University of Tehran

Department of Electrical and Computer Engineering (ECE)

Data and Information fusion

Assignment No.2

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Majors: Electrical and Computer Engineering

Field of Study: Artificial Intelligence and Control

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April 8, 2021

1.

Section A]

The table of our Bayesian problem is shown below.

Gas Type	Sensor 1	Sensor 2	Sensor 3
Clean	40%	25%	25%
Hazardous (Level 1)	30%	15%	25%
Hazardous (Level 2)	15%	30%	25%
Hazardous (Level 3)	15%	30%	25%

Let me show the Bayesian formula.

$$p(x_1|Y_1^1Y_1^2Y_1^3) = \frac{p(x_1|Y_1^1) * p(x_1|Y_1^2) * p(x_1|Y_1^3) * p(x_1|Y_0^1Y_0^2Y_0^3)}{p(x_1|Y_0^1) * p(x_1|Y_0^2) * p(x_1|Y_0^3)} * normalization$$

$$p(x_2|Y_1^1Y_1^2Y_1^3) = \frac{p(x_2|Y_1^1) * p(x_2|Y_1^2) * p(x_2|Y_1^3) * p(x_2|Y_0^1Y_0^2Y_0^3)}{p(x_2|Y_0^1) * p(x_2|Y_0^2) * p(x_2|Y_0^3)} * normalization$$

$$p(x_3|Y_1^1Y_1^2Y_1^3) = \frac{p(x_3|Y_1^1) * p(x_3|Y_1^2) * p(x_3|Y_1^3) * p(x_3|Y_0^1Y_0^2Y_0^3)}{p(x_3|Y_0^1) * p(x_3|Y_0^2) * p(x_3|Y_0^3)} * normalization$$

$$p(x_4|Y_1^1Y_1^2Y_1^3) = \frac{p(x_4|Y_1^1) * p(x_4|Y_1^2) * p(x_4|Y_1^3) * p(x_4|Y_0^1Y_0^2Y_0^3)}{p(x_4|Y_0^1) * p(x_4|Y_0^2) * p(x_4|Y_0^3)} * normalization$$

According to problem guidance in WhatsApp group, I use the above formula to calculate the fusion of nodes as below; but before, as you can see here, I estimate $p_{sensors-effect-avg}$ equals to the average of x measured by our sensors.

$$p(x_1|Y_0^1Y_0^2Y_0^3) = p(x_1|Y_0^1) * p(x_1|Y_0^2) * p(x_1|Y_0^3) * p_{sensors-effect-avg}(x_1) * normalization$$

$$\Longrightarrow p(x_1|Y_0^1Y_0^2Y_0^3) = 0.4 * 0.25 * 0.25 * 0.3 * normalization = 0.0075 * normalization$$

$$p(x_2|Y_0^1Y_0^2Y_0^3) = p(x_2|Y_0^1) * p(x_2|Y_0^2) * p(x_2|Y_0^3) * p_{sensors-effect-avg}(x_2) * normalization$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^2Y_0^3) = 0.3 * 0.15 * 0.25 * 0.233 * normalization$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^2Y_0^3) = 0.00262125 * normalization$$

$$p(x_3|Y_0^1Y_0^2Y_0^3) = p(x_3|Y_0^1) * p(x_3|Y_0^2) * p(x_3|Y_0^3) * p_{sensors-effect-avg}(x_3) * normalization$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^2Y_0^3) = 0.15 * 0.3 * 0.25 * 0.233 * normalization$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^2Y_0^3) = 0.00262125 * normalization$$

$$p(x_4|Y_0^1Y_0^2Y_0^3) = p(x_4|Y_0^1) * p(x_4|Y_0^2) * p(x_4|Y_0^3) * p_{sensors-effect-avg}(x_4) * normalization$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^2Y_0^3) = 0.15 * 0.3 * 0.25 * 0.233 * normalization$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^2Y_0^3) = 0.00262125 * normalization$$

Then we solve the solution of fusion nodes by omitting normalization terms and then normalize the probabilities.

$$\Longrightarrow p(x_1|Y_0^1Y_0^2Y_0^3) = \frac{0.0075}{0.0075 + 0.00262125 + 0.00262125 + 0.00262125} = 0.4881620698$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^2Y_0^3) = \frac{0.00262125}{0.0075 + 0.00262125 + 0.00262125 + 0.00262125} = 0.17061264339$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^2Y_0^3) = \frac{0.00262125}{0.0075 + 0.00262125 + 0.00262125 + 0.00262125} = 0.17061264339$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^2Y_0^3) = \frac{0.00262125}{0.0075 + 0.00262125 + 0.00262125 + 0.00262125} = 0.17061264339$$

Having got the above results, we can conclude the air is approximately 49% clean. And it's approximately 17% hazardous polluted by gas type 1. And same as gas type 1, it's hazardous approximately 17% polluted by gas type 2 and also the same approximation of hazardous for gas type 3. So the total danger of gases in air is 51 percent.

Section B]

First we combine sensors one and two and the result should be combined with sensor 3.

$p(x_1|Y_0^1Y_0^2) = p(x_1|Y_0^1) * p(x_1|Y_0^2) * p_{sensors-effect-avg}(x_1) * normalization$

$\Longrightarrow p(x_1|Y_0^1Y_0^2) = 0.4 * 0.25 * 0.325 * normalization = 0.0325 * normalization$

$p(x_2|Y_0^1Y_0^2) = p(x_2|Y_0^1) * p(x_2|Y_0^2) * p_{sensors-effect-avg}(x_2) * normalization$

$\Longrightarrow p(x_2|Y_0^1Y_0^2) = 0.3 * 0.15 * 0.225 * normalization$

$\Longrightarrow p(x_2|Y_0^1Y_0^2) = 0.010125 * normalization$

$p(x_3|Y_0^1Y_0^2) = p(x_3|Y_0^1) * p(x_3|Y_0^2) * p_{sensors-effect-avg}(x_3) * normalization$

$\Longrightarrow p(x_3|Y_0^1Y_0^2) = 0.15 * 0.3 * 0.225 * normalization$

$\Longrightarrow p(x_3|Y_0^1Y_0^2) = 0.010125 * normalization$

$p(x_4|Y_0^1Y_0^2) = p(x_4|Y_0^1) * p(x_4|Y_0^2) * p_{sensors-effect-avg}(x_4) * normalization$

$\Longrightarrow p(x_4|Y_0^1Y_0^2) = 0.15 * 0.3 * 0.225 * normalization$

$\Longrightarrow p(x_4|Y_0^1Y_0^2) = 0.010125 * normalization$

Then we solve the solution of fusion nodes by omitting normalization terms and then normalize the probabilities for 1^{st} and 2^{nd} sensors.

$$\Longrightarrow p(x_1|Y_0^1Y_0^2) = \frac{0.0325}{0.0325 + 0.010125 + 0.010125 + 0.010125} = 0.51689860835$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^2) = \frac{0.010125}{0.0325 + 0.010125 + 0.010125 + 0.010125} = 0.16103379721$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^2) = \frac{0.010125}{0.0325 + 0.010125 + 0.010125 + 0.010125} = 0.16103379721$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^2) = \frac{0.010125}{0.0325 + 0.010125 + 0.010125 + 0.010125} = 0.16103379721$$

Now we combine the above results with sensor 3:

$$p(x_1|Y_0^{1-2}Y_0^3) = p(x_1|Y_0^{1-2}) * p(x_1|Y_0^3) * p_{sensors-effect-avg}(x_1) * normalization$$

$$\Longrightarrow p(x_1|Y_0^{1-2}Y_0^3) = 0.51689860835 * 0.25 * 0.383 * normalization$$

$$\Longrightarrow p(x_1|Y_0^{1-2}Y_0^3) = 0.04949304174 * normalization$$

$$p(x_2|Y_0^{1-2}Y_0^3) = p(x_2|Y_0^{1-2}) * p(x_2|Y_0^3) * p_{sensors-effect-avg}(x_2) * normalization$$

$$\Longrightarrow p(x_2|Y_0^{1-2}Y_0^3) = 0.16103379721 * 0.25 * \; 0.205 * normalization$$

$$\Longrightarrow p(x_2|Y_0^{1-2}Y_0^3) = 0.0082529821 * normalization$$

$$p(x_3|Y_0^{1-2}Y_0^3) = p(x_3|Y_0^{1-2}) * p(x_3|Y_0^3) * p_{sensors-effect-avg}(x_3) * normalization$$

$$\Longrightarrow p(x_3|Y_0^{1-2}Y_0^3) = 0.16103379721 * 0.25 * \; 0.205 * normalization$$

$$\Longrightarrow p(x_3|Y_0^{1-2}Y_0^3) = 0.0082529821 * normalization$$

$$p(x_4|Y_0^{1-2}Y_0^3) = p(x_4|Y_0^{1-2}) * p(x_4|Y_0^3) * p_{sensors-effect-avg}(x_4) * normalization$$

$$\Longrightarrow p(x_4|Y_0^{1-2}Y_0^3) = 0.16103379721 * 0.25 * \; 0.205 * normalization$$

$$\Longrightarrow p(x_4|Y_0^{1-2}Y_0^3) = 0.0082529821 * normalization$$

Now we normalize the probabilities.

$$p(x_1|Y_0^{1-2}Y_0^3) = \frac{0.04949304174}{0.04949304174 + 0.0082529821 + 0.0082529821 + 0.0082529821}$$

$$\Longrightarrow p(x_1|Y_0^{1-2}Y_0^3) = \; 0.66655510574$$

$$p(x_2|Y_0^{1-2}Y_0^3) = \frac{0.0082529821}{0.04949304174 + 0.0082529821 + 0.0082529821 + 0.0082529821}$$

$$\Longrightarrow p(x_2|Y_0^{1-2}Y_0^3) = 0.11114829808$$

$$p(x_3|Y_0^{1-2}Y_0^3) = \frac{0.0082529821}{0.04949304174 + 0.0082529821 + 0.0082529821 + 0.0082529821}$$

$$\Longrightarrow p(x_3|Y_0^{1-2}Y_0^3) = 0.11114829808$$

$$p(x_4|Y_0^{1-2}Y_0^3) = \frac{0.0082529821}{0.04949304174 + 0.0082529821 + 0.0082529821 + 0.0082529821}$$

$$\Longrightarrow p(x_4|Y_0^{1-2}Y_0^3) = 0.11114829808$$

$$\Longrightarrow \begin{cases} p(x_1|Y_0^{1-2}Y_0^3) = \ 0.66655510574 \\ p(x_2|Y_0^{1-2}Y_0^3) = 0.11114829808 \\ p(x_3|Y_0^{1-2}Y_0^3) = 0.11114829808 \\ p(x_4|Y_0^{1-2}Y_0^3) = 0.11114829808 \end{cases}$$

In this part, results show the hazardous gases occupy approximately 33% of the air and the rest is occupied by clean-air. Also it shows the clean air have the highest percentage yet but this number is approximately 18% higher than the number was obtained in section A. And each of the hazardous gases percentages is reduced approximately 6% compared with the results of section A. If we ignore these difference between the resulting section A and this part of section B, we conclude this fact which both of the results are showing the same to some extent but we have some faults compared with results of A yet.

Second we combine 1st and 3rd sensors and the result should be combined with sensor 2.

$$p(x_1|Y_0^1Y_0^3) = p(x_1|Y_0^1) * p(x_1|Y_0^3) * p_{sensors-effect-avg}(x_1) * normalization$$

$$\Longrightarrow p(x_1|Y_0^1Y_0^3) = 0.4 * 0.25 * 0.325 * normalization = 0.0325 * normalization$$

$$p(x_2|Y_0^1Y_0^3) = p(x_2|Y_0^1) * p(x_2|Y_0^3) * p_{sensors-effect-avg}(x_2) * normalization$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^3) = 0.3 * 0.25 * 0.275 * normalization$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^3) = \ 0.020625 * normalization$$

$$p(x_3|Y_0^1Y_0^3) = p(x_3|Y_0^1) * p(x_3|Y_0^3) * p_{sensors-effect-avg}(x_3) * normalization$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^3) = 0.15 * 0.25 * 0.2 * normalization$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^3) = \ 0.0075 * normalization$$

$$p(x_4|Y_0^1Y_0^3) = p(x_4|Y_0^1) * p(x_4|Y_0^3) * p_{sensors-effect-avg}(x_4) * normalization$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^3) = 0.15 * 0.25 * 0.2 * normalization$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^3) = 0.0075 * normalization$$

Then we solve the solution of fusion nodes by omitting normalization terms and then normalize the probabilities for 1st and 3nd sensors.

$$\Longrightarrow p(x_1|Y_0^1Y_0^3) = \frac{0.0325}{0.0325 + 0.020625 + 0.010125 + 0.010125} = 0.44293015332$$

$$\Longrightarrow p(x_2|Y_0^1Y_0^3) = \frac{0.020625}{0.0325 + 0.020625 + 0.010125 + 0.010125} = 0.2810902896$$

$$\Longrightarrow p(x_3|Y_0^1Y_0^3) = \frac{0.0075}{0.0325 + 0.020625 + 0.0075 + 0.0075} = 0.11009174311$$

$$\Longrightarrow p(x_4|Y_0^1Y_0^3) = \frac{0.0075}{0.0325 + 0.020625 + 0.0075 + 0.0075} = 0.11009174311$$

Now we combine the above results with sensor 2:

$$p(x_1|Y_0^{1-3}Y_0^2) = p(x_1|Y_0^{1-3}) * p(x_1|Y_0^2) * p_{sensors-effect-avg}(x_1) * normalization$$

$$\Longrightarrow p(x_1|Y_0^{1-3}Y_0^2) = 0.44293015332 * 0.25 * 0.34646507666 * normalization$$

$$\Longrightarrow p(x_1|Y_0^{1-3}Y_0^2) = 0.03836495738 * normalization$$

$$p(x_2|Y_0^{1-3}Y_0^2) = p(x_2|Y_0^{1-3}) * p(x_2|Y_0^2) * p_{sensors-effect-avg}(x_2) * normalization$$

$$\Longrightarrow p(x_2|Y_0^{1-3}Y_0^2) = 0.2810902896 * 0.15 * 0.2155451448 * normalization$$

$$\Longrightarrow p(x_2|Y_0^{1-3}Y_0^2) = 0.00908814707 * normalization$$

$$p(x_3|Y_0^{1-3}Y_0^2) = p(x_3|Y_0^{1-3}) * p(x_3|Y_0^2) * p_{sensors-effect-avg}(x_3) * normalization$$

$$\Longrightarrow p(x_3|Y_0^{1-3}Y_0^2) = 0.11009174311 * 0.3 * 0.20504587155 * normalization$$

$$\Longrightarrow p(x_3|Y_0^{1-3}Y_0^2) = 0.00677215722 * normalization$$

$p(x_4|Y_0^{1-3}Y_0^2) = p(x_4|Y_0^{1-3}) * p(x_4|Y_0^2) * p_{sensors-effect-avg}(x_4) * normalization$

$\Longrightarrow p(x_4|Y_0^{1-3}Y_0^2) = 0.11009174311 * 0.3 * \ 0.20504587155 * normalization$

$\Longrightarrow p(x_4|Y_0^{1-3}Y_0^2) = 0.00677215722 * normalization$

Now we normalize the probabilities.

$$p(x_1|Y_0^{1-3}Y_0^2) = \frac{0.03836495738}{0.03836495738 + 0.0082529821 + 0.00677215722 + 0.00677215722}$$

$\Longrightarrow p(x_1|Y_0^{1-3}Y_0^2) = \ 0.63769149059$

$$p(x_2|Y_0^{1-3}Y_0^2) = \frac{0.00908814707}{0.03836495738 + 0.00908814707 + 0.00677215722 + 0.00677215722}$$

$\Longrightarrow p(x_2|Y_0^{1-3}Y_0^2) = 0.14899232189$

$$p(x_3|Y_0^{1-3}Y_0^2) = \frac{0.00677215722}{0.03836495738 + 0.00908814707 + 0.00677215722 + 0.00677215722}$$

$\Longrightarrow p(x_3|Y_0^{1-3}Y_0^2) = 0.11102366859$

$$p(x_4|Y_0^{1-3}Y_0^2) = \frac{0.00677215722}{0.03836495738 + 0.00908814707 + 0.00677215722 + 0.00677215722}$$

$\Longrightarrow p(x_4|Y_0^{1-3}Y_0^2) = 0.11102366859$

$$\Longrightarrow \begin{cases} p(x_1|Y_0^{1-3}Y_0^2) = \ 0.63769149059 \\ p(x_2|Y_0^{1-3}Y_0^2) = 0.14899232189 \\ p(x_3|Y_0^{1-3}Y_0^2) = 0.11102366859 \\ p(x_4|Y_0^{1-3}Y_0^2) = 0.11102366859 \end{cases}$$

As the results show, the hazardous gases occupy approximately 36% of the air and the rest is occupied by clean-air. Also in this case as what we saw previous, the clean-air has the highest percentage in the air but the number is approximately 14% higher than the number was obtained in section A. And the same as previous result, each of the hazardous gases percentages is reduced approximately 6%. If we ignore the difference between the resulting section A and this part, we'll conclude both of the results are showing the same conclusions to some extent but actually we'll not ignore the faults compared with results in section A.

And at the end we combine 2nd and 3rd sensors and the result should be combined with sensor 1.

$$p(x_1|Y_0^2Y_0^3) = p(x_1|Y_0^2) * p(x_1|Y_0^3) * p_{sensors-effect-avg}(x_1) * normalization$$

$$\Longrightarrow p(x_1|Y_0^2Y_0^3) = 0.25 * 0.25 * 0.25 * normalization = 0.015625 * normalization$$

$$p(x_2|Y_0^2Y_0^3) = p(x_2|Y_0^2) * p(x_2|Y_0^3) * p_{sensors-effect-avg}(x_2) * normalization$$

$$\Longrightarrow p(x_2|Y_0^2Y_0^3) = 0.15 * 0.25 * 0.2 * normalization$$

$$\Longrightarrow p(x_2|Y_0^2Y_0^3) = 0.0075 * normalization$$

$$p(x_3|Y_0^2Y_0^3) = p(x_3|Y_0^2) * p(x_3|Y_0^3) * p_{sensors-effect-avg}(x_3) * normalization$$

$$\Longrightarrow p(x_3|Y_0^2Y_0^3) = 0.3 * 0.25 * 0.275 * normalization$$

$$\Longrightarrow p(x_3|Y_0^2Y_0^3) = 0.020625 * normalization$$

$$p(x_4|Y_0^2Y_0^3) = p(x_4|Y_0^2) * p(x_4|Y_0^3) * p_{sensors-effect-avg}(x_4) * normalization$$

$$\Longrightarrow p(x_4|Y_0^2Y_0^3) = 0.3 * 0.25 * 0.275 * normalization$$

$$\Longrightarrow p(x_4|Y_0^2Y_0^3) = 0.020625 * normalization$$

Then we solve the solution of fusion nodes by omitting normalization terms and then normalize the probabilities for 1st and 3nd sensors.

$$\Longrightarrow p(x_1|Y_0^2Y_0^3) = \frac{0.015625}{0.015625 + 0.0075 + 0.020625 + 0.020625} = 0.2427184466$$

$$\Longrightarrow p(x_2|Y_0^2Y_0^3) = \frac{0.0075}{0.015625 + 0.0075 + 0.020625 + 0.020625} = 0.11650485436$$

$$\Longrightarrow p(x_3|Y_0^2Y_0^3) = \frac{0.020625}{0.015625 + 0.0075 + 0.020625 + 0.020625} = 0.32038834951$$

$$\Longrightarrow p(x_4|Y_0^2Y_0^3) = \frac{0.020625}{0.015625 + 0.0075 + 0.020625 + 0.020625} = 0.32038834951$$

Now we combine the above results with sensor 1:

$$p(x_1|Y_0^1 Y_0^{2-3}) = p(x_1|Y_0^1) * p(x_1|Y_0^{2-3}) * p_{sensors-effect-avg}(x_1) * normalization$$

$$\Rightarrow p(x_1|Y_0^1 Y_0^{2-3}) = 0.4 * 0.2427184466 * 0.3213592233 * normalization$$

$$\Rightarrow p(x_1|Y_0^1 Y_0^{2-3}) = 0.03119992459 * normalization$$

$$p(x_2|Y_0^1 Y_0^{2-3}) = p(x_2|Y_0^1) * p(x_2|Y_0^{2-3}) * p_{sensors-effect-avg}(x_2) * normalization$$

$$\Rightarrow p(x_2|Y_0^1 Y_0^{2-3}) = 0.3 * 0.11650485436 * \ 0.20825242718 * normalization$$

$$\Rightarrow p(x_2|Y_0^1 Y_0^{2-3}) = 0.0072787256 * normalization$$

$$p(x_3|Y_0^1 Y_0^{2-3}) = p(x_3|Y_0^1) * p(x_3|Y_0^{2-3}) * p_{sensors-effect-avg}(x_3) * normalization$$

$$\Rightarrow p(x_3|Y_0^1 Y_0^{2-3}) = 0.15 * 0.32038834951 * \ 0.235194175 * normalization$$

$$\Rightarrow p(x_3|Y_0^1 Y_0^{2-3}) = 0.01130302103 * normalization$$

$$p(x_4|Y_0^1 Y_0^{2-3}) = p(x_4|Y_0^1) * p(x_4|Y_0^{2-3}) * p_{sensors-effect-avg}(x_4) * normalization$$

$$\Rightarrow p(x_4|Y_0^1 Y_0^{2-3}) = 0.15 * 0.32038834951 * \ 0.235194175 * normalization$$

$$\Rightarrow p(x_4|Y_0^1 Y_0^{2-3}) = 0.01130302103 * normalization$$

Now we normalize the probabilities.

$$p(x_1|Y_0^1 Y_0^{2-3}) = \frac{0.03119992459}{0.03119992459 + 0.0072787256 + 0.01130302103 + 0.01130302103}$$

$$\Rightarrow p(x_1|Y_0^1 Y_0^{2-3}) = \ 0.51076502869$$

$$p(x_2|Y_0^1 Y_0^{2-3}) = \frac{0.0072787256}{0.03119992459 + 0.0072787256 + 0.01130302103 + 0.01130302103}$$

$$\Rightarrow p(x_2|Y_0^1 Y_0^{2-3}) = 0.11915793191$$

$$p(x_3|Y_0^1 Y_0^{2-3}) = \frac{0.01130302103}{0.03119992459 + 0.0072787256 + 0.01130302103 + 0.01130302103}$$

$$\Rightarrow p(x_3|Y_0^1 Y_0^{2-3}) = 0.18503851969$$

$$p(x_4|Y_0^1 Y_0^{2-3}) = \frac{0.01130302103}{0.03119992459 + 0.0072787256 + 0.01130302103 + 0.01130302103}$$

$$\Rightarrow p(x_4|Y_0^1 Y_0^{2-3}) = 0.18503851969$$

$$\Rightarrow \begin{cases} p(x_1|Y_0^1 Y_0^{2-3}) = \ 0.51076502869 \\ p(x_2|Y_0^1 Y_0^{2-3}) = 0.11915793191 \\ p(x_3|Y_0^1 Y_0^{2-3}) = 0.18503851969 \\ p(x_4|Y_0^1 Y_0^{2-3}) = 0.18503851969 \end{cases}$$

The results in this part are so similar to the results of 1st section (A). In this case, having approximately 49% clean air, the hazardous gasses occupy about 49% of the air. In its 49%, something about 11% belongs to level 1 hazardous gas and something about 18.5% belongs to level 2 hazardous gas and the level 3 hazardous gas has the same percentage as level 2 hazardous gas. Therefore, we see an increase in clean-air and level 2 & 3 hazardous gases percentages. And we have a reduction in level 1 hazardous gas percentage as we see in the clean air which has the highest percentage in the air. Its number is approximately 2% higher than it was obtained in section A. And see an increase sth about 1.5 percent in both level 2 & 3 hazardous gases and see a reduction something about 5 percent for level 1 hazardous gas. Also these results show the importance of data fusing and the effect of fusion sequences. In fact, we fuse the data and information based on the sensors priority in each fusion and these results come from these facts. For clearing the conclusions we notice the fact again, when we fuse measurements of 1st and 2nd sensor, we convert these two measurements to a higher priority measurement. So the fact that these fusion sequences are important and gives a priority to each sensors, effects on our results as what we saw here.

2.

A] If a system be faulty, we can express seven fault modes for this faulty system. First, assume that only h1 occurs. This can be generalize to h2 and h3. So we have 3 cases of faults in this strategy which only one faulty mode occurs.

Now assume that both faulty modes of the faults set occur simultaneously, then in these cases we have 3 kind of modes for faulty system again. First {h1, h2}, second {h2, h3} and third {h1, h3}.

Last case occurs when all faults event in the system simultaneously. It can be represented as a faulty system with h1, h2 & h3 faults.

So we have 7 modes of fault for the faulty system:

1st: {h1} 2nd:{h2} 3rd:{h3}

4th:{h1,h2} 5th:{h1,h3} 6th:{h2,h3}

7th:{h1,h2,h3}

B]

Belief and plausibility measures based on Dempster Shafer theory (DST) are a way of measuring uncertainty in an information system. In fact, belief and plausibility measures are approaches for representing partial, uncertainty, and imprecise information. Now we have a brief view in which belief and plausibility are. But, the difference between these two definitions is the belief represents the lowest probability and plausibility represents the highest probability and these formulas are shown below.

$$bel(A) = \sum_{B \subseteq A; B \neq \phi} m(B)$$ [1]

$$pl(A) = \sum_{B \cap A \neq \phi} m(B)$$

C] In this section, we should measure belief and plausibility asked in the problem.

$m(A_k)$	$bel(A_k)$	$pl(A_k)$	2^Ω	$m(B_k)$	$bel(B_k)$	$pl(B_k)$
0.2	0.2	0.9	$\{h_1\}$	0.2	0.2	0.8
0.1	0.1	0.8	$\{h_2\}$	0.0	0	0.2
0.0	0	0.1	$\{h_3\}$	0.2	0.2	0.8
0.6	0.9	1	$\{h_1, h_2\}$	0.0	0.2	0.8
0.0	0.2	0.9	$\{h_1, h_3\}$	0.4	0.8	1
0.0	0.1	0.8	$\{h_2, h_3\}$	0.0	0.2	0.8
0.1	1	1	$\{h_1, h_2, h_3\}$	0.2	1	1

[1] *m*(B) is the proportion of available evidence that supports the claim that actual state belongs to B but not any subset of B

D]

According to belief and plausibility definitions, the table of the problem completed as shown below.

$\cap$	A_1	A_2	A_3	A_4	A_5	A_6	A_7
B_1	h_1	ϕ	ϕ	h_1	h_1	ϕ	h_1
B_2	$\emptyset$	h_2	ϕ	h_2	ϕ	h_2	h_2
B_3	$\emptyset$	ϕ	h_3	ϕ	h_3	h_3	h_3
B_4	h_1	h_2	ϕ	$\{h_1 h_2\}$	h_1	h_2	$\{h_1 h_2\}$
B_5	h_1	ϕ	h_3	h_1	$\{h_1 h_3\}$	h_3	$\{h_1 h_3\}$
B_6	$\emptyset$	h_2	h_3	h_2	h_3	$\{h_2 h_3\}$	$\{h_2 h_3\}$
B_7	h_1	h_2	h_3	$\{h_1 h_2\}$	$\{h_1 h_3\}$	$\{h_2 h_3\}$	$\{h_1 h_2 h_3\}$

E]

According to what is asked in this section, we should omit these rows & cols as shown below.

$\cap$	A_1	A_2	A_3	A_4	A_5	A_6	A_7
B_1	h_1	ϕ	ϕ	h_1	h_1	ϕ	h_1
B_2	$\emptyset$	h_2	ϕ	h_2	ϕ	h_2	h_2
B_3	$\emptyset$	ϕ	h_3	ϕ	h_3	h_3	h_3
B_4	h_1	h_2	ϕ	$\{h_1 h_2\}$	h_1	h_2	$\{h_1 h_2\}$
B_5	h_1	ϕ	h_3	h_1	$\{h_1 h_3\}$	h_3	$\{h_1 h_3\}$
B_6	$\emptyset$	h_2	h_3	h_2	h_3	$\{h_2 h_3\}$	$\{h_2 h_3\}$
B_7	h_1	h_2	h_3	$\{h_1 h_2\}$	$\{h_1 h_3\}$	$\{h_2 h_3\}$	$\{h_1 h_2 h_3\}$

Therefore the table is changed as shown below.

$\cap$	A_1	A_2	A_4	A_7
B_1	h_1	ϕ	h_1	h_1
B_3	$\emptyset$	ϕ	ϕ	h_3
B_5	h_1	ϕ	h_1	$\{h_1h_3\}$
B_7	h_1	h_2	$\{h_1h_2\}$	$\{h_1h_2h_3\}$

F]

First, we should calculate m(z_i)s.

$m(z_1) = m(A_1) * m(B_1) = 0.04$

$m(z_2) = m(A_4) * m(B_1) = 0.12$

$m(z_3) = m(A_7) * m(B_1) = 0.02$

$m(z_4) = m(A_7) * m(B_3) = 0.02$

$m(z_5) = m(A_1) * m(B_5) = 0.08$

$m(z_6) = m(A_4) * m(B_5) = 0.24$

$m(z_7) = m(A_7) * m(B_5) = 0.04$

$m(z_8) = m(A_1) * m(B_7) = 0.04$

$m(z_9) = m(A_2) * m(B_7) = 0.02$

$m(z_{10}) = m(A_4) * m(B_7) = 0.12$

$m(z_{11}) = m(A_7) * m(B_7) = 0.02$

Then we calculate measure of conflict parameter as shown below.

$$1-k=\sum_{i=1}^{11} m(z)=0.76$$

Finlay, the results are calculated as shown below :

$$\Rightarrow \begin{cases} m(h_1)=\dfrac{m(z_1)+m(z_2)+m(z_3)+m(z_5)+m(z_6)+m(z_8)}{1-k}=\dfrac{0.54}{0.76}=0.71 \\ m(h_2)=\dfrac{m(z_9)}{1-k}=\dfrac{0.02}{0.76}=0.026 \\ m(h_3)=\dfrac{m(z_4)}{1-k}=\dfrac{0.02}{0.76}=0.026 \\ m(\{h_1h_2\})=\dfrac{m(z_{10})}{1-k}=\dfrac{0.12}{0.76}=0.158 \\ m(\{h_1h_3\})=\dfrac{m(z_4)}{1-k}=\dfrac{0.04}{0.76}=0.053 \\ m(\{h_1h_2h_3\})=\dfrac{m(z_{11})}{1-k}=\dfrac{0.02}{0.76}=0.026 \end{cases}$$

$$\Rightarrow \begin{cases} m(h_1)=0.71 \\ m(h_2)=0.026 \\ m(h_3)=0.026 \\ m(\{h_1h_2\})=0.158 \\ m(\{h_1h_3\})=0.053 \\ m(\{h_1h_2h_3\})=0.026 \end{cases}$$

G]

First, let’s complete the decision table:

Decision Table	m	bel	pl	Uncertainty
$\{\boldsymbol{h_1}\}$	0.71	0.71	0.947	0.237
$\{\boldsymbol{h_2}\}$	0.026	0.026	0.21	0.184
$\{\boldsymbol{h_3}\}$	0.026	0.026	0.105	0.079
$\{\boldsymbol{h_1},\boldsymbol{h_2}\}$	0.158	0.894	0.973	0.079
$\{\boldsymbol{h_1},\boldsymbol{h_3}\}$	0.053	0.789	0.973	0.184
$\{\boldsymbol{h_2},\boldsymbol{h_3}\}$	0	0.052	0.289	0.237
$\{\boldsymbol{h_1},\boldsymbol{h_2},\boldsymbol{h_3}\}$	0.026	1	1	0

The above decision table represents mass value, belief and plausibility and uncertainty parameters. Considering to it, the $\boldsymbol{h_1}$ fault mode is needed more attention by operator. Actually, because of this reason which $m(h_1)$ has the highest probabilities among other fault modes, it's more important than the other. Let's consider Bayesian approach to solve this problem. Citing an article (Context Assumptions for Threat Assessment Systems - Steven A Israel and Erik Blasch, May 2016) , we understand how Bayesian method acts as DST. So comparing these methods, it results the similarity of these two methods and the approaches of probabilities estimation. In fact, the mass values of variables in this problem are the most effective parameters in order to make a good decision. Therefore, Bayesian approaches has the same result and we'll conclude the highest probability of $\boldsymbol{h_1}$ fault mode. Especially, its probability is higher than the other.

Finally, In order to compare these two methods, considering to this fact which Bayesian theory requires a more explicit formulation of conditioning and the prior probabilities of events, and on the other hand, Dempster-Shafer theory embeds conditioning information into its belief function and does not rely on prior knowledge, each of these methods are suitable for its own situations which means when we have prior knowledge, we consider to Bayesian method and when we don't want to rely on these knowledge, we would rather using Dempster-Shafer theory.