Bulk ship routing and scheduling: solving practical problems may provide better results

Scheduling shipments of bulk cargoes when multiple cargoes may be on board a vessel simultaneously is an especially complicated task. Most research on such bulk ship routing and scheduling problems has focused on solving a simplified version of the real problem, which we denote as the basic problem. However, practical problems often pose additional complexities and opportunities that are not considered in the basic problem. We present the basic bulk ship routing and scheduling problem and three practical extensions to it: (1) flexible cargo quantities, (2) split cargoes, and (3) sailing speed optimization. Consolidating results from various sources we show that, although the problems become harder to solve when introducing these practical extensions, significantly better solutions can be obtained by using advanced heuristics. Each of these extensions may increase profit contribution by 5–25% compared to solutions of the basic problem. These methods have been incorporated into a commercial software product used by several shipping companies. Thus, such fleet operators have the opportunity to significantly improve their financial results.

1. Introduction

Bulk shipping carries the majority of the world seaborne trade. During 2009, 60.7% of the cargo weight loaded on ships consisted of oil and main bulk commodities (iron ore, grain, coal, bauxite/alumina, and phosphate), for a total of 4762 million tons [1]. The world fleet consists of more than 7000 tankers (over 10 000 dwt) and a similar number of bulk carriers [2]. Efficient operation of this fleet is in the best interest of the world economy and vessel operators, and reduces bunker fuel consumption and environmental impact. In the short to intermediate run, routing and scheduling of this fleet determines the efficiency of its operation.

Bulk vessels are either controlled by the shipper of the cargo (namely, an industrial operation) or by tramp operators. Tramp operators carry cargoes under Contracts of Affreightment (CoA) and spot (optional) cargoes. A cargo is a shipment from a single origin to a single destination (also known as an order) with which a loading and/or discharging time window may be associated. When the cargo size is smaller than the vessel size, the load on a vessel may consist of several cargoes (this is a common situation where minor bulk commodities are concerned, such as chemicals, minerals, and petroleum products). Vessel operators are interested in finding the best possible fleet schedule that satisfies all constraints, and either minimizes the cost of operating the fleet over a planning horizon (when there is a specified set of cargoes to be carried) or maximizes profit contribution (when optional cargoes are available or the cargo sizes are somewhat flexible).

The ship routing and scheduling literature focuses mainly on mathematically optimal solutions to "solvable" problems [3, 4]. However, practical problems often have characteristics that render them to be not optimally "solvable" with current technology within a reasonable operational time frame (e.g. [5]). Therefore, in the more advanced operations, a simplified version of the scheduling problem is solved "optimally" and the scheduler sometimes has to modify that "optimal" schedule in order to take into account the additional practical aspects or complexities that may not be addressed by that "optimal" solution.

In this paper, we view some of the complexities posed by real-life problems as opportunities. In particular, we address the cases of

(1) *flexible cargo sizes*, where the vessel operator can decide upon the cargoes' quantity (within the given intervals),
(2) *split cargoes*, where we allow each cargo to be split and transported by more than one vessel, and
(3) *sailing speed optimization*, where we determine the optimal sailing speed on each sailing leg simultaneously with the fleet routing and scheduling decisions.

Consolidating results from several sources shows that advanced heuristics that take advantage of the above-listed special features of these problems provide better solutions than optimal (or close to optimal) ones to their simplified versions (that do not account for these practical aspects), and do it fairly quickly.

The remainder of this paper is organized as follows: the next section describes the basic version of the bulk ship routing and scheduling problem. This is the (simplified) version of the problem that is usually studied in the research literature. We describe its characteristics and provide analysis of 13 real-life cases. Following that, we present the three extensions to the basic problem in order to better address complexities and opportunities that exist in many real problems: (1) flexible cargo quantities, (2) split cargoes, and (3) sailing speed optimization. For each of these extensions, we present results from multiple real-life cases and show the potential for increased profits by utilizing these complexities (or opportunities) in routing and scheduling the vessels. We close with summary and conclusions. The mathematical models of the various problems are relegated to the Appendix.

2. The basic bulk ship routing and scheduling problem

In this section, we describe the basic bulk ship routing and scheduling problem and present the computational results. A more detailed analysis of this problem is available in [6, 7].

A heterogeneous fleet of vessels that may differ in their load capacity, speed, fuel consumption, cost, and initial position has to be assigned to carry a set of cargoes. Each cargo is uniquely defined by its size, loading and discharging ports, and time windows for these operations. For a tramp operation, revenue is associated with each cargo, and some cargoes may be optional (spot cargoes). The problem is to determine which cargoes to carry (that is prescribed in an industrial operation), on which vessels, in what sequence (route), and at what time (schedule). We assume that changing the controlled fleet composition is not an option within a short planning horizon of a few months. Thus, costs that are not affected by the schedule, such as capital and crew costs, are fixed and may be ignored. In a tramp operation, the objective is to maximize the profit contribution (income from transported cargoes minus operating cost, such as fuel and port fees). In an industrial operation, where the set of cargoes to be carried is mandatory, the objective is to minimize the operating costs.

The problem is framed by a set of constraints that must be satisfied by any solution (schedule):

- all contracted cargoes must be transported,
- optional cargoes may be transported,
- routing constraints,
- time constraints,
- ship–port compatibility, and
- ship capacity constraints.

If the ship size is larger than the cargo sizes, it may be possible for multiple cargoes to be on the ship at the same time. These cargoes may be loaded at different ports and destined to another set of ports. Although we focus here on tramp operations, only minor modifications are necessary for an industrial operation. The mathematical model of this problem, which we denote the basic bulk ship routing and scheduling problem, is provided in the Appendix.

Solving this model for real-life problems within reasonable time (a few minutes) is impractical due to its structure and the large number of binary variables. Domain knowledge facilitates casting this problem into a much better structured model, a set partitioning model (e.g. [6, 8–11]) where for each vessel all feasible schedules (columns) are generated and the solution process selects one schedule for each vessel in a manner that satisfies all the constraints and gives the best value to the objective. However, even the set partitioning model of this basic problem cannot always be solved to optimality within reasonable time if there are too many columns (approaching or even exceeding a million). In such cases, we must usually resort to advanced heuristics.

Brønmo *et al.* [6] provide mathematically optimal solutions obtained by solving a set partitioning model for seven out of eight cases derived from real-life operations. The results of a multi-start heuristic are compared to the optimal solutions. Korsvik *et al.* [7] provide a tabu search heuristic that beats the multi-start heuristic. Table 1 provides a summary of the eight real-life cases presented in [6]. Case 1 is from a shipping operation in northern Europe transporting dry bulk commodities. Cases 2–6 are from a company shipping chemical commodities between Europe and the Caribbean. Case 7 is from shipping petroleum products among ports in northern Europe, and case 8 is from short-sea shipping of dry bulk commodities in northern Europe. The table provides indication of the size of each problem (no. of cargoes, no. of ships, and planning horizon), the number of columns (schedules) generated, and the execution times. In seven of the cases (all except case 6), the optimal solution has been found by the set partitioning approach but in most of them the computation time is far too long (hours instead of the desired minutes). In cases 1–6, the cargoes are smaller than the vessel sizes, thus multiple cargoes may be on a vessel at the same time, whereas in cases 7 and 8, only a single cargo may be on board a vessel at a time.

Table 1. Real-life cases with known optimal solutions.

Case no.	1	2	3	4	5	6	7	8
No. of cargoes	18	9	17	14	17	50	30	15
No. of vessels	6	3	6	7	13	13	13	4
Planning horizon (days)	23	75	75	40	35	150	20	35
No. of columns[a]	1 306 704	327	2 609 685	26 512 819	19 240 874	Too large	62 877	34 454
Average no. of cargoes per column[a]	6.0	4.1	7.0	7.6	6.9	NA	4.4	6.0
Column generation time (s)[a]	6482	<1	9300	13 325	27 469	NA	235	454
Set partitioning time (s)[a]	5	<1	<1	<1	2	NA	91	4
Tabu search average execution time (s)[b]	85	11	18	15	27	219	175	32
Tabu search average optimality gap (%)[bc]	0.0	0.0	0.4	0.2	0.3	0.7	0.6	0.0

Notes: [a]From [6].
[b]Variant III from [7].
[c]Gap from the best known solution.

Therefore, these last two cases are simpler and the solution time is reasonable. In addition, we present in this table typical results of the tabu search heuristic (from [7]) that allow us to evaluate its quality. For each case, the tabu search was run 10 times and average results are reported. We observe that the tabu search provides very close to optimal solutions within an acceptable time frame.

In this basic problem, the decision variables are binary (e.g. should vessel x carry cargo y? yes/no). In the more complex problems (namely, flexible cargo quantities, split cargoes, and variable sailing speed) that follow in the next sections, there are additional decision variables that are continuous (e.g. what share of cargo x should be carried by vessel y?). This makes the column generation process much more complicated (e.g. [12]). In these more complex problems, some of the constraints that are in the basic problem (explicit or implicit ones) are relaxed, and we should expect at least as good solutions or better ones.

3. Extension I: flexible cargo sizes

Bulk cargoes are frequently shipped on a recurrent basis (e.g. under CoA). In such cases, the exact cargo size is not that important and there is flexibility in the size of the cargo. Normally, there is a target cargo size with allowed variability around it (e.g. 20 000 tons ±10%). This is also known as a More Or Less Owner's Option contract. Under such a contract, the vessel operator is paid per unit delivered. Such a contract provides the operator with additional flexibility in assigning cargoes to vessels and in utilizing the vessel capacity. Reducing the size of some cargoes may also make it possible to free enough ship capacity to carry additional spot cargoes by the controlled fleet.

The mathematical model for this extension is provided in the Appendix. Solving this modified model to optimality for real-life problems within a reasonable amount of time is impractical. The additional continuous variables for the cargo sizes complicate finding an optimal solution. Brønmo *et al.* [13] developed an exact solution method where all columns (representing feasible ship routes with optimized cargo quantities) are generated *a priori*, and the problem is solved as a set partitioning problem. They presented eight small to medium size cases, the first four from a company shipping dry bulk commodities in northern Europe, and the other four from a company shipping chemicals between Europe and the Caribbean. They found optimal solutions to six of these cases. Brønmo *et al.* [12] further explored the column generation approach where the columns are dynamically generated when they are needed. That made it possible to solve somewhat larger problems. However, they had to discretize the cargo quantities, which turned the solution method into a heuristic column generation approach. They expanded the set of test problems to 10 cases from the same companies (these included six cases of the initial set). Korsvik and Fagerholt [14] developed a tabu search algorithm for the same problem, in which the optimal cargo sizes are calculated for each cargo on all vessel routes each time a local search move is evaluated. To test the tabu search, they used the same 10 real-life cases as in [12] and additional five (larger) cases.

Table 2 provides comparative results from the 10 cases. It provides the problem size indicators, execution times, and optimality gaps. The tabu search was run 10 times for each problem, and the average run times and optimality gaps are provided. The optimality gaps are from the best known solutions that were found in all 10 and eight out of 10 cases by the tabu search heuristic [14] and the column generation

Table 2. Flexible cargo sizes—comparative results.

Case no.	1[a]	2[a]	3[a]	4	5	6[a]	7[a]	8[a]	9	10
No. of cargoes	14	17	17	24	31	15	12	15	20	25
No. of vessels	4	4	4	4	6	6	4	7	5	7
Planning horizon (days)	15	15	17	20	17	120	75	75	120	120
LP_2[b] optimality gap (%)[d]	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.1
LP_2[b] execution time (s)	23	15	7	6000	2424	33	9	103	1001	760
Tabu_H[c] average optimality gap (%)[d]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.2	0.2
Tabu_H[c] average execution time (s)	25	9	7	73	29	6	6	10	20	15
Average increase in profit contribution (%)[e]	17.9	18.5	8.2	15.9	9.4	10.0	9.7	9.1	9.3	9.6

Notes: [a]Optimal solution known.
[b]From [12].
[c]From [14].
[d]From the best known solution.
[e]Compared to the best solution known for the fixed cargo size case.

approach [12], respectively. The standard deviations of the solution quality and run times were relatively small. The tabu search (Tabu_H) provides solutions of similar quality to the column generation approach (LP_2), as measured by the optimality gap, at (generally) much shorter execution times that are within a few minutes.

The advantage in considering flexible cargo sizes (in comparison to fixed ones as in the basic problem) is presented in the last row of Table 2. The fixed cargo size cases were run for the middle point of the allowed cargo size range, and around it a variation of $\pm 10\%$ was allowed (that is common in many shipping contracts). The difference in the profit contribution (between the fixed and variable cargo sizes) ranges from close to 20% in the smaller cases to about 10% in the medium size cases. Thus, considering flexibility in cargo sizes is definitely advantageous, and a shipping company may lose a lot of opportunities and profit by not dealing with this aspect explicitly in the schedule planning stage.

4. Extension II: splitting cargoes between vessels

Sometimes, splitting a cargo between two vessels (with the permission of the cargo shipper) may make a lot of sense, although at a first glance it may sound counter-productive (because it may require additional sailing time and port entries). Consider a simple example: two vessels each of 45 000 dwt have to carry three cargoes each of 30 000 tons, originating in one region and destined to a different region (e.g. US east coast to Western Europe). Without splitting a cargo, each cargo will have to be carried on a separate voyage, totaling three voyages. Splitting a cargo will facilitate carrying the three cargoes in two voyages at, most likely, reduced cost. Splitting cargoes may facilitate reduction in shipping costs and environmental impact, and may also result in carrying additional spot cargoes by the existing fleet. The mathematical model for this extension is provided in the Appendix.

Andersson *et al.* [15] used a path flow formulation where all feasible columns (routes) were generated *a priori* and solved as a set covering model with a number of additional constraints. Only small problems with up to four vessels and 16 cargoes were solved to optimality. In addition, fixed service times in ports were assumed, independent of cargo size. Later, Korsvik *et al.* [16] analyzed the ship routing and

Table 3. Split cargoes—comparative results.

Case no.	No. of cargoes	No. of vessels	Planning horizon (days)	Set covering[b] gap (%)[c]	Set covering[b] run time (s)	LNS-1000[d] average gap (%)[c]	LNS-1000[d] average run time (s)	Profit contribution increase (%)[e]
1[a]	8	3	90	0	1	0	10	0
2[a]	8	3	60	0	<1	0	10	0
3[a]	10	4	90	0	2	0	21	0.5
4[a]	10	4	60	0	2	0	13	5.5
5[a]	12	4	90	0	31	0	42	23.1
6[a]	12	4	60	0	16	0.0	40	7.9
7[a]	14	4	90	0	1059	0	44	9.4
8[a]	14	4	60	0	197	0.0	44	11.0
9[a]	16	4	90	0	4437	0.8	112	16.8
10[a]	16	4	60	0	9	0.4	47	3.1
11	18	4	90	0.0	3665	0	54	4.4
12	18	4	60	0	4537	0.6	52	12.4
13	20	4	90	0	2084	0.0	63	1.9
14	20	4	60	2.9	3681	1.6	77	11.2
15	22	5	90	–	–	0.0	138	3.4
16	22	5	60	0	3762	0.9	115	10.1
17	24	5	90	2.3	5963	0.4	259	19.2
18	24	5	60	4.7	3868	2.8	177	20.8
Average	–	–	–	0.58	1960	0.46	69.4	8.9

Notes: [a]Optimal solution known.
[b]From [15].
[c]From the best known solution.
[d]From [16].
[e]Compared to no splitting.

scheduling problem with split cargoes. They developed a large neighborhood search heuristic. This heuristic also accommodates (more realistically) port service times that depend on the cargo size (in contrast to the exact method in [15]).

Table 3 provides comparative results for the two approaches; 18 test cases are presented. These were randomly generated to reflect deep-sea bulk shipping. The set covering model in [15] found the optimal solutions to the first 10 cases (the smaller ones). Indicators of the problem size as well as run times and the gap from the best known solution for each case are presented in the table. For the set covering model a limit of 3600 s was used for the solution stage (no time limit was used for the column generation stage, but for cases 11–18 heuristic rules were used in order to limit the number of columns generated). The heuristic in [16] ("LNS-1000") was run 10 times for each case and therefore averages are provided. We see that the run times for the set covering model are acceptable only for the smaller size problems. In addition, for case 15, no solution was identified. The heuristic has acceptable run times for most of the cases and provides high-quality solutions, as measured by the average gap from the best known solution. That gap is found by the set covering model [15] for the cases where it is reported as zero, and by the LNS-1000 [16] elsewhere. Table 3 shows that the average gap for the LNS-1000 is only 0.46% compared with 0.58% for the set covering model, with much lower run times for the LNS-1000. Thus, the heuristic that takes advantage of the cargo splitting opportunities provides high-quality solutions fairly quickly for practical size problems, whereas the optimization model run times are excessive for operational purposes.

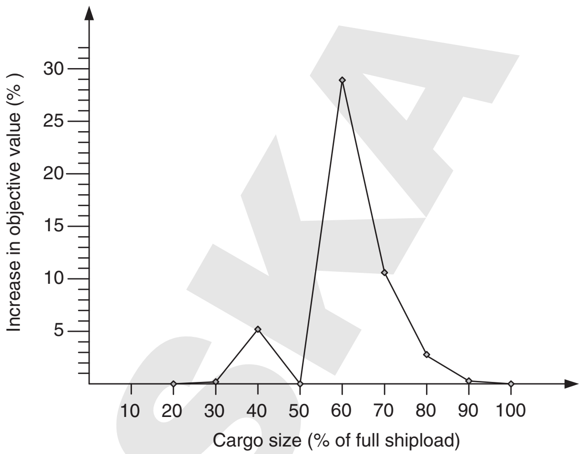

Figure 1. Impact of relative cargo size on profit contribution.
Note: From [16].

The last column in Table 3 displays the increase in profit contribution due to splitting the cargoes. We can see that significant increases in the profit contribution are possible from splitting cargoes, but no pattern is evident in them. The increase in profit contribution from splitting cargoes depends to a large extent on the relation between cargo size and ship size. This relationship is depicted in Figure 1 (from [16]). When the cargo sizes are 60–70% of the vessel sizes the profit contribution can be increased by 15–25% by splitting cargoes.

5. Extension III: sailing speed optimization

Oil price increases during recent years have brought the issue of sailing speed determination to the forefront. Bunker fuel cost is a major component of the variable operating cost of a vessel, and when fuel prices are high it may amount to the majority of the operating costs. A vessel may consume well over 10 tons of bunker fuel per day (up to 100 tons per day, depending on the vessel size), and at 500–600 USD per ton that translates into many thousands of USD per day. The bunker fuel consumption of a vessel per time unit is often estimated to be proportional to the third power of its speed (within normal operating speeds) [17]. Bunker fuel consumption per unit of distance is thus proportional to the second power of the speed. Thus, reducing the sailing speed by 10% will reduce the bunker fuel consumption for a sailing leg by close to 20%. From these facts, it should be evident that reducing sailing speed provides a significant potential for cost savings (or profit increase). The mathematical model for this extension is provided in the Appendix.

Ronen [17] provided three alternate models for determining the optimal sailing speed for a single leg. Fagerholt *et al.* [18] optimized sailing speeds on all legs of a given single route where all port calls have time windows by minimizing bunker fuel consumption. Norstad *et al.* [19] incorporated sailing speed optimization into the bulk ships routing and scheduling decision where all port calls have time windows. Their model allocates contracted cargoes to vessels, determines which spot cargoes to carry (and by which vessel), and determines the best ship routes, schedules, and speed

Table 4. Characteristics of test cases for speed optimization.

Case no.	1	2	3	4	5	6	7	8	9
No. of contracted cargoes	18	8	17	12	15	41	28	12	16
No. of spot cargoes	0	1	0	2	2	9	2	3	2
No. of vessels	6	3	6	7	13	13	13	4	6
Planning horizon (days)	23	75	75	40	35	150	20	35	90

on each sailing leg. It assumes that all port calls have hard time windows. They maximize revenue minus bunker fuel costs and additional variable sailing costs, assuming that other costs cannot be changed during the planning horizon. The problem (as formulated in the Appendix) is not solvable to optimality within any reasonable time frame for any problems of realistic size. Norstad *et al*. [19] used a multi-start local search heuristic similar to the one in [6]. Nine real-life cases coming from six international bulk shipping companies were used to evaluate their approach. The first eight cases are the same cases as in [6] and [7] that are presented in Table 1. All nine of these cases are presented in Table 4 where the cargoes are separated into contracted (mandatory) ones and (optional) spot cargoes. In cases 1–6, the vessels can carry multiple cargoes simultaneously, whereas in cases 7–9, a vessel can carry one cargo at a time.

Five different approaches were used in [19] to evaluate the impact of speed optimization using the multi-start local search heuristic. Relative results for these approaches (deviations in percents from the baseline provided by approach 1) are presented in Table 5:

(1) First, each case was solved with fixed speed at the service speed of 17 knots. This is the baseline solution that corresponds to the solution obtained by solving the basic bulk ship routing and scheduling problem.
(2) The baseline solution for each case was taken and each of the resulting ship routes in it was optimized separately (as in [18], "Service speed + speed opt."). This approach retains the income but reduces fuel consumption.
(3) Next, each case was solved at the maximal speed of 20 knots ("Max speed"). Obviously, the maximal speed increases fuel consumption but may facilitate carrying additional spot cargoes, thus increasing the income (and possibly the profit contribution).
(4) Then, maximal speed with speed optimization of each route separately was examined (as in [18], "Max speed + speed opt."). This approach retains the income but reduces fuel consumption.
(5) Finally, each case was run with variable speed (in the range of 14–20 knots), where the speed on each sailing leg was optimized simultaneously with the fleet schedule.

We see that speed optimization (approaches 2, 4, and 5) provides a better solution (profit) in each and every case compared to the corresponding approach without speed optimization, and "Variable speed" (approach 5) trumps them all. The size of the difference in the profit contribution depends on multiple factors such as bunker fuel prices and the relation between fleet capacity and the sizes of the available cargoes. In some cases, speeding up facilitates carrying more spot cargoes.

Table 5. Speed optimization—profit and fuel cost differences (in percents) from fixed service speed.

Approach	Case no.	1	2	3	4	5	6	7	8	9	Average
Service speed + speed opt. (approach 2)	Profit	0.4	0.5	2.0	4.0	1.7	6.8	0.8	0.4	8.7	2.7
	Fuel	−2.4	−3.1	−14.0	−24.6	−9.7	−43.9	−10.4	−7.9	−11.1	−14.1
Max speed (3)	Profit	−1.6	1.9	−7.0	−7.9	−6.1	−6.3	−1.9	−1.7	−25.6	−6.2
	Fuel	31.7	53.7	45.6	46.2	35.4	40.1	24.9	31.8	74.7	42.7
Max speed + speed opt. (4)	Profit	1.8	6.4	2.0	4.0	2.0	7.2	1.1	0.4	26.3	5.7
	Fuel	−15.3	27.6	−14.0	−24.4	−11.5	−45.2	−14.2	−7.9	8.5	−10.7
Variable speed (5)	Profit	2.2	7.7	2.0	6.6	2.1	10.9	1.3	0.5	30.0	7.0
	Fuel	−21.3	20.1	−14.0	−38.3	−12.4	−70.0	−18.0	−8.8	3.9	−17.6

Note: Results from [19].

Running times were not reported for these nine cases. In addition to these cases, Norstad *et al*. [19] generated four different sets of problems with 10 cases for each set. Each case had 13 vessels and only spot cargoes. Loading and discharging ports, cargo sizes, time windows, and the revenues from the cargoes were generated randomly. Run times for these cases were reported. For the two sets of the smaller to medium size cases (with a planning horizon of up to 60 days and up to 25 cargoes) run times were under a minute. For the larger cases (120 days and 70 cargoes) run times were over 10 min.

The non-linearity of the bulk fleet scheduling problem with speed optimization precludes finding optimal solutions within any reasonable time frame for realistic size problems. However, advanced heuristics can be used to find close to optimal solutions for small to medium size problems and thus save significant amounts of bunker fuel and improve profits. As shown in [19], incorporating speed optimization in the planning of fleet schedules may also provide very different (and improved) solutions regarding the assignment of cargoes to vessels.

6. Summary and conclusions

The ship routing and scheduling literature usually focuses on mathematically optimal solutions to "solvable" problems, while practical problems often have characteristics that render them to be not optimally "solvable" with current technology within reasonable time. Therefore, often a simplified version of the scheduling problem (which we have denoted the basic bulk ship routing and scheduling problem) is solved "optimally." The scheduler sometimes then has to modify that "optimal" schedule in order to take into account the additional practical aspects that may not be addressed by that "optimal" solution. In this paper, we have addressed some of the complexities posed by real-life problems as opportunities, and demonstrated that taking advantage of these opportunities may often result in much better solutions than solving the simplified version to optimality. In particular, we considered the cases of:

(1) *flexible cargo sizes*, where the vessel operator can decide upon the cargoes' quantity (within the given intervals),
(2) *split cargoes*, where we allow each cargo to be split and transported by more than one vessel, and
(3) *sailing speed optimization*, where we determine the optimal sailing speed on each sailing leg simultaneously with the fleet routing and scheduling decisions.

We have consolidated results from several sources and showed that advanced heuristics that take advantage of the above special features of the problems provide better solutions than optimal (or close to optimal) solutions to their simplified versions that do not account for these practical aspects. The cited examples indicated that flexible cargo sizes may increase profit contribution by up to 20%, splitting cargoes between vessels may result in a similar increase, and sailing speed optimization may increase profits by 7% on the average (depending on the problem characteristics). This demonstrates that it is much more important to model and solve the right problem, considering the opportunities that often arise in practical problems, than to strive for optimal solutions to simplified versions of the problem.

Scheduling bulk ships is a dynamic business with frequent changes. Often schedulers have to decide whether to accept or reject spot cargoes within a few minutes (possibly while on the phone). Thus, a tool that quickly provides close to optimal schedules is indispensable. Versions of the advanced heuristics discussed in this paper have also become available as solvers in TurboRouter®, a commercial decision support system for ship routing and scheduling [20, 21] that is being used by several shipping companies.

Shipping companies often prefer to maximize profits per ship day rather than for a given planning horizon. This also overcomes the issue of using the ships for different durations during a planning horizon. However, maximizing profit per day makes the objective a non-linear one and the problem becomes even harder to solve. Most of the advanced heuristics discussed in this paper can easily deal with such non-linear objective.

Further research is warranted to improve the response time for larger size problems as well as to address additional practical aspects such as soft time windows, multiple time windows, alternate routing, multiple products, and also into combinations of these practical aspects. We hope that the availability of advanced decision support tools that can provide high-quality solutions to complex real-life bulk ship routing and scheduling problems will encourage bulk ship operators to improve their bottom line while (possibly) saving bunker fuel and reducing emissions.

Appendix

Mathematical models

The basic problem. Let each cargo be represented by an index i. Associated with cargo i is a loading port node i and a discharging port node $n+i$, where n is the number of cargoes that might be transported during the planning horizon. Note that different nodes may correspond to the same physical port. Let $\mathcal{N}^P = \{1, 2, \ldots, n\}$ be the set of loading (pickup) nodes and $\mathcal{N}^D = \{n+1, n+2, \ldots, 2n\}$, the set of delivery nodes. The set of loading nodes is partitioned into two subsets, $\mathcal{N}^C$ and $\mathcal{N}^O$, where $\mathcal{N}^C$ is the set of loading nodes for the mandatory contracted cargoes and $\mathcal{N}^O$ is the set of loading nodes for the optional spot cargoes.

Let $\mathcal{V}$ be the set of ships. A network $(\mathcal{N}_v, \mathcal{A}_v)$ is associated with each ship v. Here, $\mathcal{N}_v$ is the set of nodes that can be visited by ship v, including the origin and an artificial destination for ship v, $o(v)$ and $d(v)$, respectively. Geographically, the origin can be either a port or a point at sea, while the artificial destination is the last planned delivery port for ship v. If the ship is not used, $d(v)$ will represent the same location as $o(v)$. From these calculations, we can extract the sets $\mathcal{N}_v^P = \mathcal{N}^P \cap \mathcal{N}_v$ and $\mathcal{N}_v^D = \mathcal{N}^D \cap \mathcal{N}_v$ consisting of the pickup and delivery nodes that ship v may visit, respectively. The set $\mathcal{A}_v$ contains all feasible arcs for ship v, which is a subset of $\mathcal{N}_v \times \mathcal{N}_v$.

For each arc let T_{ijv}^S be the sailing time from node i to node j and T_{iv}^P the service time in the port at node i with ship v. The variable transportation costs C_{ijv} consist of the sum of the sailing costs from node i to node j and the port costs of node i for ship v. Let $[\underline{T}_{iv}, \bar{T}_{iv}]$ denote the time window for ship v associated with node i, where $\underline{T}_{iv}$ is the earliest time for start of service and $\bar{T}_{iv}$ is the latest. Each cargo i has a weight Q_i and generates a revenue R_i per unit if it is transported. Let K_v be the capacity of ship v.

The binary variable x_{ijv} is assigned the value 1 if ship v sails directly from node i to node j, and 0 otherwise. The variable t_{iv} represents the time for start of service at node i. The variable l_{iv} is the load (weight) on board ship v when leaving node i.

To ease the reading of the model, we assume that each ship is empty when leaving the origin and when arriving at the artificial destination, i.e., $l_{o(v)v} = l_{d(v)v} = 0$. Finally, the binary variable y_i is equal to 1 if the optional spot cargo i is transported, and 0 otherwise.

The basic tramp ship routing and scheduling problem can now be formulated as follows:

$$\max \sum_{i \in \mathcal{N}^C} R_i Q_i + \sum_{i \in \mathcal{N}^O} R_i Q_i y_i - \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_v} C_{ijv} x_{ijv}, \tag{A.1}$$

$$\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}_v} x_{ijv} = 1, \quad i \in \mathcal{N}^C, \tag{A.2}$$

$$\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}_v} x_{ijv} - y_i = 0, \quad i \in \mathcal{N}^O, \tag{A.3}$$

$$\sum_{j \in \mathcal{N}_v} x_{o(v)jv} = 1, \quad v \in \mathcal{V}, \tag{A.4}$$

$$\sum_{j \in \mathcal{N}_v} x_{ijv} - \sum_{j \in \mathcal{N}_v} x_{jiv} = 0, \quad v \in V, i \in \mathcal{N}_v \backslash \{o(v), \mathrm{d}(v)\}, \tag{A.5}$$

$$\sum_{i \in \mathcal{N}_v} x_{id(v)v} = 1, \quad v \in \mathcal{V}, \tag{A.6}$$

$$l_{iv} + Q_j - l_{jv} - K_v(1 - x_{ijv}) \le 0, \quad v \in \mathcal{V}, j \in \mathcal{N}_v^P, (i,j) \in \mathcal{A}_v, \tag{A.7}$$

$$l_{iv} - Q_j - l_{n+j,v} - K_v(1 - x_{i,n+j,v}) \le 0, \quad v \in \mathcal{V}, j \in \mathcal{N}_v^P, (i, n+j) \in \mathcal{A}_v, \tag{A.8}$$

$$\sum_{j \in \mathcal{N}_v} Q_i x_{ijv} \le l_{iv} \le \sum_{j \in \mathcal{N}_v} K_v x_{ijv} \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.9}$$

$$0 \le l_{n+i,v} \le \sum_{j \in \mathcal{N}_v} (K_v - Q_i) x_{n+1,jv} \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.10}$$

$$t_{iv} + T_{iv}^P + T_{ijv}^S - t_{jv} - M_{ijv}(1 - x_{ijv}) \le 0, \quad v \in \mathcal{V}, (i,j) \in \mathcal{A}_v, \tag{A.11}$$

$$\sum_{j \in \mathcal{N}_v} x_{ijv} - \sum_{j \in \mathcal{N}_v} x_{n+i,jv} = 0, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.12}$$

$$t_{iv} + T_{iv}^P + T_{i,n+i,v}^S - t_{n+i,v} \le 0, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.13}$$

$$\underline{T}_{iv} \le t_{iv} \le \bar{T}_{iv}, \quad v \in \mathcal{V}, i \in \mathcal{N}_v, \tag{A.14}$$

$$l_{iv} \ge 0, \quad v \in \mathcal{V}, i \in \mathcal{N}_v, \tag{A.15}$$

$$x_{ijv} \in \{0, 1\}, \quad v \in \mathcal{V}, (i,j) \in A_v, \tag{A.16}$$

$$y_i \in \{0, 1\}, \quad i \in \mathcal{N}^O. \tag{A.17}$$

The objective function (A.1) maximizes the profit from operating the fleet. The three terms are: the revenue gained by transporting the mandatory contracted cargoes, the revenue from transporting the optional spot cargoes, and the transportation costs. The fixed revenue for the contracted cargoes can be omitted, but is included here to obtain a more complete picture of the profit. Constraints (A.2) state that all mandatory contract cargoes are transported. The corresponding requirements for the optional spot cargoes are given by constraints (A.3). Constraints (A.4)–(A.6) describe the flow on the sailing route used by ship v. Constraints (A.7) and (A.8) keep track of the load onboard at the loading and discharging nodes, respectively. Constraints (A.9) and (A.10) represent the ship capacity constraints at the loading and discharging nodes, respectively. Constraints (A.11) ensure that the time of starting service at a node j must be greater than or equal to the departure time from the previous node i, plus the sailing time between the nodes. The big M coefficient in constraints (A.11) can be calculated as $M_{ijv} = \max(0, \bar{T}_{iv} + T^P_{iv} + T^S_{ijv} - \underline{T}_{jv})$. Constraints (A.12) ensure that the same ship v visits both loading node i and the corresponding discharging node $n + i$. Constraints (A.13) force node i to be visited before node $n + i$, while constraints (A.14) define the time window within which service must start. If ship v is not visiting node i, we will get an artificial starting time within the time windows for that (i, v) combination. The non-negativity requirements for the load onboard the ship are given by constraints (A.15). Constraints (A.16) and (A.17) impose the binary requirements on the flow variables and the spot cargo variables, respectively.

Extension I: flexible cargo sizes. In order to represent flexible cargo sizes, the mathematical formulation of the basic model (above) has to be modified as follows. Instead of specifying the weight of cargo i as Q_i, the weight that can be transported is now flexible within the interval $[\underline{Q}_i, \bar{Q}_i]$, where $\underline{Q}_i$ is the minimum weight that must be transported (if it is serviced at all) while $\bar{Q}_i$ is the maximum weight. Let T^Q_{iv} be the time required to load or discharge one unit of cargo i with ship v. We now need an additional set of variables, q_{iv}, that represents the quantity of cargo i (within its interval) that is transported by ship v.

To include the flexible cargo weights, we need the following adjustments of the model for the basic tramp ship routing and scheduling problem:

$$\max \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}^C \cup \mathcal{N}^O} R_i q_{iv} - \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_v} C_{ijv} x_{ijv}, \tag{A.1$'$}$$

$$l_{iv} + q_{jv} - l_{jv} - K_v(1 - x_{ijv}) \le 0, \quad v \in \mathcal{V}, j \in \mathcal{N}^P_v, (i,j) \in \mathcal{A}_v, \tag{A.7$'$}$$

$$l_{iv} - q_{jv} - l_{n+j,v} - K_v(1 - x_{i,n+j,v}) \le 0, \quad v \in \mathcal{V}, j \in \mathcal{N}^P_v, (i, n+j) \in \mathcal{A}_v, \tag{A.8$'$}$$

$$q_{iv} \le l_{iv} \le \sum_{j \in \mathcal{N}_v} K_v x_{ijv}, \quad v \in \mathcal{V}, i \in \mathcal{N}^P_v, \tag{A.9$'$}$$

$$0 \leq l_{n+i,v} \leq \sum_{j \in \mathcal{N}_v} K_v x_{n+1,jv} - q_{iv}, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.10$'$}$$

$$t_{iv} + T_{iv}^Q q_{iv} + T_{ijv}^S - t_{jv} - M_{ijv}\big(1 - x_{ijv}\big) \leq 0, \quad v \in \mathcal{V}, (i,j) \in \mathcal{A}_v, \tag{A.11$'$}$$

$$t_{iv} + T_{iv}^Q q_{iv} + T_{i,n+i,v}^S - t_{n+i,v} \leq 0, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P. \tag{A.13$'$}$$

$$\sum_{j \in \mathcal{N}_v} \underline{Q}_i x_{ijv} \leq q_{iv} \leq \sum_{j \in \mathcal{N}_v} \bar{Q}_i x_{ijv}, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.18}$$

The change in the objective function (A.1′) compared with the original one (A.1) is that the revenue now depends on the quantity transported of each cargo. The only difference between the new constraints (A.7′)–(A.10′) and the original constraints (A.7)–(A.10) is that the cargo quantity parameter has been replaced with the new quantity variable q_{iv}. Time constraints (A.11′) and (A.13′) are similar to the original constraints (A.11) and (A.13), except for that the service time in port now depends on the quantity loaded or discharged. Constraints (A.18) are new compared to the original formulation and specify the upper and lower bounds for the new quantity variables.

Extension II: splitting cargoes between vessels. Also, here (same as for the former extension for flexible cargo sizes) we need quantity variables, q_{iv}, specifying how much of cargo i ship v is carrying. In addition, we need the following adjustments to the original model (A.1)–(A.17):

$$\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}_v} x_{ijv} \geq 1, \quad i \in \mathcal{N}^C, \tag{A.2$''$}$$

$$\sum_{v \in \mathcal{V}} \sum_{j \in \mathcal{N}_v} x_{ijv} - y_i \geq 0, \quad i \in \mathcal{N}^O, \tag{A.3$''$}$$

$$l_{iv} + q_{jv} - l_{jv} - K_v\big(1 - x_{ijv}\big) \leq 0, \quad v \in \mathcal{V}, j \in \mathcal{N}_v^P, (i,j) \in \mathcal{A}_v, \tag{A.7$''$}$$

$$l_{iv} - q_{jv} - l_{n+j,v} - K_v\big(1 - x_{i,n+j,v}\big) \leq 0, \quad v \in \mathcal{V}, j \in \mathcal{N}_v^P, (i, n+j) \in \mathcal{A}_v, \tag{A.8$''$}$$

$$q_{iv} \leq l_{iv} \leq K_v \sum_{j \in \mathcal{N}_v} x_{ijv}, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.9$''$}$$

$$0 \leq l_{n+i,v} \leq \sum_{j \in \mathcal{N}_v} K_v x_{n+1,jv} - q_{iv}, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.10$''$}$$

$$\sum_{v \in \mathcal{V}} q_{iv} = Q_i, \quad i \in \mathcal{N}^C, \tag{A.19}$$

$$\sum_{v \in \mathcal{V}} q_{iv} - Q_i y_i = 0, \quad i \in \mathcal{N}^O, \tag{A.20}$$

$$q_{iv} \geq 0, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P. \tag{A.21}$$

Constraints (A.2″) replace the original constraints (A.2) and state that all mandatory contract cargoes are transported. Each of these cargoes can be transported by one or several ships since it is now possible to split each cargo. The corresponding requirements for the optional cargoes are given by constraints (A.3″). Constraints (A.7″)–(A.10″) are similar to the original constraints (A.7)–(A.10) except that the cargo quantity parameter has been replaced with the quantity variable q_{iv}. It can be noted that constraints (A.7″)–(A.10″) are similar to constraints (A.7′)–(A.10′) for the flexible cargo case. Constraints (A.19)–(A.21) are new and specific for the split problem. Constraints (A.19) ensure that the total weight of contracted cargo i is lifted by one or several ships. A similar requirement for optional cargoes is given by constraints (A.20), while constraints (A.21) impose non-negativity requirements on the load variables.

Extension III: sailing speed optimization. The following amendments are necessary to the basic model (that was presented above) in order to incorporate sailing speed optimization. Each ship v has in practice a feasible sailing speed interval defined by $[\underline{S}_v, \bar{S}_v]$. Let D_{ij} be the sailing distance from node i to node j. The variable s_{ijv} defines the speed of travel from node i to node j with ship v. The time it takes to sail along arc (i,j) is D_{ij}/s_{ijv}. The non-linear function $C_v(s)$ defined in the interval $[\underline{S}_v, \bar{S}_v]$ represents the sailing costs per unit of distance for ship v sailing at speed s. The cost of sailing an arc (i,j) with ship v at speed s_{ijv} is then $D_{ij}C_v(s_{ijv})$.

The model for the basic tramp ship routing and scheduling problem (A.1)–(A.17) can now be adjusted as follows:

$$\max \sum_{i \in \mathcal{N}^C} R_i Q_i + \sum_{i \in \mathcal{N}^O} R_i Q_i y_i - \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_v} D_{ij} C_v(s_{ijv}) x_{ijv}, \tag{A.1‴}$$

$$t_{iv} + T_{iv}^P + D_{ij}/s_{ijv} - t_{jv} - M_{ijv}\left(1 - x_{ijv}\right) \le 0, \quad v \in \mathcal{V}, (i,j) \in \mathcal{A}_v, \tag{A.11‴}$$

$$t_{iv} + T_{iv}^P + D_{ij}/s_{ijv} - t_{n+i,v} \le 0, \quad v \in \mathcal{V}, i \in \mathcal{N}_v^P, \tag{A.13‴}$$

$$\underline{S}_v \le s_{ijv} \le \bar{S}_v \quad v \in \mathcal{V}, (i,j) \in \mathcal{A}_v. \tag{A.22}$$

The objective function (A.1‴) has now become a non-linear function because of the non-linear relationships between fuel consumption and speed. Constraints (A.11‴) and (A.13‴) correspond to constraints (A.11) and (A.13) in the original formulation. These constraints are also non-linear now because the sailing time depends on the speed variable. The new constraints (A.22) define the upper and lower bounds for the speed variables.