University of Tehran

Department of Electrical and Computer Engineering (ECE)

Data and Information fusion

Assignment No.3

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Majors: Electrical Engineering and Computer Engineering

Fields: Artificial Intelligence in CE and Control in EE

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April 27, 2021

Part 1

❖ Section A

First, let me calculate λ parameter by $\mathrm{g}(x_i)$ values.

$$\lambda + 1 = \prod_{i=1}^{4} \left(1 + \lambda * \mathrm{g}(x_i)\right)$$

$$\Rightarrow \lambda + 1 = (1 + 0.3\lambda) * (1 + 0.6\lambda) * (1 + 0.7\lambda) * (1 + 0.3\lambda)$$

Let us solve above equation with the help of matlab tool. The result is shown below.

```
close;
clear;
clc;
```

Definition of variable lambda

```
syms x
```

Part 1-a

```
C = coeffs(expand((1+0.3*x)*(1+0.6*x)*(1+0.7*x)*(1+0.3*x) - x - 1),'All');
digits(6)
r = vpa(roots(C))
```

```
r =

                    0
            -0.914477
 - 4.42371 - 2.54302i
 - 4.42371 + 2.54302i
```

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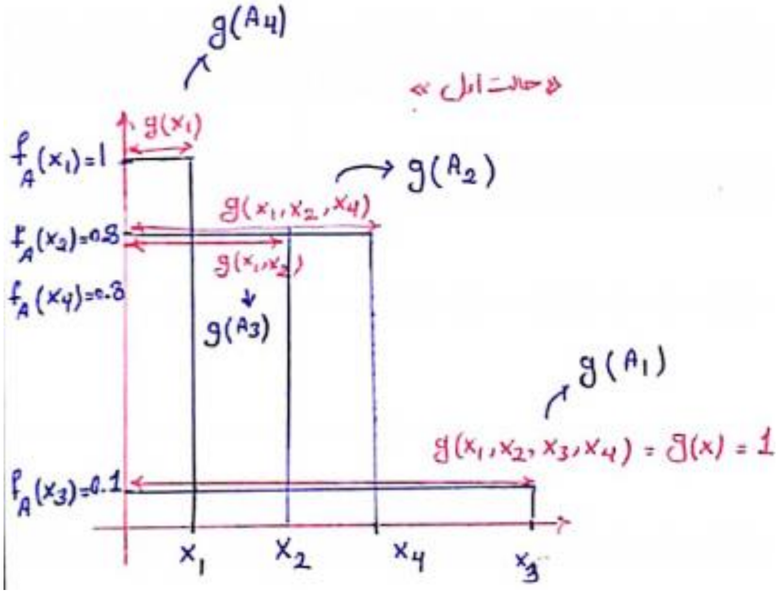
Paying attention to the sum of $\mathrm{g}(x_i)$'s, which is higher than one, we conclude the acceptable value of Lambda is approximately equal to -0.914 .

Assigning Sugeno technic to movie A



$\mathrm{g}(A_4) = \mathrm{g}(x_1) = 0.3$

$\mathrm{g}(A_3) = \mathrm{g}(x_1, x_2) = 0.3 + 0.6 + (-0.914) * (0.3) * (0.6) = 0.735$

$\mathrm{g}(A_2) = 0.735 + 0.3 + (-0.914) * (0.735) * (0.3) = 0.833$

$\mathrm{g}(A_1) = 1$

$S_A = \max[\min(0.1\,, 1)\,, \min(0.8\,, 0.833)\,, \min(0.8\,, 0.735)\,, \min(1\,, 0.3)] = 0.8$

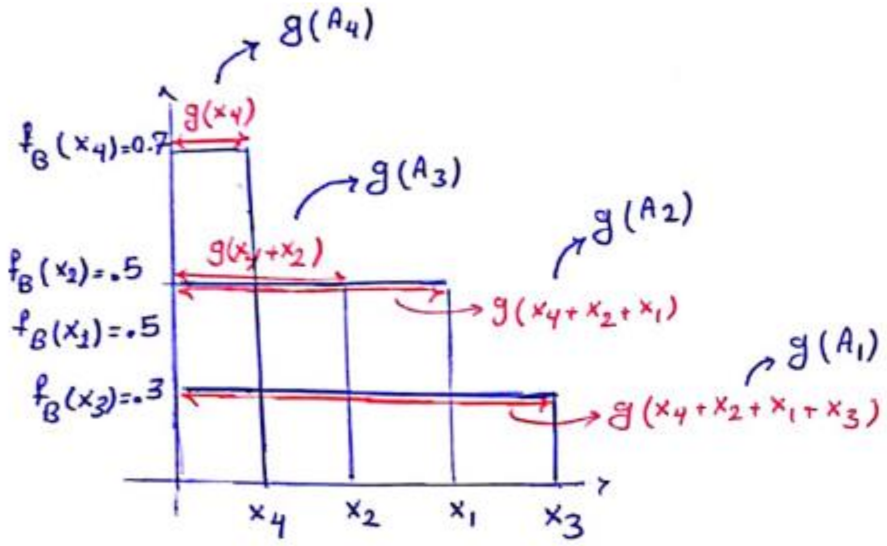
Assigning Choquet technic to movie A

$$C = \sum_{i=1}^{4} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.1 - 0] * 1 + [0.8 - 0.1] * 0.833 + [0.8 - 0.8] * 0.735 + [1 - 0.8] * 0.3$

$\Rightarrow C_A = 0.743$

Assigning Sugeno technic to movie B



$\mathrm{g}(A_4) = \mathrm{g}(x_4) = 0.3$

$\mathrm{g}(A_3) = \mathrm{g}(x_4, x_2) = 0.3 + 0.6 + (-0.914) * (0.3) * (0.6) = 0.735$

$\mathrm{g}(A_2) = 0.735 + 0.3 + (-0.914) * (0.735) * (0.3) = 0.833$

$\mathrm{g}(A_1) = 1$

$S_B = \max[\min(0.3\,, 1)\,, \min(0.5\,, 0.833)\,, \min(0.5\,, 0.735)\,, \min(0.7\,, 0.3)] = 0.5$

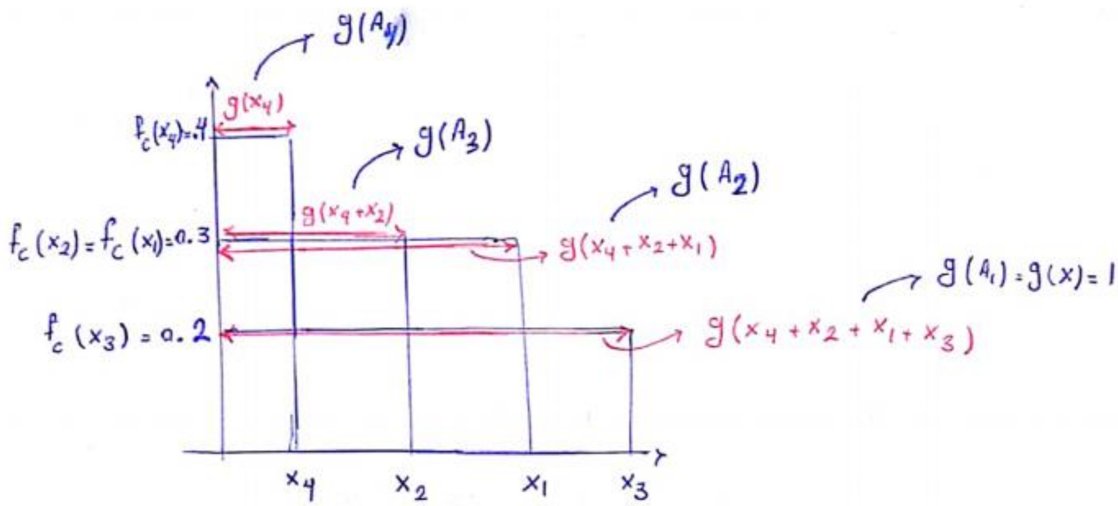
Assigning Choquet technic to movie B

$$C = \sum_{i=1}^{4} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.3 - 0] * 1 + [0.5 - 0.3] * 0.833 + [0.5 - 0.5] * 0.735 + [0.7 - 0.5] * 0.3$

$\Rightarrow C_B = 0.5266$

Assigning Sugeno technic to movie C



$\mathrm{g}(A_4) = \mathrm{g}(x_4) = 0.3$

$\mathrm{g}(A_3) = \mathrm{g}(x_4, x_2) = 0.3 + 0.6 + (-0.914) * (0.3) * (0.6) = 0.735$

$\mathrm{g}(A_2) = 0.735 + 0.3 + (-0.914) * (0.735) * (0.3) = 0.833$

$\mathrm{g}(A_1) = 1$

$S_C = \max[\min(0.2\ , 1)\ , \min(0.3, 0.833)\ , \min(0.3\ , 0.735)\ , \min(0.4\ , 0.3)] = 0.3$

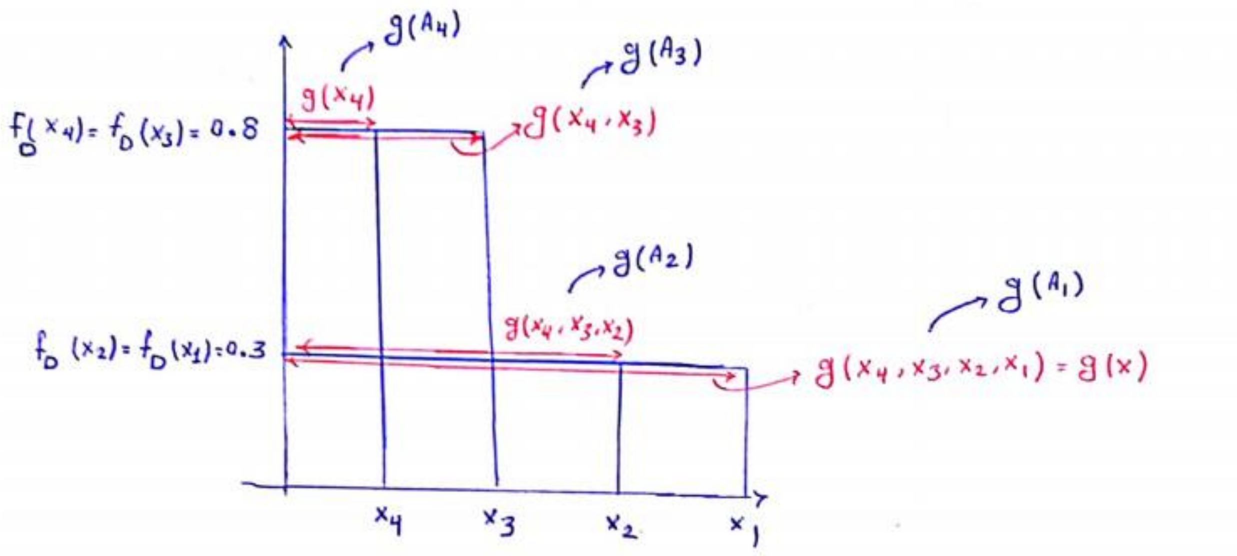
Assigning Choquet technic to movie C

$$C = \sum_{i=1}^{4} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.2 - 0] * 1 + [0.3 - 0.2] * 0.833 + [0.3 - 0.3] * 0.735 + [0.4 - 0.3] * 0.3$

$\Rightarrow C_C = 0.3133$

Assigning Sugeno technic to movie D



$g(A_4) = g(x_4) = 0.3$

$g(A_3) = g(x_4, x_3) = 0.3 + 0.7 + (-0.914) * (0.3) * (0.7) = 0.808$

$g(A_2) = 0.808 + 0.6 + (-0.914) * (0.808) * (0.6) = 0.965$

$g(A_1) = 1$

$S_D = \max[\min(0.3\,, 1)\,, \min(0.3, 0.965)\,, \min(0.8\,, 0.808)\,, \min(0.8\,, 0.3)] = 0.8$

Assigning Choquet technic to movie D

$$C = \sum_{i=1}^{4} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$\Rightarrow C = [0.3 - 0] * 1 + [0.3 - 0.3] * 0.965 + [0.8 - 0.3] * 0.808 + [0.8 - 0.8] * 0.3$

$\Rightarrow C_D = 0.704$

Comparison

	Movie A	Movie B	Movie C	Movie D
Sugeno	0.8	0.5	0.3	0.8
Choquet	0.743	0.5266	0.3133	0.704

Ranking:

- Sugeno
  - ✓ Rank 1: Movie A & Movie D
  - ✓ Rank 2: Movie B
  - ✓ Rank 3: Movie C
- Choquet
  - ✓ Rank 1: Movie A
  - ✓ Rank 2: Movie D
  - ✓ Rank 2: Movie B
  - ✓ Rank 3: Movie C

- **Section B**

Now, let us calculate λ parameter for this section by $g(x_i)$ values.

$$\lambda + 1 = \prod_{i=1}^{2}\left(1 + \lambda * g(x_i)\right)$$

$$\Rightarrow \lambda + 1 = (1 + 0.3\lambda) * (1 + 0.6\lambda)$$

In order to solve above equation, I get help from matlab tool as shown below.

Part 1-b

```
C_b = coeffs(expand((1+0.3*x)*(1+0.6*x) - x - 1),'All');
digits(6)
r_b = vpa(roots(C_b))
```

```
r_b =
 
        0
 0.555556
```

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Paying attention to the sum of $\mathrm{g}(x_i)$'s, which is lower than one, we conclude the acceptable value of Lambda is approximately equal to 0.555 .

Assigning Sugeno technic to movie A

$\mathrm{g}(A_2) = \mathrm{g}(x_1) = 0.3$

$\mathrm{g}(A_1) = 1$

$S_A = \max[\min(0.8\,,1)\,,\min(1\,,0.3)] = 0.8$

Assigning Choquet technic to movie A

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.8 - 0] * 1 + [1 - 0.8] * 0.3$

$\Rightarrow C_A = 0.86$

Assigning Sugeno technic to movie B

$\mathrm{g}(A_2) = \mathrm{g}(x_2) = 0.6$

$\mathrm{g}(A_1) = 1$

$S_B = \max[\min(0.5\,,1)\,,\min(0.5\,,0.6)] = 0.5$

Assigning Choquet technic to movie B

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.5 - 0] * 1 + [0.5 - 0.5] * 0.6$

$\Rightarrow C_B = 0.5$

Assigning Sugeno technic to movie C

$\mathrm{g}(A_2) = \mathrm{g}(x_2) = 0.6$

$\mathrm{g}(A_1) = 1$

$S_C = \max[\min(0.3\,,1)\,,\min(0.3, 0.6)] = 0.3$

Assigning Choquet technic to movie C

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$$\Rightarrow C = [0.3 - 0] * 1 + [0.3 - 0.3] * 0.6$$

$$\Rightarrow C_C = 0.3$$

Assigning Sugeno technic to movie D

$$g(A_2) = g(x_2) = 0.6$$

$$g(A_1) = 1$$

$$S_D = \max[\min(0.3\,, 1)\,, \min(0.3, 0.6)] = 0.3$$

Assigning Choquet technic to movie D

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$$\Rightarrow C = [0.3 - 0] * 1 + [0.3 - 0.3] * 0.6$$

$$\Rightarrow C_D = 0.3$$

Comparison

	Movie A	Movie B	Movie C	Movie D
Sugeno	0.8	0.5	0.3	0.3
Choquet	0.86	0.5	0.3	0.3

Ranking:

- Sugeno
  - ✓ Rank 1: Movie A
  - ✓ Rank 2: Movie B
  - ✓ Rank 3: Movie C & Movie D
- Choquet
  - ✓ Rank 1: Movie A
  - ✓ Rank 2: Movie B
  - ✓ Rank 3: Movie C & Movie D

- **Section C**

Now, let us calculate λ parameter for this section by $\mathrm{g}(x_i)$ values.

$$\lambda + 1 = \prod_{i=1}^{2} \left(1 + \lambda * \mathrm{g}(x_i)\right)$$

$$\Rightarrow \lambda + 1 = (1 + 0.7\lambda) * (1 + 0.3\lambda)$$

In order to solve above equation, I get help from matlab tool as shown below.

Part 1-c

```
C_c = coeffs(expand((1+0.7*x)*(1+0.3*x) - x - 1),'All');
digits(6)
r_c = vpa(roots(C_c))
```

```
r_c =

 0
 0
```

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Paying attention to the sum of $\mathrm{g}(x_i)$'s, which is equal to one, we conclude the acceptable value of Lambda is equal to zero .

Assigning Sugeno technic to movie A

$$\mathrm{g}(A_2) = \mathrm{g}(x_4) = 0.3$$

$$\mathrm{g}(A_1) = 1$$

$$S_A = \max[\min(0.1\,,1)\,,\min(0.8\,,0.3)] = 0.3$$

Assigning Choquet technic to movie A

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$$\Rightarrow C = [0.1 - 0] * 1 + [0.8 - 0.1] * 0.3$$

$$\Rightarrow C_A = 0.31$$

Assigning Sugeno technic to movie B

$$\mathrm{g}(A_2) = \mathrm{g}(x_4) = 0.3$$

$g(A_1) = 1$

$S_B = \max[\min(0.3\,, 1)\,, \min(0.7\,, 0.3)] = 0.3$

Assigning Choquet technic to movie B

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$\Rightarrow C = [0.3 - 0] * 1 + [0.7 - 0.3] * 0.3$

$\Rightarrow C_B = 0.42$

Assigning Sugeno technic to movie C

$g(A_2) = g(x_4) = 0.3$

$g(A_1) = 1$

$S_C = \max[\min(0.2\,, 1)\,, \min(0.4, 0.3)] = 0.3$

Assigning Choquet technic to movie C

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$\Rightarrow C = [0.2 - 0] * 1 + [0.4 - 0.2] * 0.3$

$\Rightarrow C_C = 0.26$

Assigning Sugeno technic to movie D

$g(A_2) = g(x_4) = 0.3$

$g(A_1) = 1$

$S_D = \max[\min(0.8\,, 1)\,, \min(0.8, 0.3)] = 0.8$

Assigning Choquet technic to movie D

$$C = \sum_{i=1}^{2} [f(\widetilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$\Rightarrow C = [0.8 - 0] * 1 + [0.8 - 0.8] * 0.3$

$\Rightarrow C_D = 0.8$

Comparison

	Movie A	Movie B	Movie C	Movie D
Sugeno	0.3	0.3	0.3	0.8
Choquet	0.31	0.42	0.26	0.8

Ranking:

- Sugeno
  - ✓ Rank 1: Movie D
  - ✓ Rank 2: Movie A & Movie B & Movie C
- Choquet
  - ✓ Rank 1: Movie D
  - ✓ Rank 2: Movie B
  - ✓ Rank 3: Movie A
  - ✓ Rank 4: Movie C

- **Section D**

Now, let us calculate λ parameter for this section by $g(x_i)$ values.

$$\lambda + 1 = \prod_{i=1}^{2}\left(1 + \lambda * g(x_i)\right)$$

$$\Rightarrow \lambda + 1 = (1 + 0.6\lambda) * (1 + 0.7\lambda)$$

In order to solve above equation, I get help from matlab tool as shown below.

Part 1-c

```
C_d = coeffs(expand((1+0.6*x)*(1+0.7*x) - x - 1),'All');
digits(6)
r_d = vpa(roots(C_d))
```

```
r_d =

         0
 -0.714286
```

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Paying attention to the sum of g(x_i)'s, which is higher than one, we conclude the acceptable value of Lambda is approximately equal to -0.714 .

Assigning Sugeno technic to movie A

$\mathrm{g}(A_2) = \mathrm{g}(x_2) = 0.6$

$\mathrm{g}(A_1) = 1$

$S_A = \max[\min(0.1\,,1)\,,\min(0.8\,,0.6)] = 0.6$

Assigning Choquet technic to movie A

$$C = \sum_{i=1}^{2} [f(\tilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.1 - 0] * 1 + [0.8 - 0.1] * 0.6$

$\Rightarrow C_A = 0.52$

Assigning Sugeno technic to movie B

$\mathrm{g}(A_2) = \mathrm{g}(x_2) = 0.6$

$\mathrm{g}(A_1) = 1$

$S_B = \max[\min(0.3\,,1)\,,\min(0.5\,,0.6)] = 0.5$

Assigning Choquet technic to movie B

$$C = \sum_{i=1}^{2} [f(\tilde{x_i}) - f(\widetilde{x_{i-1}})] * \mathrm{g}(A_i)$$

$\Rightarrow C = [0.3 - 0] * 1 + [0.5 - 0.3] * 0.6$

$\Rightarrow C_B = 0.42$

Assigning Sugeno technic to movie C

$\mathrm{g}(A_2) = \mathrm{g}(x_2) = 0.6$

$\mathrm{g}(A_1) = 1$

$S_C = \max[\min(0.2\,,1)\,,\min(0.3, 0.6)] = 0.3$

Assigning Choquet technic to movie C

$$C = \sum_{i=1}^{2} [f(\tilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$$\Rightarrow C = [0.2 - 0] * 1 + [0.3 - 0.2] * 0.6$$

$$\Rightarrow C_C = 0.26$$

Assigning Sugeno technic to movie D

$$g(A_2) = g(x_3) = 0.7$$

$$g(A_1) = 1$$

$$S_D = \max[\min(0.3\,, 1)\,, \min(0.8, 0.7)] = 0.7$$

Assigning Choquet technic to movie D

$$C = \sum_{i=1}^{2} [f(\tilde{x_i}) - f(\widetilde{x_{i-1}})] * g(A_i)$$

$$\Rightarrow C = [0.3 - 0] * 1 + [0.8 - 0.3] * 0.7$$

$$\Rightarrow C_D = 0.65$$

Comparison

	Movie A	Movie B	Movie C	Movie D
Sugeno	0.6	0.5	0.3	0.7
Choquet	0.52	0.42	0.26	0.65

Ranking:

- Sugeno
  - Rank 1: Movie D
  - Rank 2: Movie A
  - Rank 2: Movie B
  - Rank 2: Movie C
- Choquet
  - Rank 1: Movie D
  - Rank 2: Movie A
  - Rank 3: Movie B
  - Rank 4: Movie C

Part 2

We define two functions named Sugeno and Choquet and tset them as shown below:

- ✓ Sugeno tests

```
✓ clear;close;clc;

colNames = {'Movie_A' 'Movie_B' 'Movie_C' 'Movie_D'};
rowNames = {'1st_section', '2nd_section', '3rd_section3', '4th_section4'};
Table = array2table([Sugeno([0.3,0.6,0.7,0.3],[1.0,0.8,0.1,0.8]),...
    Sugeno([0.3,0.6,0.7,0.3],[0.5,0.5,0.3,0.7]),...
    Sugeno([0.3,0.6,0.7,0.3],[0.3,0.3,0.2,0.4]),...
    Sugeno([0.3,0.6,0.7,0.3],[0.3,0.3,0.8,0.8]);...
    Sugeno([0.3,0.6],[1.0,0.8]),...
    Sugeno([0.3,0.6],[0.5,0.5]),...
    Sugeno([0.3,0.6],[0.3,0.3]),...
    Sugeno([0.3,0.6],[0.3,0.3]);...
    Sugeno([0.7,0.3],[0.1,0.8]),...
    Sugeno([0.7,0.3],[0.3,0.7]),...
    Sugeno([0.7,0.3],[0.2,0.4]),...
    Sugeno([0.7,0.3],[0.8,0.8]);...
    Sugeno([0.6,0.7],[0.8,0.1]),...
    Sugeno([0.6,0.7],[0.5,0.3]),...
    Sugeno([0.6,0.7],[0.3,0.2]),...
    Sugeno([0.6,0.7],[0.3,0.8]),...
    ],'VariableNames',colNames,'RowNames',rowNames)


function res = Sugeno(mu, f)
syms x

n = size(mu,2);

lambdaCalFunc = 1;
for i=1:1:n
     lambdaCalFunc = lambdaCalFunc*(1+ x*mu(1,i));
end
C = coeffs(expand(lambdaCalFunc - x - 1),'All');
digits(6)
r = vpa(roots(C));
mr = size(r,1);

lambda = 0;
if sum(mu) == 1
    lambda = 0;
elseif sum(mu) > 1
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>-1 && r(i,1)<0
            lambda = r(i,1);
        end
```

```
        end
    else
        for i=1:1:mr
             if isreal(r(i,1)) && r(i,1)>0
                  lambda = r(i,1);
             end
        end
    end

    F = [mu' f'];
    A = zeros(n,2);
    for i=1:1:n
        [a, index]=max(F(:,2));
        A(i,:) = [F(index,1) a];
        F(index,:)=[];
    end

    GAi = zeros(n,2);
    Gi = 0;
    for i=1:1:n

        Gi = Gi + A(i,1) + lambda*Gi*A(i,1);

        GAi(i,:) = [Gi, A(i,2)];
    end

    minComparison = ones(n,1);
    for i=1:1:n
       minComparison(i,1) = min(minComparison(i,1),min(GAi(n-i+1,:)));
    end
    res = max(minComparison);

    end
```

✓

```
Table =

  4x4 table

                      Movie_A    Movie_B    Movie_C    Movie_D
                      _______    _______    _______    _______

    1st_section         0.8        0.5        0.3        0.8
    2nd_section         0.8        0.5        0.3        0.3
    3rd_section3        0.3        0.3        0.3        0.8
    4th_section4        0.6        0.5        0.3        0.7
```

✓ *Published with MATLAB® R2018a*

As you see above, all the answers of sugeno approach for solve the problem of all sections are exactly the same as what we manually calculated.

✓ Choquet tests

```
✓ clear;close;clc;

colNames = {'Movie_A' 'Movie_B' 'Movie_C' 'Movie_D'};
rowNames = {'1st_section', '2nd_section', '3rd_section3', '4th_section4'};
Table = array2table([Choquet([0.3,0.6,0.7,0.3],[1.0,0.8,0.1,0.8]),...
    Choquet([0.3,0.6,0.7,0.3],[0.5,0.5,0.3,0.7]),...
    Choquet([0.3,0.6,0.7,0.3],[0.3,0.3,0.2,0.4]),...
    Choquet([0.3,0.6,0.7,0.3],[0.3,0.3,0.8,0.8]);...
    Choquet([0.3,0.6],[1.0,0.8]),...
    Choquet([0.3,0.6],[0.5,0.5]),...
    Choquet([0.3,0.6],[0.3,0.3]),...
    Choquet([0.3,0.6],[0.3,0.3]);...
    Choquet([0.7,0.3],[0.1,0.8]),...
    Choquet([0.7,0.3],[0.3,0.7]),...
    Choquet([0.7,0.3],[0.2,0.4]),...
    Choquet([0.7,0.3],[0.8,0.8]);...
    Choquet([0.6,0.7],[0.8,0.1]),...
    Choquet([0.6,0.7],[0.5,0.3]),...
    Choquet([0.6,0.7],[0.3,0.2]),...
    Choquet([0.6,0.7],[0.3,0.8]),...
    ],'VariableNames',colNames,'RowNames',rowNames)


function res = Choquet(mu, f)
syms x

n = size(mu,2);

lambdaCalFunc = 1;
for i=1:1:n
     lambdaCalFunc = lambdaCalFunc*(1+ x*mu(1,i));
end
C = coeffs(expand(lambdaCalFunc - x - 1),'All');
digits(6)
r = vpa(roots(C));
mr = size(r,1);

lambda = 0;
if sum(mu) == 1
    lambda = 0;
elseif sum(mu) > 1
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>-1 && r(i,1)<0
             lambda = r(i,1);
        end
   end
else
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>0
             lambda = r(i,1);
        end
```

```
        end
    end

    F = [mu' f'];
    A = zeros(n,2);
    for i=1:1:n
        [a, index]=max(F(:,2));
        A(i,:) = [F(index,1) a];
        F(index,:)=[];
    end

    GAi = zeros(n,2);
    Gi = 0;
    for i=1:1:n

        Gi = Gi + A(i,1) + lambda*Gi*A(i,1);

        GAi(i,:) = [Gi, A(i,2)];
    end

    result = 0;
    previousVal = 0;
    for i=1:1:n
       result = result+(GAi(n-i+1,2) - previousVal)*GAi(n-i+1,1);
       previousVal = GAi(n-i+1,2);
    end
    res = result;

    end
```

✓

```
Table =

  4x4 table
```

	Movie_A	Movie_B	Movie_C	Movie_D
1st_section	0.74355	0.52673	0.31336	0.70398
2nd_section	0.86	0.5	0.3	0.3
3rd_section3	0.31	0.42	0.26	0.8
4th_section4	0.52	0.42	0.26	0.65

✓ *Published with MATLAB® R2018a*

As you see above, all the answers of sugeno approach for solve the problem of all sections are approximately the same as what we manually calculated.

It's time to compare the results of 1st part vs 2nd part.

Comparison Table

	Movie	Approach	Result of part 1	Result of matlab
Section A	A	Sugeno	0.8	0.8
		Choquet	0.743	0.74355
	B	Sugeno	0.5	0.5
		Choquet	0.5266	0.52673
	C	Sugeno	0.3	0.3
		Choquet	0.3133	0.31336
	D	Sugeno	0.8	0.8
		Choquet	0.704	0.70398
Section B	A	Sugeno	0.8	0.8
		Choquet	0.86	0.86
	B	Sugeno	0.5	0.5
		Choquet	0.5	0.5
	C	Sugeno	0.3	0.3
		Choquet	0.3	0.3
	D	Sugeno	0.3	0.3
		Choquet	0.3	0.3
Section C	A	Sugeno	0.3	0.3
		Choquet	0.31	0.31
	B	Sugeno	0.3	0.3
		Choquet	0.42	0.42
	C	Sugeno	0.3	0.3
		Choquet	0.26	0.26
	D	Sugeno	0.8	0.8
		Choquet	0.8	0.8
Section D	A	Sugeno	0.6	0.6
		Choquet	0.52	0.52
	B	Sugeno	0.5	0.5
		Choquet	0.42	0.42
	C	Sugeno	0.3	0.3
		Choquet	0.26	0.26
	D	Sugeno	0.7	0.7
		Choquet	0.65	0.65

As we see the above table, all the results of Sugeno are exactly the same in both manual and matlab calculation. Also, this is true for Choquet approach but matlab calculates the answers of these approach with more accuracy.

- Report codes
  - Sugeno

First, I define variable x as the lambda parameter in the equation $\lambda + 1 = \prod_{i=1}^{n}\left(1 + \lambda * g(x_i)\right)$. So now we have $x + 1 = \prod_{i=1}^{4}\left(1 + x * g(x_i)\right)$ instead of previous equation.

```
function res = Sugeno(mu, f)
syms x

```

Then found the column size of f (or mu) array by the command as shown below.

```
n = size(mu,2);

```

It's time to form the phrase $\prod_{i=1}^{n}\left(1 + x * g(x_i)\right)$.

```
lambdaCalFunc = 1;
for i=1:1:n
      lambdaCalFunc = lambdaCalFunc*(1+ x*mu(1,i));
end
```

Then we calculated all the answers of the equation $x + 1 = \prod_{i=1}^{4}\left(1 + x * g(x_i)\right)$ and also calculate the size of answer vector in order to find the correct parameter of lambda by calculation g's summation.

```
C = coeffs(expand(lambdaCalFunc - x - 1),'All');
digits(6)
r = vpa(roots(C));
mr = size(r,1);
```

We are ready to calculate the parameter lambda.

```
lambda = 0;
if sum(mu) == 1
    lambda = 0;
elseif sum(mu) > 1
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>-1 && r(i,1)<0
            lambda = r(i,1);
        end
   end
else
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>0
            lambda = r(i,1);
        end
   end
end
```

It’s exactly the same as what T.A told us. Here I remind it.

$$\sum_{i=1}^{n} g_{\lambda}(x_i) = 1 \Rightarrow \lambda = 0$$

$$\sum_{i=1}^{n} g_{\lambda}(x_i) < 1 \Rightarrow \lambda > 0$$

$$\sum_{i=1}^{n} g_{\lambda}(x_i) > 1 \Rightarrow 0 > \lambda > -1$$

As we see here, if summation of g’s is equal to one, lambda parameter should be exactly equal to zero.

```
16 -  if sum(mu) == 1
17 -      lambda = 0;
```

Else if the summation is higher than one, lambda should be between -1 and 0.

```
18 -  elseif sum(mu) > 1
19 -      for i=1:1:mr
20 -          if isreal(r(i,1)) && r(i,1)>-1 && r(i,1)<0
21 -              lambda = r(i,1);
22 -          end
23 -      end
```

Finally, if the summation is lower than one, the parameter should be higher than zero.

```
24 -  else
25 -      for i=1:1:mr
26 -          if isreal(r(i,1)) && r(i,1)>0
27 -              lambda = r(i,1);
28 -          end
29 -      end
30 -  end
```

Adding the condition isreal() help us throw out the answers which are not real.

Sorting fs (matrix F(:,2)) and keeping it’s g, place in a new matrix named A. I use max function for this section in order to find the highest f and it’s index for find it’s g then delete this item in line 37 and act on the rest of matrix. Then create and place the vector in the new matrix (A).

```
32 -  F = [mu' f'];
33 -  A = zeros(n,2);
34 -  for i=1:1:n
35 -      [a, index]=max(F(:,2));
36 -      A(i,:) = [F(index,1) a];
37 -      F(index,:)=[];
38 -  end
39
```

Now we can calculate $G(A_i)$s as shown below.

```
40  GAi = zeros(n,2);
41  Gi = 0;
42  for i=1:1:n
43      Gi = Gi + A(i,1) + lambda*Gi*A(i,1);
44      GAi(i,:) = [Gi, A(i,2)];
45  end
46
```

GAi indicates $G(A_i)$s and Gi is the previous value of GAi which equal to $G(A_{i-1})$ for $i>1$ and is equal to zero for $i=1$.

Finally we are able to use Sugeno formula as below.

```
47  minComparison = ones(n,1);
48  for i=1:1:n
49      minComparison(i,1) = min(minComparison(i,1),min(GAi(n-i+1,:)));
50  end
51  res = max(minComparison);
52
```

As you see here, I create a vector named minComparison which valued 1 for all of its elements which is the highest possible amount. And then compare it to f_i and it's $G(A_i)$ and then place it in minComparison vector.

Now we find the maximum of minComparison element by using max command and then put it in the res variable of Sugeno function.

```
51  res = max(minComparison);
```

- ✓ Choquet

Explanation of code line 1 to code line 46 is the same as what we said in Sugeno code. So we repeat these lines again.

First, I define variable x as the lambda parameter in the equation $\lambda + 1 = \prod_{i=1}^{n}\left(1 + \lambda * g(x_i)\right)$. So now we have $x + 1 = \prod_{i=1}^{n}\left(1 + x * g(x_i)\right)$ instead of previous equation.

```
1  function res = Choquet(mu, f)
2  syms x
3
```

Then found the columns size of f (or mu) array by the command as shown below.

```
4  n = size(mu,2);
5
```

It’s time to form the phrase $\prod_{i=1}^{4}\left(1 + \mathrm{x} * \mathrm{g}(x_i)\right)$.

```
lambdaCalFunc = 1;
for i=1:1:n
      lambdaCalFunc = lambdaCalFunc*(1+ x*mu(1,i));
end
```

Then we calculated all the answers of the equation $\mathrm{x} + 1 = \prod_{i=1}^{n}\left(1 + \mathrm{x} * \mathrm{g}(x_i)\right)$ and also calculate the size of answer vector in order to find the correct parameter of lambda by calculation g’s summation.

```
C = coeffs(expand(lambdaCalFunc - x - 1),'All');
digits(6)
r = vpa(roots(C));
mr = size(r,1);
```

We are ready to calculate the parameter lambda.

```
lambda = 0;
if sum(mu) == 1
    lambda = 0;
elseif sum(mu) > 1
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>-1 && r(i,1)<0
            lambda = r(i,1);
        end
   end
else
   for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>0
            lambda = r(i,1);
        end
   end
end
```

It’s exactly the same as what T.A told us. Here I remind it.

$$\sum_{i=1}^{n} g_\lambda(x_i) = 1 \implies \lambda = 0$$

$$\sum_{i=1}^{n} g_\lambda(x_i) < 1 \implies \lambda > 0$$

$$\sum_{i=1}^{n} g_\lambda(x_i) > 1 \implies 0 > \lambda > -1$$

As we see here, if summation of g’s is equal to one, lambda parameter should be exactly equal to zero.

```
if sum(mu) == 1
    lambda = 0;
```

Else if the summation is higher than one, lambda should be between -1 and 0.

```
elseif sum(mu) > 1
    for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>-1 && r(i,1)<0
            lambda = r(i,1);
        end
    end
```

Finally, if the summation is lower than one, the parameter should be higher than zero.

```
else
    for i=1:1:mr
        if isreal(r(i,1)) && r(i,1)>0
            lambda = r(i,1);
        end
    end
end
```

Adding the condition isreal() help us throw out the answers which are not real.

Sorting fs (matrix F(:,2)) and keeping it's g, place in a new matrix named A. I use max function for this section in order to find the highest f and it's index for find it's g then delete this item in line 37 and act on the rest of matrix. Then create and place the vector in the new matrix (A).

```
32  F = [mu' f'];
33  A = zeros(n,2);
34  for i=1:1:n
35      [a, index]=max(F(:,2));
36      A(i,:) = [F(index,1) a];
37      F(index,:)=[];
38  end
39
```

Now we can calculate $G(A_i)$s as shown below.

```
GAi = zeros(n,2);
Gi = 0;
for i=1:1:n
    Gi = Gi + A(i,1) + lambda*Gi*A(i,1);
    GAi(i,:) = [Gi, A(i,2)];
end
```

GAi indicates $G(A_i)$s and Gi is the previous value of GAi which equal to $G(A_{i-1})$ for i>1 and is equal to zero for i=1.

Finally we are able to use Choquet formula as below.

```
    result = 0;
    previousVal = 0;
    for i=1:1:n
        result = result+(GAi(n-i+1,2) - previousVal)*GAi(n-i+1,1);
        previousVal = GAi(n-i+1,2);
    end
    res = result;

end
```

As you see here, I use a parameter named result which valued 0 at begin. And then I calculated the answer similar to the formula shown below.

$$C = \int f.dg = \sum_{i=1}^{n} [f(\bar{x}_i) - f(\bar{x}_{i-1})].g(A_i)$$

Also I use previousVal for keeping the value of previous $g(A_i)$ which is equal to zero at begin. Then I put the result in the output variable (res) of Choquet function.

```
    res = result;
```