mascheroni (γ) · Logarithm of 2 · Twelfth root of two ($\sqrt[12]{2}$) · Apéry's ($\zeta(3)$) · Sophomore's dream · Plastic (ρ) · Square root of 2 ($\sqrt{2}$) · Erdős–Borwein (E) · Golden ratio (φ) · Square root of 3 ($\sqrt{3}$) · Square root of 5 ($\sqrt{5}$) · Silver ratio (δ_S) · Second Feigenbaum (α) · **Euler's** (e) · Pi (π) · First Feigenbaum (δ)

Schizophrenic · Transcendental · Trigonometric

Categories: Transcendental numbers | Mathematical constants | E (mathematical constant) | Real transcendental numbers | Leonhard Euler