Question 01

$$X = \begin{bmatrix} 1/3 & 2/3 & 4/3 \\ 8 & 2 & 2 \end{bmatrix}; Y = \begin{bmatrix} -2 & -2 & 1 \\ -5 & 0 & 5 \end{bmatrix}; z = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Compute the following:
(a) $X'X =$

(b) $X'z =$

(c) $XY' =$

(d) $z'z =$

Question 02 Describe a multiple linear regression in **vector-probabilistic** form. Define any symbols you use as parts of this equation.

Question 03 Given $(x,y)_i$ for $i = 1$ to 100 and predictive model $f(x;\beta)$, the sum squares error equals

$$SSE(\beta) = \sum_{i=1}^{100} \left(y_i - [f(x_i;\beta)]\right)^2$$

(a) Why is the SSE a function of β?

(b) Let $f(x_i;\beta) = \beta_0 + \beta_1 x_i$ and compute $\dfrac{dSSE(\beta)}{d\beta_0}$

(c) Set $\dfrac{dSSE(\beta)}{d\beta_0} = 0$ and solve for β_0

Question 04 In your own words,

(a) Describe the difference between Holdout validation and Cross-Validation

(b) Given K folds, define an expression for the cross-validation error

Question 05

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}; b = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}; c = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$$

(a) Compute $a'b$

(b) Compute $a'c$

(c) Are a and b orthogonal?

(d) Compute the orthogonal projection of a onto c

Question 06 Consider a $N \times 1$ vector of ground truth observations y and a $N \times M$ matrix X of explanatory data. You decide to fit a multiple linear regression model.
(a) What is the formula for computing the optimal vector of βs?

(b) If I specify my model as $f(X;\beta) = X\beta + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$, how would I compute the vector of predictions $\hat{y}$ (Hint: This matrix puts the "hat" on y.)

Question 07 The Bias-variance trade off is a fundamental concept in statistical learning.
(a) The Bias-Variance has three terms. What are the 3 terms involved in the Bias-Variance tradeoff?

(b) Describe a model that has high bias and low variance.

(c) Describe a model that has low bias and high variance

Question 08 A regression model is valid when the data to be modeled follows **four** assumptions. Please list and briefly explain the four assumptions.

Question 09 Should I compare model fits on my training data? on a test set? Please explain in a few (3-4) brief sentences.

Question 10 In your own words, please describe the steps to computing a K-fold cross-validation.