Chapter 1

Damping Ratio

#Damping-Ratio

1.1 Definition:

$$\xi = a/\omega_n$$

- ξ is the damping ratio
- a is the rate of exponential decay
    - For second order systems: e^{-at}
- ω_n is the natural frequency of oscillation
- Measures the relative rate at which oscillation magnitudes are **reduced**, normalized by the natural frequency of the system
- $\zeta < 1$ -> underdamped
    - Oscillatory response decays exponentially
    - Lots of periods to eliminate the motion following a disturbance
- $\zeta = 0$
    - Indefinitely oscillating response
- $\lim_{\zeta \to 1} \zeta$
    - Oscillations dissipate in relatively few periods of oscillatory motion
- $\zeta = 1$
    - Critically damped system, with no motion

1.1.1 Percent Overshoot Method

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

- Response with too few peaks or too small to accurately extract from the time response
- $\%OS$: Percent overshoot to a step input
  - Applying it to a system with additional poles and/or zeros is a good-way of determining "effective" damping ratio
  - **Large amount of overshoot but small residual oscillations**
    - Could be caused by presence of uncanceled zeros in the mid-frequency range (e.g. 1-10 rad/sec)
    - Perceived as lightly damped mode by pilot, especially in turbulence

> Doesn't exist for impulse resulting in a steady state value of zero

1.1.2 Logarithmic Decrement

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

- Where δ (the natural logarithm of the ratio of successive peaks with period T overreacted in measurement $y(t)$:
  - $\delta = \ln\left[\frac{y(t)}{y(t+T)}\right]$
- Significant residual oscillations exists
- Used peak times and magnitude to determine natural frequency and decay frequency of the response, from which ζ is calculated
- Usually calculated from an impulse inputs and rate output (e.g., pitch rate)
  - But step input should work
- At least 3 oscillations required

> From Wikipedia: The method of logarithmic decrement **becomes less and less precise as the damping ratio increases past about 0.5**; it does not apply at all for a damping ratio greater than 1.0 because the system is overdamped