Chapter 5

Satellite navigation

5.1 Introduction

It is surprising that the space technology that we rely on so heavily today had its origins over 50 years ago when, in the early 1950s, with the shock launching by the USSR of a man-made satellite into low orbit, the United States space programme was born. Although a tiny vehicle by present day standards, the USSR's 'Sputnik' had a radio transmitter on board, the frequency of which exhibited a pronounced Doppler shift when observed from any fixed point on the earth's surface. The Doppler phenomenon was well documented but this was the first time the effect had been produced by and received from a man-made orbiting satellite. Space engineers soon recovered from the initial shock and were quick to see that the effect could be exploited to create a truly accurate global positioning system, free from many of the constraints of the existing earth-bound hyperbolic navigation systems.

The first commercially available system to be developed, the Navy Navigation Satellite System (NNSS), made good use of the Doppler effect and provided the world's shipping with precise position fixing for decades. However, nothing lasts forever. The technology became old and the system was dropped on 31 December 1996 in favour of the vastly superior Global Positioning System (GPS). Although a number of NNSS Nova satellites are still in orbit, the system is no longer used for commercial navigation purposes.

5.2 Basic satellite theory

Whilst it is not essential to understand space technology, it is helpful to consider a few of the basic parameters relating to satellite orbits and the specific terminology used when describing them. A satellite is placed in a pre-determined orbit, either in the nose of an expendable launch vehicle or as part of the payload of a space shuttle flight. Either way, once the 'bird' has been delivered into the correct plane, called the 'inclination', that is the angle formed between the eastern end of the equatorial plane and the satellite orbit, it is subject to Kepler's laws of astrophysics.

Figure 5.1 shows orbits of zero inclination for the equatorial orbit, 45°, and for a polar orbit, 90°. The final desired inclination partly determines the launching site chosen. In practice it is difficult to achieve an inclination which is less than the latitude of the launching site's geographical location. A zero inclination orbit is most effectively produced from a launch pad situated on the equator, but this is not always possible and a compromise is often made. Launch normally takes place in an easterly direction because that way it is possible to save fuel, and thus weight, by using the earth's rotational speed to boost the velocity of the accelerating rocket. For an easterly launch from a site on the equator, the velocity needed to escape the pull of gravity, is 6.89 km s^{-1}, whereas for a westerly launch it is 7.82 km s^{-1}. Launch velocities also vary with latitude and the direction of the flight path.

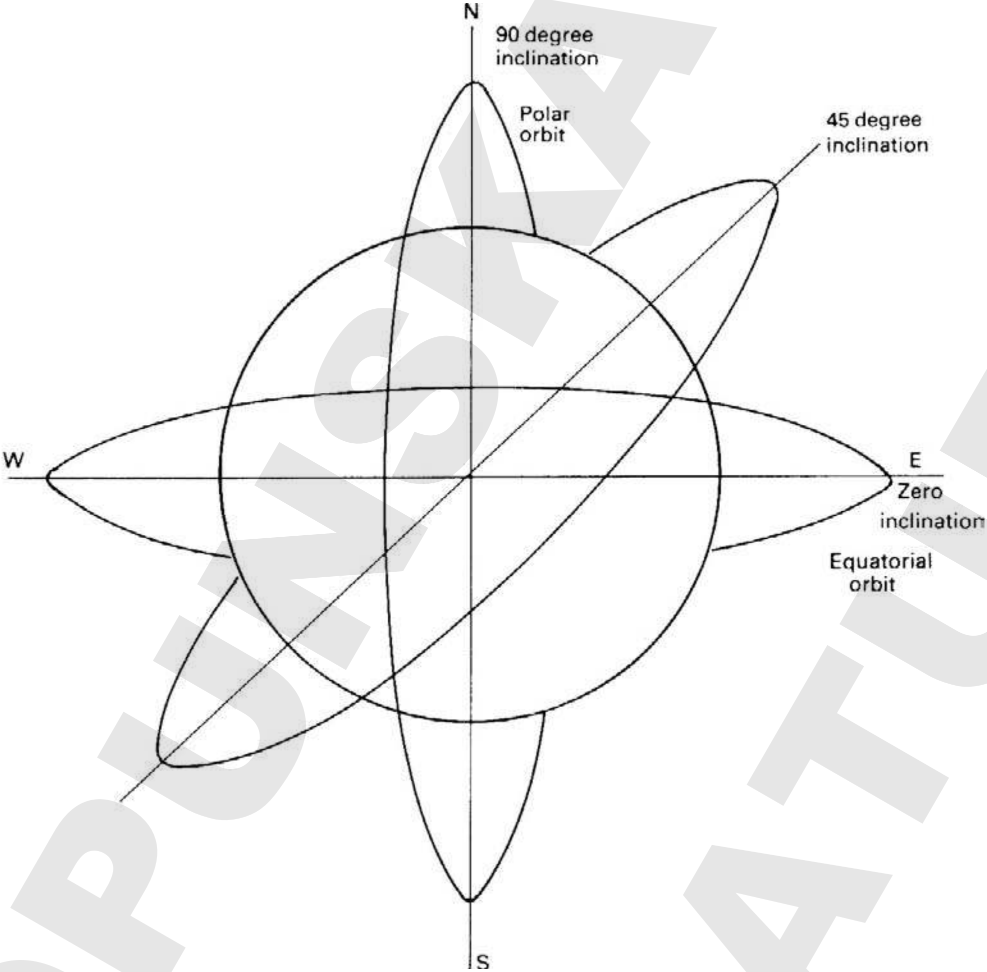

Figure 5.1 Illustration of orbital inclination.

5.2.1 Kepler's Laws

Essentially, an artificial earth-orbiting satellite obeys three laws that were predicted in the late 16th century by Johannes Kepler (1571–1630) who also developed theories to explain the natural orbits of the planets in our solar system. When applied to artificial orbiting satellites, Kepler's laws may be summarized as follows.

- A satellite orbit, with respect to the earth, is an ellipse.
- Vectors drawn from the satellite orbit to the earth describe equal areas in equal times.
- The square of the period of the orbit is equal in ratio to the cube of its mean altitude above the earth's surface.

True to Kepler, artificial earth satellites follow elliptical orbits. In some cases the ellipse eccentricity is large and is a requirement of the first stage of a launch to the higher geostationary orbit, but in most

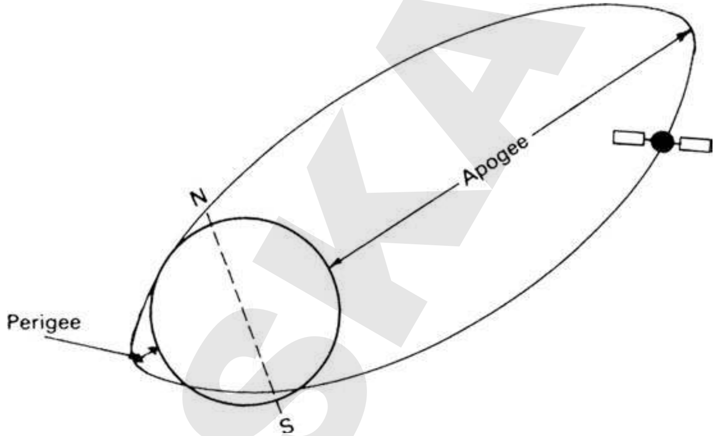

Figure 5.2 Illustration of apogee and perigee.

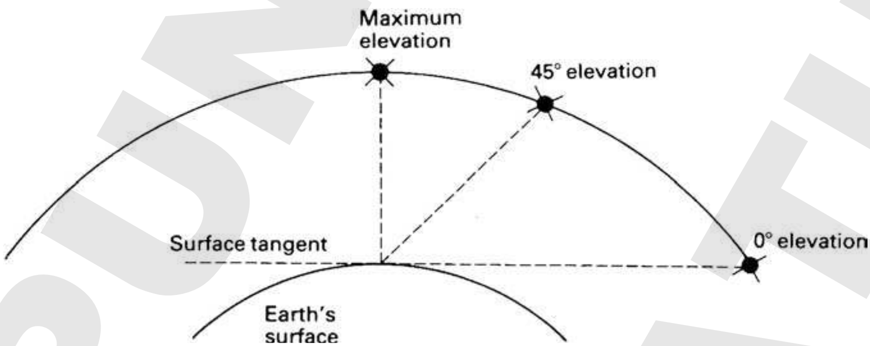

Figure 5.3 Showing the changing angle of elevation during a satellite pass. The angle reaches a maximum at the closest point of approach to the earth bound observer.

cases it is created because the earth is not a perfect sphere. The closest point of approach to the earth of any elliptical orbit is called the 'perigee' and the furthest distance away is the 'apogee', as shown in Figure 5.2. The direction vector to the satellite from a fixed point on the earth is called the 'azimuth' and is quoted in degrees. The angle between the satellite, at any instant, and the earth's surface tangent is the 'elevation' and again is quoted in degrees (see Figure 5.3).

5.2.2 Orbital velocity

A satellite can only remain in orbit if its velocity, for a given altitude, is sufficient to defeat the pull of gravity ($9.81\ \text{ms}^{-1}$) and less than that required to escape it. The velocity must be absolutely precise for the orbital altitude chosen. Eventually, drag will slow the satellite causing it to drop into a lower orbit and possibly causing it to re-enter the atmosphere and burn-up. The nominal velocity for a satellite at any altitude can be calculated by using the formula:

$$V = \frac{K}{(r + a)^{1/2}}\ \text{kms}^{-1}$$

where V = orbital velocity in kms^{-1},
a = altitude of the satellite above the earth's surface in km,
r = the mean radius of the earth (approximately 6370 km), and
K = 630 (a constant derived from a number of parameters).

The earth is not a perfect sphere and therefore its radius with respect to orbital altitude will vary. However, to derive an approximate figure for velocity, an earth radius figure of 6370 km is close enough. The velocity of a satellite with an altitude of 200 km would be:

$$V = \frac{630}{(6370 + 200)^{1/2}} = 7.77\,kms^{-1}$$

Orbital paths can be transferred to a Mercator projection chart as shown in Figure 5.4. The inclination will be the same in both northern and southern hemispheres and corresponds to latitude. The six orbits shown are for Navstar (GPS) satellites with an orbital inclination of 55°.

5.2.3 Orbital period

The time period for one complete orbit of a satellite can be readily calculated using the simple formula below:

$$P = K\left(\frac{r + a}{r}\right)^{3/2}$$

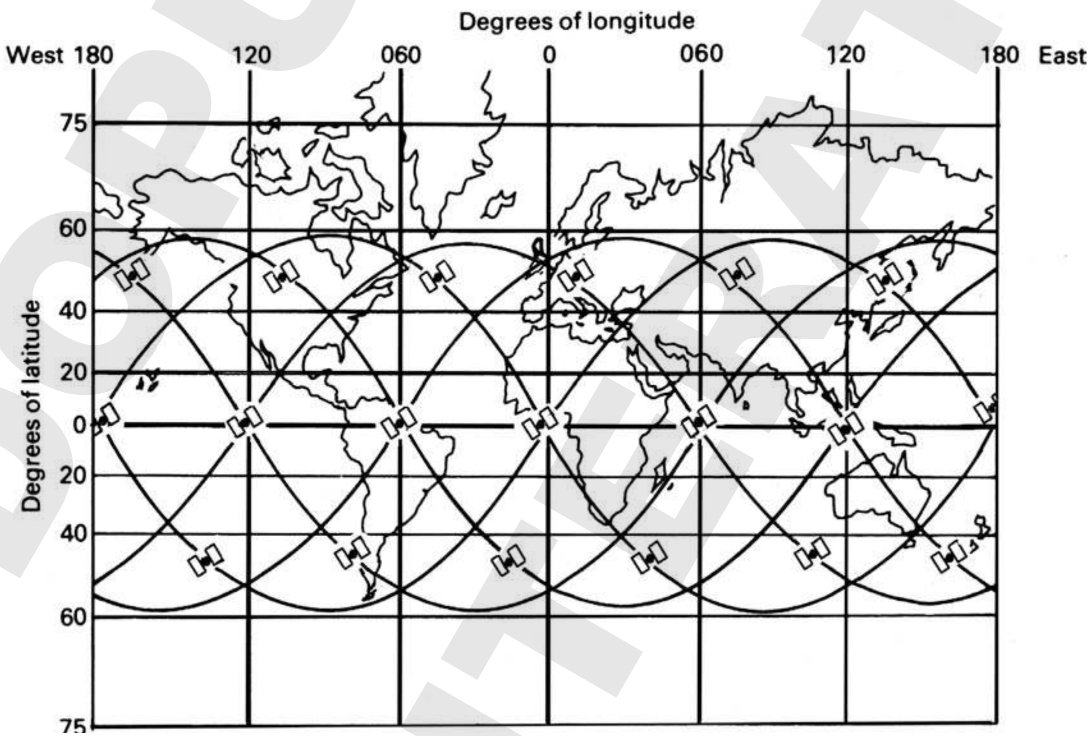

Figure 5.4 Mercator presentation of the orbital inclination paths described by satellite orbits.

where P = the period of one orbit in min,
a = the altitude of the orbit above the earth's surface in km,
r = the mean radius of the earth in km, and
K = 84.49 (a constant derived from a number of parameters).

The orbital period for a satellite at an altitude of 200 km is:

$$P = 84.49\left(\frac{6371 + 200}{6371}\right)^{3/2} = 88.45 \text{ min}$$

5.3 The Global Positioning System (GPS)

In 1973 a combined US Navy and US Air force task-force set out to develop a new global satellite navigation system to replace the ageing Navy Navigation Satellite System (NNSS).

The original test space vehicles (SVs) launched in the new programme were called Navigation Technology Satellites (NTS) and NTS1 went into orbit in 1974 to became the embryo of a system that has grown into the Global Positioning System (GPS). GPS was declared to be fully operational by the US Air Force Space Command (USAFSC) on 27April 1995, and brought about the demise of the NNSS which finally ceased to provide navigation fixes at midnight on 31 December 1996.

The GPS, occasionally called NAVSTAR, shares much commonality with the Russian Global Navigation System (GLONASS), although the two are in no way compatible. The GPS consists of three segments designated Space, Control and User.

5.3.1 The space segment

Satellite constellation calls for 24 operational SVs, four in each of six orbital planes, although more satellites are available to ensure the system remains continuously accessible (see Figure 5.5). SVs orbit the earth in near circular orbits at an altitude of 20 200 km (10 900 nautical miles) and possess an inclination angle of 55°.

Based on standard time, each SV has an approximate orbital period of 12 h, but when quoted in the more correct sidereal time, it is 11 h 58 min. Since the earth is turning beneath the SV orbits, all the satellites will appear over any fixed point on the earth every 23 h 56 min or, 4 min earlier each day. This, totally predictable, time shift is caused because a sidereal day is 4 min shorter than a solar day and all SVs complete two orbits in one day. To maintain further orbital accuracy, SVs are attitude-stabilized to within 1 m by the action of four reaction wheels, and on-board hydrazine thrusters enable precision re-alignment of the craft as required.

This orbital configuration, encompassing 24 SVs, ensures that at least six SVs, with an elevation greater than 9.5°, will be in view of a receiving antenna at any point on the earth's surface at any time. When one considers the problems of rapidly increasing range error caused by the troposphere at low SV elevations, 9.5° has been found to be the minimum elevation from which to receive data when using a simple antenna system.

The original satellites, numbered 1–11 and designated Block I, have ceased operation. Currently, the GPS constellation is based on the next generation of SVs, designated Block II. Block II (numbers 13–21) and block IIA (numbers 22–40) satellites, manufactured by Rockwell International, were launched from Cape Canaveral between February 1989 and November 1997. Each SV holds four atomic clocks, two rubidium and two caesium, and has selective availability (SA) and anti-spoofing (A-S) capabilities, although the US Government has now given an assurance that the system

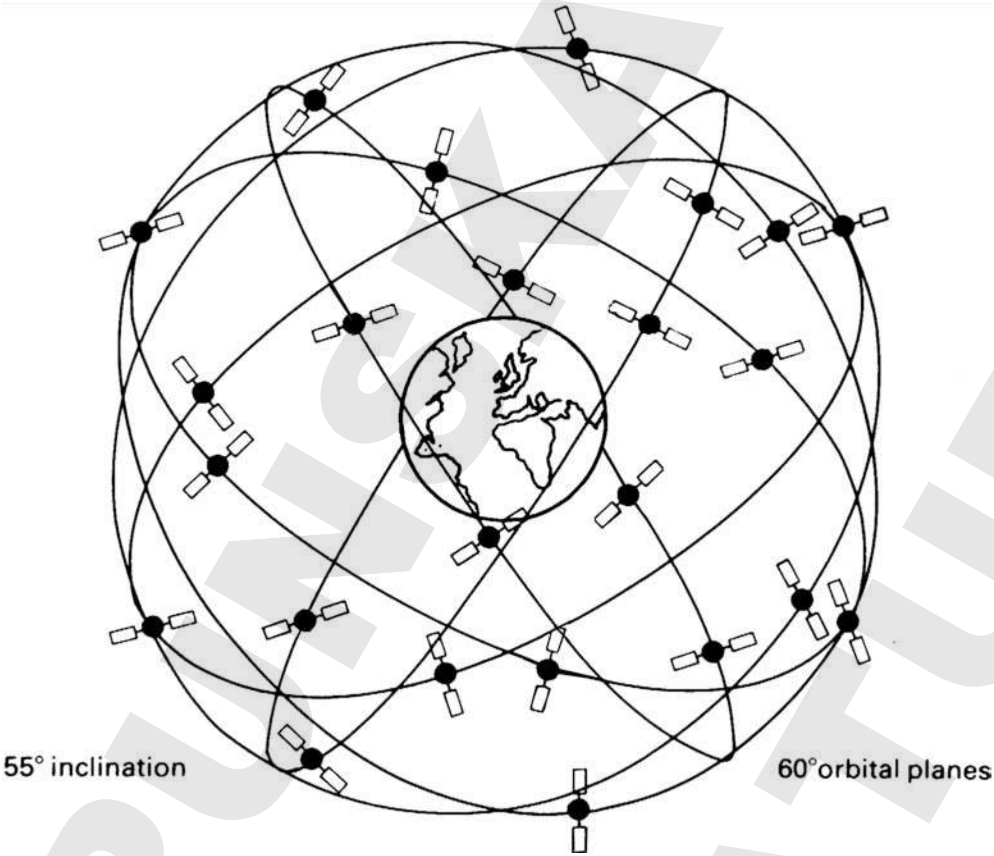

Figure 5.5 GPS satellite coverage. Twenty-four satellites provide global coverage; four in each of six orbital planes.

downgrading functions, SA and A-S, will no longer be implemented in the GPS. Block IIR SVs (numbers 41–62) are replenishment satellites and have been designed for an operational life of 7.8 years.

All SVs transmit a navigation message comprising orbital data, clock timing characteristics, system time and a status message. They also send an extensive almanac giving the orbital and health data for every active SV, to enable a user to locate all SVs once one has been acquired and the data downloaded.

5.3.2 The control segment

The GPS is controlled from Schriever Air Force Base (formerly Falcon AFB) in Colorado. It is from there that the SV telemetry and upload functions are commanded. There are five monitor stations (see Figure 5.6), which are situated in the Hawaii Islands in the Pacific Ocean, on Ascension Island in the Atlantic, on Diego Garcia in the Indian Ocean, on Kwajalein Island, again in the Pacific, and at Colorado Springs on mainland US territory. SV orbital parameters are constantly monitored by one or more of the ground tracking stations, which then pass the measured data on to the Master Control Station (MCS) at Schriever. From these figures the MCS predicts the future orbital and operational

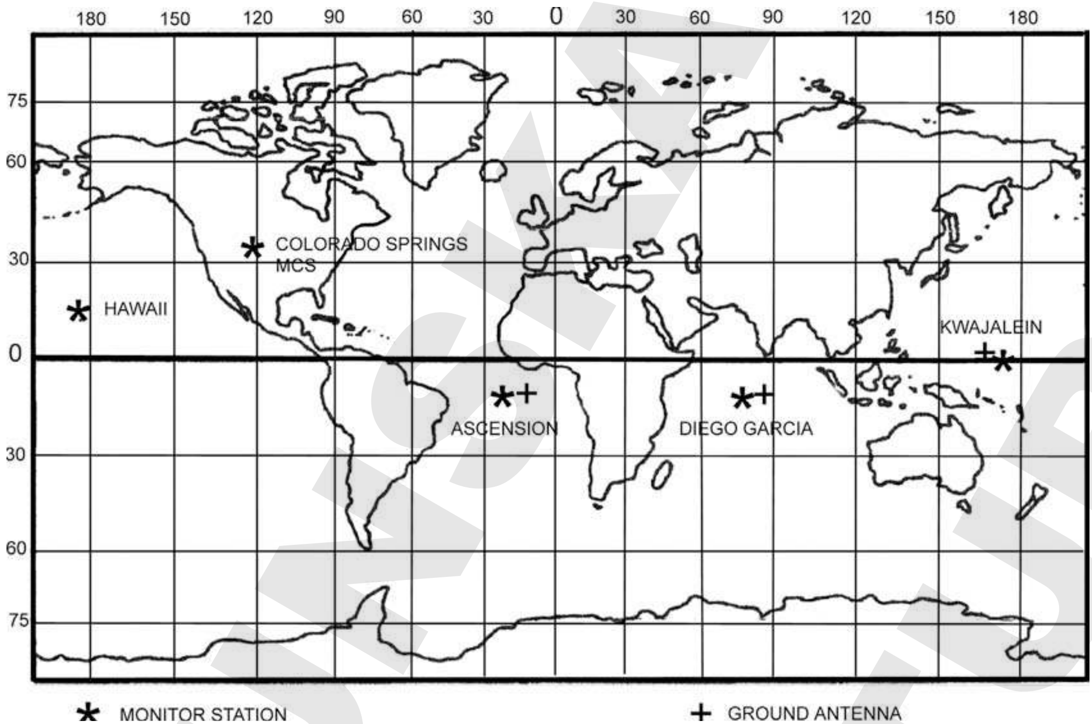

Figure 5.6 GPS control segment stations.

parameters to be fed to the Upload Stations (ULS) on Ascension, Diego Garcia and Kwajalein Islands. All ground station locations have been precisely surveyed with respect to the World Geodetic System 1984 (WGS-84). Data are transmitted to each SV from a ULS, to be held in RAM and sequentially transmitted as a data frame to receiving stations.

Signal parameters

Navigation data are transmitted from the SV on two frequencies in the L band (see Table 5.1). In practice the SV clock is slightly offset to a frequency of 10.229 999 995 45 MHz to allow for the effects of relativity. SV clock accuracy is maintained at better than one part in 10^{12} per day. Dual frequency transmission from the SV ensures that suitably equipped receivers are able to correct for signal delay (range error) caused by the ionosphere. Ionosopheric delays are proportional to $1/f^2$ hence the range error produced will be different on each frequency and can be compensated for in the receiver.

The C/A (Coarse and Acquire) code, see Figure 5.7, is a PRN (pseudo random noise) code stream operating at 1.023 megabits/s and is generated by a 10-bit register. C/A code epoch is achieved every 1 ms (1023 bits) and quadrature phase modulates the L_1 carrier only. This code has been designed to be easily and rapidly acquired by receivers to enable SPS fixing. Each SV transmits a unique C/A code that is matched to the locally generated C/A code in the receiver. A unique PRN is allocated to each SV and is selected from a code series called Gold codes. They are specifically designed to minimize the possibility that a receiver will mistake one code for another and unknowingly access a wrong satellite. Navigation data is modulated onto the L_1 C/A code at a bit rate of 50 Hz.

Table 5.1 SV transmission frequencies

Band	*Derivation (MHz)*	*Frequency (MHz)*	*Wavelength (cm)*	*Code*
L_1	154×10.23	1575.42	19	C/A
L_2	120×10.23	1227.60	24.5	C/A & P

Both carriers are derived from the SV clock frequency 10.23 MHz

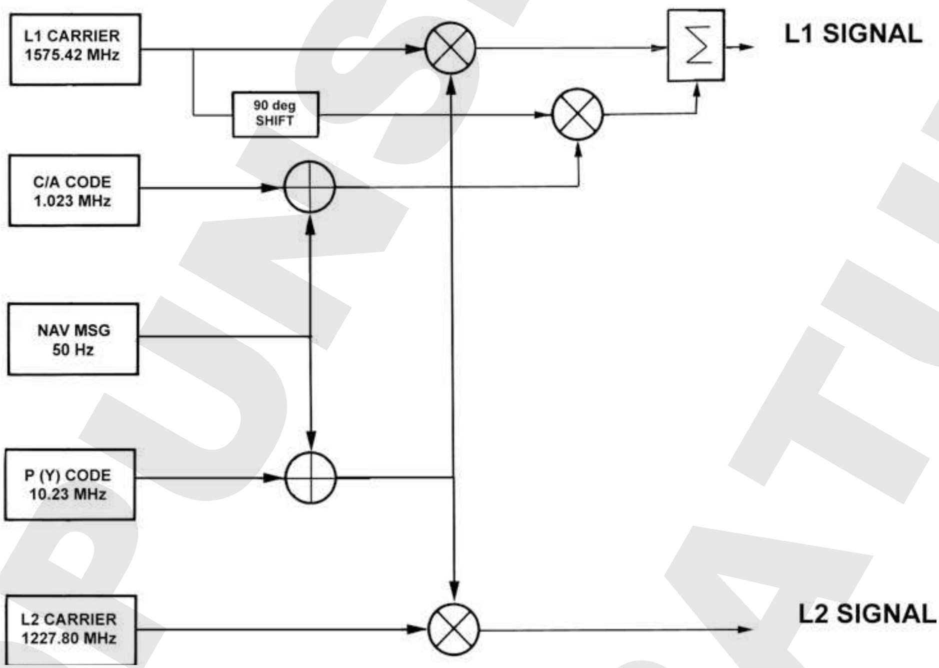

Figure 5.7 Schematic diagram of a SV modulation circuit.

The P (Precise) code, operating at 10.23 MHz, is a PRN code produced as the modulo 2 sum of two 24-bit registers, in the SV, termed X1 and X2. This combination creates a PRN code of 2^{48-1} steps equating to a complete code cycle (before code repetition occurs) of approximately 267 days. Each SV employs a unique and exclusive 7-day long phase segment of this code. At midnight every Saturday, GPS time, the X1 and X2 code generators are reset to their initial state (epoch) to re-initiate the 7-day phase segment at another point along the 267-day PRN code cycle. Without prior knowledge of the code progression, it is not possible to lock into it.

The navigation data message

A 50-Hz navigation message is modulated onto both the P code and C/A codes. One data frame is 1500 bits and takes 30 s to complete at the bit rate of 50 bit s^{-1}. Navigation data are contained in five subframes each of 6 s duration and containing 300 bits. Table 5.2 shows the data format structure.

Table 5.2 Data format structure

Five words 300 bits each with a total of 6 s			
	30 bits	*30 bits*	*240 bits*
01	TLM	HOW	Data block 1: Clock correction data. Accuracy and health of the signal.
02	TLM	HOW	Data block 2: Ephemeris data. Precise orbital parameters to enable a receiver to compute the position of an SV.
03	TLM	HOW	Data block 3: Ephemeris. Continued.
04	TLM	HOW	Data block 4: Almanac. Orbital data, low-precision clock data, simple health and configuration status for every SV, user messages, ionosopheric model data and UTC calculations.
05	TLM	HOW	Data block 5. Almanac. Continued.

Subframes 4 and 5 hold low precision data, common to all SVs, and less critical for a satellite to acquire quickly.

As shown in Figure 5.8, each of the five subframes commences with a 14-bit TLM word (telemetry) containing SV status and diagnostic data. This is followed by a 17-bit handover word (HOW). HOW data enables a receiver, which has knowledge of the code encryption, to acquire the P code. Data subframe block 1 contains frequency standard corrective data enabling clock correction to be made in the receiver. Data blocks 2 and 3 hold SV orbit ephemeris data. The two blocks contain such data as orbit eccentricity variations and Keplerian parameters. Message block 4 passes alphanumeric data to the user and is only used when the ULS has a need to pass specific messages. Block 5 is an extensive almanac that includes data on SV health and identity codes.

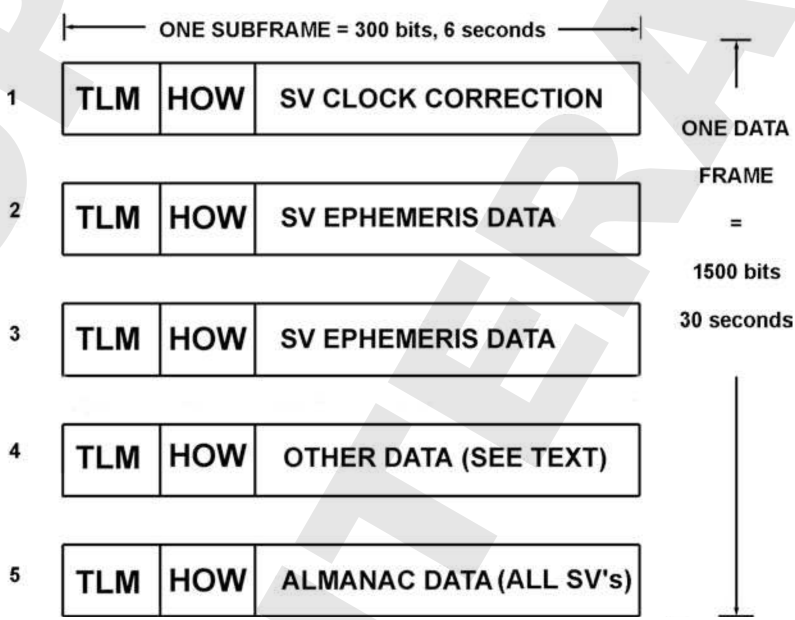

Figure 5.8 Navigation data format.

Table 5.3 Summary of data in a 30-s frame

A	SV orbital parameters
B	SV clock error data
C	Sidereal correction figures
D	Almanac of all operational SVs
E	Polar wander data (Earth axis wander)
F	SV performance status
G	Time of last data inject
H	Data to enable P code acquisition (HOW)
I	Telemetry data (TLM)
J	SV number
K	Specific messages as required (i.e. an indication that an SV is off station)
L	Receiver clock correction data

At the 50-Hz transmission rate, it takes 6 s to download a subframe, 30 s for one data frame (see Table 5.3) and a full 12.5 min to access all 25 frames.

The L_1 signal carrier is BPSK-modulated by both the P and C/A PRN codes and the navigation message. Modulation possesses both in-phase and quadrature components as shown in Figure 5.9.

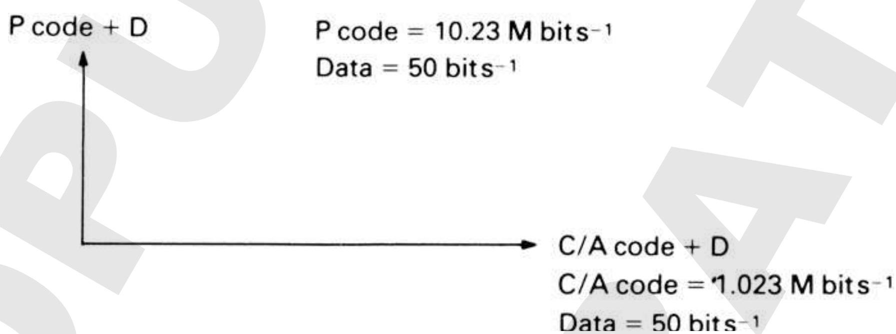

Figure 5.9 Phase relationship between the P and C/A codes.

P code amplitude is –3dB down (half the power level) on the C/A code signal strength, thus the slower C/A code provides a better signal-to-noise ratio at the antenna. This makes the C/A code easier to access. The L_2 carrier is BPSK-modulated by the P code and the navigation message. The use of BPSK modulation causes a symmetrical spread of the code bandwidth around the carrier frequency. The frequency spectrum produced by both P and C/A codes on the L_1 carrier is shown in Figure 5.10. The bandwidth of the C/A code is 2.046 MHz and that of the P code is 20.46 MHz. The C/A code component of the L_1 signal possesses a power of –160 dBW (with respect to 1 watt), the L_1 P code a power of –163 dBW, and the L_2 P code signal has a power level of –166 dBW.

It should be noted that data modulation at 50 bit s^{-1} produces a bandwidth of 100 Hz that is impossible to illustrate on this scale. Signal bandwidth, code matching and data stripping are further explained in the GPS receiver pages later in this chapter.

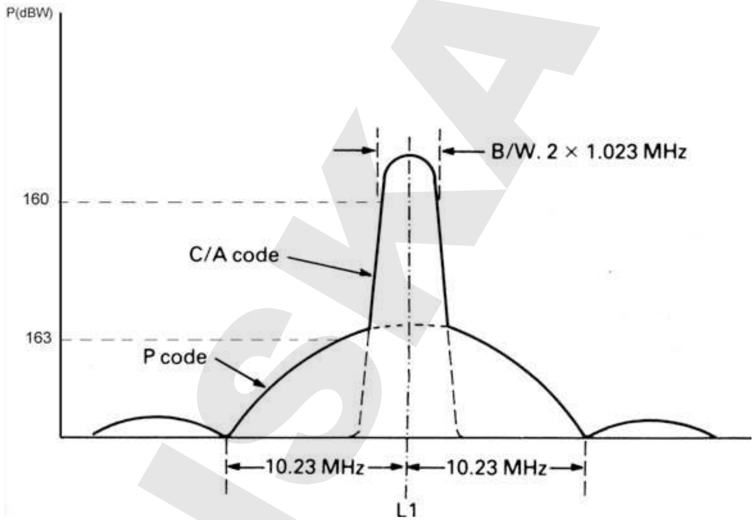

Figure 5.10 Bandwidth power distribution curves for the P and C/A codes.

Frequency stability

SV clock frequency stability is of major importance in any system that relies upon the accurate measurement of range for its operation. Stability is not easy to maintain in an electronic unit that is subjected to constantly varying ambient temperatures. The SV is travelling through a hostile environment where temperatures can vary by as much as 300°C. In addition, at the high altitudes of any SV, there is little protection from the sun's radiation. For these reasons the clock oscillators in SVs are under constant scrutiny

Since the early days of radiocommunication development, oscillator stability has been a major problem and it is one that has been compounded with the need to send clock oscillators into space. Older SVs, such as the Transit and Nova range on which the earlier NNSS sat-nav system was based, used quartz-controlled clock oscillators to give a short-term stability of 10^{-11} with a 24-h change less than 10^{-9}. Timation SVs, the first to provide navigation capability by the calculation of the range between satellite and receiver, carried a quartz clock oscillator with a stability of 1 part in 10^{-11} per day. Timation SVs carried a new frequency standard unit formed by a quartz oscillator locked to an atomic resonance line of rubidium.

The technology used in rubidium and caesium clock oscillators is beyond the scope of this book. However, it should be noted that use of this type of oscillator in NTS1 produced the two transmission signals (UHF and L band) to an accuracy of 1 part in 10^{-12} per day. Caesium/quartz units offer even greater frequency stability and in 1975 the second generation of NTS vehicles was launched into orbit. NTS2 carried a caesium frequency standard unit from which were produced the carrier frequencies (SHF, L_1 and L_2) with an accuracy of 1 part in 10^{-13} per day. These oscillators are still in orbit and still being tested by the armed forces. Caesium clocks, however, require regular updating from the ground and in an effort to further improve and maintain stability for extended periods, clock units using hydrogen maser technology are being considered.

The clock oscillators used in current Navstar SVs are caesium/quartz with rubidium/quartz back-up units.

System time

GPS system time is locked to the Master Clock (MC) at the USNO and further synchronized to UTC from which it will never deviate by more than 1 μs . Actual system time is given by its Composite Clock (CC) or, as it is often called a ‘paper’ clock, which had its epoch at 0000 UTC on 17 June 1990. Information about the GPS time difference and rate of system time against UTC (USNO) is contained in the navigation message transmitted to all users. Once a satellite has been accessed the user equipment clock is corrected.

5.4 The position fix

The GPS provides two levels of service known as Precise Positioning Service (PPS) and Standard Positioning Service (SPS), the accuracy of which were defined in the 1994 US Federal Radionavigation Plan. The PPS predictable accuracy is given in Table 5.4.

Table 5.4 PPS predictable accuracy

Horizontal accuracy	21 m
Vertical accuracy	27.7 m
Time transfer accuracy	197 ns

Based on a 95% Rayleigh distribution probability

PPS fixes are based on range measurement and the acquiring and integrating of the C/A code and the complex P code transmitted on both the L_1 and L_2 carrier frequencies. The method provides highly accurate positioning, timing and velocity figures for users authorized by the US Government. PPS users were generally the US military, government agencies and approved allied forces, but since 1 May 2000, when selective availability was ended, PPS fix accuracy is available to anyone with suitable equipment.

Selective availability (SA) was the name given to a process employed by the US Department of Defence to deny PPS accuracy to civilian users. SA was applied by offsetting SV clock frequency (dithering), and/or manipulating navigation orbit data (epsilon). To guard against the fake transmission of SV data, a system called anti-spoofing (A-S) was used whereby the P code was encrypted becoming the Y code. By Presidential order, on 1 May 2000, the US Government ceased to apply SA to the GPS and thus there is now little difference between SPS and PPS fix accuracy (see Table 5.5).

Table 5.5 SPS predictable accuracy

	Prior to 1 May 2000	*Subsequent to 1 May 2000*
Horizontal error	100 m	25 m
Vertical error	156 m	30 m
Time transfer error	340 ns	200 ns

Based on a 95% Rayleigh distribution probability
Note: On 1 May 2000, Selective Availablity (S/A) was set to zero and SPS accuracy was thus improved by a factor of almost 10. The figures in column 3 are an approximation.

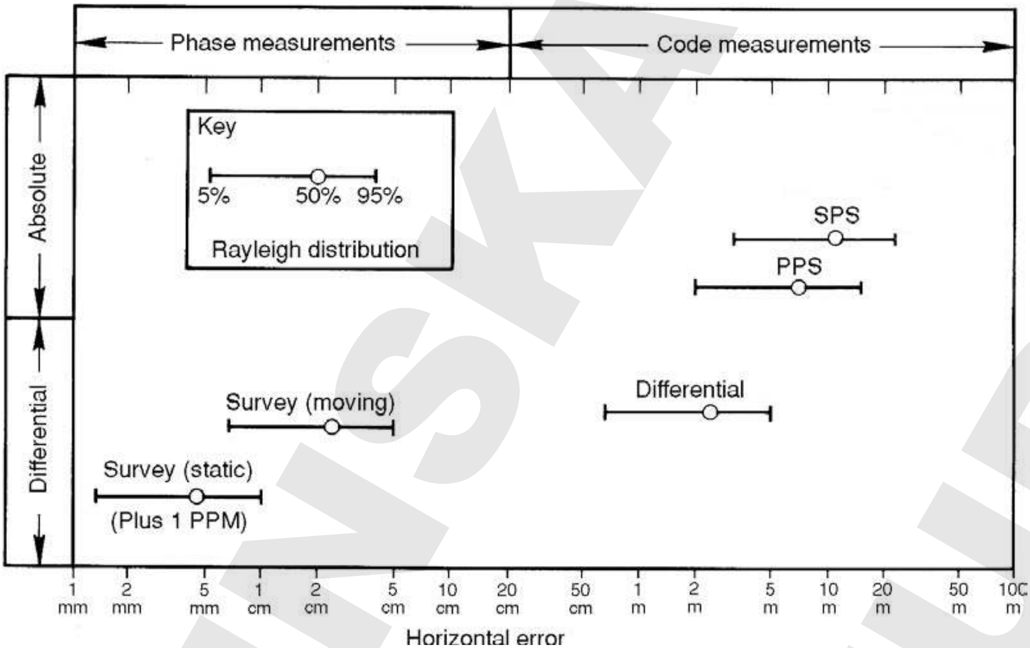

Figure 5.11 Levels of GPS accuracy. (Reproduced courtesy of Magnavox.)

The decision to remove SA from the GPS was taken because it would have minimal impact on national security. Based on threat assessment analysis, it is possible for the US Government to selectively deny GPS signals on a regional basis if national security is threatened.

SPS fixes are based on acquiring and integrating the C/A code data transmitted on the L_1 carrier frequency, measuring ranges and decoding the navigation message. SPS fix accuracy can be extensively improved by using Differential GPS (see Figure 5.11). Data is received, at both a mobile and a ground station, from multiple SVs and, after the computation of correction figures at the fixed station, is retransmitted to the mobile receiver. The process is achieved in real time although because of the relatively short distances travelled by a ship between fixes it is possible to apply corrections to subsequent computations.

The upper part of Figure 5.11 shows the anticipated levels of accuracy of a standard position fix without the aid of differential techniques, whereas the lower half shows fix accuracy for receivers with a differential input. It also demonstrates that the use of phase measurement in addition to code measurement improves the fix still further. All fix lines are shown as Rayleigh distribution data.

GPS position fixes are achieved by the precise measurement of the distance between a number of SVs and a receiver at an instant in time and/or by phase measurement. It is possible for a receiver, with a precise clock and with a knowledge of altitude above the earth reference spheroid, to fix its position in three dimensions by interrogating a minimum of three SVs. But in practice, modern equipment provides for more precise position fixing using the data from four or more SVs. By interrogating multiple SVs it is possible to obtain accurate fixes in three dimensions (XYZ) plus time. All fixes computed by a receiver are known as earth-centred-earth-fixed (ECEF) locations and therefore navigation fixes are often quoted as ECEF XYZ positions.

To measure the precise distance between the transmitter and the receiver requires highly accurate time clocks in both vehicles. The satellite clock is monitored from the ground and is corrected by atomic standard time. During calculations, it is accepted therefore, that this clock, which is used to generate the transmission frequencies, is accurate and the receiver clock may be in error.

For this reason range measurements are termed false or 'pseudo-ranges', and must be corrected in the receiver. The pseudo-range measurement for a receiver with an imprecise clock is given as:

$$PsR = Rt + C\Delta td + C(\Delta tu - \Delta ts)$$

where range figures are in metres and time in seconds, PsR = pseudo-range between satellite and receiver, Rt = true range, C = speed of light (3×10^8 ms^{-1}), Δts = satellite clock error from GPS time, Δtu = receiver clock error from GPS time, and Δtd = propagation delays due to both the ionosphere and the troposphere.

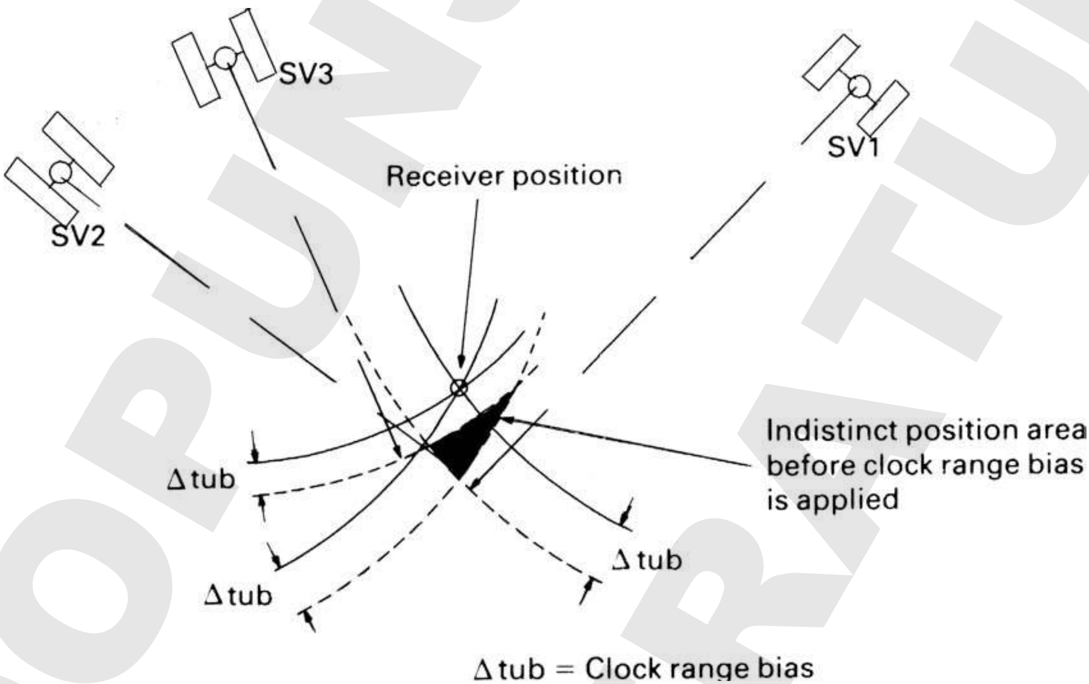

Figure 5.12 Showing the indistinct position fix obtained from three SVs before clock range bias is applied.

The GPS receiver calculates the pseudo-range time taken for the transmission by measuring the phase shift of the P code and comparing it with a locally generated code in the receiver computer. Figure 5.12 illustrates that the pseudo-ranges calculated for three satellites will not converge at a specific point unless the receiver clock error is corrected.

The computed position in XYZ co-ordinates is converted as a function of the receiver algorithm to geodetic latitude, longitude and altitude above the reference ellipsoid. The ship's position is solved with reference to Cartesian co-ordinates as shown in Figure 5.13 with reference to a minimum of three celestial 'fixed' points (the SVs).

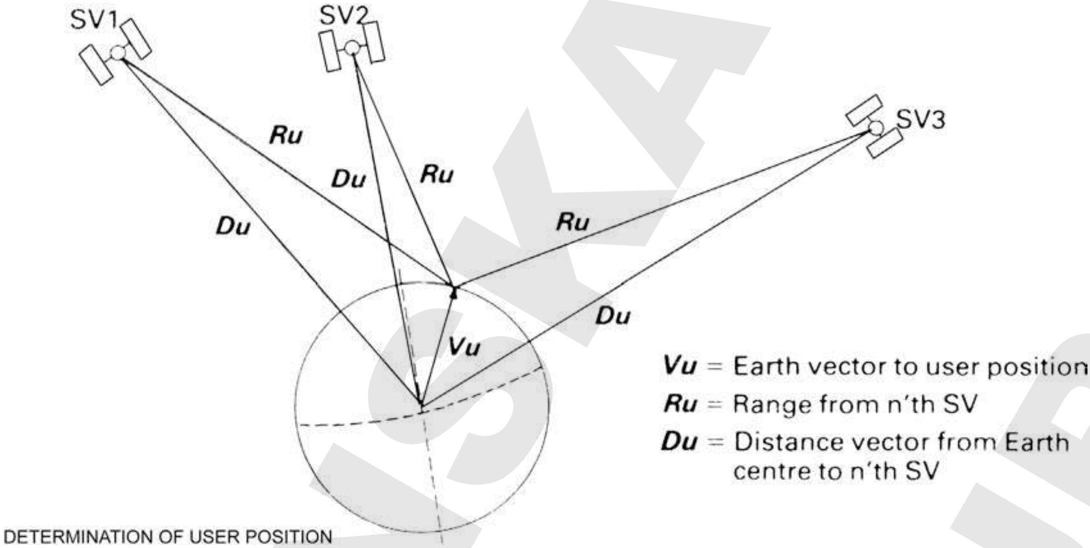

Figure 5.13 Using Cartesian co-ordinates to determine an earth centred position fix.

5.5 Dilution of Precision (DOP)

Dilution of Precision (DOP) is a term used for expressing the mathematical quality of a solution. DOP can exist in one dimension only. Examples are; time DOP (TDOP); horizontal DOP; vertical DOP and geometric DOP, referring to SV geometry. But it is the position dilution of precision, PDOP, that is of most value to a navigator. PDOP in the GPS has an optimum value of unity. If the figure is higher the solution is degraded (diluted). The PDOP will approach unity when a solution is made with a satellite overhead and three other satellites evenly spaced at low elevation angles. Alternatively, if all satellites are in the same plane, PDOP would be near infinity and the navigation fix solution would be unsound. The PDOP figure has a direct bearing on user range error (URE). For example, for a URE of 50 m and a PDOP of unity, the best fix accuracy is 50 m. If the PDOP is 2, the accuracy drops to 100 m. Modern GPS receivers may be programmed to reject a position solution if the PDOP level is high.

The geometry of the satellite orbital cage can seriously affect the accuracy of a position fix. With 24 satellites in six orbits there is a better than average chance that as many as six will be in view of a receiver at any given time. When pseudo-ranges are measured from SVs that are close together in the sky (Figure 5.14(a)), the result is an enlarged area of improbability resulting in a bad GDOP, as shown above. Alternatively if the SVs are well spaced, the improbability area will be smaller. Modern GPS receivers pick the optimum SVs from those available before correcting timing errors.

5.6 Satellite pass predictions

The system is so well documented and controlled that it has become increasingly easy to predict satellite passes at a given location. Trimble Navigation Limited, one of the biggest manufacturers of GPS equipment, operates a world wide web site that will be of interest to students. It is called GPS Mission Planning and is accessed on http://www.trimble.com/cgi/satview.cgi. It is also interactive and

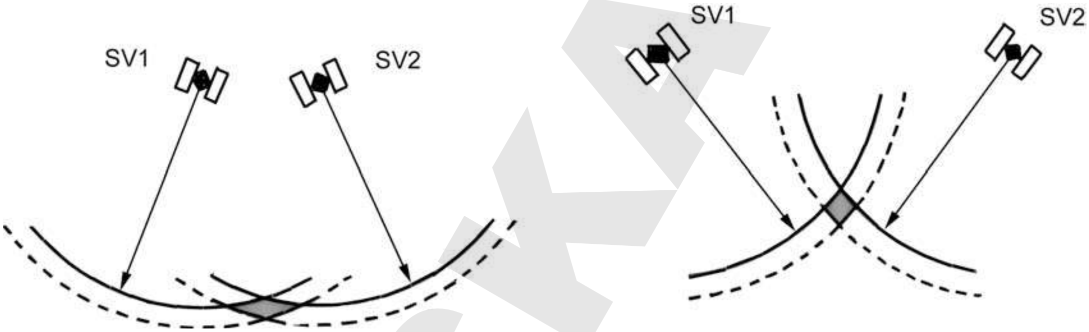

Figure 5.14 Fix accuracy can be improved by selecting appropriate SVs. (a) Two SVs giving a poor GDOP and (b) two SVs providing a much better solution.

provides six different charts of predictions. User parameters for all the plots are input into boxes as shown below. Latitudes south of the equator and longitudes west of the Greenwich Meridian are identified with a minus sign. The time input in GMT is in two figures between 00 and 23.

Using this system it is easy to predict SV passes at a given location and consequently it is simple to select the appropriate SVs to give a good GDOP.

Latitude:	32.43	Date:	00–00–2000
Longitude:	–117.10	Starting hour GMT:	00 hours
Mask:	15.0 degs.	Duration:	4 hours

The six plots are as follows.

- *Azimuth Plot.* Use this plot to locate SVs with optimum azimuth angle for a given location.
- *DOP Plot.* A low DOP indicates a high probability of accuracy, whereas a high DOP shows a low probability. The plot shown in Figure 5.15 is the result of calculations evaluating the geometry of four available SVs that will provide the most accurate fix. The plot has three data lines corresponding to HDOP, VDOP and PDOP predictions.
- *Elevation Plot.* This plot (Figure 5.16) shows the paths of all the satellites in view for a specified time period at a specific location. An SV reaching an elevation of 90° will pass directly overhead.
- *Sky Plot.* This plot (Figure 5.17) is oriented so that the GPS receiver is in the centre of concentric rings spaced at 15° intervals. The outer ring represents the horizon. Using this plot it is easy to see if a SV could suffer signal block from buildings or trees because it is low on the horizon.
- *Total-in-View Plot.* This is a graph showing the total SVs in view over a specified elevation angle. It is particularly useful for checking if sufficient satellites will be in view to make a good fix.
- *Visibility Periods Plot.* Another form of presentation showing the time periods when satellites will be in view above the angle of elevation specified.

5.7 System errors

Errors in any system arise from a number of sources. They can be predictable or not and avoidable or not. The GPS is no exception. If suffers from error-inducing factors which will downgrade its

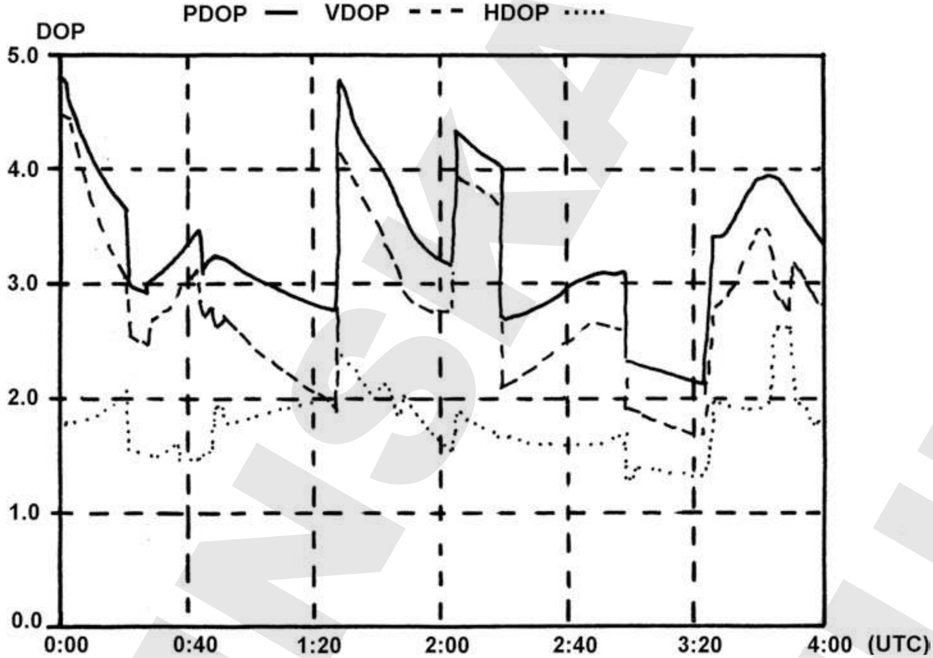

Figure 5.15 Trimble mission planning DOP graph taken over 4 hours. A low DOP indicates a high level of accuracy.

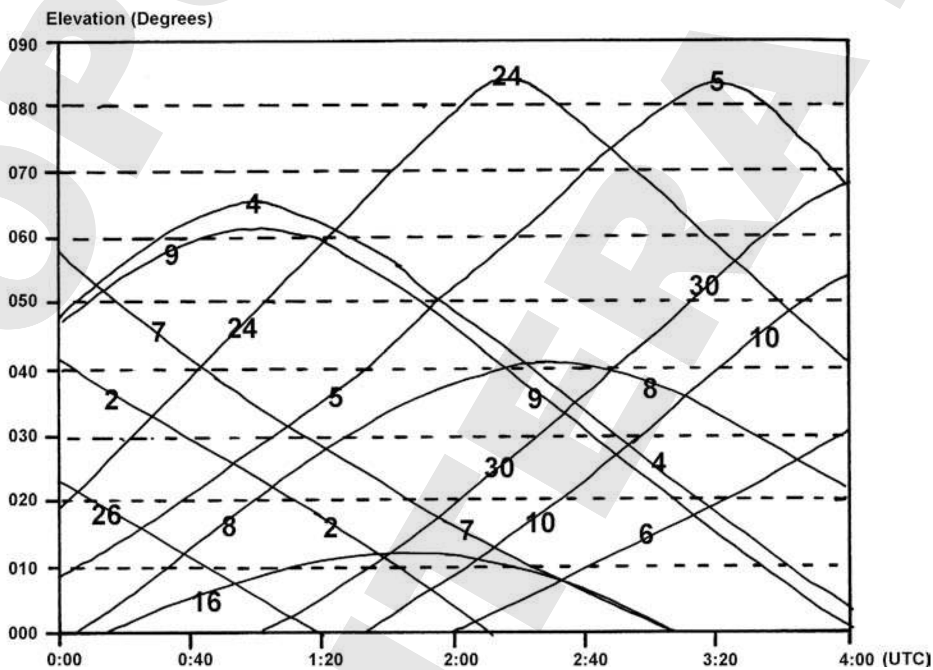

Figure 5.16 Trimble SV elevation plot. A 4-h plot showing all SVs in view.

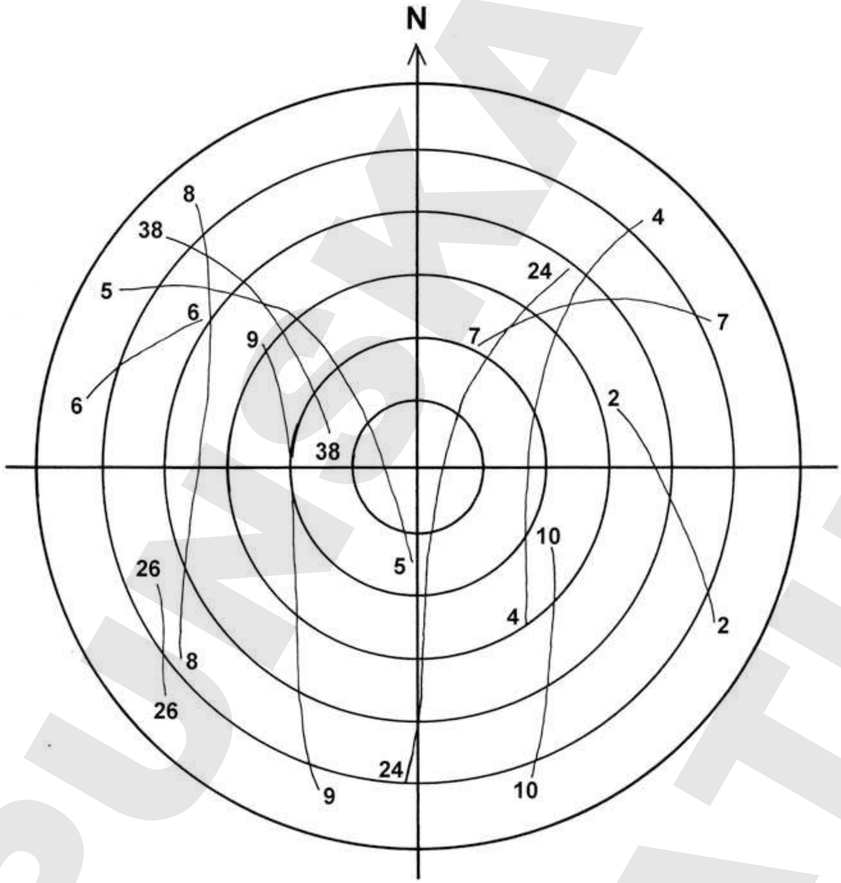

Figure 5.17 Trimble SV sky plot presentation. A GPS receiver is in the centre of concentric circles. The outer ring represents the horizon or zero elevation.

performance as a position fixing system. However, the total error produced by a combination of all error-producing factors is very small. Assuming that the system is free from operator error (corrupt data inputting), the error most likely to downgrade system accuracy is an error in the SV clock, which in turn will cause range measurement error.

GPS accuracy is promulgated in a number of ways as indicated below.

- *Circular Error Probable (CEP).* This represents an accuracy figure achievable 50% of the time in two dimensions only. This is a fix error in latitude and longitude.
- *Spherical Error Probable (SEP).* An accuracy that is achievable 50% of the time in all three dimensions.
- *Root Mean Square Radial Distance error (d_{RMS}).* A circle around the true position containing 95% of the fix calculations.
- *User Equivalent Range Error (UERE).* This is determined by summing the squares of the individual range errors and then taking the square root of the total.

The following errors affect the accuracy of GPS position fixes.

Satellite clock error

It has already been stated that a satellite clock oscillator is a precision instrument, but it is still necessary to re-adjust it from the ground support network. Error introduced by SV clock error is unlikely to exceed 1 m and regular uplinking of clock data reduces it to a minimum. Block IIA and Block IIR satellites, the latest SVs, carry better clock oscillators and will consequently provide higher accuracy fixes.

Ionospheric delay error

As the two transmitted carriers must pass through the ionosphere, a speed reduction caused by refraction of the radio wave occurs. The extent of the delay, and consequently the error introduced into the pseudo-range measurement calculation, is dependent upon the electron density the radio wave encounters along the signal path. Electron density is itself dependent upon three main factors:

- the time of day
- the SV elevation
- the latitude of the receiver.

Fortunately, ionospheric error is inversely proportional to the square of the carrier frequency. GPS SVs transmit on two frequencies so that the delay may be quantified in the receiver, an error correction figure calculated and applied to the final fix solution. After all corrective data has been applied to the solution in a single frequency GPS receiver system, fix error due to the ionosphere is unlikely to exceed 10 m.

Tropospheric delay error

Extending from the earth's surface to an altitude of 70 km, the troposphere also introduces a delay into the pseudo-range calculation. Unfortunately the error is independent of frequency, but it is predictable. GPS receivers hold a software solution in the form of a mathematical model to eliminate the effect of this delay. Figures for relative humidity, pressure and temperature are interfaced with the processor computer to produce corrective data which is then applied to fix calculation. Error from this source is unlikely to exceed 1 m.

Both ionospheric and tropospheric errors are reduced if ranges are measured from SVs showing a high elevation from the receiver. Modern receivers are capable of automatically selecting SVs with the highest elevation or those exceeding pre-set limits.

Multipath error

This results from the reception of the same SV signal from more than one source. A major contributor to this error is the reflected wave from an object close to the receiving antenna. Each receiver position is unique and therefore the error is not consistent. Final fix errors in the region of 1 metre can be produced by this effect. Careful positioning of the antenna will eliminate this error.

Relativity error

A commonly referred error is that produced by the effects of relativity. It is entirely predictable and is effectively cancelled in the GPS but it is briefly described here.

Albert Einstein stated that time is compressed by the mass of the earth. Time on the surface of the globe is compressed by $1.4 \times 10^{-9}\,\text{ms}^{-2}$ compared to time in free space. It is evident that as one travels further away from the earth's surface towards free space, the compression of time is of less significance. At the altitude of a GPS SV, time compression is calculated to be $0.4 \times 10^{-9}\,\text{ms}^{-2}$. An effective rate range time error of 1 ns therefore exists between the time on board the SV and that in the receiver. At the accepted propagation velocity of radio waves, i.e. $300 \times 10^{6}\,\text{ms}^{-1}$, an error of 1 ns corresponds to a range error of 0.3 m. In addition, a second time error is produced by time compression caused as the SV moves at $26.61\,\text{kms}^{-1}$ through space. To compensate for all relativity errors, the SV clock oscillator frequency is slightly offset. By the time that the radio wave arrives at the receiving antenna the effects of relativity will have been cancelled and the pseudo-range can be more accurately calculated.

These are by no means the only factors that affect the accuracy of the GPS system but they are often referred to in papers on this subject. A combined position error produced by all the above factors is unlikely to exceed 12 m.

User Range Error (URE)

This is a parameter for the estimated error in range calculation due to unknown factors. These include multipath, unmodelled atmospheric effects, operator error and unpredictable orbital errors. The URE figure is sent from SVs to GPS receivers and may be displayed in metres.