

# Homework 5: Confidence Intervals

STAT 343: Mathematical Statistics

## Problem 1

Let  $X_1, \dots, X_n$  be an i.i.d. sample from an exponential distribution with the density function

$$f(x|\tau) = \frac{1}{\tau} e^{-x/\tau}, \quad 0 \leq x < \infty.$$

- (a) Find the MLE of  $\tau$ .
- (b) What is the exact sampling distribution of the MLE?
- (c) Use the Central Limit Theorem to find a normal approximation of the sampling distribution.
- (d) Show that the MLE is unbiased, and find its exact variance. (*Hint: The sum of the  $X_i$  follows a gamma distribution.*)
- (e) Is there any other unbiased estimate with smaller variance?
- (f) Find the form of an approximate  $(1 - \alpha) * 100\%$  confidence interval for  $\tau$ .
- (g) Find the form of an exact confidence interval for  $\tau$ .