Quiz 3: Asymptotics and Efficiency
STAT 343: Mathematical Statistics

Problem 1:

Suppose $X_1, ..., X_n \overset{iid}{\sim} Normal(\mu, \theta)$, where μ in known and θ is unknown.

- $f(x_i|\mu, \theta) = \frac{1}{\sqrt{2\pi\theta}} \exp\left\{-\frac{1}{2\theta}(x_i - \mu)^2\right\}$

(a) What is $\ell(\theta|x)$?

(b) What is $\frac{d}{d\theta}\ell(\theta|x)$?

(c) What is $\frac{d^2}{d\theta^2}\ell(\theta|x)$?

(d) What is the (expected) Fisher information $I(\theta_0)$?

(e) We have shown previously that the sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of θ. We also have shown that $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$. Using these two pieces of information, determine whether S^2 attains the Cramer-Rao lower bound.

(f) What does your result in (e) mean in terms of efficiency of the estimator S^2?