Fleet Operations Optimisation and Fleet Deployment

1. Introduction

Optimising the operation (i.e. minimising operating costs, if revenues are fixed) of a single merchant ship is not difficult to do and can be achieved by a few simple calculations, which can point out the minimum of the operating cost curve as a function of the speed, for example. Texts such as Stopford[1] can provide useful info on the above. Optimising an entire fleet of generally different ships, however, is definitely not as simple, and requires certain levels of computer, probability, optimisation and other mathematical skills, as we will see in the following.

Deployment of merchant shipping fleets covers a wide range of problems, concerned with fleet operations, scheduling, routing, and fleet design. Many use some kind of economic criterion such as profitability, income or costs on which to base decisions. (Benford,[2] Marbury,[3] Fischer and Rosenwein,[4] and Perakis [5]). Others use non-economic criteria such as utilisation or service; these are more common in fleet deployment models used in the liner trades.[6] Reviews of various fleet deployment models and problems are given in.Ronen,[7] Ronen,[8] and Perakis.[9]

An aspect of fleet deployment not covered extensively in the literature until the early 1980s was "slow-steaming" analysis and optimisation. "Slow steaming" is the practice of operating a ship or fleet of ships at a speed less than design or maximum (sustained) operating speed, in order to take advantage of improved fuel economy and reduced operating costs, but, most importantly, to reduce fleet overcapacity (if done by a large number of ship owner).

Managers of merchant ship fleets, especially bulk carriers and tankers, frequently find themselves with excess transport capacity, and hence must decide which ships to use (and at what speeds) and which to keep idle (or perhaps make available to another fleet by sale or charter). Moreover, if fuel prices become relatively high, excess transport capacity offers the potentially profitable strategy of slow steaming some or all of their ships. Such a strategy not only substantially reduces the operating costs of the fleet, but furthermore reduces the supply of tonne-miles of the existing total bulker fleet, thereby improving the depressed freight rates. On the other hand, a sharp drop in fuel prices could make it advisable to "fast-steam" ships built during the expensive fuel era, although this would be limited by their design speed and associated operating margin.

The remainder of this chapter is organised as follows: Section 1 discusses the correct solution of a simple fleet deployment problem, which has been earlier suboptimally solved in the literature. It is shown that its correct solution can save over 15% of the annual fleet operating costs, or $7.93 million per year, for the same 10-ship fleet of the previously published example. In Section 2, more realistic, single-origin, single-destination, and even multi-origin, multi-destination fleet deployment problems for (liquid or dry) bulk shipping are formulated and solved, using both nonlinear and a series of linear programmes. In Section 3, fleet deployment problems for Liner Shipping Fleets are solved, and specific examples are given from the fleet of a major liner company. Section 4 provides a summary and conclusions, as well as recent developments in this area. An alphabetical list of references is given at the end of this chapter.

2. A Simple fleet Deployment Problem

The ships of a fleet could be assumed to belong to N different groups, each consisting of n(i) sister ships, i = 1, …, N, of equal cargo carrying capacity, speed and fuel consumption (or in general, operating costs). Design speed, cargo capacity and operating costs will in general be different among different ship groups. This is both an efficient and general model, since the case of no two ships in a fleet being identical is obviously covered by setting n(i) = 1, i = 1, …, N. The mission of a fleet is assumed, for our purposes, to be the movement of one commodity between two given ports.

A simple (but realistic) bulker fleet deployment problem was defined in Benford (see endnote 2).Some of the assumptions inherent in the solution approach, such as no-cost lay-up of unneeded vessels, a contract to move a given quantity of a given commodity between one origin and one destination port, availability of more than enough ships (tonnage) suited to the trade, etc., were not unrealistic. However, the method proposed for its solution did not give the optimal answer, primarily because of an artificial constraint that all vessels must be operated at a speed resulting in the same unit cost of operation per ton of cargo delivered, imposed for ease of solution, but not a natural constraint of the problem. Table 1 below presents the approach adopted in Benford (see endnote 2) and its results.

In Perokis (see endnote 5), the problem was correctly solved analytically, without the above equal unit cost constraint, using Lagrange multipliers. The results (see Table 2) showed an improvement of at least 15% over those of Table 1, thus verifying once more that ‘constraints impair performance.’ More realistic and complicated versions of the problem solved in that paper were subsequently formulated and solved.

Comparing Tables 1 and 2, we see an annual cost reduction of $7.93 million, or over 15%. This represents a considerable improvement. The difference in costs could be even greater if lay-up charges are levied against ships I and J in the solution presented in Table 1. On the other hand, this difference could possibly be reduced if ships I and J could be chartered or sold to a third party at a particular price.

Perhaps it is now appropriate to clarify that in the above problem, the annual demanded transport capacity is assumed to be a given output (constant). This is the case for vessels operating under relatively long-term charters, which normally specify, among other

Table 1

Ship group	Annual transport capacity (÷ 10^6 tonnes)	Operating cost per tonne	Annual operating cost (÷ 10^6)
1. A, B, C	4.902	$4.562	$22.36
2. D, E	2.884	$4.546	$13.11
3. F, G, H	3.726	$4.562	$17.00
4. I, J	–	–	–
Total	11.500		$52.47

Table 2

Ship	Annual transport capacity ($\div 10^6$ tonnes)	Operating cost per tonne	Annual operating cost ($\div 10^6$)
1. A, B, C	4.158	\$3.531	\$14.684
2. D, E	2.450	\$3.692	\$ 9.046
3. F, G, H	3.241	\$4.023	\$13.038
4. I, J	1.651	\$4.707	\$ 7.771
Total	11.500		\$44.539

things, the freight rate and the amount of cargo to be carried annually. In a normal market environment, long-term charters are the overwhelming majority of fixtures, whereas vessels operating in the spot market constitute less than 10% of the available capacity.

The conclusion from the above is that, in contrast to past practices where significant effort has been directed toward the optimisation of the design and operation of individual ships, an owner of a fleet of ships (usually non-uniform in terms of age, size and operating speed) should operate each ship in a manner generally quite different from that dictated by single-ship optimisation. Adoption of the results of this and subsequent research should result in significant cost savings in the operations of several shipping companies.

3. More Realistic Bulk Shipping Fleet Deployment Models

Perakis and Papadakis,[10,11] and Papadakis and Perakis,[12] presented far more realistic and complicated fleet deployment problems and their "optimal" solutions. The problem of single-origin, single-destination fleet deployment was first studied. A computer programme was developed to solve the problem and to help the fleet operator to make slow steaming policy decisions. A detailed discussion of the problem solution and a sensitivity analysis are presented in Perakis and Papadakis (see endnote 10). Sensitivity analysis provides the user with an understanding of the influence on the total fleet operating cost of its various components. For small to moderate changes of one or more cost components, the user can get an extremely accurate estimate of his new total operating cost without having to re-run the computer programme. Some interesting conclusions were made on the basis of the sensitivity results.

The fleet deployment problem with time-varying cost components was also formulated and solved. A computer programme was developed to implement the solution of this problem (Perakis, Papadakis and Pogoulates).[13] The relevant algorithms are briefly described there as well. The problem of fleet deployment when the cost coefficients are random variables with known probability density functions was formulated in detail (see endnote 11), where analytical expressions for the basic probabilistic quantities were presented. A shorter description of the above is included in this presentation.

3.1 Objective function and constraints

A fleet, consisting of a given number of ships, is available to move a fixed amount of cargo between two ports, over a given period of time, for a fixed price. Each vessel in the fleet is assumed to have known operating cost characteristics. The problem objective is to determine each vessel's full load and ballast speeds such that the total fleet operating cost is minimised *and* all contracted cargo is transported.

A first constraint imposes upper and lower bounds on the vessel full load and ballast speeds. These speed constraints are necessary to ensure a feasible solution to the problem; which is, that each speed is less than or equal to its maximum and greater than or equal to its minimum operating limits. In practice, the minimum speed is non-zero and is determined by the lower end of the normal operating region of the vessel's main engine. The minimum speed should also be adequate for purposes of ship safety in maneuverability and control. The equality constraint must be satisfied to insure all contracted cargo is transported.

This formulation is based on the following assumptions, some of which use state-of-the-art empirical formulae taken from published articles, cited in the list of refs of the detailed papers and reports of ours (see endnote 10)[14]:

1. A vessel carries a full load of cargo from load port to unload port.
2. When the vessel is operating in restricted waters, it has a known and constant restricted speed which is usually the maximum allowable speed in the region in question, hence requiring a known, fixed power and fuel rate.
3. The number of days a vessel spends in port per round trip is known and constant.
4. The charges incurred at the load port and unload port per round trip are known and constant.
5. The amount of fuel burned per day in the load port and unload port is known and constant.
6. The annual costs of manning, stores, supplies, equipment, capital, administration, maintenance and repair, and make ready for sail are known and constant.
7. The power of vessel i (in HP) may be expressed by:
   $P_i = a_i \cdot X_i^{b_i}$ for the full load and by:
   $P_{bi} = a_{bi} \cdot Y_i^{b_{bi}}$
   for the ballast condition, where X_i and Y_i are the full load and ballast speeds of ship i respectively and the rest are appropriate constants.
8. The all-purpose fuel rate for a fully loaded vessel i may be expressed by:
   $(R_f)_i = g_i \cdot p_i^2 + s_i \cdot p_i + d_i$ for the full load and by
   $(R_f)_{bi} = g_{bi} \cdot p_{bi}^2 + s_{bi} \cdot p_{bi} + d_{bi}$
   for the ballast condition where p_i and p_{bi} are the normalised (percent) p_i and p_{bi} respectively, and the rest are appropriate constants.
9. The *total* annual cost of laying up vessel i is known for all i = 1, …, z.
10. The number of days per year vessel i is out of service for maintenance and repair is known and constant.
11. This problem formulation and solution is for a single stage, "one-shot" decision.

In the literature, the number of tonnes carried per year is assumed to be a linear function of a ship's full load and ballast speeds. In our research, we have shown that this assumption can be quite unrealistic. This function is quite nonlinear in nature. A derivation of this function may be found in Perakis and Papadakis (see endnote 10).

Operating costs, developed in detail in Perakis and Papadakis (see endnote 14)[15], are considered to fit into one of two categories, those that do not vary with ship speed, or *daily running costs*, and those that vary with ship speed, or *voyage costs*. Typical plots for the total (*not* per tonne) operating costs per year for a particular ship, for various ballast speeds, are given in Perakis and Papadakis (see endnote 10).

A typical plot of $F(X_i, Y_i)$ is also shown in Perakis and Papadakis (see endnote 10), as a function of the full load and ballast speeds. It is seen that F is a smooth convex curve or surface with a single minimum. There is also a finite speed range in which F is not very different from its minimum value, a property which allows approximate solutions to the problem using very different speeds for individual ships to produce total fleet costs very close to one another and to the optimum cost itself. For X_i and/or Y_i going towards either 0 or 8, F approaches infinity. Figures 1 and 2 are for the same ship and for constant route data.

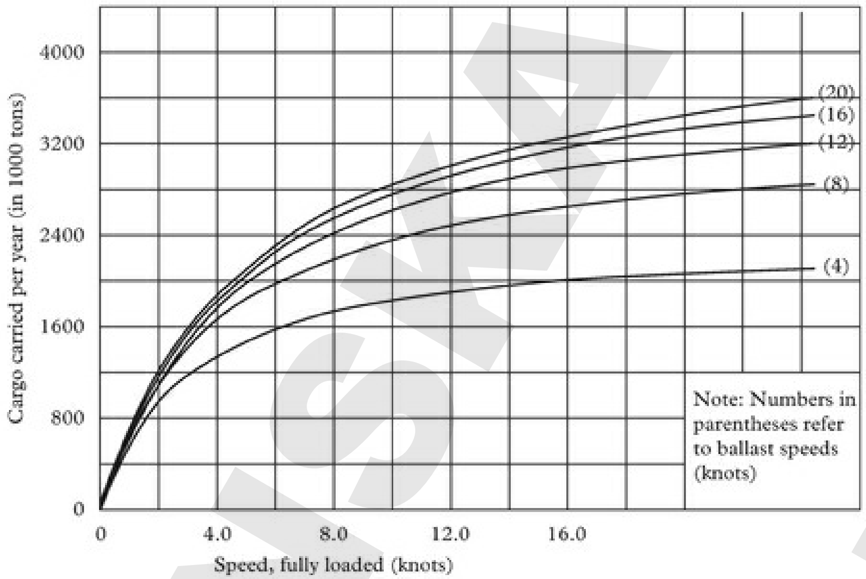

Figure 1: Typical plot of cargo carried per year as a function of ship speeds

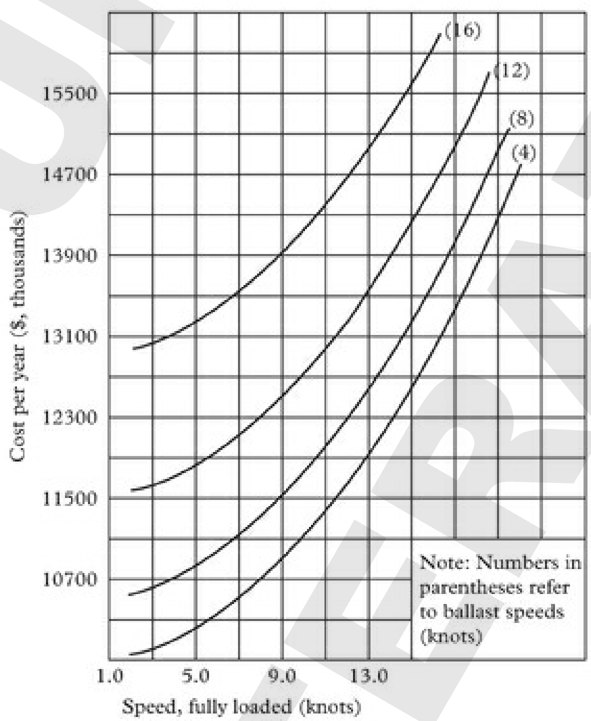

Figure 2: Typical plot for the total operating cost per year as a function of ship full load and ballast speeds

Introducing the linear inequality constraints on the speeds complicates the problem solution considerably. In the first part of this research, an External Penalty Technique (EPT) has been combined with the Nelder and Mead Simplex Search Technique to solve our optimisation problem. The purpose of a penalty function method is to transform a constrained problem into an unconstrained problem which can be solved using the coupled unconstrained technique.

A computer programme has been written to solve this problem using the techniques and the formulation mentioned above. The solution returned consists of the ship speeds, for those vessels specified for analysis, that will minimise the total mission operating costs and fulfil the cargo transport obligation.

For the lay-up option, it is shown in the technical report that for even moderate numbers of ships in a fleet, it is rather too time-consuming to use an exhaustive enumeration scheme. Instead, a dynamic programming-like sequential optimisation approach is developed, significantly reducing the computational burden. If Z is the number of ships in the fleet, the maximum number of fleets we will have to examine using this approach is $M_{max} = Z(Z + 1)/2 – 1$. The *actual* number of fleets which we will have to consider will be significantly smaller than M_{max}, due to the elimination of several fleets as infeasible and much smaller than the upper limit of total possible cases. The above scheme has been implemented and referred to in the following as the operating cost without re-running the programme. This property holds for a given fleet and not in cases when one or more ships are laid-up or chartered to a third party (on top of the changes in the cost components).

The fleet deployment problem with time-varying cost components was also studied. A time horizon in this formulation is any interval within which cost components are constant but at least one of them is different than its value in another interval. In other words, our cost components are given 'staircase' functions of time. In the case of rapidly changing costs, resulting in rather short intervals where these costs are constant, the problem of non-integer number of round trips per interval could be crucial. A heuristic approach was developed to find the nearest integer solution corresponding to the non-'integer' solution generally provided by the SIMPLEX algorithm.

Further details may be found in Perakis and Papadakis (see endnotes 10, 11, 15), and the associated user's documentation, Perakis, Papadakis and Pogoulatos (see endnote 13), where a more extensive multi-page flow-chart is presented.

The fleet deployment problem for the case when some of the cost components are random variables with known probability density functions was finally considered (Perakis and Papadakis (see endnote 11)). We note that the minimum of the possible mean values of the total annual operating costs, C_{min} and the variance of C_{min} can be found relatively "easily". However, this approach has not yet been implemented on a computer and probably will not prove very useful: The inputs to the problem (i.e. the user-supplied probability density functions) can have any particular theoretical or experimental form, thus discouraging the development of any general computer code for this problem.

3.2 The multi-origin, multi-destination fleet deployment problem

The problem of minimum-cost operation of a fleet of ships which has to carry a specific amount of cargo from several origin ports to several destination ports during a specified time interval was next examined. During the season any vessel can be loaded in any source port (S) and unloaded at any destination port (D) provided that these ports belong to a subset I or J (respectively) of the total set of ports, such that draft and other constraints for the corresponding vessel are satisfied. Under this assumption for each vessel the number of possible routes (number of possible sequences of S–D ports) is quite large. The full load and ballast characteristics of each ship on each route are assumed to be known.

This nonlinear optimisation problem consists of a nonlinear objective function and a set of five linear and two integer constraints. The objective function to be minimised is the total fleet operating cost during the time interval (shipping season) in question. The following constraints have to be satisfied:

a. for each vessel, the total time spent in loading, travelling from origins to destinations, unloading and travelling from destinations to origins plus the lay-up time has to be equal to the total amount of time available for each ship in the shipping season,
b. the total amount shipped to a particular destination j must be equal to the amount of cargo to be delivered to j during the shipping season (in tonnes),
c. the total amount of cargo loaded from a particular source i must be less or equal to the cargo available at i,
d. for each ship, the number of trips to destination j must be equal to the number of trips out of j,
e. same as (d) for all source ports;
f. the full-load and ballast operating speeds have to be between given upper and lower limits,
g. the numbers of full load and ballast trips for each vessel, origin and destination combination must all be non-negative integers.

The constraints presented above are linear except constraints (a) and (g). The maximum number of unknown variables (if all source and destinations ports are accessible by any ship of the given fleet) is $(4\cdot I\cdot J+1)\cdot Z$. The number of the associated constraints is $2Z+(1+J)\ (Z+1)+4\cdot I\cdot J\cdot Z$. For a case with $I=4$, $J=6$ and $Z=10$ we have 970 variables and 1,090 constraints. Using today's personal computers, it is clear that we cannot use any classical nonlinear optimisation technique, since the expected computation time would be too long.

In the case of the multi-origin, multi-destination fleet deployment problem, it was seen that the linear programming approaches to the literature do not take into account significant nonlinearities of the relevant cost functions and may lead to very suboptimal decisions. The iterative procedure we developed uses a linear programming software in an algorithmic scheme that takes into account these nonlinearities and produces accurate results. This approach is ideally suited for a personal computer due to the reasonable running times of the LP software for almost any practical situation. A second, nonlinear approach to solve the multi-origin, multi-destination problem was also implemented, using the available MINOS nonlinear optimisation package.

In [endnote 12], the fleet deployment problem for a fleet of vessels operating between a set of *several loading and unloading ports* under certain time and cargo constraints was examined. Full load and ballast voyage costs were treated as nonlinear functions of the ship full load and ballast speeds, respectively. An optimisation model, appropriate for bulk carrier fleets, minimising the total operating cost, was formulated. The existence of a coupling between the optimal speed selection and the optimal vessel allocation on the available routes was demonstrated, and conditions leading to the decoupling of these problems were established. Considerations referring to the structure of the optimal solution resulted in a substantial reduction of the dimensionality of the problem. We found that in cases of low-to-moderate fleet utilisation, linear programming may be applied to derive the optimal solution, while in cases of higher fleet utilisation, use of nonlinear optimization may become necessary. The potential benefits of our approach were demonstrated by several examples.

Finally, we would like to note that the algorithms and the computer codes, developed for both the one-origin one-destination and for the multi-origin, multi-destination fleet deployment problem can be easily used to find not only the optimal fleet deployment policy within the given time horizon, but also to help the fleet operator to make decisions in case unexpected events like strikes or accidents occur. In such a case the programs can be re-run for the remaining time interval and an optimal decision can still be obtained. Other plans, such as renewing or improving a part of the fleet and selling or chartering decisions may also be evaluated.

4. Fleet Deployment Models for Liner Shipping

In Perakis and Jaramillo,[16] we have reviewed the relevant work on liner shipping deployment and described current industry practices. Our objectives and assumptions were then presented. A model for the optimisation of the deployment of a liner fleet composed of both owned and chartered vessels was formulated. The determination of the operating costs of the ships in every one of the routes in which the company operates was carried out by a means of a realistic model, providing the coefficients representing voyage cost and time required for the input of the linear program presented in Jaramillo and Perakis.[17] A method for determining the best speeds and service frequencies was also presented; the fixing of those two groups of variables was required to linearize the deployment problem as formulated there. The overall optimisation method was described in detail, and a real-life case study was presented, based on the co-author's company (FMG, Flota Mercante Grancolombiana) operations, (Jaramillo and Perakis (see endnote 17)).

In Powell and Perakis,[18] we extended and improved on the above. An Integer Programming (IP) model was developed to minimise the operating and lay-up costs for a fleet of liner ships operating on various routes. The IP model determines the optimal deployment of the existing fleet, given route, service, charter, and compatibility constraints. Two case studies were carried out, with the same as above extensive actual data provided by FMG. The optimal deployment was determined for their existing ship and service frequency requirements.

The inputs to the optimisation model (presented in Powell and Perakis (see endnote 18) are based on the existing cost estimation model provided in Perakis and Jaramillo (see endnote 16), including ship daily running costs, voyage costs, costs at sea, costs at port, daily lay-up costs.

The optimisation model in Perakis and Jaramillo (see endnote 16) is given as:

$$\text{Minimise}\left(\sum_{k=1}^{K}\sum_{r=1}^{R} C_{Kr}X_{Kr} + \sum_{k=1}^{K} e_k Y_k\right)$$

where:

C_{kr} = operating cost per voyage for a type k ship on route r

X_{kr} = number of voyages per year of a type k ship on route r

e_k = lay-up cost for a type k ship

Y_k = number of lay-up days per year for a type k ship

In Perakis and Jaramillo (see endnote 16) and Jaramillo and Perakis (see endnote 17), a Linear Programming (LP) approach was used to solve this optimisation problem. Using an LP formulation required the rounding of the number of ships allocated to each route. The rounding led to some variations in targeted service frequencies and to sub-optimal results. An Integer Programming formulation is used in Powell and Perakis (see endnote 18), to eliminate any rounding errors in the previous LP solution.

4.1 Integer programming problem formulation

4.1.1 Decision variables

N_{kr} = the number of a type k ship operating on route r

Y_k = the number of lay-up days per year of a type k ship

for k = 1 to K and r = 1 to R; K is the number of ship types and R is the number of routes.

4.1.2 Objective function

The objective function in the model minimises the sum of the operating costs and the lay-up costs. The objective function in terms of the decision variables is:

$$\text{Minimise}\left(\sum_{k=1}^{K}\sum_{r=1}^{R} C'_{Kr} N_{Kr} + \sum_{k=1}^{K} Y_k e_k\right)$$

where:

$C'_{kr} = C_{kr} X_{kr}$, are the operating costs of a type k ship operating on route r

e_k = daily lay-up cost for a type k ship

4.1.3 Constraints

Ship availability. The maximum number of ships of type k operating cannot be greater than the maximum number of ships of type k available. Therefore:

$$\sum_{r=1}^{R} N_{kr} \le N_k^{max}\text{, for each type k ship}$$

where:

N_k^{max} = maximum number of type k ships available

Service frequency. Service frequency is the driving force in liner shipping. With all rates being set by conferences, the main product differentiation is on service. To ensure that minimum service frequencies are met, the following constraint is included:

$$\sum_{r=1}^{R} t'_{kr} N_{kr} \ge M_r^{max} \quad \text{for all r,}$$

where:

t'_{kr} = yearly voyages of a type k ship on route r and:

$t'_{kr} = t_{kr}/T_k$

T_k = shipping season for a type k ship

M_r = number of voyages required per year in route r

By finding the highest load level for any given leg of route r and comparing this with given ship capacity, we find the minimum required number of voyages per year for a specific route.

Ship/route incompatibility. Some ships may be unable to operate on a given route due to cargo constraints, government regulations, and/or environmental constraints. It is necessary to eliminate these ships from the model. Therefore:

$$N_{kr} = 0\text{, for given (k,r) pairs}$$

Lay-up Time. The lay-up time in our models is equal to the time a ship is not operating during the year. This includes dry-docking and repair time:

$$Y_k = 365 N_k^{max} - T_k \sum_{r=1}^{R} N_{kr}$$

Non-negativity. The decision variables N_{kr} must be non-negative.

4.1.4 Software application

The software package used to run the above example was “A Mathematical Programming Language” (AMPL) (Holmes,[19] and Fourer, Gay and Kerninham.[20]) and OSL, a mathematical programme solver. See Powell and Perakis (see endnote 18) for more details. The output file from AMPL gives the following information:

i. optimal value of objective function;
ii. value of objective function with LP relaxation;
iii. number of iterations to find solution; and
iv. values of variables at the optimal solution.

The values of the N_{kr} variables will show how many type k ships should be allocated to each route r. The Y_k variable will indicate the number of days for which type k ships must be laid-up.

4.1.5 Optimisation examples

The following two examples are for the fleet deployment for FMG. The fleet consists of six types of owned ships and five types of chartered ships (one long-term charter and four short-term charters). The data used to calculate the coefficients for the optimisation model is taken from Jaramillo and Perakis (see endnote 17). The cost and time coefficients used are transformed from per voyage units to per ship values.

Example 1: The first example optimises the FMG fleet deployment for their current shipping conditions. This example uses FMG’s existing service frequencies and the number of ships available of each type. The current allocation is shown in Table 3.

Table 3: Current ship allocation

		Route							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
	1	3			3				6
	2						2		2
(Owned)	**3**				2		1		3
	4							1	1
	5		1						1
	6			1					1
Ship Type									
	7						1		1
	8		1	1					2
(Chartered)	**9**								0
	10								0
	11					2			2
	Total	3	2	2	5	2	4	1	19

Example 1 results: The IP optimal allocation is given in Table 4. The minimum objective function yields a total operating cost of $91,831,000. This is compared with $93,148,000 for the current allocation. This corresponds to a reduction in total operating costs of 1.4% (a savings of $1,317,000 per year). Analysing the resulting allocation shows that all owned ships (k = 1 to 6) and the long-term charter (k = 7) are in use for the entire shipping season. This is due to the high lay-up costs associated with these ship types.

None of ship type 9 are allocated. This ship type has the highest operating cost of any of the short-term charters.

Example 2: Example 2 uses the frequency constraints of the LP model presented in Jaramillo and Perakis (see endnote 17). The resultant allocation of the LP model is contained in Table 5.This example compares the results and highlights the advantages of the IP model versus the results of the LP model.

Example 2 results: The IP optimal allocation of ships is given in Table 6. The minimum objective function gives a total operating cost of $99,400,000

The resulting allocation of the IP optimisation model maintains all of the target frequencies. Routes 1, 3, and 5 exactly meet the target frequencies while on routes 2, 4, 6, and 7 the frequency is improved. The improvement ranges from 1.3 days to 3.3 days.

For the LP comparison example presented, the optimal objective function of the IP model is $99,400,000. Although the cost produced by the LP model is substantially

Table 4: Resultant ship allocation (Example 1)

		Route							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
	1	3			3				6
	2			1				1	2
(Owned)	**3**				1		2		3
	4		1						1
	5		1						1
	6						1		1
Ship Type									
	7			1			1		2
	8								0
(Chartered)	**9**								0
	10					2			2
	11				1				1
	Total	3	2	2	5	2	4	1	19

Table 5: Linear programming allocation

		Route							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
	1	1			5				6
	2			1				1	2
(Owned)	**3**						3		3
	4		1						1
	5		1						1
	6						1		1
Ship Type									
	7			1					1
	8								0
(Chartered)	**9**								0
	10					2			2
	11	2							2
	Total	3	2	2	5	2	4	1	19

Table 6: Integer programming allocation (Example 2)

		Route							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
	1	1			1		4		6
	2				1			1	2
(Owned)	**3**				3				3
	4		1						1
	5		1						1
	6	1							1
Ship Type									
	7	1			1				2
	8		1						1
(Chartered)	**9**								0
	10					2			2
	11			2					2
	Total	3	3	2	6	2	4	1	21

Table 7: Comparison of frequencies

	Route						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Target Frequency	14	14	21	15	30	23	35
IP Model	14	10.7	21	12.9	29.2	20.4	33.7
Difference	0.0	–3.3	0.0	–2.1	0.0	–2.6	–1.3
LP Model	14.7	16.1	18.9	16.1	29.2	19.2	33.7
Difference	0.7	2.1	–2.1	1.1	–0.8	–3.8	–1.3

smaller, it is important to note that the service frequencies are compromised in the 1991 LP solution, which leads to sub-optimal allocation. Table 7 shows the comparison between service frequencies of the IP optimisation model and the LP model.

Since service is a priority in liner shipping, it is necessary to meet the target frequencies. The IP optimisation model ensures that all target frequencies are met. The LP model violates the target frequency for routes 1, 2 and 4. This is an average increase in service time of 1.3 days or 9.1%.

Using Integer Programming to solve integer problems always produces the optimal solution for the given constraints. No manipulation of results is necessary. Using Linear Programming to solve IPs requires manipulation of the results to make the decision variables integer numbers. This leads to sub-optimal solutions and constraints being violated.

Substantial savings may be achieved by applying our IP optimisation model for the fleet deployment of a liner shipping company. The first example in Powell and Perakis (see endnote 18) compares our IP model against the existing fleet deployment of a liner shipping company.

This example shows a reduction in operating costs of 1.5%. The second example compares our IP model with the LP model contained in Perakis and Jaramillo (see endnote 16). The results of the IP model are optimal and meet all service frequency constraints. The LP model violates the service constraints in three routes by an average of 9.1%.

The solution indicates that all owned and long-term charter ship types should be operated for their entire shipping season, due to the high lay-up cost associated with these ship types. Short-term charters should only be used if the owned ships and long-term charters cannot meet the cargo and service frequency constraints.

5. Fleet Deployment and Operations Optimisation: An Update

Recent years have been quite turbulent for the ocean shipping industry, especially considering prices and freight rates. The huge swings in the price of oil in the space of only a few months, from the all-time high of $147 per barrel in summer 2008 to its collapse a few months later, with the help of the world economic crisis, and the possibility of higher fuel prices in the future, when the world economy recovers, and especially when China and other high-growth emerging economies expand their demand for raw materials, and especially fuel, has made the original fleet deployment idea, (i.e. the determination of the optimal speeds of each individual ship in a fleet for a specific mission) reappear back on centre stage. Interestingly, none of the recent fleet optimisation references are in this classic “fleet deployment” form. Regardless, it may be worthwhile to look at some of them and their main results:

Christiansen and Fagerholt,[33] investigate the robustness of ship schedules using time windows. A main objective is to minimise the idle time of ships in port. It has to be ensured that ships arrive at times which are well before port closure, if any (e.g. Friday night). Such arrivals are considered to be “risky” and will be considered using penalties. Depending on the size of the penalty cost, the optimal solution will prefer arrival times at less risky (and hence more robust) times.

Agrawal and Ergun[34] combine the problems of ship scheduling and containerised cargo routing to identify the most profitable routes. The paper develops a model that maximises profit from satisfying a set of demand patterns between sets of origin and destination ports on given days of the week. An important consideration of this model is the capturing of the weekly frequency requirement for liner services. A mixed-integer program is formulated and three solution approaches are provided: a Greedy heuristic algorithm, column generation and Benders decomposition. The solutions are confirmed with application on a set of real data as observed by OOCL and APL in 2005.

Andersen *et al.*[35] present an optimisation model for the tactical design of scheduled service transportation system networks (in general) and focus on the importance of neighbouring systems in a multimodal transportation system. The main challenge that the paper addresses is how two different transportation modes can be coordinated to minimize fleet costs and throughput time. The resulting model minimises cost and waiting time at nodes subject to cover, count, vehicle balance and node balance constraints such that the demands at the final destination nodes are satisfied. Finally, real data are applied to verify the model.

Christiansen *et al.*[36] give a summary of the developments in ship routing and scheduling within the decade (approximately) before its publication. The reviewed papers are divided in categories: Strategic ship planning (optimal fleets, maritime supply, chain); Tactical/operational ship scheduling (optimal assignment of cargoes to ships and ships to schedules); Liner network design and fleet deployment and a group including all other references such as Navy applications.

In Bronmo *et al.*[37] a ship scheduling model is initially formulated, which requires an input of available combinations of ships and routes–these are then considered through binary variables in an integer LP. A number of initial solutions are generated by a constructive heuristic and then improved by a local search. Finally a computational study is demonstrated to verify the solution methodology.

Fagerholt presents TurboRouter, a decision support system for ship fleet scheduling which is based on interaction with the user rather than analytical methods for optimisation.[38]

Gunnarson *et al.*[39] aim to generate a simultaneous model for terminal location and the ship routing problem, in intermodal transportation. Using the case of a pulp supplier, they formulate a mixed integer linear programming model minimising total distribution cost, subject to a significant number of network flow constraints, production capabilities and demand from various customers. They also attempt to provide a solution methodology, as their set of columns is very large. A heuristic method, similar to column generation, is used. Fremont, gives a practical (without any operations research applications) approach to the advantages and disadvantages of the hub-and-spoke network contrasted to the direct port-to-port network. The paper uses extensively the example of Maersk's geographical coverage through the hub-and-spoke network.[41]

Bronmo *et al*.,[42] present a Danzig-Wolfe procedure for ship scheduling with flexible cargo sizes. This is a problem similar to the pickup and delivery problem with time windows, but the cargo sizes are defined by intervals instead of by fixed values. The authors found it computationally hard to find exact solutions to the subproblems, hence their method cannot guarantee finding the optimum over all solutions. To be able to show how good the solutions are, the authors generated bounds on differences between the true optimal objectives and the objectives in their solutions.

6. Summary and Conclusions

The optimisation of the operations of a fleet of ships is mathematically far more complicated than the optimisation of the operations of a single ship. However, trying to optimise the various full load and ballast speeds of each different ship in the fleet can be even more complicated than that, and necessitate the use of nonlinear programming algorithms and software, as opposed to the largely linear-integer algorithms in cases when the speeds are fixed and not optimised (slow-steaming).

Due to the very different degree of competition in the bulker and liner markets, and also due to the very dissimilar constraints on their respective operations, optimal fleet deployment is quite different for each one. Over the past several years, we have provided "exact" and "approximate" algorithms for realistic, single or multi-origin and destination problems for bulker fleet deployment, including optimal slow-steaming lay-up decisions, under conditions of certainty or uncertainty for the various cost components. We then also solved problems in optimal strategic planning and ship-route allocation for a major liner company, presenting independent models for fixing both the service frequencies in the different routes and the speeds of the ships, using at first linear and integer programming. Several insights from a review and comparative study of the above were presented here, starting from the proper problem definition (constraints artificially imposed have resulted in 15% higher costs in early literature on this problem) and ending with the benefits of optimal integer solutions to the liner fleet deployment problems we studied.

Length limitations prohibit us from discussing our extensive work in other areas of fleet optimisation, such as the operational, day-to-day decisions for a major oil company fleet, which we modelled and solved in Bremer and Peraklis (see endnote 21) and Perakis and Bremer (see endnote 22), or go into more details on the research modelling and results we did discuss in this paper. Our work with Bremer was an example of an operational (as opposed to long-term or strategic) optimisation.

We will also not discuss the mathematical details of some recent work of ours (Cho and Perakis (see endnote 23)) where we were able to re-formulate a complicated bulk cargo ship scheduling problem formulation Ronen (see endnote 24), from a nonlinear-integer to an equally accurate integer-linear problem, with far fewer variables, using a generalisation of the “capacitated facility location problem”, a classic result of optimisation theory. That problem was referring to a single loading port, several unloading ports, and fixed speeds (no slow-steaming allowed).

A major shortcoming of the classic ship scheduling problem, addressing uncertainty, received more attention after 2002. Shipping networks are prone to a very volatile behaviour. Uncertainty can be considered into two categories: internal and external in regards to the marine transportation network. Internal uncertainty refers to schedule deviation that can occur due to ship operation (within the transportation network), (i.e. bad weather conditions, mechanical faults, speed variation, etc). External uncertainty refers to schedule deviation which occur due to network-related issues, such as the effect of market volatility and inventory variation through supply and demand at origin and destination ports.

In an attempt to attack internal uncertainty, Christiansen and Fagenholt (see endnote 33), introduced the concept of time windows, a technique which has been used in the airline industry. The concept of a time window in general is that departures and arrivals are not considered as a single point in time, but as a time window. In the airline industry this provided with an advantage of allocating flights more efficiently within the network. In maritime operations, time windows account for deviation in arrival times and hence slight delays or early arrivals should not affect the network. In this way robustness is introduced in the model. Time windows also simplify the model as they reduce the number of available time slots. However, a challenge with this technique is to define the “size” of a time window. Too small time windows may not reflect the ship’s actual ability to follow the schedule, and the advantage of a time window will be cancelled out. Too large time windows might generate a more realistic but inefficient model as the ship will always be on time, but the terminal’s capacity will be inefficiently used.

To address the internal uncertainties, Christiansen and Fagenholt (see endnote 33) use time windows to attack the issue of risky arrivals. In cases where ports have restricted operating hours (e.g. no operation overnight or during weekends), available operating hours can be modelled as time windows. Furthermore, the concept of risky arrivals is introduced, which imply arrivals that are close to weekends and could cause the ship to stay idle for several days. On the basis of how risky an arrival is, each time window is associated with a penalty. Risky arrivals in the network will probably not be eliminated, but will be penalised and therefore reduced. However, it has to be noted that this methodology does not address uncertainty to a sufficient extent. It does not directly tackle the issue of delay propagation in a ship’s schedule, but instead it tries to ensure that delays will not be extended due to the port’s operating timetable.

External uncertainties have received considerably more attention than internal throughout the literature since 2002, due to the fact that they are usually better defined. An important uncertainty is in the supply and demand on origin and destination ports respectively. It is often the case that the vessel has arrived at its loading port, but has to wait for the cargo to pile up. Christiansen addressed this problem by considering a combination of the ship scheduling problem and the inventory management problem. The objective of the model is to identify the sequence of port calls for each ship with minimum cost while ensuring that inventories at ports are never full or empty (so that ships can always proceed). This is achieved by introducing “alarm levels” on the inventories, defined as *soft inventory constraints*. These include an upper and a lower bound which are tighter than the actual limits of the inventory. In the case where a ship's arrival would cause the inventory to shift beyond its alarm level, a penalty cost will be introduced. In this way a schedule including such a port call will be avoided. The benefits of this model not only involve the ships, but also the ports as well, since it ensures that they always have sufficient inventories. From a ship scheduling point of view the results from this paper have a considerable effect on robustness by reducing the possibility of port-related delays, which nowadays is the most common source of delays.

Hwang *et al.*(see endnote 40) deal with external uncertainty. Market fluctuations are the most important source of uncertainty in the maritime industry. Unlike other scheduling models, Hwang considered profit variability in the objective function of the problem, by analysing the profit from assigning a cargo to a schedule. The paper assumes that charter rates are linearly related to a single spot rate and therefore a single market random variable can be used to account for freight rate volatility. However, this is not entirely correct, as rates in different shipping sectors may not have such a strong correlation. Furthermore, unlike other models, the shipper had the option of chartering ships in and out of his fleet through time or voyage charters. Assuming that an operator aims for more sustainable profits, the model developed reduces profit variability at the smallest possible cost. The paper makes an important contribution, as there is very little research considering market volatility, which is a driving factor in shipping

For a detailed exposition of liner shipping economics, the textbook by Janson and the late D. S. Schneerson (see endnote 25), is highly recommended. In the recent liner logistics research, Rana and Vickson (see endnotes 26, 27) presented nonlinear programming models, aiming to maximise total profit by finding an optimal sequence of ports of call for each ship. For solution methods, they used Lagrangean relaxation[27] and decomposition methods. Their first paper develops only a one-ship model, while their second is rather complicated by its non-linearities in both objective function and constraints. The model of Perakis and Jaramillo (see endnote 16) and its subsequent more accurate integer solution Powell and Perakis (see endnote 18) is easier to use for a realistic situation, but does not take into account the cargo demand forecasts that arise between pairs of ports in the model.

We have addressed that in Cho and Perakis (see endnote 28), where we have suggested two optimisation models. The first is a linear programming model of profit maximisation, providing an optimal routeing mix for each ship available and optimal service frequencies for each candidate route. The second is a mixed integer programming model with binary variables, providing not only optimal routeing mixes and service frequencies, but also best capital investment alternatives to expand fleet capacity, and is a cost minimisation model. In both models, we have suggested and used the concept of "flow-route incidence matrix", and discuss its usefulness for similar route-ing and scheduling problems (see endnote 28). The most important merit of using the flow-route incidence matrix is that it links various cargo demands to route utilisation in a simple, systematic way. These models can help improve existing network of routes or service frequencies, and their solution can be easily implemented with standard linear or integer programming packages.

Other examples of complicated operational models we have studied in detail are Ship Weather Routing problems, but since we are restricting this chapter to fleet, not individual ship, optimisation, we will not discuss them here. The models described in this chapter are all strategic. However, at the request of the reviewer of this book chapter, we cite a few key recent references in that area (see endnotes 29–31).

The results of our research in Fleet Deployment have been cited in graduate courses at Michigan, MIT, and elsewhere in the US, but also in universities around the world, such as in the recent textbook used at the Maritime Studies Dept. in Dalian, China (see endnote 32). Further dissemination of these results in this chapter will hopefully result in more students and practitioners being exposed to the significant benefits of proper optimisation and the pitfalls, and their heavy price in higher fleet operating costs, of suboptimal policies.